Capacity of Gaussian Relay Channel with Orthogonal Components and Non-causal Interference at the Source

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Abstract

In this work, we consider a relay channel model, with an out-of-band relay, in which the interference signal is available non-causally only at the source. The case in which the relay operates over different frequency bands with respect to the direct link is investigated. In particular, the source-destination (S-D), the source-relay (S-R) and the relay-destination (R-D) links are designed to be pairwise orthogonal. This may model a communication scenario in which the direct link (S-D) is parallel to the multihop channel (S-R-D). For this channel model, we establish the capacity of the Gaussian relay channel by showing that the developed upper and lower bounds are equal. In particular, the capacity results are obtained in which the relay may operate in either a full-duplex mode or a half-duplex mode.

Keywords: Relay Channel, Decode and Forward, Orthogonal Transmission Bands, Gaussian Channel.

1. INTRODUCTION

Relay channel (RC) plays a main role in extending and improving the signal coverage in wireless networks. Basically, the RC is composed of a source-destination pair augmented by an intermediate node, the relay. This channel was initially established in [Meulen,1971] and then discussed in many different scenarios like decode and forward [Cover & El Gamal,1979], and compute-and-forward [Wei & Chen, 2012]. Practically, a wireless node cannot simultaneously receive and then transmit over a given frequency band. Therefore, RC with orthogonal components was also developed and studied in many different scenarios. For example, the authors in [Liang & Veeravalli, 2005] investigated a case in which the transmission period is divided into two phases wherein the second phase is exploited by only the relay to forward its message to the destination. Another solution is to use a full-duplex relay with orthogonal components. For instance, the authors in [El Gamal & Zahedi, 2005] studied a case in which the source may transmit to both the relay and the destination over different frequency bands.

However, the transmission over the RC may be affected by a transmission from another user, the interferer. In particular, this interference limited scenario was investigated in many different cases. For instance, the authors in [Khomuji etl,2013] considered the case in which the interference signal is available at both source and relay. Furthermore, the case in which the interference signal is only available at the source was studied in [Bakanoglu etl, 2013], [Zaidi etl, 2013]. For example, the authors in [Bakanoglu etl, 2013] proposed some encoding schemes and also derived the associated achievable rates of the RC with orthogonal components in which the interference signal is known only to the transmitter. In addition, the case in which the interference signal is only known to the relay was investigated by many authors like [Deng & Langl, 2014].

In this paper, we consider the transmission over the RC, as depicted in Fig.1 . In this channel model, a source, S, wants to communicate with a destination, D, with the help of a relay, R. Indeed, the received signal at the destination is affected by another signal from another source, the interferer, I. This interference signal is assumed to be non-causally available only at the source. This communication scenario may appear in the case that a node is cognitive and the other is not. In order to avoid the effect of this interference, the source may need to spend part of its power to let the relay learn about the interference signal [Bakanoglu etl, 2013], [Zaidi etl, 2013]. Another solution is to let the source transmit to both the relay and the destination over different frequency bands. Thus, the relay may easily decode the desired signal. After that, the relay forwards the source's signal to the destination. Thus, if the relay uses the same frequency band that is used over the S-D link, then, this transmission is not an interference-free. Consequently, the source may help the relay by allocating part of its power to help in decoding the relay signal at the destination. This can be performed by employing the generalized DPC [Kotagiri & Laneman, 2008] at the source. Another solution is to let the relay transmit to the destination over a frequency band that is different from the one used over the S-D link. We mainly derive lower and upper bounds on the achievable capacity of the additive white Gaussian noise (AWGN) RC with orthogonal components and interference which is available only at the source. These bounds are shown to be equal such that the capacity of the proposed model is derived. In addition, these results show that an interference-free achievable capacity is obtained.

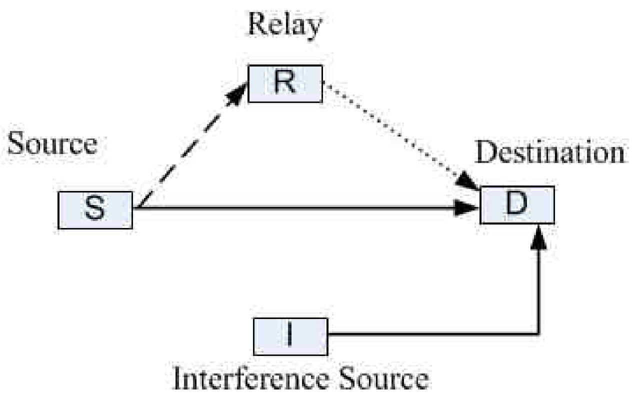

Figure 1. Transmission over relay channel which is affected by another source, the interferer. This interferer's signal is known non-causally at only the source. Furthermore, different link formats are used to indicate orthogonal frequency bands.

Next, in Section 2, the channel model and system preliminaries are introduced. Then, the capacity of the developed AWGN channel model is derived in Section 3. These derivations are obtained in the case that the relay may operate in either half-duplex mode or full-duplex mode. Finally, the paper is concluded in Section 4.

2. CHANNEL MODEL AND PRELIMINARIES

The state-dependent RC, in which the source wants to transmit a message, W , to the destination over n uses of the channel, is investigated. This message is uniformly selected from the set $\mathcal{W} \in \{1, 2, \cdots, 2^{nR_T}\}$, at a data rate R_T with probability of error goes to 0 for sufficiently large n. This channel model is composed of a source $X = (X_D, X_R) \in \mathcal{X} = \mathcal{X}_D \times \mathcal{X}_R$, a relay sender $X_2 \in \mathcal{X}_2$ and three channel outputs, $Y_R \in \mathcal{Y}_R$ at the relay, $Y_1 \in \mathcal{Y}_1$ and $Y_2 \in \mathcal{Y}_2$ at the destination. The channel outputs Y_R and Y_2 are mainly controlled by the signals X_R from the source, and X_2 from the relay, respectively. In addition, the channel output, Y_1, is controlled by both the channel input X_D and the channel state, X_I. We note that the interference source is not able to cooperate with our source to reduce its effect at the destination. Indeed, the state-dependent discrete memoryless RC is described by the conditional probability mass function given by

$$p(y_R, y_1, y_2 | x, x_2, x_I) = p(y_1 | x_D, x_I)\ p(y_2 | x_2)\ p(y_R | x_R, x_2).$$

Further, the AWGN channel, as depicted in Fig. 2, is also considered. The input-output relations are given by

$$\begin{aligned} Y_R &= h_{SR} X_R + Z_R, \\ Y_1 &= h_{SD} X_D + h_I X_I + Z_D, \\ Y_2 &= h_{RD} X_2 + Z_2, \end{aligned} \tag{1}$$

where the noise signals Z_R, Z_D, and Z_2 are assumed to be independent Gaussian random variables with 0 mean and variances normalized to 1. Additionally, h_{RD} and h_I are the channel gains from the relay and the interference source to the destination, respectively. h_{SD} and h_{SR} are the channel gains from the source to the destination and the relay, respectively. All of these channel gains are assumed to be globally known. The signals,

X_D and X_R are the transmitted by the source to the destination and the relay, respectively. These signals, X_D and X_R, are constrained by an average power of βP_S and $(1-\beta) P_S = \bar{\beta} P_S$, respectively, where P_S is the average total power at the source. Further, the factor β is the power allocation factor at the source. Indeed, the relay signal, X_2 and the interference signal X_I are subject to an average power of P_R, and Q, respectively. In addition, the following notations $H(\cdot)$ and $I(\cdot;\cdot)$ are used to represent the entropy of a random variable and the mutual information between two random variables, respectively. Further, we define $C(x) = \frac{1}{2}\log_2(1+x)$.

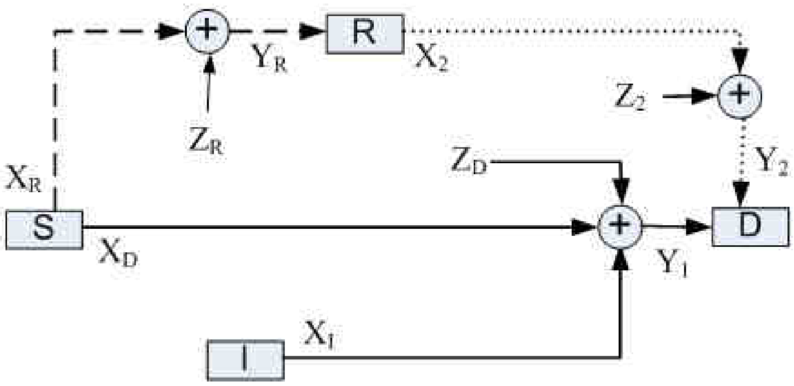

Figure 2. AWGN relay channel in which the transmission is carried out over different frequency bands. The received signal at the destination is also affected by the interference signal.

3. CAPACITY OF GAUSSIAN RELAY CHANNEL WITH ORTHOGONAL COMPONENTS AND NONCAUSAL INTERFERENCE AT THE SOURCE

In this section, the capacity of the AWGN RC, with orthogonal components and non-causal states only at the source, is derived. Our derivation is mainly based on the capacity bounds for the RC [Cover & El Gamal, 1979], the achievable rate of the multiple access channel (MAC) with states only at one of the two encoders [Kotagiri & Laneman, 2008], and the capacity of the dirty paper channel [Costa, 1983].

Theorem: The capacity, C^*, of the AWGN RC with different frequency bands and input-output relations given in (1), is given by

$$C^* = \max_{0\le\beta\le 1} C(\beta P_S) + \min\left\{C\left(|h_{SR}|^2 \bar{\beta} P_S\right), C\left(|h_{RD}|^2 P_R\right)\right\}$$

Remark: The maximum data rate is obtained in the case of $C\left(|h_{SR}|^2 \bar{\beta} P_S\right) = C\left(|h_{RD}|^2 P_R\right)$ or equivalently $|h_{SR}|^2 \bar{\beta} P_S = |h_{RD}|^2 P_R$. In addition, the optimum value of β which maximizes the capacity is given by $\beta^* = 1 - \frac{|h_{RD}|^2 P_R}{|h_{SR}|^2 P_S}$.

Proof: The proof is based on the two bounds of the capacity of the discrete memoryless RC from [Cover & El Gamal, 1979], the achievable rate of the MAC with states available non-causally at one of the two encoders [Kotagiri & Laneman, 2008], and the capacity of the dirty paper channel [Costa, 1983]. Consequently, the lower bound may be given as

$$C \ge \max\min\left\{I(U;Y_R|X_2) + I(X;Y|X_2,U),\quad I(U_D,X_2;Y_1,Y_2)\quad I(U_D;X_I)\right\}. \tag{2}$$

This bound is achieved by using the generalized block Markov encoding in which the relay is assumed to decode part of the new message, U , and also can cooperatively forward enough information with the source, X , such that the receiver can reliably obtain the previous message [Cover & El Gamal, 1979]. In addition, the term $I(U;Y_R|X_2)+I(X;Y|X_2,U,X_I)$ is the broadcast term from the source to both the destination and the relay, in which only the destination is affected by the states. Further, the term $I(U_D,X_2;Y_1,Y_2)-I(U_D;X_I)$ represents the total achievable rate of estimating the signals from both of the source and the relay in which only the source has a non-causal version of the state [Kotagiri & Laneman, 2008]. Moreover, U_D is an auxiliary random variable and is given by $U_D = X_D + \zeta X_I$, where the scaling factor ζ is given by $\frac{\beta P_S}{\beta P_S + 1}$ [Costa, 1983].

In addition, as in [Cover & El Gamal, 1979], the upper bound may be obtained by using the "max-flow min-cut" as

$$C \leq \max \min \left\{ I(X;Y,Y_R|X_2),\, I(U_D,X_2;Y_1,Y_2) \quad I(U_D;X_I) \right\}. \tag{3}$$

We note that, as in [2], the MAC term appears in both the lower and upper bounds. To go forward, we substitute $X=(X_D,X_R), U=X_R$ and $Y=(Y_1,Y_2)$ in (2) such that we get

$$\begin{aligned}
R_{BC1} &= I(U_1;Y_1|X_2)+I(X;Y|X_2,U,X_I),\\
&= I(X_R;Y_R|X_2)+I(X_D,X_R;Y_1,Y_2|X_2,X_R,X_I),\\
&=^a I(X_R;Y_R|X_2)+I(X_D;Y_1,Y_2|X_2,X_1),\\
&= I(X_R;Y_R|X_2)+H(Y_1,Y_2|X_2,X_I)-H(Y_1,Y_2|X_2,X_I,X_D),\\
&= I(X_R;Y_R|X_2)+H(Y_1,Z_2|X_I)-H(Y_1,Z_2|X_I,X_D),\\
&= I(X_R;Y_R|X_2)+H(Y_1|X_I)+H(Z_2|X_I,Y_1)-H(Y_1|X_I,X_D)-H(Z_2|X_I,X_D,Y_1),\\
&= I(X_R;Y_R|X_2)+H(Y_1|X_I)+H(Z_2)-H(Y_1|X_I,X_D)-H(Z_2),\\
&= I(X_R;Y_R|X_2)+H(Y_1|X_I)-H(Y_1|X_I,X_D),\\
&= I(X_R;Y_R|X_2)+I(X_D;Y_1|X_I),\\
&= C\left(|h_{SR}|^2\,\bar{\beta}\,P_S\right)+C(\beta P_S),
\end{aligned} \tag{4}$$

where R_{BC1} is the broadcast term in the lower bound. Moreover, step (a) follows by the fact that $X_R \rightarrow X_2 \rightarrow Y_2$ forms a Markov chain. In a similar manner, the broadcast term in the upper bound can be obtained as

$$\begin{aligned}
R_{BC2} &= I(X;Y_R,Y|X_2,X_I),\\
&= I(X_D,X_R;Y_R,Y_1,Y_2|X_2,X_I),\\
&= I(X_D,X_R;Y_R|X_2,X_I)+I(X_D,X_R;Y_1,Y_2|X_2,X_I,Y_R),
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
I(X_D,X_R;Y_R|X_2,X_I) &= I(X_R;Y_R|X_2,X_I)+I(X_D;Y_R|X_2,X_I,X_R),\\
&=^b I(X_R;Y_R|X_2,X_I),\\
&= H(Y_R|X_2,X_I)-H(Y_R|X_2,X_I,X_R),\\
&=^c H(Y_R|X_2)-H(Y_R|X_2,X_R),\\
&= I(X_R;Y_R|X_2),
\end{aligned} \tag{6}$$

where step (b) is strict since $I(X_D;Y_R|X_2,X_I,X_R)=0$. Further, step (c) follows by the fact that Y_R and X_I are independent random variables. Further, the second term in (5) can be simplified as

$$
\begin{aligned}
I(X_D, X_R; Y_1, Y_2 | X_2, X_I, Y_R) &= H(Y_1, Y_2 | X_2, X_I, Y_R) - H(Y_1, Y_2 | X_2, X_I, Y_R, X_D, X_R), \\
&=^d H(Y_1, Y_2 | X_2, X_I) - H(Y_1, Y_2 | X_2, X_I, X_D, X_R), \\
&= H(Y_1, Z_2 | X_I) - H(Y_1, Z_2 | X_I, X_D, X_R), \\
&= I(X_D; Y_1 | X_I),
\end{aligned} \tag{7}
$$

where step (d) is obtained since $Y_R \to X_2 \to Y_2$ forms a Markov chain. Therefore, by substituting the results from (6) and (7) into (5), we may have

$$
\begin{aligned}
R_{BC2} &= I(X_R; Y_R | X_2) + I(X_D; Y_1 | X_I) \\
&= C\left(|h_{SR}|^2 \bar{\beta} P_S\right) + C(\beta P_S)
\end{aligned} \tag{8}
$$

Further, from (4) and (8), it is clear that $R_{BC1} = R_{BC2}$.

Next, the MAC term may be given by

$$
\begin{aligned}
R_{MAC} &\le I(U_D, X_2; Y_1, Y_2) - I(U_D; X_I), \\
&= I(U_D; Y_1, Y_2) + I(X_2; Y_1, Y_2 | U_D) - I(U_D; X_I), \\
&= I(U_D, Y_1) + I(Y_1; Y_2 | U_D) + I(X_2; Y_2 | U_D) + I(X_2; Y_1 | U_D, Y_2) - I(U_D; X_I), \\
&=^e \{I(U_D; Y_1) - I(U_D; X_I)\} + I(X_2; Y_2 | U_D), \\
&= \{I(U_D; Y_1) - I(U_D; X_I)\} + H(Y_2 | U_D) - H(Y_2 | X_2, U_D), \\
&=^f \{I(U_D; Y_1) - I(U_D; X_I)\} + H(Y_2) - H(Y_2 | X_2), \\
&= \{I(U_D; Y_1) - I(U_D; X_I)\} + I(X_2; Y_2), \\
&=^g I(X_D; Y_1 | X_I) + I(X_2; Y_2), \\
&= C(\beta P_S) + C\left(|h_{RD}|^2 P_R\right)
\end{aligned} \tag{9}
$$

where the result in step (e) is obtained since $I(Y_1; Y_2 | U_D) = 0$ and $I(X_2; Y_1 | U_D, Y_2) = 0$. In addition, since Y_2 and U_D are independent, step (f) is obtained. Further, we use the result reported in [Costa, 1983] to obtain the result in step (g).

Finally, from the obtained results in (4), (8), and (9), the achievable capacity, as described by the ***Theorem***, is obtained.

Next, the capacity in the case that the relay may operate in the half-duplex mode is attained.

Lemma: The capacity of the relay channel with orthogonal components and non-causal interference at only the source is given by

$$
C = \max_{\beta, \tau} \min\left\{ \tau C(\theta_1 \beta P_S) + \tau C\left(\frac{|h_{DS}|^2 \bar{\beta} P_S}{\tau}\right), \bar{\tau} C(\theta_2 \beta P_S) + \bar{\tau} C \frac{|h_{RD}|^2 P_R}{\bar{\tau}} \right\}
$$

where θ_1 and θ_2 determine the power allocation during the first and second phases such that $\tau \theta_1 + \bar{\tau} \theta_2 = 1$.

Proof: The proof is obtained by developing a two-phase transmission scheme. In this first phase, which lasts for a τn channel uses, the source may transmit to both the relay and the destination. Therefore, the achievable rate, R_1 in this phase is given as

$$\begin{aligned} R_1 &= \tau\, I(X_D;Y_1|X_I) + \tau\, I(X_R;Y_R|X_2) \\ &= \tau\, C(\theta_1\, \beta\, P_S) + \tau\, C\left(\frac{|h_{DS}|^2 \bar{\beta}\, P_S}{\tau}\right) \end{aligned} \tag{10}$$

Then, in the second phase, which lasts for $(1-\tau)n = \bar{\tau}\, n$, both the source and the relay may transmit to the destination such that the achievable rate, R_2 in this phase is given by

$$\begin{aligned} R_2 &= \bar{\tau}\, I(X_D;Y_1|X_I) + \bar{\tau}\, I(X_2;Y_2) \\ &= \bar{\tau}\, C(\theta_2\, \beta\, P_S) + \bar{\tau}\, C\left(\frac{|h_{RD}|^2 P_R}{\bar{\tau}}\right) \end{aligned} \tag{11}$$

Finally, the minimum of (10) and (11) represents the achievable capacity as given in the *Lemma*.

4. CONCLUSION

In this paper, the transmission over the AWGN RC with orthogonal components, in which the interference signal is only available at the source, has been investigated. An upper and lower bounds on the achievable capacity have been obtained. These bounds have shown to be equal such that the capacity is attained. Further, the capacity results have been obtained in which the relay may operate in either half-duplex mode or full-duplex mode.

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