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# BEM-FEM of Coupling for Prosthetic Socket

Hasan Saad*, Mohammad Qasim Abdullah, Hatem R. Wasmi

Department of Mechanical Engineering, College of Engineering, University of Baghdad, Iraq

## Abstract
In this study, the investigating of the mechanical interaction between the residual limb and its prosthetic socket, and in computer-aided design and computer-aided manufacturing of prosthetic sockets is solving numerically by using FEM-BEM coupling approach.

This work tried to improve the deflection of the loading of Socket. The BEM techniques has been used to estimate a deflection at each distance for the main part of loading that applied on Socket .In addition, a simplified stiffener has been added to optimize a moment of inertia of this main part thereby the deflection will be better , for this purpose a case study has been considered according to international standard for the Socket specification that investigated to illustrate the goal of this work

Moreover, a physical problem governed by a linear elliptic partial deferential equation but with a socket (where this paper approximate Socket to shell ) discontinuity in the domain cannot be efficiently solved using the traditional boundary element method. This paper also shows how the Laplace equation can be solved in an interior region containing shell discontinuities by recasting it as an integral equation known as a boundary and shell integral equation and applying collocation to derive a method termed the boundary and Socket element method. Direct and indirect methods are derived and applied to a test problem. The results show that using stabilized method enables us to get stable and accurate numerical approximations consistent with the physical configuration of the problem over rough mesh by using ANSYS Workbench 14.0 ,AutoCAD and COMSOL 5.2.

**Keywords:** Finite element model; boundary element method; discontinuity; Socket

## 1. Introduction
The boundary element method (BEM) is the most potent tool for the solution of linear Socket partial differential equations (PDEs) in an exterior domain. The standard BEM also serves as an attractive alternative to methods such as the finite-element and finite-difference methods for interior linear elliptic problems. The BEM has thus become an established computational method in recent years. However, apart from suffering the limitation of being inappropriate for nonlinear problems, the BEM is unable to cope directly with discontinuities in the variables in the domain of the PDE. The discontinuity will be assumed to have the topology of a shell-an open surface in three-dimensional problems, a line in two dimensions. The socket is a basic component for prosthetic performance. Below-knee amputees generally demonstrate some gait abnormalities such as lower walking speed , increased energy cost , and asymmetries between legs of unilateral amputees in stance phase cycle, step length and maximum vertical force . Successful fitment of prosthesis may be achieved by understanding the biomechanical structure of socket and its material, weight, thickness in particular to fulfill the desirable load distribution in soft tissues and bone of residual limb. Most commonly used socket design in developing countries is patellar tendon bearing (PTB) socket developed following the World War II at the University of California, Berkeley in the late 1950 s . The Finite Element Method (FEM) has been used widely in biomechanics to obtain stress, strain and deformation in complicated systems and have been identified as an important tool in analyzing load transfer in prosthesis . The finite element analysis (FEA) models have been used to study the effects of the inertial loads and contact conditions on the interface between prosthetic socket and stump of an amputee during gait cycle [1]. The traditional BEM is derived from a boundary integral equation (BIE) formulation of the PDE by dividing the boundary into boundary elements and applying an integral equation method (usually collocation) to obtain the method of solution. Boundary integral equation reformulations of the PDEs on each subdomain can now be obtained and, through coupling these equations across common boundaries, the solution throughout the domain can be obtained. A similar method to the traditional BEM for the solution of a PDE in the infinite domain exterior to a shell discontinuity can be derived through recasting the PDE as an integral equation termed a shells integral equation (SIE). A numerical method can then be derived in a way similar to the BEM[2]. However, the main value of shell elements is in their use in conjunction with boundary elements. This paper shows how the PDE in the domain of Fig.1 can be reformulated in a straightforward way as an integral equation termed a boundary and shell integral equation (BSIE) and thus how a numerical method termed the boundary and Socket element method (BSEM) can be derived. Boundary element methods have traditionally fallen into two distinct classes, direct BEMs and indirect BEMs, based on direct and indirect integral equation formulations. In this paper direct and indirect BSIE formulations are given for the interior two-dimensional Laplace equation. The BSIEs are a hybrid of the corresponding direct or indirect BIE with the SIE. risk of degenerative tissue ulcer in the stump because of cyclic or constant peak pressure applied by the prosthetic socket[3,4]. The pressure also

can lead to various skin deases such as follicular hyperkeratosis, allergic contact dermatitis, infection and veracious hyperplasia. Despite significant scrutiny in the field of prosthetics in the previous decades, still many amputees experience pressure ulcers with the use of prostheses. Sometimes, skin problems lead to chronic infection, which may necessitate re-amputation. This will obviate the long-term use of prosthesis, which indicatively reduces the routine activities of prosthesis users and the quality of life. Many studies have concentrated on interface pressure magnitude between the socket and stump during level walking. This paper will address finite element analysis of the socket prosthetic prescription. Specifically, the contribution of Mechanical properties of prostheses mechanical characteristics of socket that include stress, total deformation and safety factor will be discussed [5]. For domains in the form of Fig.3, the traditional BEM can be applied by dividing the domain into subdomains, as illustrated in Fig.4.

**2. Geometry**

To create an FE model, first the geometry of the modelled objects needs to be obtained, including the shapes of the free residual limb, pylon, the socket, foot and the adapter if involved. Although some models were based on ideal and simplified geometry, an accurate description should be based on an actual geometry as shown in Fig.1. The main object of this study is the prosthetic socket. The simplified geometry of his residual limb was modeled in Pro-engineer and then it is being imported (from AutoCAD) and modified in ANSYS Workbench 14.0 [1].

**3. Defining the Analysis Type and Applying Load**

The term ‘load’ includes the boundary conditions (constraints, supports, or boundary field specification), as well as other externally and internally applied loads. The load which is used in the ANSYS workbench version 14 software will be fixed support at the adapter of socket. While, the interface pressure is distributed according to particular positions Fig. (2) Shown values with positions of pressure distributions of present experimental case study. In fatigue solution, the fatigue tool is used to find the equivalent stress, maximum shear stress, total deformation, safety factor, and life at particular loads.

Fig**. (1):** Prosthetic hold Socket [1].

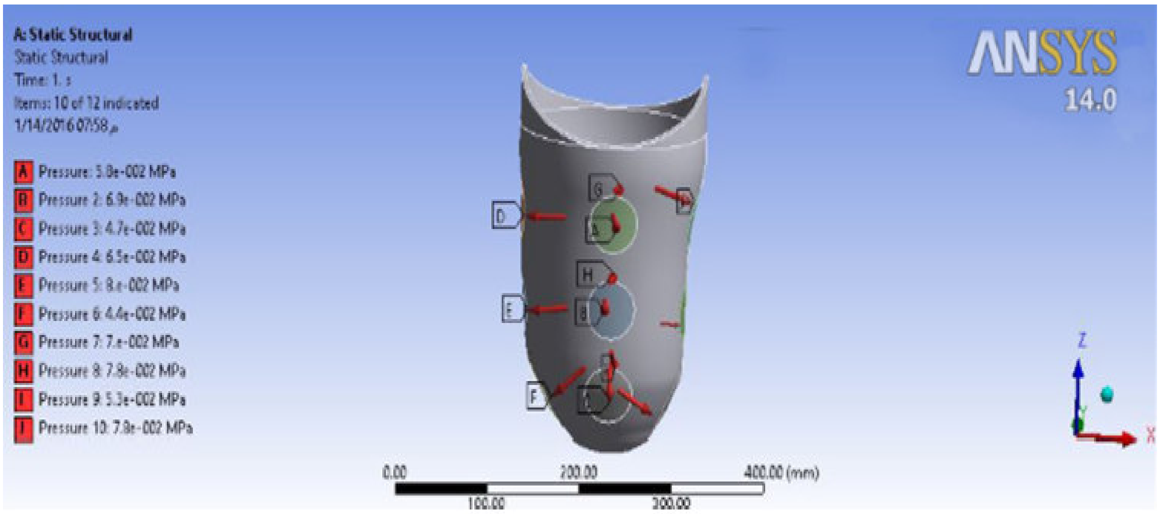


Fig. (3): Applying the pressure forces values and locations to the model in ANSYS workbench version 14 [1].

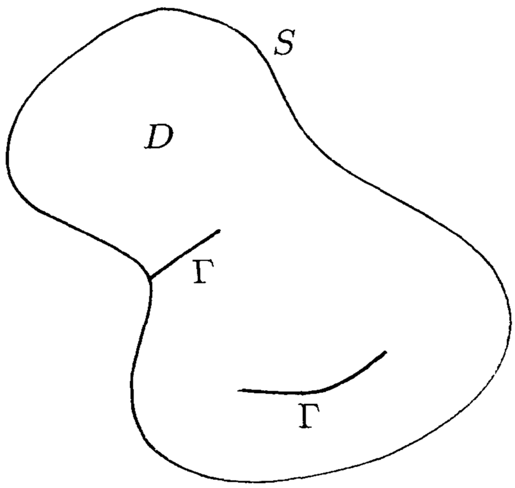


Fig. (3): Show the general domain.

Hence, new integral equation-based methods for the solution of linear elliptic PDEs with discontinuities in its domain are introduced in this paper. The methods are demonstrated on the two-dimensional interior.

$$\nabla^2\varphi(P) = 0 \quad (1)$$

It consists of a region $D$ with boundary S and with shell discontinuities I. In order to specify the problem fully, conditions for points on the boundary and on the shell must be stated. These are the boundary condition and the shell condition. Extension of the methods to exterior problems, three-dimensional problems, and to other PDEs is straightforward through the appropriate choice of integral formulation and Green's function.

In [6],' the SIEs are derived by first assuming that the shells have finite thickness and hence the standard BIE formulation is valid. The shell thickness is then allowed to approach zero. A similar limiting process can be used to derive the BSIE by assuming S to be fixed and taking the limit as the thickness of the shells approaches zero. The integral equation formulations of the interior Laplace problem are stated in Section 2.

In order to derive a particular method, the boundary and shell are divided into uniform elements and the functions defined on the boundary and shell are approximated by a constant on each element. The integral equation method is then derived through collocation. The methods are demonstrated through their application to a test problem where the domain is the unit square and a discontinuity lies between $(\frac{1}{2},\frac{1}{2})$ and $(\frac{1}{2},1)$.

can be regarded as hybrids of their respective standard BIE formulations as given, for example, in [2] and SIE formulations as given in [5].

 

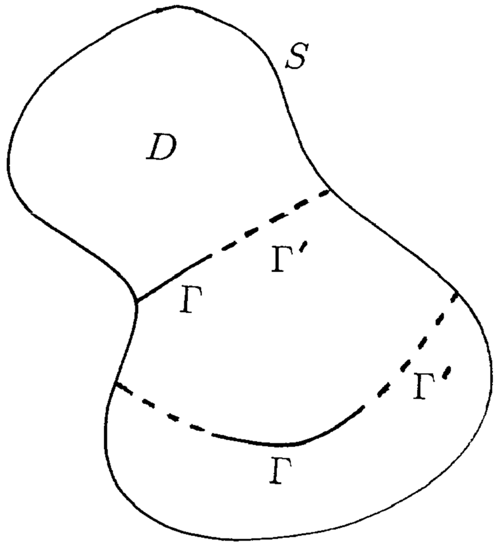


Fig. 4: Preparation for application of the BEM.

**4. Mathematical Model**

In this section the direct and indirect BSIE formulations of the interior Laplace equation are given. The BSIEs can be regarded as hybrids of their respective standard.

BIE formulations as given, for example, in [2] and SIE formulations as given in [5] and Hamdi4 and [6].

**4.1 Notation**

Let the function $v(p)$ for $(p \in S)$ be define as following:

$$v(p) = \lim_{\in \to 0+} \frac{\partial \varphi}{\partial n_p}(p + \in n_p) \quad (p \in S) \tag{2}$$

Where np is the unit outward normal to S at **p.** Each shell is assumed to have two sides or surfaces; let $\Gamma_+$be the upper surface and let $\Gamma_-$ be the lower surface. The potential $\varphi$ and its derivatives are generally discontinuous at the shell; however they take limiting values on $\Gamma_+$and $\Gamma_-$ Let the functions$\varphi + (p), \varphi - (p), v + (p)$ , $v - (p)$, $(p \epsilon \Gamma)$ be defined as following:

$$\varphi + (p) = \lim_{\in \to 0+} \varphi(p + \in n_p) \tag{3}$$

$$\varphi - (p) = \lim_{\in \to 0+} \varphi(p - \in n_p) \tag{4}$$

$$v + (p) = \lim_{\in \to 0+} \frac{\partial \varphi}{\partial n_p}(p + \in n_p) \tag{5}$$

$$v - (p) = \lim_{\in \to 0+} \frac{\partial \varphi}{\partial n_p}(p - \in n_p) \tag{6}$$

It is helpful to introduce the functions$\delta(p)$, $(p)$ , $v(p)$, and V(p) for $(p \epsilon \Gamma)$, which are defined as follows :

$$\delta(p) = \varphi + (p) - \varphi - (p) \quad (p \epsilon \Gamma) \tag{7}$$

$$v(p) = v + (p) - v - (p) \quad (p \epsilon \Gamma) \tag{8}$$

$$\Phi(p) = c(p)\varphi + (p) + (1 - c(p))\varphi - (p) \quad (p \epsilon \Gamma) \tag{9}$$

 

$$V(p) = c(p)v + (p) - (1 - c(p))v - (p) \qquad (p\epsilon\, \Gamma) \tag{10}$$

The geometrical function **c(p)**$(p\varepsilon\ s\ \Gamma)$ is defined to be the angle subtended by the points in the interior region at **p** for points on S and the angle subtended by points in the interior on the face I+ for points on the Socket $\Gamma$ , each divided by $2\pi$.

### 4.2 *Boundary and shell conditions*

The boundary condition has the form:

$$\alpha(p)\varphi(p) + \beta(p)v(p) = \gamma(p) \quad (p\varepsilon S) \tag{11}$$

$$a(p)\delta(p) + b(p)v(p) = f(p) \qquad (p\varepsilon\, \Gamma)$$

The shell conditions are assuming to have the following general form:

$$A(p)\Phi(p) + B(p)V(p) = F(p) \qquad (p\varepsilon\, \Gamma) \tag{12}$$

$$\Phi(p) = \{L\sigma\}_S(p) + \{M\delta\}_\Gamma(P) - \{Lv\}_\Gamma(P) \qquad (P \in \Gamma) \tag{13}$$

$$V(p) = \{M^t\sigma\}_S(p) + \{N\delta\}_\Gamma(P) - \{M^t v\}_\Gamma(P) \qquad (P \in \Gamma) \tag{14}$$

### 4.3 Application of the BSEMs to the test problem

To demonstrate the direct and indirect BSEMs, the test problem with the domain of the unit square and with a discontinuity between $(\frac{1}{2},\frac{1}{2})$ and $(\frac{1}{2},1)$ is introduced. The boundary conditions are such that $\varphi(p) = 1$ for 0< $p_1$< $\frac{1}{2}$ and $p_2 = 1$ , $\varphi(p) = -1$ for $0 < p_1 < 1$ and $p_2 = 1$ on the remainder of the boundary. The shell condition is such that v(p) = V(p) = 0 for all points on the shell. The test problem is illustrating in Fig.5. The discrete forms of the integral operators are computed using a subroutine described in the discrete forms of the integral operators are computed using a subroutine described in [7]. These numerical integrations are computing to sufficient accuracy so the error does not contribute significantly to the overall error in the integral equation methods.

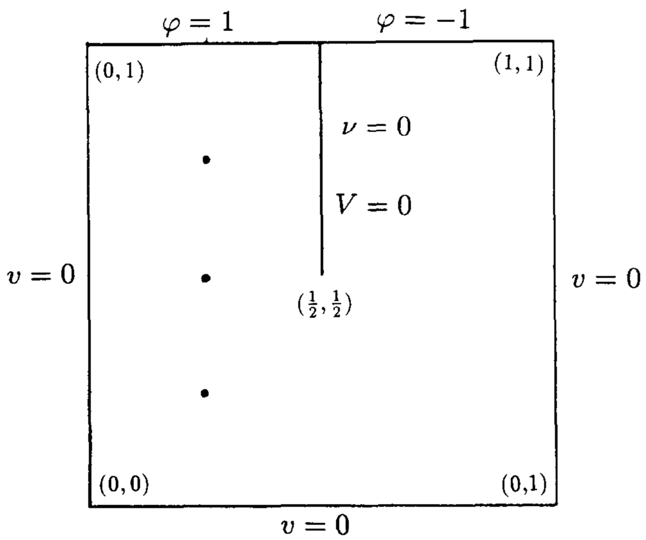


Fig. 5: Illustration of the domain of the test problem.

### 4.4 *Indirect BSEM*

The following linear system of equations follows from equations

$$\begin{bmatrix} I_{ss} & 0_{ss} & -L_{ss} & 0_{s\Gamma} & -M_{s\Gamma} \\ 0_{ss} & I_{ss} & -({M^t}_{ss}+\frac{1}{2}I_{ss}) & 0_{s\Gamma} & -N_{s\Gamma} \\ {D^\alpha}_{ss} & {D^\beta}_{ss} & 0_{ss} & 0_{s\Gamma} & 0_{s\Gamma} \\ 0_{\Gamma s} & 0_{\Gamma s} & -L_{\Gamma s} & I_{\Gamma\Gamma} & -M_{\Gamma\Gamma} \\ 0_{\Gamma s} & 0_{\Gamma s} & -{M^t}_{\Gamma s} & 0_{\Gamma\Gamma} & -N_{\Gamma\Gamma} \end{bmatrix} \begin{bmatrix} \underline{\hat{\varphi}_s} \\ \underline{\hat{\sigma}_s} \\ \underline{\hat{\sigma}_s} \\ \underline{\hat{\Phi}_\Gamma} \\ \underline{\hat{\delta}_\Gamma} \end{bmatrix} = \begin{bmatrix} \underline{0_s} \\ \underline{0_s} \\ \underline{\gamma_\Gamma} \\ \underline{0_\Gamma} \\ \underline{0_\Gamma} \end{bmatrix}$$

The test problem illustrated in *Fig.6*. The direct formulation is the Green's formula; the indirect formulation is the one arising through writing cp as a single layer potential. The boundary functions are discretized in a way similar to the method in the previous section. The number of elements in the experiments is 6, 12, 24, 48, 96, 192, and 384 of uniform size *h*. The results from the application of the standard direct and indirect BEMs are given in *Tab*le 1, where Fig.7 shown BEM solved by COMSOL.

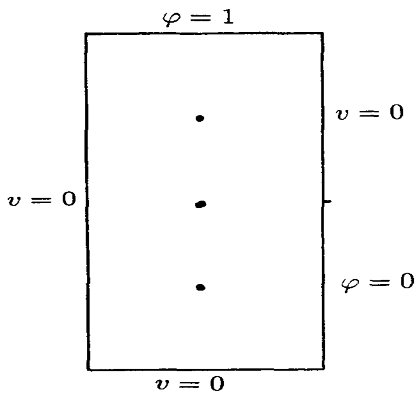


Fig. 6: Equivalent problem to which the BEM is applied.

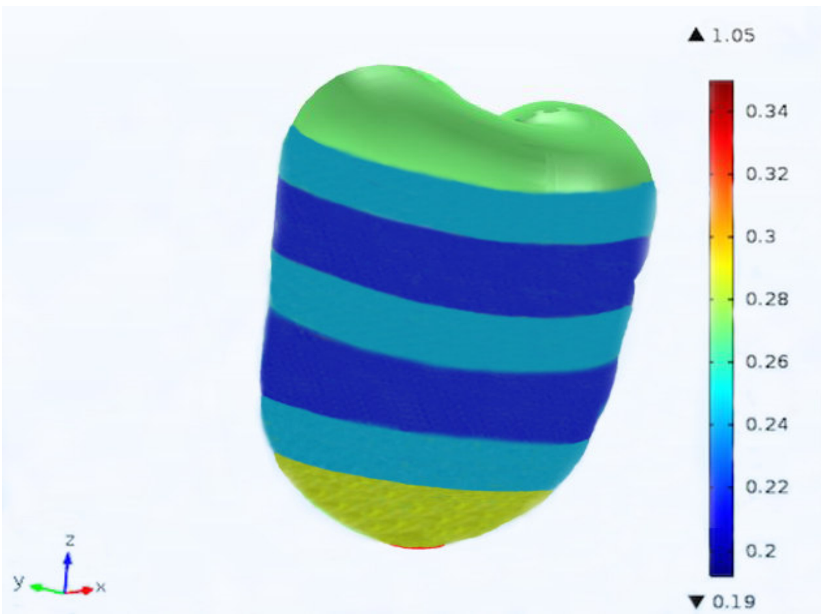


Fig. 7: Equivalent BEM by using COMSOL.

 

Table 1: Solution via the direct BEM.

| H | P=(0.3,0.3) | P=(0.3,0.7) | P=(0.3,0.75) |
|---|---|---|---|
| $\frac{1}{2}$ | 0.325928 | 0.525842 | 0.821443 |
| $\frac{1}{4}$ | 0.287250 | 0.500075 | 0.792895 |
| $\frac{1}{8}$ | 0.225842 | 0.496164 | 0.6977145 |
| $\frac{1}{16}$ | 0.218533 | 0.447135 | 0.769262 |
| $\frac{1}{32}$ | 0.203802 | 0.396208 | 0.765348 |
| $\frac{1}{64}$ | 0.201621 | 0.373434 | 0.763403 |
| $\frac{1}{128}$ | 0.200413 | 0.362574 | 0.762427 |

After completed the hand calculations for BEM for the mention model used in this study the authors found high degree of agreement in deflection values of both BEM and FEM when a comparison has been made between these two techniques, this comparison has made for deflection values at (free edge of loading). An important point is that the BEM is nearer to exact results with very small difference from FEM, so this support the opinion that high degree of accuracy had performed by BEM. Values of deflections and von misses stresses has illustrated in Table 2.

Table 2: Comparison between BEM & FEM results.

| Method | Deflection |
|---|---|
| Boundary Element Method | 0.98 |
| Finite Element Method | 1.03 |

**Conclusion**

In this paper, FEM have described state of the art mesh generation procedure using finite element method. It obtained in the simulation show good agreement with the measured loads in various regards. In addition, the results summarized that assimilating local submissive properties within socket wall can be an effective methods to distribute maximum stress areas and to relief contact pressure between the socket and stump. In addition, a new numerical method termed the boundary and shell element method has been described and demonstrated on the two-dimensional interior Laplace problem. Similar equations and methods can be derived for other linear elliptic problems through appropriate change of Green's function, in exterior as well as interior domains, and they are applicable to three-dimensional problems as well as two. The BSEM is a useful generalization of the standard BEM, for the application of the standard BEM in many cases result in an inefficient method when the shells r' need to be large with respect to the size of S and r. Moreover, the way in which r' is joined to the boundary S could result in numerical problems with the overall method.

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