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On Fixed Point Theorems in Intuitionistic Fuzzy Metric Space using E.A Property

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Abstract
The aim of this paper is to prove some common fixed point theorems for intuitionistic fuzzy metric space using E.A property.

Introduction
In 1957, Fuzzy set was defined by Zadeh [15], Kramosil and Michalek [9] introduced fuzzy metric space. In 1986, Jungck [6] introduced the notion of compatible mappings and utilized the same to improve commutativity conditions in common fixed point theorems . This concept has been frequently employed to prove existence theorems on common fixed points. However, the study of common fixed points of non-compatible mappings is also equally interesting which was initiated by Pant [12] . Recently, Aamri and Moutawakil [1] and Liu et al. [11] respectively, defined the property (E.A) and proved some common fixed point theorems in metric spaces. Imdad et.al. [5] extended the results of Aamri and Moutawakil [1] to semi metric spaces and Kubiaczyk and Sharma[10] defined the property (E.A) in PM spaces and used it to prove results on common fixed points wherein authors claim to prove their results for strict contractions which are merely valid upto contraction. In this paper , we prove the fixed point theorems for weakly compatible mappings using an implicit relation in intuitionistic fuzzy metric space satisfying the common property(E.A) .

2 Preliminaries

Definition 2.1 [13]: A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ is satisfying the following conditions:

(i) $*$ *is commutative and associative*

(ii) $*$ *is continuous;*

(iii) $a * 1 = a$ *for all* $a \epsilon [0,1]$;

(iv) $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *for all* $a, b, c, d \epsilon [0,1]$.

Definition 2.2 [13]: A binary operation $\lozenge: [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $\lozenge$ is satisfying the following conditions:

(i) $\lozenge$ is commutative and associative;

(ii) $\lozenge$ is continuous;

(iii) $a \lozenge 0 = a$ *for all* $a \epsilon [0,1]$;

(iv) $a \lozenge b \leq c \lozenge d$ *whenever* $a \leq c$ *and* $b \leq d$ *for all* $a, b, c, d \epsilon [0,1]$

Definition 2.3 [2]: A 5 – tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-space) if X is an arbitrary set, $*$ is a continuous t- norm , $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \epsilon X$ and $t, s > 0$,

(i) $M(x,y,t) + N(x,y,t) \leq 1$; for all $x, y \epsilon X$ and $t > 0$

(ii) $M(x,y,0) = 0$; for all $x, y \epsilon X$;

(iii) $M(x,y,t) = 1$ *for all* $t > 0$ *if and only if* $x = y$;

(iv) $M(x,y,t) = M(y,x,t)$; for all $x, y \epsilon X$ and $t > 0$;

(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$ *for all* $x, y, z \epsilon X$ *and* $s, t > 0$;

(vi) $M(x,y,.): [0,\infty) \to [0,1]$ *is continuous;*

(vii) $lim_{t\to\infty} M(x,y,t) = 1$ *for all* x, y *in* X *and* $t > 0$;

(viii) $N(x,y,0) = 1$ for all $x, y \epsilon X$;

(ix) $N(x,y,t) = 0$ *for all* $x, y \epsilon X$ *and* $t > 0$ *if and only if* $x = y$;

(x) $N(x,y,t) = N(y,x,t)$ *for all* $x, y \epsilon X$ *and* $t > 0$;

(xi) $N(x,y,t) \lozenge N(y,z,s) \geq N(x,z,t+s) for all\ x,y,z \in X\ and\ s,t > 0;$

(xii) $N(x,y,.): [0,\infty) \to [0,1]\ is\ continuous\ for\ all\ x,y \in X\ ;$

(xiii) $lim_{t\to\infty} N(x,y,t) = 0\ for\ all\ x,y \in X\ and\ t > 0;$

Then (M,N) is called an intuitionistic fuzzy metric space. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark (2.1): Every fuzzy metric space $(X,M,*)$ is an intuitionistic fuzzy metric space of the form $(X,M,1-M,*,\lozenge)$ such that t- norm $*$ and t-conorm $\lozenge$ are associated as

$$x \lozenge y = 1 - \big((1-x)*(1-y)\big)\quad for\ all\ x,y \in X$$

Example (2.1): Let (X,d) be a metric space, define t-norm $a*b = Min\{a,b\}$ and t-conorm $a \lozenge b = Max\{a,b\}$ and for all $x,y \in X$ and $t > 0$,

$$M_d(x,y,t) = \frac{t}{t+d(x,y)}\ , N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$$

Then $(X,M,N,*,\lozenge)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M,N) induced by the metric d the standard intuitionistic fuzzy metric.

Definition 2.4 [13]: Two self mappings A and B of an intuitionistic fuzzy metric space $(X,M,N,*,\lozenge)$ is said to be non-compatible if there exists at least one sequence $\{x_n\}$ such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Bx_n = z\ for\ some\ z\ in\ X\ but\ neither$$
$$lim_{n\to\infty} M(ABx_n, BAx_n, t) \neq 1\ , and\ lim_{n\to\infty} N(ABx_n, BAx_n, t) \neq 0$$

Or the limit does not exists.

Definition 2.5 [4]: Let $(X,M,N,*,\lozenge)$ be an intuitionistic fuzzy metric space. Let A and B be self maps on X. Then a point x in X is called a coincidence point of A and B iff $Ax = Bx$. In this case, $w = Ax = Bx$ is called a point of coincidence of A and B.

In 1996, Jungck [6] introduced the notion of weakly compatible maps as follows.

Definition 2.6 [7]: A pair of self mappings (A,B) of a intuitionistic fuzzy metric space $(X,M,N,*,\lozenge)$ is said to be weakly compatible if they commute at their coincidence points i.e $Ax = Bx$ for some x in X, then $ABx = BAx$.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Definition 2.7: Let $(X,M,N,*,\lozenge)$ be an intuitionistic fuzzy metric space. Two self-mapping $f,g: X \to X$ are said to be compatible if and only if

(a) *Sequence* $\{x_n\}$ *in* X *is said to be Cauchy sequence if, for all* $t > 0$ *and* $p > 0$,

$$M(x_{n+p}, x_n, t) \to 1\ , N(x_{n+p}, x_n, t) \to 0\ for\ n \to \infty\ .$$

(b) *A Sequence* $\{x_n\}$ *in* X *is said to be Convergent to a point* $x \epsilon X$ *if ,for all* $t > 0$,

$$M(x_n, x, t) \to 1,\ N(x_n, x, t) \to 0\, for\ n \to \infty.$$

Definition 2.8: *Let* $(X,M,N,*,\lozenge)$ *is an intuitionistic fuzzy metric space.* Two self-mappings $f,g: X \to X$ are said to be compatible if and only if $M(fgx_n, gfx_n, t) \to 1$ and $N(fgx_n, gfx_n, t) \to 0$ for all $t > 0$ whenever $\{x_n\}$ in X such that $fx_n, gx_n \to z$ for some $z \in X$.

Definition 2.9: Let $(X,M,N,*,\lozenge)$ be a intuitionistic fuzzy metric space. Two self-mappings $f,g: X \to X$ are said to satisfy the (E.A) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$$

Definition 2.10: Two pairs $\{f,g\}$ and $\{p,q\}$ of self-mappings of a intuitionistic fuzzy metric space $(X,M,N,*,\lozenge)$ are said to satisfy the common property $(E.A)$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X and some z in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} py_n = \lim_{n\to\infty} qy_n = z$$

Example 2.2: Let $(X,M,N,*,\lozenge)$ be a intuitionistic fuzzy metric space with $X = [-1,1]$ and

$$M(x,y,t) = \begin{cases} e^{\frac{-|x-y|}{t}} & if\ t > 0\ for\ all\ x,y \in X \\ 0 & if\ t = 0. \end{cases}$$

Define self- mappings f, g, p *and* q on X as $fx = \frac{x}{2}$; $gx = \frac{-x}{2}$; $px = \frac{x}{4}$ and $qx = \frac{-x}{4}$; for all $x \epsilon X$.
Then with sequences $\{x_n\} = \frac{1}{n}$ and $\{y_n\} = \frac{-1}{n}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \lim_{n\to\infty} py_n = \lim_{n\to\infty} qy_n = 0$$

This shows that the pairs $\{f, g\}$ and $\{p, q\}$ share the common property $(E.A)$.

Definition 2.11: Two self mappings f and g of a Fuzzy intuitionistic metric space $(X, M, N, *, \Diamond)$ sare said to be weakly compatible if the mappings commute at their coincidence points i.e $fx = gx$ for some $x \in X$ implies

$$fgx = gfx$$

We shall call $w = fx = gx$ a point of coincidence of f and g.

Definition 2.12: Implicit Relation

Let Φ be the set of all real continuous functions $\phi(\mathbb{R}^7): [0,1] \to \mathbb{R}$, non-decreasing in the argument satisfying the following condition:
For all $x, y \in X$ and: $\phi: [0,1]^7 \to [0,1]$ such that $\phi(t, 1, 1, t, t, 1, t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of p, f, q and g.

Lemma 2.13 [8]: Let X be a set f and g be owc self maps of X. If f and g have a unique point of coincidence, $w = fx = gx$ then w is the unique common fixed point of f and g.

3 Main Results

Theorem 3.1: Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space and let p, q, f *and* g be self – mappings of X. Let the pairs $\{p, f\}$ and $\{q, g\}$ be owc. If there exists $w \in (0,1)$ for all $x, y \in X$ and $t > 0$

$$M(px, qy, wt) \geq \Phi\begin{pmatrix} M(fx, gy, t), M(fx, px, t), M(qy, gy, t), M(px, gy, t), \\ M(qy, fx, t), M(px, qy, t), M(fx, gy, t) * M(px, px, t) \end{pmatrix}$$

and

$$N(px, qy, wt) \leq \Phi\begin{pmatrix} N(fx, gy, t), N(fx, px, t), N(qy, gy, t), N(px, gy, t), \\ N(qy, fx, t), N(px, qy, t), N(fx, gy, t) * N(px, px, t) \end{pmatrix} \quad ...(1)$$

Proof: Let the pairs $\{p, f\}$ and $\{q, g\}$ be owc, so there are points $x, y \in X$ such that $px = fx$ and $qy = gy$. We claim that $px = qy$. If not, by inequality (1)

$$M(px, qy, wt) \geq \Phi\begin{pmatrix} M(fx, gy, t), M(fx, px, t), M(qy, gy, t), M(px, gy, t), \\ M(qy, fx, t), M(px, qy, t), M(fx, gy, t) * M(px, px, t) \end{pmatrix}$$

$$M(px, qy, wt) \geq \Phi\begin{pmatrix} M(px, qy, t), M(px, px, t), M(qy, qy, t), M(px, qy, t), \\ M(qy, px, t), M(px, qy, t), M(px, qy, t) * M(px, px, t) \end{pmatrix}$$

$$= \Phi\begin{pmatrix} M(px, qy, t), 1, 1, M(px, qy, t), \\ M(qy, px, t), M(px, qy, t), M(px, qy, t) \end{pmatrix}$$

$$> M(px, qy, t)$$

$$N(px, qy, wt) \leq \Phi\begin{pmatrix} N(fx, gy, t), N(fx, px, t), N(qy, gy, t), N(px, gy, t), \\ N(qy, fx, t), N(px, qy, t), N(fx, gy, t) * N(px, px, t) \end{pmatrix}$$

$$N(px, qy, wt) \geq \Phi\begin{pmatrix} N(px, qy, t), N(px, px, t), N(qy, qy, t), N(px, qy, t), \\ N(qy, px, t), N(px, qy, t), N(px, qy, t) * N(px, px, t) \end{pmatrix}$$

$$= \Phi\big(N(px, qy, t), 0, 0, N(px, qy, t), N(qy, px, t), N(px, qy, t), N(px, qy, t)\big)$$

$$< N(px, qy, t)$$

A contradiction, therefore $px = qy$, ie $px = fx = qy = gy$. Suppose that there is another point z Such that $pz = fz$ then by (1). We have $pz = fz = qy = gy$, so $px = pz$ and $w = px = fx$ is the unique point of coincidence of p and f. Similarly there is a unique point $z \in X$ such that $z = qz = gz$.
Thus z is a common fixed point of p, q, f and g. The uniqueness of the fixed point holds from (1).

Theorem 3.2: Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric space and let p, q, f and g be self-mappings of X. Let the pairs $\{p, f\}$and $\{q, g\}$ be owc. If there exists $w \in (0,1)$ for all $x, y \in X$ and $t > 0$

$$M(px,qy,wt) \geq \begin{pmatrix} M(fx,gy,t) * M(fx,px,t) * M(qy,gy,t) * M(px,gy,t) \\ * M(qy,fx,t) * M(px,qy,t) * M(fx,gy,t) \end{pmatrix}$$

and

$$N(px,qy,wt) \leq \begin{pmatrix} N(fx,gy,t) * N(fx,px,t) * N(qy,gy,t) * N(px,gy,t) \\ * N(qy,fx,t) * N(px,qy,t) * N(fx,gy,t) \end{pmatrix}$$

...(2)

Then there exists a unique common fixed point of p, q, f and g.

Proof: Let the pairs $\{p, f\}$ and $\{q, g\}$ be owc, so there are points $x, y \in X$ such that $px = fx$ and $qy = gy$. We claim that $px = qy$. If not, by inequality (2) We have

$$M(px,qy,wt) \geq \begin{pmatrix} M(fx,gy,t) * M(fx,px,t) * M(qy,gy,t) * M(px,gy,t) \\ * M(qy,fx,t) * M(px,qy,t) * M(fx,gy,t) \end{pmatrix}$$

$$= \begin{pmatrix} M(px,qy,t) * M(px,px,t) * M(qy,qy,t) * M(px,qy,t) \\ * M(qy,px,t) * M(px,qy,t) * M(px,qy,t) \end{pmatrix}$$

$$= \left(M(px,qy,t) * 1 * 1 * M(px,qy,t) * M(qy,px,t) * M(px,qy,t) * M(px,qy,t)\right)$$

$$> M(px,qy,t)$$

$$N(px,qy,wt) \leq \begin{pmatrix} N(fx,gy,t) * N(fx,px,t) * N(qy,gy,t) * N(px,gy,t) \\ * N(qy,fx,t) * N(px,qy,t) * N(fx,gy,t) \end{pmatrix}$$

$$= \begin{pmatrix} N(px,qy,t) * N(px,px,t) * N(qy,qy,t) * N(px,qy,t) \\ * N(qy,px,t) * N(px,qy,t) * N(px,qy,t) \end{pmatrix}$$

$$= \left(N(px,qy,t) * 0 * 0 * N(px,qy,t) * N(qy,px,t) * N(px,qy,t) * N(px,qy,t)\right)$$

$$< N(px,qy,t)$$

Thus we have, $px = qy$, ie $px = fx = qy = gy$ .Suppose that there is another point z Such that $pz = fz$ then by (2) we have $pz = fz = qy = gy$, so $px = pz$ and $w = px = fx$ is the unique point of coincidence of p and f. Similarly there is a unique point $z \in X$ such that $z = qz = gz$. Thus z is a common fixed point of p, q, f and g.

Corollary 3.3 Let $(X, M, N ,*,\Diamond)$ be a complete fuzzy metric space and let p, q, f and g be self- mappings of X. Let the pairs $\{p, f\}$ and $\{q, g\}$ be owc. If there exists $w \in (0,1)$ for all $x, y \in X$ and $t > 0$

$$M(px,qy,wt) \geq \begin{pmatrix} M(fx,gy,t) * M(fx,px,t) * M(qy,gy,t) * M(px,gy,t) \\ * M(qy,fx,2t) * M(px,qy,t) * M(fx,gy,t) \end{pmatrix}$$

and

$$N(px,qy,wt) \leq \begin{pmatrix} N(fx,gy,t) * N(fx,px,t) * N(qy,gy,t) * N(px,gy,t) \\ * N(qy,fx,2t) * N(px,qy,t) * N(fx,gy,t) \end{pmatrix}$$

...(3)

Then there exists a unique common fixed point of p, q, f and g.

Proof: We have

$$M(px,qy,wt) \geq \begin{pmatrix} M(fx,gy,t) * M(fx,px,t) * M(qy,gy,t) * M(px,gy,t) \\ * M(qy,fx,2t) * M(px,qy,t) * M(fx,gy,t) \end{pmatrix}$$

$$M(px,qy,wt) \geq \begin{pmatrix} M(fx,gy,t) * M(fx,px,t) * M(qy,gy,t) * M(px,gy,t) \\ * M(fx,gy,t) * M(gy,qy,t) * M(px,qy,t) * M(fx,gy,t) \end{pmatrix}$$

$$\geq \begin{pmatrix} M(fx,gy,t) * M(fx,px,t) * M(qy,gy,t) * M(px,gy,t) \\ * M(px,qy,t) * M(fx,gy,t) \end{pmatrix}$$

$$\geq \begin{pmatrix} M(px,qy,t) * M(px,px,t) * M(qy,qy,t) * M(px,qy,t) \\ * M(qy,px,t) * M(px,qy,t) * M(px,qy,t) \end{pmatrix}$$

$$= \begin{pmatrix} M(px,qy,t) * 1 * 1 * M(px,qy,t) * M(qy,px,t) \\ * M(px,qy,t) * M(px,qy,t) \end{pmatrix}$$

$$> M(px,qy,t)$$

$$N(px,qy,wt) \le \begin{pmatrix} N(fx,gy,t) * N(fx,px,t) * N(qy,gy,t) * N(px,gy,t) \\ * N(qy,fx,2t) * N(px,qy,t) * N(fx,gy,t) \end{pmatrix}$$

$$N(px,qy,wt) \le \begin{pmatrix} N(fx,gy,t) * N(fx,px,t) * N(qy,gy,t) * N(px,gy,t) \\ * N(fx,gy,t) * N(gy,qy,t) * N(px,qy,t) * N(fx,gy,t) \end{pmatrix}$$
$$\le \begin{pmatrix} N(px,qy,t) * N(px,px,t) * N(qy,qy,t) * N(px,qy,t) \\ * N(px,qy,t) * N(px,qy,t) \end{pmatrix}$$
$$\le \begin{pmatrix} N(px,qy,t) * N(px,px,t) * N(qy,qy,t) * N(px,qy,t) \\ * N(qy,px,t) * N(px,qy,t) * N(px,qy,t) \end{pmatrix}$$
$$= \begin{pmatrix} N(px,qy,t) * 0 * 0 * N(px,qy,t) * N(qy,px,t) \\ * N(px,qy,t) * N(px,qy,t) \end{pmatrix}$$
$$< N(px,qy,t)$$

And therefore from theorem 3.2, p, q, f and g have a common fixed point.

Corollary 3.4: Let $(X, M, N, *, \Diamond)$ be a complete fuzzy metric space and let p, q, f and g be self –mappings of X. Let the pairs $\{p, f\}$and $\{q, g\}$ be owc. If there exists $w \in (0,1)$ for all $x, y \in X$ and $t > 0$

$$M(px,qy,wt) \ge M(fx,gy,t)$$

and

$$N(px,qy,wt) \le N(fx,gy,t) \qquad ...(4)$$

Then there exists a unique common fixed point of p, q, f and g.

The proof follows from Corollary 3.3.

Theorem 3.5: Let $(X, M, N, *, \Diamond)$ be a complete fuzzy metric space. Then continuous self- mappings f and g of X have a common fixed point in X if and only if there exists a self mappings p of X such that the following conditions are satisfied

(i) $p(X) \subset g(X) \cap f(X)$

(ii) The pairs $\{p, f\}$ and $\{p, g\}$ are weakly compatible,

(iii) There exists a point $w \in (0,1)$ such that for all $x, y \in X$ and $t > 0$

$$M(px,py,wt) \ge M(fx,gy,t) * M(fx,px,t) * M(py,gy,t) * M(px,gy,t) * M(py,fx,t)$$

and

$$N(px,py,wt) \le N(fx,gy,t) * N(fx,px,t) * N(py,gy,t) * N(px,gy,t) * N(py,fx,t) \qquad ...(5)$$

Then p, f and g have a unique common fixed point.

Proof: Since compatible implies owc, the result follows from Theorem 3.2.

Theorem 3.6: Let $(X, M, N, *, \Diamond)$ be a complete fuzzy metric space and let p and q be self- mappings of X. Let p and q are owc. If there exists $w \in (0,1)$ for all $x, y \in X$ and $t > 0$

$$M(fx,fy,wt) \ge \alpha\, M(px,py,t) + \beta \min\{M(px,py,t), M(fx,px,t), M(fy,py,t), M(fx,py,t)\}$$

and

$$N(fx,fy,wt) \le \alpha\, N(px,py,t) + \beta \min\{N(px,py,t), N(fx,px,t), N(fy,py,t), N(fx,py,t)\} \qquad ...(8)$$

For all $x, y \in X$ where $\alpha, \beta > 0$, $\alpha + \beta > 1$. Then p and f have a unique common fixed point.

Proof: Let the pairs $\{p, f\}$ be owc, so there are points $x \in X$ such that $px = fx$. Suppose that there exist another point $y \in X$for which $py = fy$. We claim that $fx = fy$. By inequality (8)

$$M(fx,fy,wt) \ge \alpha\, M(px,py,t) + \beta \min\{M(px,py,t), M(fx,px,t), M(fy,py,t), M(fx,py,t)\}$$

$$= \alpha M(fx,fy,t) + \beta \min\{M(fx,fy,t), M(fx,fx,t), M(fy,fy,t), M(fx,fy,t)\}$$

$$= (\alpha + \beta)M(fx, fy, t)$$
$$N(fx, fy, wt) \leq \alpha\, N(px, py, t) + \beta \min\{N(px, py, t), N(fx, px, t), N(fy, py, t), N(fx, py, t)\}$$
$$= \alpha N(fx, fy, t) + \beta \min\{N(fx, fy, t), N(fx, fx, t), N(fy, fy, t), N(fx, fy, t)\}$$
$$= (\alpha + \beta)N(fx, fy, t)$$

A contradiction, since $(\alpha + \beta) > 1$. Therefore $fx = fy$. Therefore $px = py$ and px is unique.
From lemma 2.13 , p and f have a unique fixed point.

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