Building Project Activities/Tasks Time Scheduling using a Linear Programming Model

P.E. Amiolemhen* J. Akpomwomwo
Department of Production Engineering, University of Benin, Nigeria

Abstract
The problem with the management of construction works is the ability to create a good work breakdown structure (WBS); determine inter-relationship among tasks; determine duration for each of the tasks and to create a workable schedule. Most locally handled construction work always experience such problems as: increase cost of project work; project work not delivered within the expected time (schedule slippage); resource constraint (limitation) etc. This study involves data collection from project supervisor; identification of the activities/tasks in the project; precedence relationship amongst the activities/tasks; developed a network diagram; formulate the Linear programming Model of both the earliest start time (EST) and the latest start time (LST); solve the model using TORA to Obtain the EST and LST of each activity/Task of the Project; determine the activities/tasks which are critical to the timely completion of the project; float for activities/tasks in the project. The result shows that the expected completion time for the project is 210 days (30 weeks) and the critical activities are activity X_1-X_2-X_3-X_4-X_5-X_7-X_8-X_9-X_{10}-X_{12}-X_{19}-X_{14}-X_{17}-X_{20} {site clearing; setting out; foundation excavation; blinding; block work to DPC; filling and compaction of DPC; mass concrete slab; block work to final level; roofing/roof covering; ceiling finishing; electrical fittings; floor screeding; painting internal/external walls; commissioning} The Non critical activities are Construction of isolated column base (X_6), Installation of doors and windows frame (X_{15}), Installation of blackboards (X_{16}), Installation of doors and windows (X_{18}) and their "total float" is 4, 3, 7 and 4 days respectively.
Keywords: Task time, Precedence relationship, Project scheduling, linear programming method, work breakdown structure

1. INTRODUCTION.

The successful achievement and management of building project requires careful planning, scheduling and co-ordination of numerous interrelated activities. Any building project will involve the completion of a number of smaller tasks. Some of these responsibilities can be started straight away while some need to await the completion of other tasks or done in parallel before they eventually commence. Therefore, the delay experienced in the completion of building project may arise if the specific tasks are not completed before the maximum possible time expected to finish the tasks. One way of overcoming such problems is through the use of network models.

Network models are conventional means of finding most skillful way to link a number of activities directly or indirectly in order to satisfy demand and supply requirement at different activity locations and project scheduling. The need for networking arises in building construction to programme and monitor the progress of the stages involved so that the building project is completed in the minimum time. In doing this, it pin-points the part of the project that are crucial which if delayed beyond the allotted time would increase the completion time of the project as a whole. It further assists in allocating resources such as labour and equipment and thus helps to make the total cost of the building project a minimum by finding the optimum balance between various costs and time involved [1].

Tarek and Tolga [2] presented a practical approach for the modeling and optimization of overall construction schedules using a simplified spreadsheet-based model. The spreadsheet model integrates critical-path network scheduling with time-cost trade off analysis, resource allocation, resource leveling and cash flow management. The model uses the total project cost as the objective function to be minimized.

Mohamed and Celik [3] present an integrated knowledge-based system for estimating scheduling construction costs. Other researchers focus on applying graph techniques to analyze the tasks involved in completing a given project, especially the time needed to complete each task, and identifying the minimum time needed to complete the entire project. For example, the critical path method is the most common technique employed today for drawing up robust schedules.

Koo et al. [4] present a formal identification and re-sequencing process using the Critical Path Method (CPM) which support the rapid development of sequencing alternatives in construction schedules.

Pontrandolfo [5] emphasis the complexity of construction projects and the uncertainty that surrounds any estimation of the length of certain activity, resulting in a higher criticality of project scheduling. Addressing this problem, he uses PERT-state and PERT-path techniques, focusing on network complexity and time uncertainty.

Biruk and Jaskowski [6] present a new methodology for project scheduling with repetitive processes using a Petri-nets based approach.

This study determined the earliest and latest start times through identification of critical path using Linear Programming method.

2. METHODOLOGY.

2.1 Work Breakdown Structure (WBS)

Table 1: Description of activities involved for the construction process of a School building at Isaba, Warri

S/N	Activity	Description of activities
1	X_1	Site Clearing
2	X_2	Setting out
3	X_3	Foundation excavation
4	X_4	Blinding
5	X_5	Block work to DPC
6	X_6	Construction of isolated column base (20 Numbers)
7	X_7	Filling and compaction of DPC
8	X_8	Mass concrete slab
9	X_9	Block work (internal and external walls to final level)
10	X_{10}	Roofing/Roof covering
11	X_{11}	Conduit piping
12	X_{12}	Ceiling finishing
13	X_{13}	Plastering/Rendering
14	X_{14}	Floor screeding
15	X_{15}	Installation of doors and windows frame
16	X_{16}	Installation of blackboards
17	X_{17}	Painting internal/external walls
18	X_{18}	Installation of doors and windows
19	X_{19}	Electrical fittings
20	X_{20}	Commissioning

2.2 Method of data collection

Data were collected from the project supervisor and the Network diagram was drawn with all the project activities properly located in their various positions on the Network diagram. Site clearing have no predecessor to start and finish before their immediate successor can start as shown in Table 2.

Table 2: Project activity, description of activities, predecessor, duration of a School building at Isaba, Warri

S/N	ACTIVITY	DESCRIPTION OF ACTIVITIES	PREDECESSOR	DURATION (DAYS)
1	X_1	Site Clearing	-	7
2	X_2	Setting out	X_1	4
3	X_3	Foundation excavation	X_2	7
4	X_4	Blinding	X_3	4
5	X_5	Block work to DPC (Sub-structure)	X_4	11
6	X_6	Construction of isolated column base (20 number)	X_4	7
7	X_7	Filling and compaction of DPC	X_5, X_6	21
8	X_8	Mass concrete slab	X_7	11
9	X_9	Block work (internal and external walls to final level)	X_8	70
10	X_{10}	Roofing/Roof covering	X_9	21
11	X_{11}	Conduit piping	X_9	7
12	X_{12}	Ceiling finishing	X_{10}	14
13	X_{13}	Plastering/Rendering	X_{11}, X_{15}	28
14	X_{14}	Floor screeding	X_{16}, X_{18}, X_{19}	21
15	X_{15}	Installation of doors and windows frames	X_9	4
16	X_{16}	Installation of blackboard	X_{13}	4
17	X_{17}	Painting internal/external walls	X_{14}	7
18	X_{18}	Installation of doors and windows	X_{13}	7
19	X_{19}	Electrical fittings	X_{13}, X_{12}	11
20	X_{20}	Commissioning	X_{17}	1

2.3 Linear Programming Model (LPM) for the Earliest Start Time(EST)

The project scheduling problem was formulated as linear programming problem that seeks to determine the project completion time subject to meeting the precedence relationship between activities.

The linear programming model is given as thus:

$$\text{Minimize } Z = \sum_j X_j \quad (1)$$

Subject to:

$$X_j \geq X_i + t_i \text{ , for all j, for all i } \epsilon \text{ P(j)} \quad (2)$$

$$X_j \geq 0, \text{ for all j} \quad (3)$$

i.e

$$\text{Min. } Z = X_1+X_2+X_3+X_4+X5+X_6+X_7+X_8+X_9+\ldots\ldots+X_{17}+X_{18}+X_{19}+X_{20} \quad (4)$$

Subject to:

$X_1 = 0$
$X_2 - X_1 \geq 7$;
$X_3 - X_2 \geq 4$
$X_4 - X_3 \geq 7$
$X_5 - X_4 \geq 4$
$X_6 - X_4 \geq 4$
$X_7 - X_5 \geq 11$
$X_7 - X_6 \geq 7$
$X_8 - X_7 \geq 21$
$X_9 - X_8 \geq 11$
$X_{10} - X_9 \geq 70$
$X_{11} - X_9 \geq 70$
$X_{12} - X_{10} \geq 21$
$X_{13} - X_{11} \geq 7$
$X_{13} - X_{15} \geq 4$
$X_{14} - X_{16} \geq 4$
$X_{14} - X_{18} \geq 7$
$X_{14} - X_{19} \geq 11$
$X_{15} - X_9 \geq 70$
$X_{16} - X_{13} \geq 28$
$X_{17} - X_{14} \geq 21$
$X_{18} - X_{13} \geq 28$
$X_{19} - X_{13} \geq 28$
$X_{19} - X_{12} \geq 14$
$X_{20} - X_{17} \geq 7$ (5)

And $X_1, X_2, X_3, \ldots, X_{20} \geq 0$ (6)

Where

x_j= earliest start time for activity j,

t_j = the duration for activity j, and

P(j) = set of immediate predecessors activity.

2.4 Linear programming model for the determination of the critical path

The CPM model that seeks the longest path between the start and finish nodes of the building network was formulated as thus:

$$\text{Maximize } Z = \sum_j t_j X_j \quad (7)$$

For each node, there is constraint that represents the conservation of flow:

Total input flow = Total output flow

All variables, X_j, are nonnegative.

Let represent the Dummy activities, D_j, in network by the same variables used for the activities for the sake of consistency: Hence, let $X_{21} = D_1$; $X_{22} = D_2$; $X_{23} = D_3$; $X_{24} = D_4$

Therefore, Objective Function:

$$\text{Maximize } Z = X_1+4X_2+7X_3+4X_4+11X_5+7X_6+21X_7+11X_8+70X_9+21X_{10}+7X_{11}+14X_{12}+28X_{13} +21X_{14}+4X_{15}+4X_{16}+7X_{17}+7X_{18}+11X_{18}+11X_{19}+X_{20} \quad (8)$$

Subject to:

$-X_1 = -1$
$X_1 - X_2 = 0$
$X_2 - X_3 = 0$
$X_3 - X_4 = 0$

$X_4 - X_5 - X_6 = 0$
$X_6 - X_{21} = 0$
$X_5 + X_{21} - X_7 = 0$
$X_7 - X_8 = 0$
$X_8 - X_9 = 0$
$X_9 - X_{10} - X_{11} - X_{15} = 0$
$X_{10} - X_{12} = 0$
$X_{15} - X_{22} = 0$
$X_{11} + X_{22} - X_{13} = 0$
$X_{13} - X_{16} - X_{18} - X_{23} = 0$
$X_{12} + X_{23} - X_{19} = 0$
$X_{16} - X_{24} = 0$
$X_{18} + X_{19} + X_{24} - X_{14} = 0$
$X_{14} - X_{17} = 0$
$X_{17} - X_{20} = 0$
$X_{20} = 0$ (9)
And $X_1, X_2, X_3, \ldots, X_{23}, X_{24} \geq 0$ (10)

3. RESULTS.

3.1 Tora software result of the LPM for EST

The optimal solution obtain after 29 iteration is
Z*(Min) = 2801 , X1 = 0, X2 = 7, X3 = 11, X4 = 18, X5 = 22, X6 = 22, X7 =33, X8 = 54, X9 = 65, X10 = 135, X11 = 135, X12 = 156, X13 = 142 , X14 = 181, X15 = 135, X16 = 170, X17 = 202, X18 = 170, X19 = 170, X20 =209. The values which are returned by the variables above are the earliest start times (EST) (in days) for the various activities they represent.

3.2 Tora software result of the LPM for the critical path/activities

The Optimal solution obtain after 34 iterations is
Z*(Max) = 210 days (the expect project duration, t_e)
$X_1 = 1$, $X_2 = 1$, $X_3 =1$, $X_4 = 1$, $X_5 =1$, $X_6 = 0$, $X_7 = 1$, $X_8 = 1$, $X_9 =1$, $X_{10} = 1$, $X_{11} = 0$, $X_{12} = 1$, $X_{13} = 0$, $X_{14} = 1$, $X_{15} = 0$, $X_{16} = 0$, $X_{17} = 1$, $X_{18} = 0$, $X_{19} = 0$, $X_{20} = 1$, $X_{21} = X_{22} = X_{23} = X_{24} = 0$. The variables above that return a numerical value of one are the critical activities of the network while the others are non-critical activities.

3.3 Float Computation

The floats were calculated from the results obtain from the linear programming models of the network formulated and presented as follows:
Slacks (s) = LST – EST or LFT – EFT
Total Float (TF) = LFT – EST – t_j
Free Float (FF) = EFT – EST – t_j
Independent Float (IF) = EFT – LST – t_j
Where: t_j = Activity Duration, EST = Earliest Start Time, EFT = Earliest Finish Time,
LST = Latest Start Time, LFT = Latest Finish Time

Table 3: Float for the Project activities

S/N	Activity	Description of Activity	EST	LST	EFT	LFT	t_j	S	TF	FF	IF
1	X_1	Site Clearing	0	0	7	7	7	0	0	0	0
2	X_2	Setting out	7	7	11	11	4	0	0	0	0
3	X_3	Foundation excavation	11	11	18	18	7	0	0	0	0
4	X_4	Blinding	18	18	22	22	4	0	0	0	0
5	X_5	Block work to DPC	22	22	33	33	11	0	0	0	0
6	X_6	Construction of isolated column base (20 number)	22	26	29	33	7	4	4	0	-4
7	X_7	Filling and compaction of DPC	33	33	54	54	21	0	0	0	0
8	X_8	Mass concrete slab	54	54	65	65	11	0	0	0	0
9	X_9	Block work (internal and external walls to final level)	65	65	135	135	70	0	0	0	0
10	X_{10}	Roofing/Roof covering	135	135	156	156	21	0	0	0	0
11	X_{11}	Conduit piping	135	135	142	142	7	0	0	0	0
12	X_{12}	Ceiling finishing	156	156	170	170	14	0	0	0	0
13	X13	Plastering/Rendering	142	142	170	170	28	0	0	0	0
14	X14	Floor screeding	181	181	202	202	21	0	0	0	0
15	X15	Installation of doors and windows frames	135	138	139	142	4	3	3	0	-3
16	X16	Installation of blackboards	170	177	174	181	4	7	7	0	-7
17	X17	Painting internal/external walls	202	202	209	209	7	0	0	0	0
18	X18	Installation of doors and windows	170	174	177	181	7	4	4	0	-4
19	X19	Electric fittings	170	170	181	181	11	0	0	0	0
20	X20	Commissioning	209	209	210	210	1	0	0	0	0

The critical path calculations involve two passes: The forward pass determines the earliest occurrence times of the events, and backward pass calculates their latest occurrence times [7]. The earliest time is calculated as follows:

$$E_j = Max(E_i + t_j) \qquad (4)$$

Where i is the starting node number for a particular activity;
j is the ending node number for particular activity; t_j is activity duration .
The backward pass is calculated as the latest time of occurrences of the last node, which is calculated as

$$L_i = Min\left(L_j - t_j\right) \qquad (5)$$

The network diagram of the project is shown in Figure 1. In the diagram, all the activities are represented by alphabets X_j, j = 1, 2… 20. The earliest and latest time required to complete an activity are shown in Table 3.

3.4 Precedence and Activities Duration

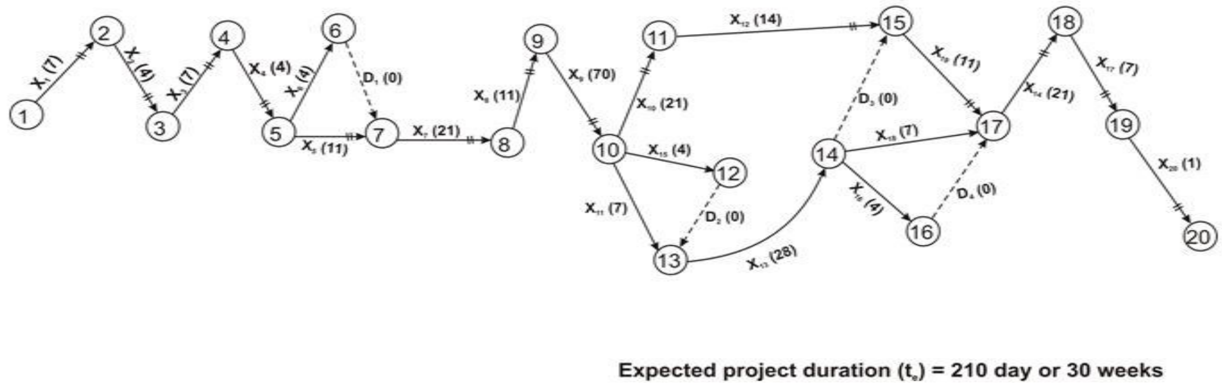

Figure 1: Activity network diagram

4. DISCUSSION.

Table 1shows the description of activities involved for the construction process of a model six classroom block with an office and a store at Isaba, Warri. The construction activities begin with activity X_1 and ends with activity X_{20}. Table 2 shows the distribution of the project activities relative to the actual number of days to complete individual activity and their precedence relationship. Table 3 shows the floats for project activities.

It is observe from the Network diagram (Figure 1) that the critical path is indicated by striking the connected arrows with two bars. The critical activities path in the Network are activity X_1-X_2-X_3-X_4-X_5-X_7-X_8-X_9-X_{10}-X_{12}-X_{19}-X_{14}-X_{17}-X_{20} {site clearing; setting out; foundation excavation; blinding; block work to DPC; filling and compaction of DPC; mass concrete slab; block work to final level; roofing/roof covering; ceiling finishing; electrical fittings; floor screeding; painting internal/external walls; commissioning} and X_1-X_2-X_3-X_4-X_5-X_7-X_8-X_9-X_{10}-X_{12}-D_3-X_{19}-X_{14}-X_{17}-X_{20} {site clearing; setting out; foundation excavation; blinding; block work to DPC; filling and compaction of DPC; mass concrete slab; block work to final level; conduit piping; plastering/rendering; dummy activity; electrical fittings; floor screeding; painting internal/external walls; commissioning} respectively. The expected project completion date is 210 days (30 weeks).

The non-critical activities are activity X_6, X_{15}, X_{16} and X_{18} {construction of isolated column base; installation of doors and windows frames; installation of blackboard; and installation of doors and windows} respectively.

Activity X_6, construction of isolated column base, have a total float of 4 days; activity X_{15}, installation of doors and windows frames, have a total float of 3 days; activity X_{16}, installation of blackboard, have a total float of 7 days and activity X_{18}, installation of doors and windows, have a total float of 4 days. Hence, each of this non critical activities can be delayed by the number of days that represent their total float (TF) without affecting the project completion date. For instance, if "construction of isolated column base", X_6, is delayed by 4 days after its start time of 22nd day from the network, its start time would now be on the 26th day and is to be completed on the 33rd day which is now the late start time for activity D_1 (Dummy). When activity X_{15}, installation of doors and windows frames, is delayed by 3 days after it earliest start time of 135th day, its time of commencement would now be 138th day i.e. its latest start time and must be completed on the 142nd day which is the earliest and latest start time of activity X_{13}, plastering/rendering. Activity X_{16}, installation of blackboards, cannot be delayed beyond 7 days after its earliest start time of commence (170th day) has passed. If delayed beyond its total float time, the expected project completion time would be extended by the same amount of days also. Lastly, activity X_{18}, installation of doors and windows, which has 4 days of total float, must be started on or before the 174th day after its earliest start date of 170th day. Any delay beyond the 174th day, would extend the expected project completion time. Also, each of the non-critical activities all have zero free float (FF) and negative values of independent float (IF). Hence, the free float and the independent float are of no use in this network.

If Linear programming model (LPM) was applied to this project with a proper scheduling of activities and the client (NDDC) plan of 18 months was not used to schedule the activities on calendar time. The disbursement of cash per period by the client can be optimally used for the activities it can handles following the order in the network and all other activities are suspended till the next period of cash disbursement. It is even possible, with proper understanding between the client and the contractor for the cash disbursement to be arranged in such a way that it flows with the network in a segmented pattern, i.e. taking a group of activities together (mostly those that are running concurrently).

5. CONCLUSION.

The Linear programming model (LPM) applied in this project is simple and effective method that can provide construction managers assistance in controlling project's activities/tasks to avoid schedule slippages and which eventually increases project overall cost. In no doubt, this procedure would eventually lower the project cost in the long run.

6. REFERENCES.

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