Dynamic Response and Flutter Prediction of Structures with Unknown System Parameters using Experimental Modal Parameters

Achuthan. C. Pankaj (Corresponding author)
Structural Technologies Division, CSIR-National Aerospace Laboratories,
Bangalore, Karnataka, India- 560017
E-mail : acpankaj@nal.res.in

Shanthini. G
Structural Technologies Division, CSIR-National Aerospace Laboratories,
Bangalore, Karnataka, India- 560017
E-mail : shanthini@nal.res.in

Shivaprasad. M.V
Structural Technologies Division, CSIR-National Aerospace Laboratories,
Bangalore, Karnataka, India- 560017
E-mail : sprasad@nal.res.in

Chandra. N
Structural Technologies Division, CSIR-National Aerospace Laboratories,
Bangalore, Karnataka, India- 560017
E-mail : nchandra@nal.res.in

Manjuprasad. M
Structural Technologies Division, CSIR-National Aerospace Laboratories,
Bangalore, Karnataka, India- 560017
E-mail : manjuprasad@nal.res.in

Abstract

This paper presents an approach for prediction of dynamic modal transient response and flutter characteristics of structures with unknown system parameters, such as stiffness and mass, using experimental modal parameters. Correlation of the results with a finite element model created using the actual material properties of the structure was studied. Close agreement was observed for the computed transient responses and flutter velocities by the proposed method, using experimental modal parameters, for which material properties are not a pre-requisite.

Keywords: Finite element; Ground vibration test; Transient response; Flutter; System identification; Experimental modal parameters

1. Introduction

Detailed modeling of the structures having full knowledge of the system in terms of distributions of mass, stiffness and inertia are essential for the purpose of dynamic analysis using theoretical/computational method like the Finite Element Method. Experimental methods are often used to validate these results. The modal parameters such as eigenvalues & vectors play a very important role in formulation of frequency, transient response analysis, unsteady/steady aerodynamics and the subsequent derived aeroelastic results like flutter speeds etc. Slight variations in these values have noticeable changes in flutter and response characteristics. However, in absence of exact details of the structural system, like aircrafts in service for which there is no access to drawings, construction details, material properties, etc. or FE model of airframe structures/components, the modal parameters (like eigenvalues and vectors) obtained from accurate experimental tests can be used as an input to predict the flutter characteristics (Manjuprasad *et.al* 2011) and the dynamic response of the structure accurately, that can substitute for the unknown distributions of stiffness and mass.

The present work is aimed at the development of a reliable method to predict the flutter and transient dynamic response characteristics of an aircraft structure of unknown configuration under an anticipated aerodynamic loading using software such as MSC Nastran /ZAERO/ Matlab and experimental modal parameters, (like mode shapes, natural frequencies and damping) from Ground Vibration Tests. The method is validated for a simple tapered plate structure by comparing its results with those from numerical methods where full details of the actual stiffness and mass distributions are used[4].

2. Procedure

2.1 Flutter

A finite element model having the same number of nodes as the test points on the structure is created. Appropriate boundary conditions and constraints corresponding to the test structure are applied on the FE model. The nodes corresponding to the response points are left free in the direction of acquired response. The eigenvalues (ω) and the mass normalized eigenvectors (∅) of the structure obtained from the test are replaced after the normal modes analysis in NASTRAN software for further use by the flutter module using unsteady doublet lattice aerodynamics in case of NASTRAN and ZONA6 aerodynamics for ZAERO software. At this stage the generalized mass matrix corresponds to unity and the generalized stiffness matrix corresponds to the eigenvalues obtained from ground vibration tests.

$$\text{Generalized Stiffness} = K = \emptyset^{\mathrm{T}} k \emptyset = \omega^2$$

$$\text{Generalized Mass} = M = \emptyset^{\mathrm{T}} m \emptyset = [1]$$

$$\text{Generalized Aerodynamic Force} = F = \emptyset^{\mathrm{T}} f$$

The generalized aerodynamic force matrix is computed using the eigenvectors obtained from ground vibration tests and further solution of the flutter problem continues by considering together the structural equations that leads to generalized equations of motion in the classical form.

$$M\ddot{\mathrm{q}} + (\rho \mathrm{VB} + C)\dot{\mathrm{q}} + (\rho \mathrm{V}^2 \mathrm{D} + K)\mathrm{q} = 0 \tag{1.0}$$

where *M*, B, *C*, D, *K* are the structural inertia, aerodynamic damping, structural damping, aerodynamic stiffness, and structural stiffness matrices in generalized coordinates q (typically modal coordinates) respectively. Thus the equation (1) can be simplified to:

$$[I]\ddot{q} + (\rho VB + C)\dot{q} + (\rho V^2 D + \omega^2)q = 0 \tag{2.0}$$

The stability of the system is explored using an eigenvalue approach (Wright & Cooper 2007). The aeroelastic equation can be expressed in state space terms as:

$$\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix} - \begin{bmatrix} 0 & I \\ -(\rho V^2 D + \omega^2) & -(\rho VB + C) \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3.0}$$

The equation can be solved by assuming the classical eigensolution form: (A-Iλ) = 0

where A is given by:

$$\begin{bmatrix} 0 & I \\ -(\rho V^2 D + \omega^2) & -(\rho V B + C) \end{bmatrix} \quad (4.0)$$

For an oscillatory system, such as the aeroelastic system considered here, the eigenvalues λ of the system matrix A occur in complex conjugate pairs and are in the form:

$$\lambda_j = -\zeta_j \omega_j \pm i\omega_j \sqrt{1-\zeta_j^2} \qquad j=1,2,\ldots\ldots,N. \quad (5.0)$$

where ω_j=1...N, are the natural frequencies and ζ_j = 1...N, are the damping ratios. Thus the upper (or lower) halves of the eigenvectors yield the mode shapes in terms of generalized coordinates. If the real part of the complex eigenvalues is positive then the system becomes unstable. A Direct Matrix Abstraction program has been written for the procedure as an input in NASTRAN bulk data through a file using the direct matrix input module available in NASTRAN.

2.2 Transient Response Analysis

A Matlab program has been written for obtaining transient response of the structure with modal parameters as input, based on Houbolt method and the results are compared with those from experiments and finite element model data using NASTRAN. The modal approach that utilizes the mode shapes of the structure to reduce and uncouple the equations of motion is used to obtain the solution through summation of the individual modal responses.

The governing equation of motion for a damped system under excitation (Paz 2004) is represented in the physical co-ordinates as:

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = \{P(t)\} \quad (6.0)$$

where $P(t)$ and $u(t)$ are the global load and displacement matrices respectively. Modal transient analysis approach, has been used to transform the physical co-ordinates to modal co-ordinates (eq.7.0) in the generalized uncoupled form as given in Eq.8.0 & 9.0.

$$\{u(t)\} = [\phi]\{q(t)\} \quad (7.0)$$

$$[\phi]^T[m][\phi]\{\ddot{q}\} + [\phi]^T[c][\phi]\{\dot{q}\} + [\phi]^T[k][\phi]\{q\} = [\phi]^T\{P(t)\} \quad (8.0)$$

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = [\phi]^T\{P(t)\} \quad (9.0)$$

where:

$[\phi]^T\{P\}$= Generalized modal force vector and

Generalized Damping = $[\phi]^T[c][\phi] = [C] = 2\zeta\omega$

At this stage the generalized mass matrix [*M*] is an identity matrix (I), the generalized stiffness matrix [*K*] corresponds to the eigenvalues ω^2. The eigenvalues ω and the mass normalized eigenvector ϕ, and the damping ratios (ζ) are obtained from the modal tests conducted on the structure. The individual modal responses are subsequently obtained by the solution of the uncoupled equations (Eq. 9.0) by the Houbolt method using Matlab. The physical response of the structure is recovered via the summation of the modal responses using Eq. (7.0).

3. Details of Test Specimen

A tapered aluminum cantilever plate is considered as test specimen to represent the wing like structure. The specimen under consideration has a thickness variation from 4 mm to 3 mm and width variation from 120 mm to 100 mm for a length of 500 mm. The candidate structure has Young's modulus of 71GPa, Poisson's ratio of 0.3 and density of 2854.32 kg/m^3. A uniform 4 mm thick aluminum plate is CNC machined to get the accurate dimensions of the required shape.

4. Ground Vibration Test

The experimental setup consists of specimen, data acquisition hardware, sensors, impulse hammer and computer with modal analysis software as shown in the Fig.1. The data acquisition system is a SCADAS

III, multichannel 24 bit with inbuilt ADC and signal conditioners for ICP type of accelerometers. Communication between data acquisition hardware and computer system is established through SCASI card. A Laptop with advanced modal analysis software LMS Test.lab5 is used for data acquisition, analysis and extracting the modal parameters such as frequency, damping and modal vectors. PCB made accelerometer with sensitivity of 100mV/g is used for response measurement. The Plate is mounted on a vibration table with four bolts to ensure proper boundary conditions (Fig.1). The specimen is marked with equally spaced 33 measurement points. The response accelerometer location has been chosen at the corner tip of the free end of the plate that gives better response for all the modes without any nodal point. Instrumented impulse hammer is used for exciting at all the 33 location marked on the plate (Fig. 2). The geometry of the specimen is generated in LMS Test Lab as per the test points chosen for measurements (Fig. 2). Channels setup is carried out with setting type of sensor, excitation voltage, units, reference point, measurement point IDs and gain settings. In scope settings maximum frequency is set to 512 Hz and spectral lines of 2048 Hz, that gives a frequency resolution of 0.25 Hz. Cross Power Spectrums, Peak Spectra, FRF, and Auto Power Spectra etc. are selected to be stored into the computer database. The test is repeated for the different points using rowing hammer technique and corresponding responses are collected and stored.

5. Finite Element Model

In the experimental set-up, responses were obtained at 33 points on the plate as shown in Fig.2. The equivalent finite element mesh model of the plate corresponds to the locations and nodes as the accelerometers in the experimental set-up (Fig.3).The mass of the accelerometer have been lumped as concentrated mass (CONM2) elements on the respective node. The model was pre-processed using Hypermesh software. The plate model (Fig.3) consists of 20 CQUAD4 elements with 33 nodes representing the measured test response points. The method has been validated with an FE model with the actual material properties of the plate structure. The geometric properties like the moment of inertia, thickness, etc of the plate were input through PSHELL entries.

6. Aerodynamic model

The aerodynamic model for the plate is a mesh consisting of flat panels based on doublet lattice method in NASTRAN and ZONA6 aerodynamics in ZAERO for the lifting surface in case of plate model, idealized by means of trapezoidal boxes lying parallel to the flow direction. Surface spline functions are used to generate the necessary interpolation matrix to estimate the displacement of aerodynamic grids based upon the displacement of structural grids.

7. Results and Discussion

7.1 Flutter

Dynamic analysis of all the FE models mentioned above have been carried out by constraining one end of the specimen in all degrees of freedom. The dynamic frequency spectrum has been obtained by invoking the Lanczos method in NASTRAN, with unit mass criteria for normalizing mode shapes. The results obtained from GVT (Table 1) has been compared with the FEM results (Table 2). The normal mode shapes for the bending and torsional modes of beam have been shown in Fig 4a. – 4d.

The flutter analysis has been carried out after taking into consideration 4 modes, i.e. up to about 195 Hz of the spectrum. The cut off frequency includes mainly the bending and torsional modes. The PK method and g-method (Chen 2000) of solution has been used in NASTRAN and ZAERO respectively for the flutter analysis. The eigenvalues and mass normalized eigenvectors obtained from the GVT are replaced in the flutter module as per the new DMAP sequence for the tapered plate model and solved for the flutter analysis. The results obtained from the finite element model with the actual properties of the structure has been compared with the proposed method by replacing the eigen values and vectors obtained from ground vibration tests in Table 3 and the corresponding flutter plots are shown in Fig.5 and 6. Coalescence of the second bending with the first torsion mode can be clearly seen in all the V-f plots. The flutter speeds evaluated by ZAERO are higher than the values computed by NASTRAN for all the cases. This variation in case of the natural frequencies, mainly the torsional mode as seen in Table 2, affects the flutter speed evaluations. The variation in the geometric, material properties and boundary conditions etc assumed in the FE model as compared to the actual test structure properties results in the marginal variation of the frequencies obtained by analyses and experimental tests.

7.2 Modal Transient Response Analysis

Modal parameters extracted by performing experimental modal analysis (Shih et.al 1998) were used for the formation of the generalized stiffness, damping and force matrix. The equation of motion (Eq. 9.0) has been solved using the Houbolt's method to estimate response. An equivalent FE model of the plate for NASTRAN was created as the test points in the experimental set-up as nodes (Fig.3). A dynamic cosine load (Fig.7) is applied at one of the free end corners of the test plate and the response obtained from NASTRAN has been validated with the results obtained from the developed in-house code. The results show a good comparison with an RMSD of 0.026m (Fig.8). This deviation is because the eigenvalues and eigenvectors obtained from the experiment include structural damping whereas NASTRAN modal parameters do not include damping. A cosine chirp load with bandwidth of 0-128 Hz is applied at node 13 and the response is calculated at node 31 with a sampling frequency of 1024 Hz. The responses from the experiment and the code are in perfect agreement with an RMSD of 0.1086g. The modal transient response of the tapered plate is also determined for a random excitation by performing a shaker test in a bandwidth of 0-512Hz and the response is captured with a sampling frequency of 1024 Hz. The response obtained using the in-house code is in sync with the experimental response with an RMSD of 0.06g. All the above results prove that the response obtained using in-house code are in agreement with the experimental responses, demonstrating the efficacy of the methodology in obtaining the transient response. It can be noted that for the present methodology neither the geometric information nor the material properties of the specimen was a pre-requisite.

8. Conclusions

The experimental modal parameters obtained from the modal vibration tests, representing the true behavior of the structure, has been used as an input through a program sequence in NASTRAN/ZAERO for predicting the flutter characteristics of a tapered plate structure and by a Matlab program for computing the modal transient response of the structure. Thus the errors that result in a finite element normal modes analysis caused due to non-availability or improper representation of the exact boundary conditions, material properties and damping has been eliminated and the accuracy of the determination of eigenvalues & eigenvectors depend on the accuracy of the experimental measurements only. The proposed method helps in a realistic prediction of flutter characteristics and transient response of the structure with the only requirement of the geometric configuration of the structure and need no material property, mass or stiffness related parameters for the finite element modeling of the structure. By conducting a single test on a structure, the responses can be subsequently evaluated for different loading distributions that could be difficult for imposition on the structure experimentally and also to avoid the expensive repetition of the experiments for other loading cases.

Acknowledgements

The authors acknowledge all the support received from colleagues of different groups in the laboratory and the Head, STTD, NAL. They also thank the Director, National Aerospace Laboratories, Bangalore, India, for his kind permission to publish the paper.

References

Chen, P. C. (2000), "Damping Perturbation Method for Flutter Solution: The g Method", *AIAA Journal*, **38**(9), pp. 1519-1524.

LMS International. (2005), *The LMS Theory and background Book- Analysis and design*, Belgium.

Manjuprasad, M., Pankaj, A. C., Shivaprasad, M. V., Chandra, N., Shanthini, G. & Nandan, A. M. (2011), "Flutter Prediction of Sweptback wing like structures using EMA based system identification and NASTRAN / ZAERO", TM-ST-11-01, National Aerospace Laboratories, Bangalore, India -560017.

Mario Paz. (2004), *Structural Dynamics - Theory & Computation* (2nd edition), New Delhi: CBS Publishers & Distributors.

MSC. Software Corporation. (2008), *MSC Nastran 2008 r1 Documentation*, USA.

Shih, C. Y., Tsuei, Y. G., Allemang, R. J. & Brown, D. L. (1988), "A frequency domain global parameter estimation method for multiple reference frequency response measurements" *Mechanical Systems and Signal Processing* **2**(4), pp. 349-365.

Wright, J. R. & Cooper, J. E. (2007), *Introduction to Aircraft Aeroelasticity and Loads*, England: Wiley.

Table 1 – Frequency and Damping (GVT)

S. No	Frequency (Hz)	Damping %	Description
1	13.286	0.16	First bending
2	72.531	0.19	Second bending
3	116.732	0.05	First torsion
4	193.894	0.11	Third bending

Table 2 – Comparison of Natural Frequencies (Hz)

MODE SHAPE	FEM (Nastran)	GVT
1^{st} bending	13.43	13.286
2^{nd} bending	72.106	72.531
1^{st} torsion	114.258	116.732
3^{rd} bending	193.27	193.894

Table 3 – Flutter Parameters

Flutter Parameters	Finite Element Model		Ground Vibration Tests	
	Nastran	ZAERO	Nastran	ZAERO
Velocity (m/s)	222.41	264.5	246.46	292.6
Frequency (Hz)	53.27	56.12	64.17	60.14

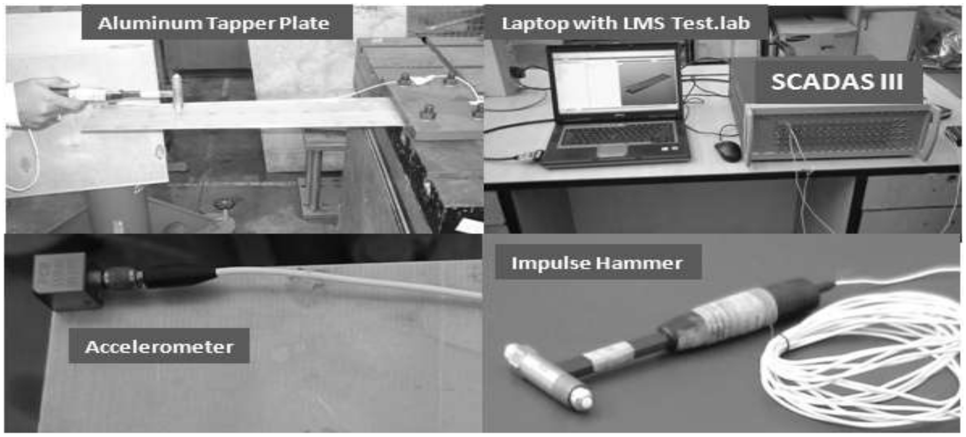


Fig.1 Experimental Setup

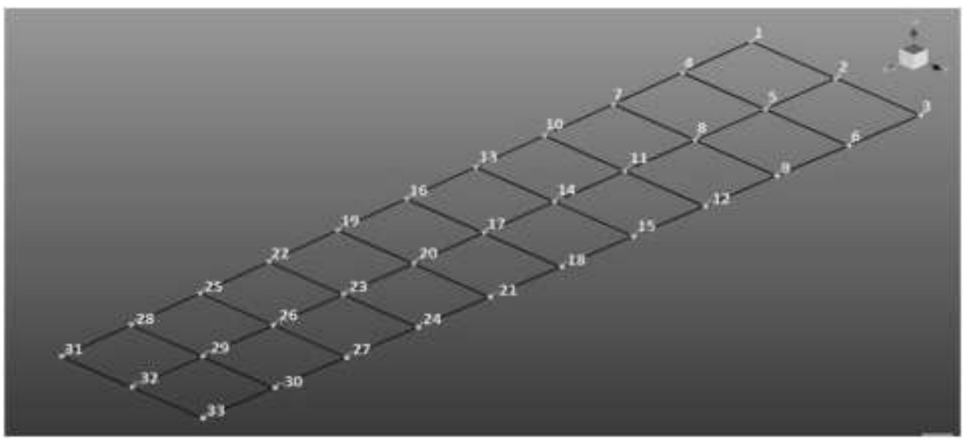


Fig.2 Geometry created in Test. Lab

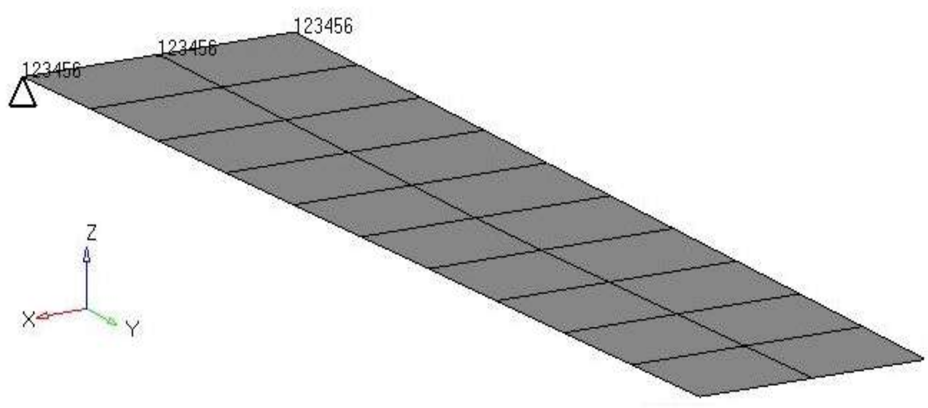


Fig. 3 FE Mesh Model of the Tapered Plate Specimen

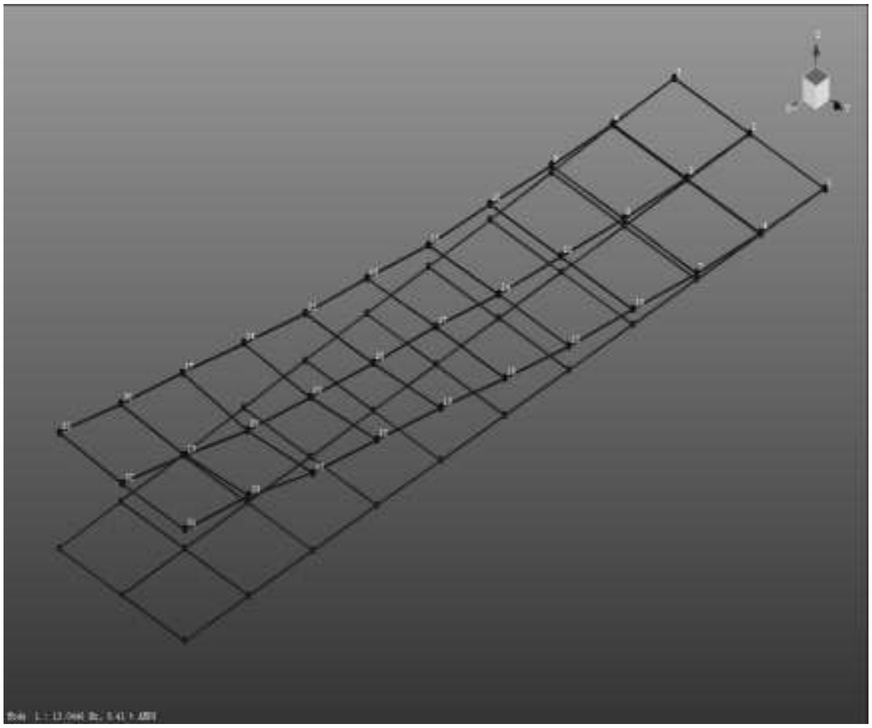


Fig. 4a First bending mode

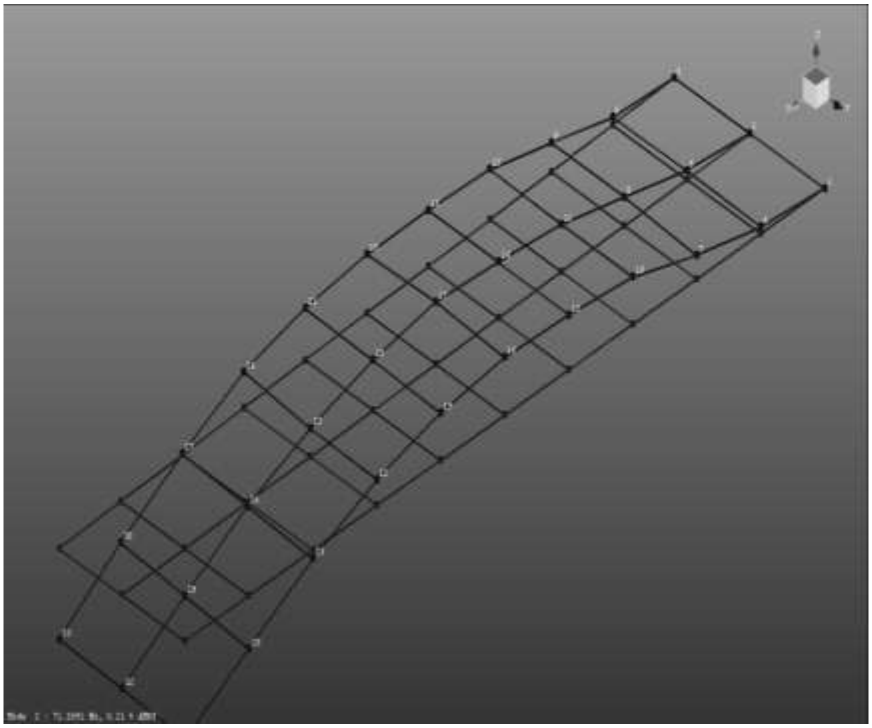


Fig. 4b Second bending mode

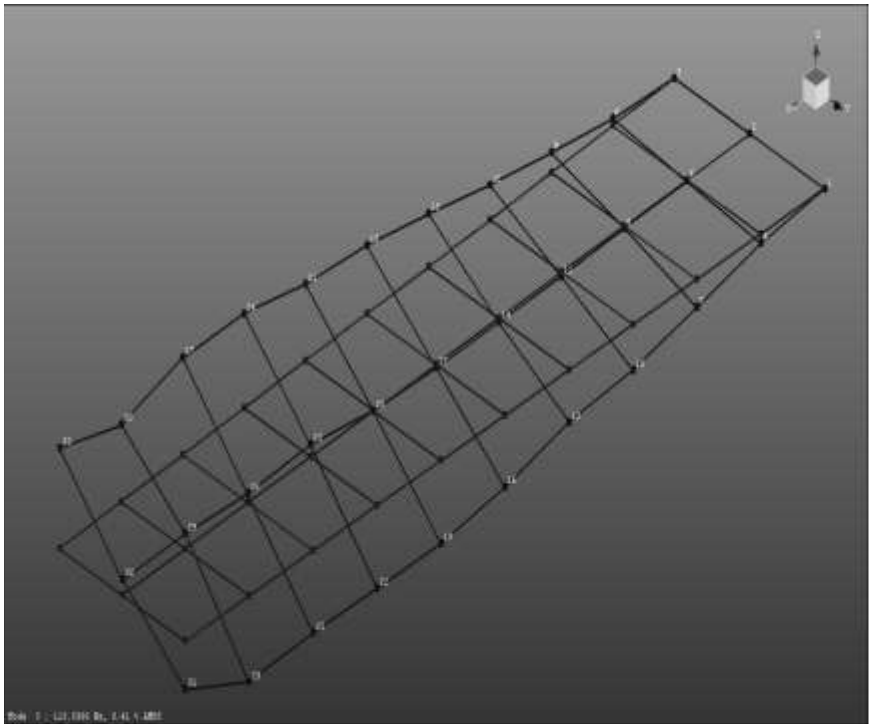


Fig. 4c- First torsion mode

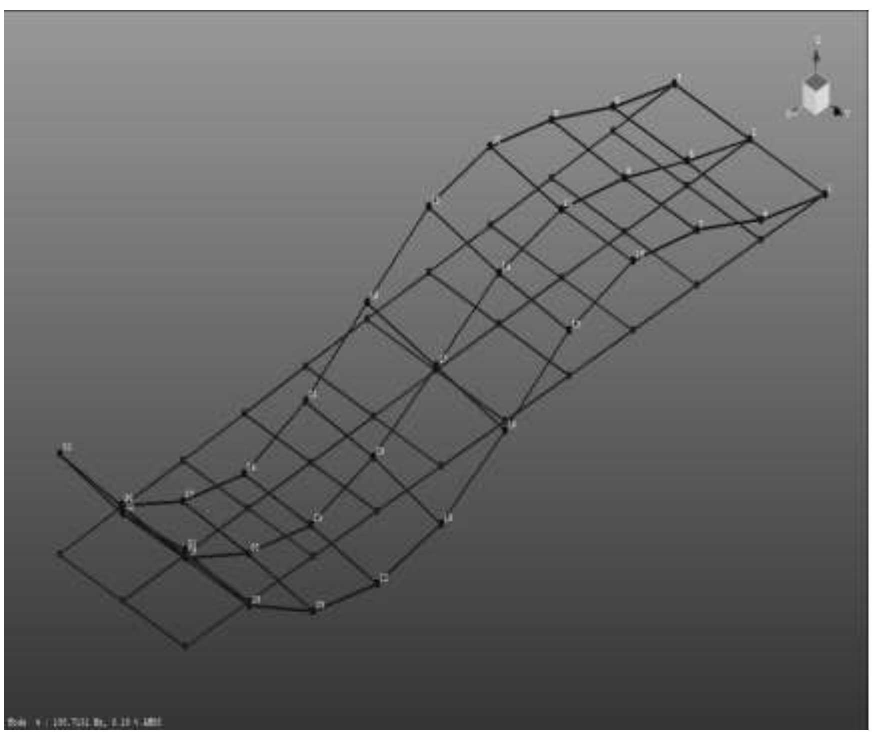


Fig. 4d- Third bending mode

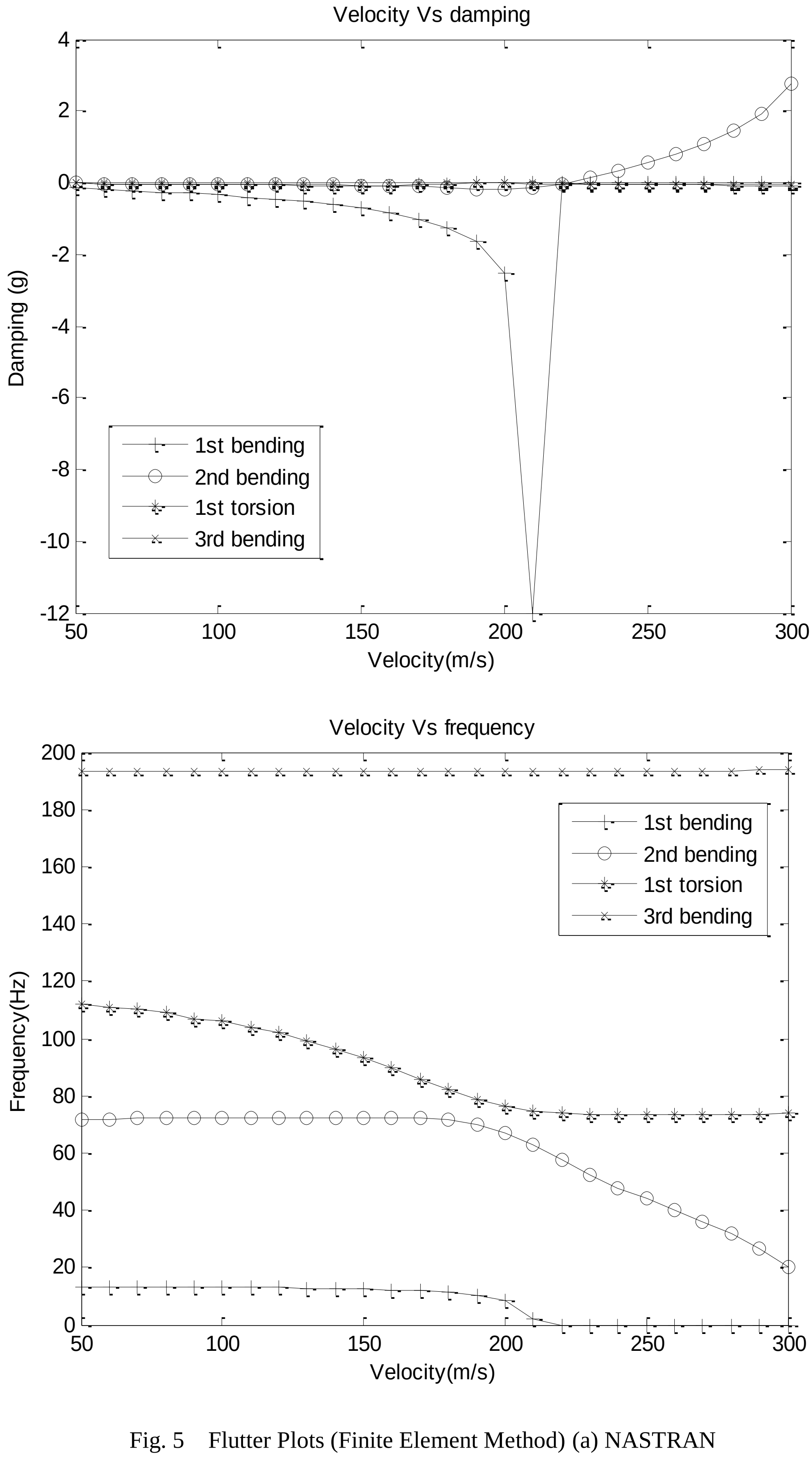


Fig. 5 Flutter Plots (Finite Element Method) (a) NASTRAN

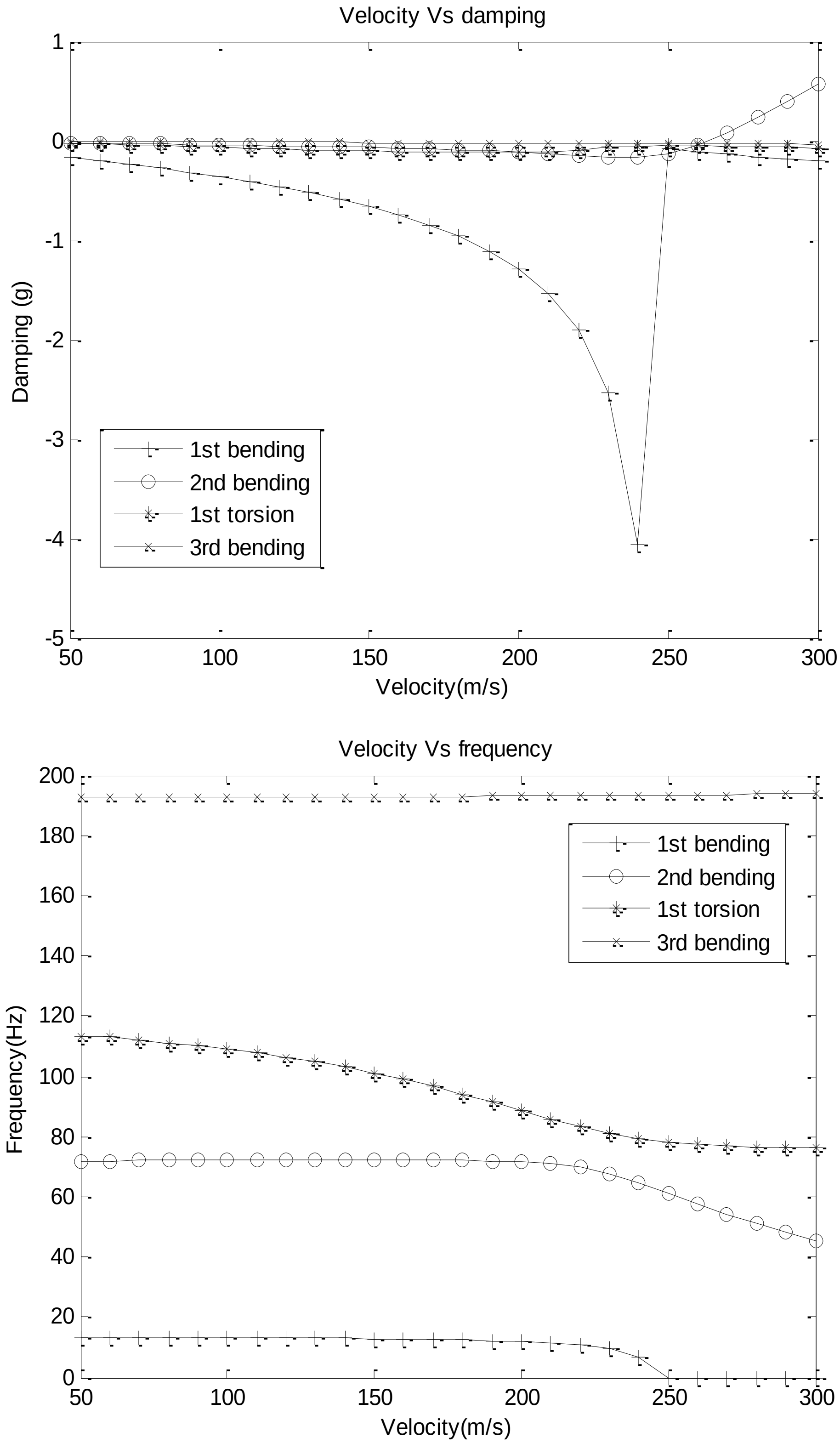


Fig. 5 Flutter Plots (Finite Element Method) (b) ZAERO

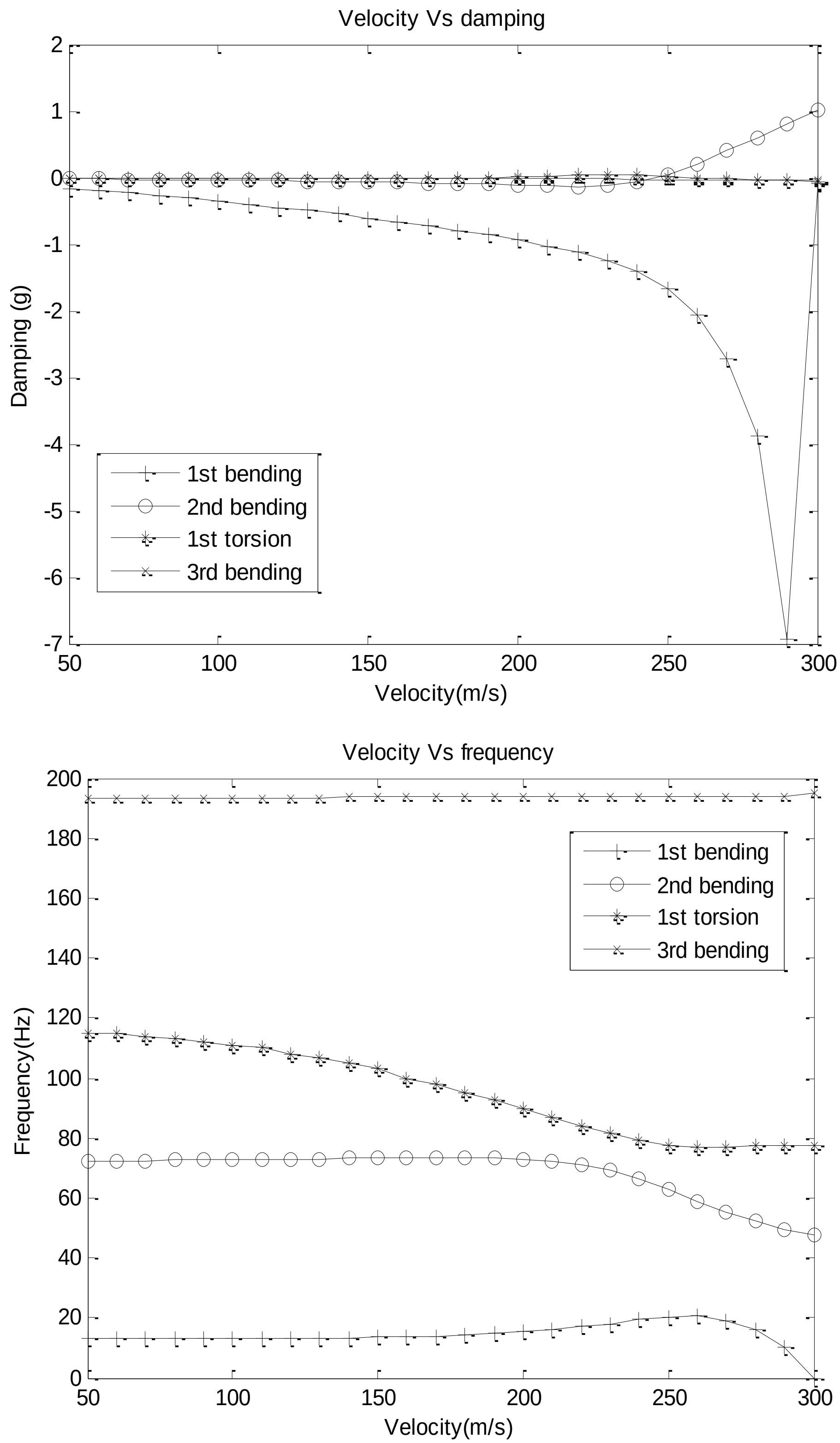
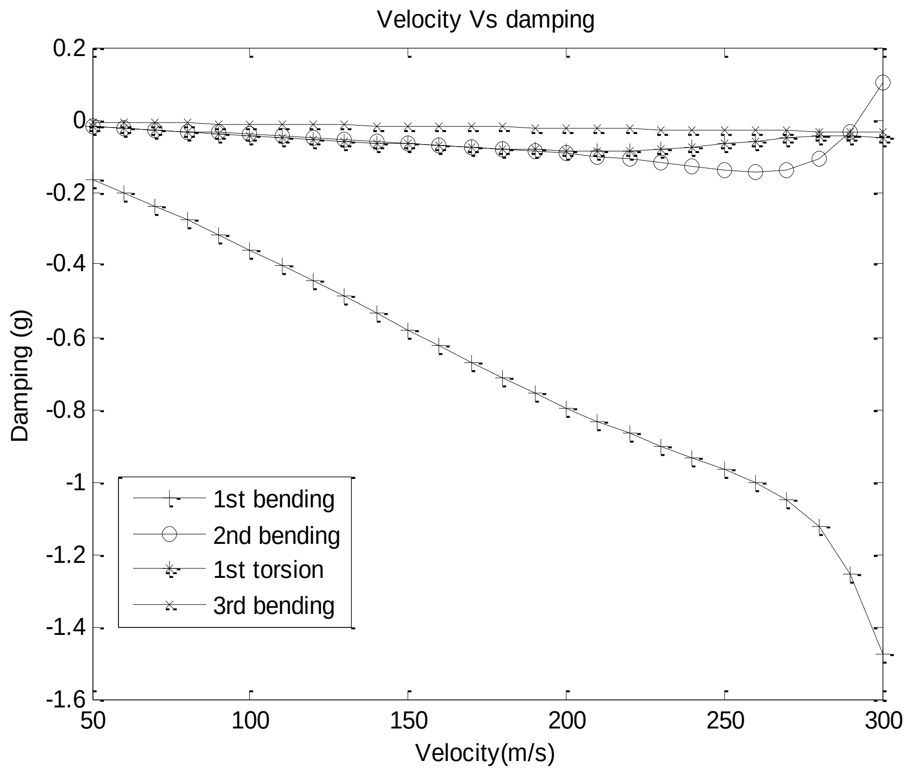


Fig. 6 Flutter Plots using Modal Parameters from GVT (a) NASTRAN


Velocity Vs damping
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
Damping (g)
50
100
150
200
250
300
Velocity(m/s)
1st bending
2nd bending
1st torsion
3rd bending

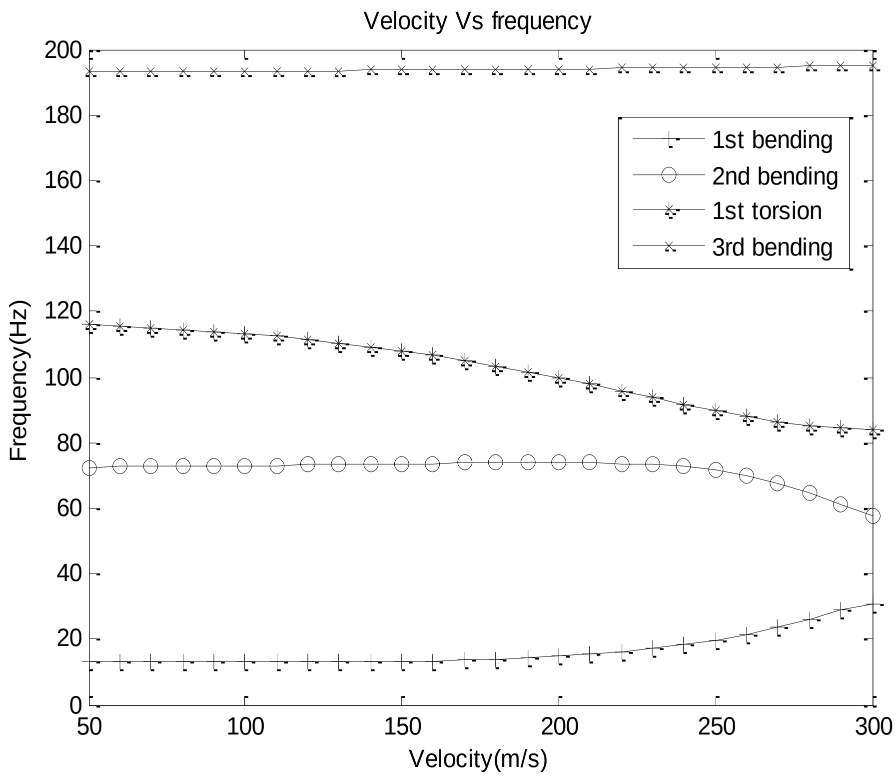


Fig. 6 Flutter Plots using Modal Parameters from GVT (b) ZAERO

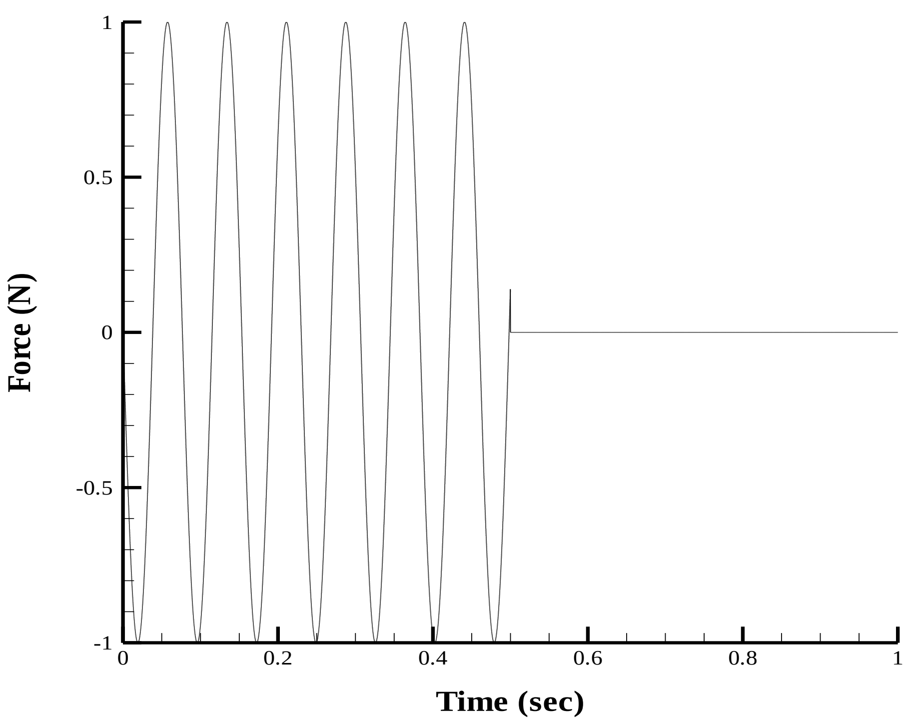


Fig. 7 Dynamic Cosine load

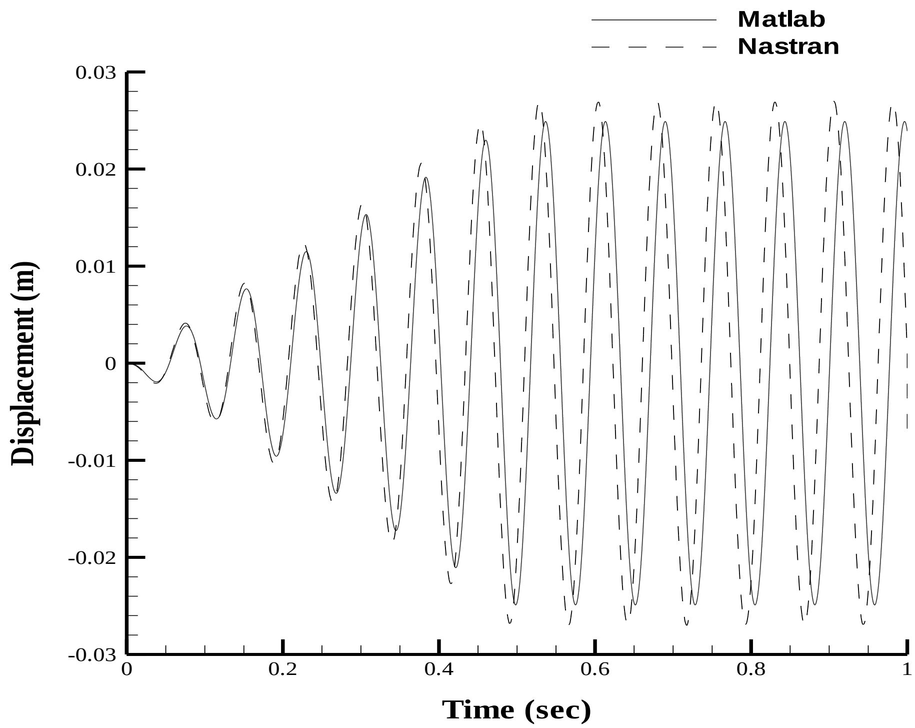


Fig. 8 Response for the dynamic Cosine load