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# Nonlinear Channel Estimation Error Effect on Capacity of MIMO System

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**Abstract**
This paper presents the effect of nonlinearity and the effect of estimation error on the channel capacity in MIMO system. We consider a nonlinear MIMO channel, and compare the capacity of Rayleigh MIMO channel model with estimation error with the nonlinear model, at different estimation errors. We consider the estimation error as Gaussian distribution. The simulation results show that the channel capacity of linear and nonlinear MIMO channels are sensitive to the channel estimation error, and due to the nonlinearity, the capacity is less than linear channel.
**Keywords:** MIMO, MIMO Modeling, channel capacity, channel estimation error, nonlinear MIMO channel.

## 1- Introduction

Wireless communications undergoes a dramatically change in recent years. More and more people are using modern communication services, thus increasing the need for more capacity in transmissions. Since bandwidth is a limited resource, the strongly increased demand in high transmission capacity has to be satisfied by a better use of existing frequency bands and channel conditions. One of the recent technical breakthroughs, which will be able to provide the necessary data rates, is the use of multiple antennas at both transmitter and receiver. These systems are referred to as multiple-input multiple-output (MIMO) wireless systems. Initial theoretical studies from Foschini [4] and Telatar [3], as well as other pioneer works, have shown the potential of such systems.
Such MIMO systems are capable of realizing higher throughput without increasing bandwidth or transmitter power. It is obvious that such a gain in transmissions rates and reliability comes at the cost of higher computational requirements. The capacity increases linearly with the number of antennas for fixed power and bandwidth [4].

The communication channel is hit by hardware impairments due to nonlinearity; the channel capacity may be affected. The characteristic of nonlinear devices can be represented either as instantaneous nonlinearities or nonlinearities with memory. Nonlinear solid-state devices, such as high power amplifiers (HPA), low noise amplifiers, and other circuit components in the transmission / reception chains, are usually modeled as memoryless devices whose nonlinearity appears when the instantaneous input signal power fluctuates and approaches the saturation level of the device [9], [10]. Other nonlinear devices such as fiber optics are usually modeled as nonlinear functions with memory, and their nonlinear effects originate from the physical limitations of the transmission channel [11].

In [13] capacity of a nonlinear optical channel with finite memory is studied.

In this paper, the estimation error effects on the channel capacity of MIMO nonlinear channel is studied and compare the results with Rayleigh channel.

The rest of this paper is organized as follows. In Section II, the system model is introduced with the capacity analysis of the Rayleigh channel, the estimation error, and the nonlinear channel model.

Finally, simulation results are presented to compare the capacity with and without the channel estimation errors of MIMO Rayleigh channel and MIMO nonlinear channel in Section III.

## 2- System Model

Consider a MIMO channel with t transmit antennas and r receive antennas as shown in Fig. 1, we consider a Rayleigh channel model. The received vector signal $y \in C^r$ is [2]

$$y = Hx + n \qquad (1)$$

where H is $r \times t$ a complex matrix, $x \in C^t$ is the transmitted vector signal and n is zero-mean complex Gaussian noise vector with independent real and imaginary parts. The matrix H characterizes the channel transition property, in which each entry $h_{ij}$ represents the path gain from j-th transmitter antenna to i-th receiver antenna. Reasonably, the noise of each receiving antenna is independent, i.e., the covariance matrix of n is $\varepsilon[nn^T]=I_r$ where ε is the expectation operator. In this paper we consider the 'channel' includes the effects of the transmitter filter, the transmission medium, and the receiver matched filter. It is convenient to denote the MIMO channel (H) in matrix notation as:

$$H=\begin{bmatrix} h11 & \cdots & h1t \\ \vdots & \ddots & \vdots \\ hr1 & \cdots & hrt \end{bmatrix} \qquad (2)$$

The channel matrix H defines the input - output relation of the MIMO system and also known as the channel transfer function. First, H is considered as Rayleigh channel and has complex elements and well known at receiver.

Second, H is considered as nonlinear channel and has complex elements and make comparison between Rayleigh channel and nonlinear channel with normal distributed error (Gaussian error), i.e. estimation error with different percentage error.
H is taken a hyperbolic tangent function as an example of nonlinear channel, and each element of H in (2) as a smooth clipping function as shown in Fig. 2 [7]

$$H_{ij}(a) = \tanh(a) \qquad (3)$$

Where a is a complex

**A. Channel Capacity**

Information theory was formally initiated in 1948 by Shannon in his pioneering work [8] in which he showed that reliable communication between a transmitter and receiver was possible even in the presence of noisy channel. Shannon defined capacity as the mutual information maximized over all possible input distributions [9].

$$c = B\log_2(1 + \frac{S}{N}) \qquad (4)$$

Where c is the channel capacity in bits/sec, B is the bandwidth of the channel in hertz, S is the average received signal power over the bandwidth, and N is the average noise power over the bandwidth.

For a given channel, the maximum information transmission rate at which the transmitter and receiver can communicate with an arbitrarily low probability of error and no delay constraints is called channel capacity.

A common representation of the channel capacity is within a unit bandwidth of the channel and can be expressed in bps/Hz. This representation is also known as spectral (bandwidth) efficiency. The definition of entropy and mutual information is the same when the channel input and output are vectors instead of scalars, as in the MIMO channel. Thus, the Shannon capacity of the MIMO AWGN channel is based on its maximum mutual information.

MIMO channel capacity depends heavily on the statistical properties and antenna elements correlations of the channel. Representating the input and output of a memoryless channel with the random variables x and y respectively, the channel capacity is defined as the maximum of the mutual information between x and y [2]

$$c = maxp(x)I(x;y) \qquad (5)$$

A channel is said to be memoryless if the probability distribution of the output depends only on the input. $p(x)$ is the probability distribution function (pdf) of the input $x$ , and $I(x;y)$ is the mutual information between input $x$ and output y.

The channel capacity for MIMO systems is higher than that in SIMO and MISO channels [9].

The capacity of MIMO system with a total transmit power constraint P and perfect channel state information (CSI) at the receiver side has been characterized in [1]. [2]. [10]. For a stationary memoryless channel, the ergodic capacity is given by [2]

$$c = \log_2 det(I_r + \frac{SNR}{t}HH^*) \qquad (6)$$

Where $()^*$refers to the conjugate - transpose of the matrix and $I_r$ is unitary matrix.
Equation (6) may be decomposed as [9]

$$c = \sum_{i=1}^{b}\log_2(1 + \frac{SNR}{t}\lambda_i) \qquad (7)$$

where b is the rank of H and $\lambda$ $(i = 1,2,\ldots.,b)$ denotes the positive eigenvalues of $HH^*$. Equation (7) expresses the spectral efficiency of MIMO channel as the sum of the capacities of b SISO channels with corresponding channel gains
$\sqrt{\lambda}$ $(i = 1,2,\ldots..,b)$ [9].

**B. Raleigh channel with estimation error**

The goal of channel estimation in a MIMO system is to obtain an accurate estimate of the current channel matrix suitable for the MIMO processing at the receiver side. It is theoretically known that MIMO system can provide higher channel capacity than a conventional wireless system. The MIMO capacity is increased as a function of signal to noise ratio (SNR) and a number of antennas at both transmitter and receiver. In this section we focus on the effect of estimation error on the Rayleigh channel and compare the results with the Rayleigh channel without error. Now, the case where H is Rayleigh channel is considered, and imperfectly known to the receiver, and $\widetilde{H}$ is the estimated channel matrix, and $\widetilde{H} = H + E$ where E is the channel estimation error matrix.
The values of E as a scaled value of $H$ (0.1, 0.3, and 0.5), the channel estimation errors is assumed as a Gaussian additive noise error model [11] with variance $\sigma^2$ , then the capacity with channel estimation error is given by [9]

$$c = \log_2 det(I_r + \frac{SNR \times HH^*}{(1+\sigma^2)\times t}) \qquad (8)$$

**C. Nonlinear Channel**

Consider a wireless MIMO system where a transmitted signal x is passed through nonlinear MIMO channel (tanh) that shown in Fig.2. The nonlinearity in the channel appears due to the nonlinear amplification of the (HPA). In this analysis, the effect of channel nonlinearity on the capacity of MIMO system is focused. The information theoretic capacity of MIMO channel under the nonlinearity can be expressed as [12]:

$$c = [\log_2( det(I_r + \frac{SNR}{t} \alpha\alpha^* H R_{xx} H^*))] \quad (9)$$

where $\alpha$ represents the distortions caused by the non-linear channel.
In [13], after convergence the authors considered the channel becomes a nearly linear one.

## 3. Simulation Results

The performance (capacity per bandwidth against SNR) of nonlinear channel (tanh) model is discussed, and compared with the Rayleigh channel, and the channel is perfect known at the receiver i.e. without estimation error as shown in Fig. 3.
The estimation errors with 0.1, 0.3, and 0.5 as scaled value of H is considered, the capacity is reduced when the error is increased, as shown in Fig. 4a, Fig. 4b and Fig. 4c respectively.

## 4. Conclusions

This paper provided a thorough investigation of the capacity achieved by MIMO Rayleigh fading channels, and nonlinear MIMO channel (tanh model), taking into consideration the effect of channel estimation errors at the receiver. Our numerical results showed that the capacity decreases with increasing the channel estimation error. Due to the nonlinearity, the channel capacity is reduced.

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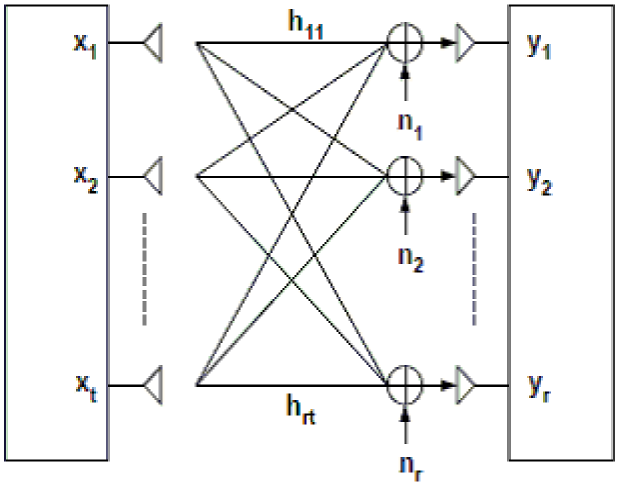



Figure 1 MIMO System with “t” transmit and “r” receive antennas

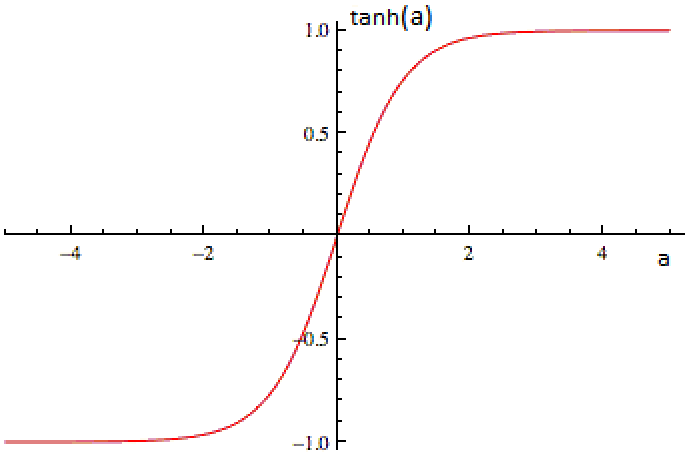



Figure 2 A simple example of nonlinear channel

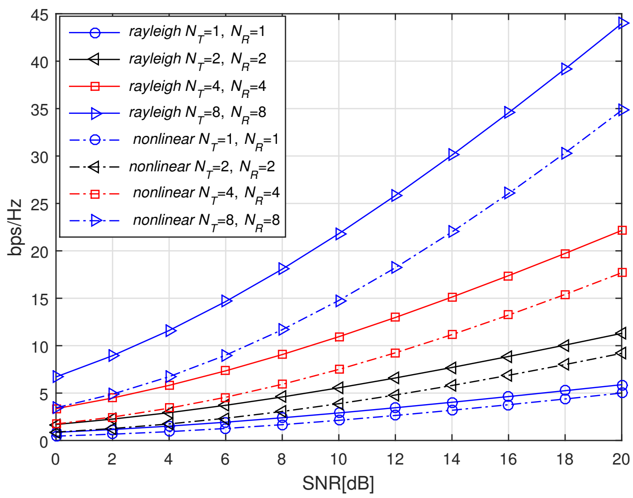
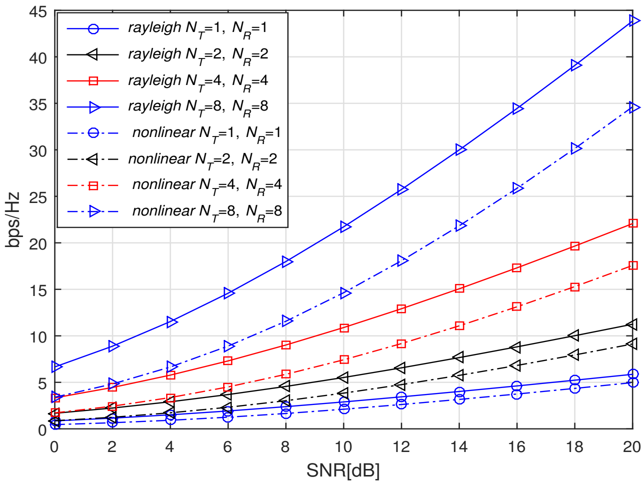
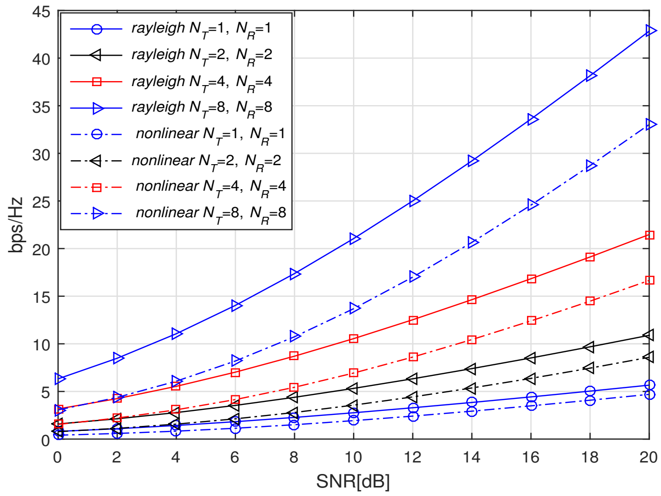


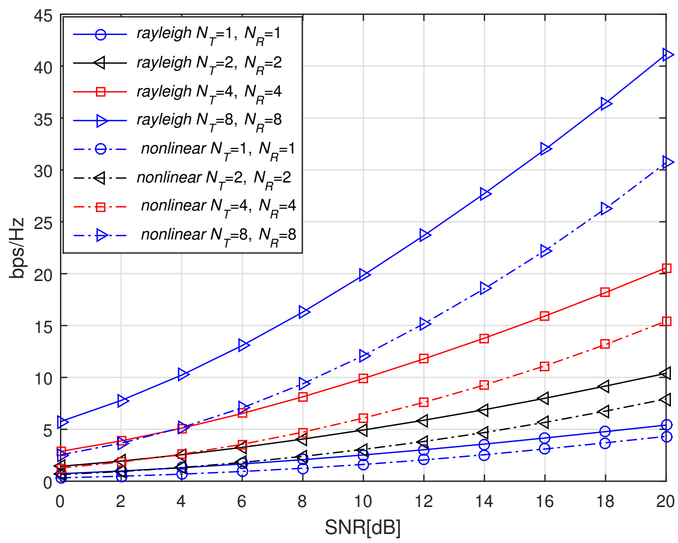

Figure 3 MIMO nonlinear channel (tanh) and Rayleigh channel without estimation error.




(a)




(b)




(c)

Figure 4 MIMO nonlinear channel (tanh) and Rayleigh channel with, a) estimation error=0.1 b) estimation error=0.3, and c) estimation error=0.5 respectively