Study of Free Vibration of Visco-Elastic Square Plate of Variable Thickness with Thermal Effect

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Abstract

Visco-elastic Square plates are widely used in various mechanical structures, aircrafts and industries. For a proper design of plate structures and efficient use of material, the behavior and strength characteristics of plates should be accurately determined. A mathematical model is presented for the use of engineers, technocrats and research workers in space technology, mechanical Sciences have to operate under elevated temperatures. Two dimensional thermal effects on frequency of free vibrations of a visco-elastic square plate of variable thickness are considered. In this paper, the thickness varies parabolically in both direction and thermal effect is varying linearly in one direction and parabolic in another direction. Rayleigh Ritz method is used to evaluate the fundamental frequencies. Both the modes of the frequency are calculated by the latest computational technique, MATLAB, for the various values of taper parameters and temperature gradient.

Keywords: Visco-elastic, Plate, Square Plate,Vibration, Thermal gradient, Frequency.

1. Introduction

In the engineering we cannot move without considering the effect of vibration because almost all machines and engineering structures experiences vibrations. As technology develops new discoveries have intensified the need for solution of various problems of vibrations of plates with elastic or visco-elastic medium. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Plates with thickness variability are of great importance in a wide variety of engineering applications.

Many researchers have analysed the free vibration of visco-elastic plates with variable thickness for many years. The aim of present investigation is to study the two dimensional thermal effect on the vibration of visco-elastic square plate. It is also considered that the temperature varies linearly in one direction and parabolic in another direction and thickness of square plate varies parabolically in both directions. It is assumed that the plate is clamped on all the four edges. Due to temperature variation, we assume that non homogeneity occurs in Modulus of Elasticity (E).For various numerical values of thermal gradient and taper constants; frequency for the first two modes of vibration are calculated. All results are shown in Graphs.

2. Equation of Motion and Analysis

Differential equation of motion for visco-elastic square plate of variable thickness in Cartesian coordinate is in equation (2.1) :

$$\begin{aligned}&[D_1\left(W,_{xxxx}+2W,_{xxyy}+W,_{yyyy}\right)+2D_{1},_{x}\left(W,_{xxx}+W,_{xyy}\right)+2D_{1,y}\left(W,_{yyy}+W,_{yxx}\right)+\\&D_{1,xx}(W,_{xx}+\nu W,_{yy})+D_{1,yy}(W,_{yy}+\nu W,_{xx})+2(1-\nu)D_{1,xy}W,_{xy}]-\rho hp^2W=0\end{aligned} \quad (2.1)$$

which is a differential equation of transverse motion for non-homogeneous plate of variable thickness.

Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = Eh^3/12(1-\nu^2) \quad (2.2)$$

and corresponding two-term deflection function is taken as

$$W=[(x/a)(y/a)(1-x/a)(1-y/a)]^2[A_1+A_2(x/a)(y/a)(1-x/a)(1-y/a)] \quad (2.3)$$

Assuming that the square plate of engineering material has a steady two dimensional temperature distribution i.e.

$$\tau=\tau_0\left(1-x/a\right)\left(1-y^2/a^2\right) \quad (2.4)$$

where, τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "*a*" is the length of a side of square plate.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this form,

$$E = E_0(1-\gamma\tau) \quad (2.5)$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of the variation of E with τ. The modulus variation (2.5) become

$$E = E_0\{(1-\alpha)(1-x/a)(1-y^2/a^2)\} \quad (2.6)$$

where, $\alpha = \gamma\tau_0 (0 \le \alpha < 1)$ thermal gradient.

It is assumed that thickness varies parabolically in both directions as shown below:

$$h = h_0(1+\beta_1 x^2/a^2)(1+\beta_2 y^2/a^2) \quad (2.7)$$

where, β_1 & β_2 are taper parameters in x- & y- directions respectively and $h=h_0$ at $x=y=0$.

Put the value of E & h from equation (2.6) & (2.7) in the equation (2.2), one obtain,

$$D_1 = E_0\left[1-\alpha\left(1-x/a\right)\left(1-y^2/a^2\right)\right]h_0^3\left(1+\beta_1 x^2/a^2\right)^3\left(1+\beta_2 y^2/a^2\right)^3]/12\left(1-\nu^2\right) \quad (2.8)$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V^* - T^*) = 0 \quad (2.9)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left.\begin{aligned} W = W,_x = 0 &, \quad x = 0, a \\ W = W,_y = 0 &, \quad y = 0, a \end{aligned}\right\} \quad (2.10)$$

Now assuming the non-dimensional variables as

$$X = x/a, Y = y/a, \overline{W} = W/a, \overline{h} = h/a \quad (2.11)$$

The kinetic energy T* and strain energy V* are

$$T^* = (1/2)\rho p^2 \overline{h}_0 a^5 \int_0^1\int_0^1 [(1+\beta_1 X^2)(1+\beta_2 Y^2)\overline{W^2}]dYdX \quad (2.12)$$

and

$$V^* = Q\int_0^1\int_0^1 [1-\alpha(1-X)(1-Y^2)](1+\beta_1 X^2)^3(1+\beta_2 Y^2)^3\{(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2\nu\overline{W},_{XX}\overline{W},_{YY} + 2(1-\nu)(\overline{W},_{XY})^2\}dYdX \quad (2.13)$$

where, $Q = E_0 h_0^3 a^3 / 24(1-\nu^2)$

Using equations (2.12) & (2.13) in equation (2.9), one get

$$(V^{**} - \lambda^2 T^{**}) = 0 \qquad (2.14)$$

where,

$$V^{**} = \int_0^1\int_0^1 [1-\alpha(1-X)(1-Y^2)](1+\beta_1 X^2)^3 (1+\beta_2 Y^2)^3 \{(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2v\overline{W},_{XX}\overline{W},_{YY} + 2(1-v)(\overline{W},_{XY})^2\} dY dX \qquad (2.15)$$

and

$$T^{**} = \int_0^1\int_0^1 [(1+\beta_1 X^2)(1+\beta_2 Y^2)\overline{W^2}] dY dX \qquad (2.16)$$

Here, $\lambda^2 = 12\rho(1-v^2)a^2 / E_0 h_0^2$ is a frequency parameter. Equation (2.10) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W. These two constants are to be determined as follows

$$\partial(V^{**} - \lambda^2 T^{**}) / \partial A_n \quad , n = 1, 2 \qquad (2.17)$$

On simplifying (2.19), one gets

$$bn_1 A_1 + bn_2 A_2 = 0 \quad , n = 1, 2 \qquad (2.18)$$

where, bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.
For a non-trivial solution, the determinant of the coefficient of equation (2.18) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} b_{12} \\ b_{21} b_{22} \end{vmatrix} = 0 \qquad (2.19)$$

With the help of equation (2.19), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1 (Mode1) & λ_2 (Mode2) for different values of taper constant and thermal gradient for a clamped plate.

3. Result and Discussion

Computation has been done for frequency of visco-elastic square plate for different values of taper constants β_1 and β_2, thermal gradient α, at different points for first two modes of vibrations have been calculated numerically.
In Fig 1: - It is clearly seen that value of frequency decreases as value of thermal gradient increases from 0.0 to 1.0 for $\beta_1 = \beta_2 = 0.0$, $\beta_1 = \beta_2 = 0.6$ and $\beta_1 = \beta_2 = 0.8$ for both modes of vibrations.
In Fig 2: - It is evident that frequency increases continuously as increasing value of taper constant β_1 from

0.0 to 1.0 and

i. β_2=0.2, α=0.0
ii. β_2=0.6, α=0.4 and
iii. β_2=0.8, α=0.6 respectively.

Conclusion

Main aim for our research is to develop a theoretical mathematical model for scientists and design engineers so that they can make a use of it with a practical approach, for the welfare of the human beings as well as for the advancement of technology.

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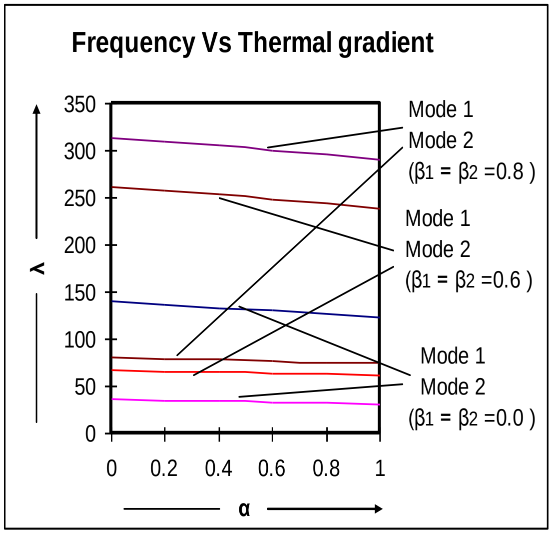

Fig 1:- Frequency Vs Thermal gradient

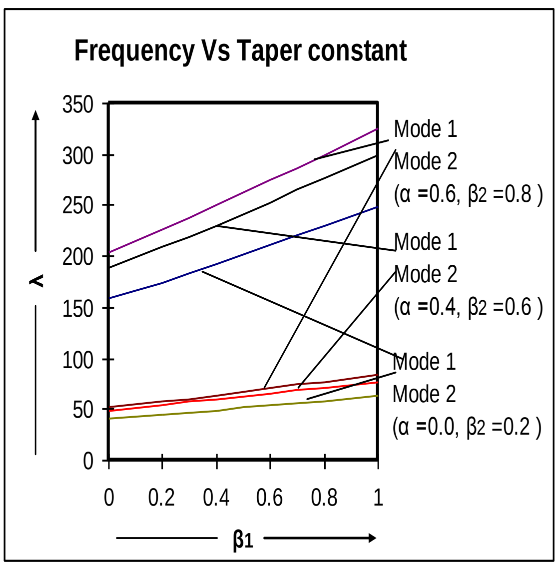

Fig 2:- Frequency Vs Taper parameter