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# Acceleration-Displacement Ratio Transform of Differentials for Transverse Plates and Constant Stiffness Buckling Solution.

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**Abstract**

The existing classical plate solution expands the load function in Fourier(trigonometric) series to be in tune with the partial differentials representing loading. In this presentation, constant point-wise values of the differentials are sought in conformity with constant rate of loading. Faster exact results are demonstrated by this new method for both lateral loading and buckling. The subject is relevant for developing technologies on account of huge constructional works in office and apartment buildings, ships, oil pipe-lines, oil reservoirs and platforms ,beams-and-plates bridges and similar events. The method is shown to lead to buckling solutions by the constant initial elastic stiffness procedure , perhaps, for the first time .

**Keywords : acceleration; displacement; ratio; envelops; beam; plate ;buckling ;stiffness matrix .**

## 1. Introduction:

Civil, environmental and specialized industrial construction-works are every-day activities and plate bending ,buckling of plates and frames in buildings, air-crafts, bridge-works, and many other areas call for fast ,accurate and simplified design-analysis. Timoshenko and Woinowsky-Krieger,(1959), studied the strength of plates and shells and their publication has continued to serve as reference compendium to researchers. The connection of beams and columns to make load-carrying structures carries with it the need to continue to study the phenomenon of buckling failure. Wood (1974),Horne(1975), Cheong-Siat- Moi (1977), Mottram(2008) give relatively recent effective opinions on buckling of frames. What is, remarkably, absent in the literature is the study of buckling by the constant elastic stiffness matrix; rather in the existing method ,the stiffness matrix is reformed by stability functions adjustments of the slope-deflection equations according to the growth of axial forces. Johnarry (2009,2011) showed how the acceleration-displacement transform of partial differentials can successfully lead to a constant stiffness solution of buckling problems in bars and frames .All solutions are ,first ,with respect to curvatures and moments relative to support values and these resolve to final solution values when any tuning support fixing curvatures/moments have been accounted for.

The method of matrix analysis of frames and the finite element analysis of continuums are well known. In the present computer implementation of acceleration-displacement transform, bending moments are interpreted as accelerations (and they are) and where gravity loads appear in the final stiffness equation, [K].{ d }= {F} , the nodal bending moments are substituted .By successive approximation, after some two or three iterations a steady-state situation is achieved and that is the objective. The relative acceleration-displacement-ratio must be point-wise constant, as much as feasible; that is resonance.

The method readily computes the fundamental frequencies of structures/frames (not shown here, Johnarry ,2009). A new and simpler buckling instability criterion is, by this method, defined.

## 2. Plate Solution by Acceleration-displacement Ratio Transform:

The flexure of the plate is described by the bi-harmonic equation,

$$D[\partial^4 w / \partial x^4 + 2\, \partial^4 w / \partial x^2 \partial y^2 + \partial^4 w / \partial y^4] = q \qquad \text{.....(2.1)}$$

$= D(w_{,xxxx} + 2\, w_{,xxyy} + w_{,yyyy}) = q$ ;alternate statement of Eq.2.1

The equation can be solved as, (D=flexural rigidity of thin plate.)

$$D.\, J.\, w = q \quad ; \text{or} \quad w = (q/D)/ J \qquad \text{..... (2.2)}$$

This is a direct opposite of the Fourier series transformation of the load method in that each of the differentials in Eq.2.1 is made to have a point-wise constant value over the domain .

Transform into acceleration-displacement ratio, -'$R_{ad} = R_{xx}.,R_{yy}.,R_{xy}.$,' thus, ($\chi = \partial^2 w/\partial x^2$ .)

$$\partial^4 w/ \partial x^4 = J_{xx}.[\, (\partial^2 w/\partial x^2) - (\partial^2 w/\partial x^2)_0] /w\, . = J_{xx}\, .R_{xx} \qquad (2.3)$$

So, multiply through by 'w' and integrate,

$$\iint (\partial^4 w/ \partial x^4)\, w\, \partial x\, \partial y = J_{xx} \iint (\chi - \chi_0)\, \partial x \partial y \qquad \text{.....(2.4)}$$

For the all-round - ss case of a plate, take

$$w = \sum \sum A_{mn} (\sin m \pi x/a)\sin n \pi y/b \qquad \ldots (2.5)$$

$$= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} S_x.S_y.$$

Introducing Eq.2.5 for Eq.2.4, we have

$$J_{xx} = A_{mn} m^3 n \pi^4 /16a^2 \qquad \ldots..(2.6)$$

Also

$$\partial^4 w/\partial y^4 = J_{yy} (\partial^2 w/\partial y^2) / w$$

$$J_{yy} = A_{mn} m n^3 \pi^4 / 16b^2$$

**2.1.1 Twist Transform :**

If it were possible,

$$w_{,xxyy}. = J_{xy}. (w,_{xy}.)_{relative} /w$$

The twisting curvature ($w_{,xy,rel}$) integrate to zero and is unsuitable ,vis-à-vis the 'LHS'. Twists are complements of normal shapes and lack independence and can be related to the normal shapes. Find an alternate equivalent twist shape function, $G,_{w,xxyy-eff}$

$$G._{w,xxyy-eff}. = K_t. [G_{w,xxxx}. + G_{w,yyyy}. ] \qquad \ldots\ldots\ldots..(2.7)$$

Choose '$K_t$' so that the maximum relative ordinates on both sides of Eq.2.8 are equal.
Relative accelerations form the basis of the analysis.

$$K_t. = [ G_{w,xxyy}]_{rel,max}. / [G_{w,xxxx} + G_{w,yyyy}, ]_{rel,max}. \qquad (2.8)$$

In the ss-case, '$K_t$' = 1/2 .More studies are necessary here.

$G_{w,xxyy,eff}$ .= $K_t$ .[$G_{w,xxxx}$ + $G_{w,yyyy}$ ] = $J_{xy}$ . ;;(constant contribution);

$$\iint G_{w,xxyy,eff}.\ w\ \partial x\partial y = J_{xy.}. \iint (1.0)\ w\ \partial x\partial y \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.8)$$

$$[(m^2)(n^2)(\pi^4)/(a^2.b^2)] \iint (S_x)^2(S_y)^2 .\partial x\partial y = (J_{xy}) \iint (S_x)(S_y)\ \partial x\partial y$$

$$J_{xy} = A_{mn} m^3 n^3 \pi^6 /(16a^2 b^2)$$

Eq.2.1 now becomes

$D [ J_{xx}( \partial^2 w / \partial x^2) / w + 2 J_{xy} . +J_{yy} (\partial^2 w / dy^2) / w] = q$ ……….. (2.9)

When reduced, we have

$A_{mn} [\pi^6/16] m n ( m^2/a^2 + n^2 / b^2)^2 = q / D$ …………. (2.10)

$A_{mn} = \sum \sum (16q /D) (\pi^6 m n (m^2 /a^2 + n^2 / b^2)^2$ . ……………………. .(2.11)

This is an exact amplitude expression in Timoshenko&Krieger (1959) and so, all results of displacements and moments are as in that citation.

$w_i = A_c S_x.S_y. = 0.00416qa^4 /D$, for $m = 1, n = 1; x = a/2_; y = b/2$

which is the exact known result , for the fundamental.

Taking additionally, m=1,n=3;m=3,n=1;m=3,n=3; $w_c. =0.00406 qa^4/D$

**2.2 The Point Load Case** :

From Eq.2.9 multiplied by 'w' for integration

$D\iint [ J_{xx} \partial^2 w / \partial x^2 + 2J_{xy} w + J_{yy} \partial^2 w/ \partial y^2] = q^* w = \iint q^* \partial x \partial y A_{mn}.S_x.S_y.$ … (2.12)

'q*' only exists at an isolated center area where the total load is 'P' with maximum displacement, $w_p .=A.^*_{mn}$ Integrate over area so that,

$\iint (q^*) \partial x \partial y (w_p) = P (A_{mn} )S_x.S_y$ ………… (2.13)

This allows potentials to be compared on both sides of Eq.2.9

$DA_{mn} .\Sigma \Sigma [ J_{xx} (\pi^2 m^2) (4ab )/(a^2 mn\pi^2 ) + J_{yy} (\pi^2 n^2) (4ab)/(b^2 mn \pi^2 ) +$
$2J_{xy} .(4) a b/(m n\pi^2 )] = \Sigma \Sigma (q^* \partial x \partial y w_{c,mn}. )(S_x.S_y.)_{norm} = (P)(S_x.S_y.)_{norm}$ …… (2.14)

$A_{mn} =(16P/D/\pi^4)S_x.S_y./[ m^3 n (4mb)/(a^2 n a) + mn^3 (8na)/(b^2.mb)+4m^2..n^2./(ab)]$ .. (2.15)

$w = \Sigma \Sigma A_{mn}. S_x.S_y.$ ;(put in $A_{mn}$. and complete the solution)

$w = [(4P/D/\pi^4) (S_x.S_y.)^2] / [m^4 b /a . + 2m^2.n^2.a/b + n^4.a^3/b^3] /a^2$ ….. (2.16)

$w_{1,1} = P a^2 ./97.4 = 0.0103 P a^2 /D$ ;;(exact as in Ref -1, for m=n=1);

$w_{1,3}. = 0.00041 = w_{3,1}.$ ;$w_{1,6}=0.00006075 =w_{1,6}$ ;$w_{3,3}=0.000127$ ;So,

$w_{1,1;1,3;3,1,1,6;6,1;3,3}. = 0.01135 Pa^2. /D$ ;(exact,0.0116-Ref-1))

$(\chi_x) = D\, \partial^2 w / \partial^2 x$ ……………. (2.17)

Alternatively find curvatures/moments from the eventual displacement relation,

$w = w_c$. Sin $\pi x/a$ .Sin $\pi y/b$. ($w_c$.= final central displacement=known) …

$(\chi_c)_x$. = $0.01135(\pi^2)$ P = 0.112

Allowing for poison's ratio of '0.3' then '$m_c$'= 0.1456 P = P/6.87; almost final but no comparable exact result source. The present method finds exact results of displacements and credible results for bending moments, if the point load does not cause immediate collapse.

**2.3 The All-round Clamped Plate Under Uniform Loading**

The deflection function, side lengths ,a,b ,in X and Y

$w_i = A_{mn}. \Sigma \;\; \Sigma (\cos m\pi x/a - 1)(\cos n\pi y/b - 1)$ (2.18)

meets all boundary conditions,Fig.1f

put , $(\cos m\pi\, x/a) = C_x$ ; $(\cos n\pi\, y/b) = C_y$ .

Transform, employing relative curvature for tuning,

$(\partial w^4 / \partial x^4) = J_{xx}. [(\partial w^2 / \partial x^2) - (\partial^2 w/\partial x^2.)_0.] / w = (J_{xx})(R_{ad})_{xx}$

The acceleration-displacement ratio,($R_{ad}$) must have a point-wise constant ratio ,hence employ relative curvatures $(\chi_r) = (\chi_i) - (\chi_0)$ to achieve it. Tuning, forces the moment diagram to emulate deflection diagram, presenting a point-wise constant ratio, ($R_{ad}$), between the two. Tuning, produces buckling-compliant curvatures.

$(\partial^2 w/\partial x^2)_{relative}. = (\chi) - (\chi_0) = (R_{xx})\, w$ …………… (2.19)

This transformation, in effect, compares the envelops of the acceleration and the deflection, Fig.1d,c .To find ($R_{ad}.=R_{xx}$) multiply both sides by 'w' and integrate over domain.

$(\partial^4 w/\partial x^4) = J_{xx}. R_{xx}$ . ………… (2.20 )

Multiply through by 'w' and integrate to find, $J_{xx}$ , for the domain ,

$\iint(\partial^4 w/\partial x^4)\, w\, \partial x\, \partial y = \iint J_{xx} R_{xx}\, w\, \partial x \partial y$ (2.21)

For the function given,

$R_{xx} = (m^2)(\pi^2)/a^2$. …………………. (2.22 )

and,

$J_{xx}.=(3/4)\ m^2(\pi^2)/(a^2)$

$J_{yy}. = (3/4)n^2(\pi^2\ )/b^2)$

**2.3.1 Twist Transform**

From $\partial^4 w/\partial x^2.\partial y^2 . = (J_{xy})$ ,

$$[(m^2)(n^2)(\pi^2)/(a^2.b^2)]C_x.C_y . = (J_{xy});;(Q_{4xy}.C_x.C_y.=J_{xy}) \qquad (2.23)$$

Invoke Eqs.2.7 & 2.8

$K_t. =1/4$

Coming from, $[C_x.C_y]_{rel,max} =2.0$ ; $[2C_x.C_y. – 2C_x – 2C_y +2]_{rel,max}= 8.0$

In this way the twist capacity, $w_{,xxyy}$, is mixed with, and carried by the tuned values of '$w_{,xxxx}$' and '$w_{,yyyy}$'. The twist is not independently tuned. So,

$\iint Q_{4xy}.\ K_t.(2C_x.C_y.-C_x-C_y).w = J_{xy}.\iint w$

$J_{xy} = 0.375\ Q_{4xy}.$

$\partial^4 w/\partial x^2.\partial y^2. = (J_{xy}) = (0.375)(m^2\ .n^2)\ \pi^4/(a^2\ .b^2)$ .

Solving Equation-1,

$D\ [J_{xx}(\partial^2 w/\partial x^2)/w + 2J_{xy}. + J_{yy}(\partial^2 w/\partial y^2)/w\ ] = q$

$A_{mn} = (q/D) / [\ J_{xx}\ R_{xx} . +2J_{xy} . + J_{yy}\ R_{yy}\ ]$

$$= [q/(D\pi^4)]/[0.75m^4/a^4.+0.75m^2.n^2./a^2.b^2+0.75n^4./b^4\ ] \ \ldots\ldots \qquad (2.24)$$

For m=n=2;x=a/2,y=b/2; $(w_{i,})_c = w_{max}= 0.001151\ q\ a^4\ /D$

Adding ,m=2,n=6;m=6,m=n=2;m=2,n=10 ; $w_c$ = 0.001243 ;(Ref-1,0.00126;Ref-8,0.001265); 'w' tends to exact because the tuning support moments in clamped plates do not add to central displacements.

**2.3.2 Curvatures/Moments**:

Fig.1b has so far been treated and only partial curvatures are expected to be available thus far; the tuning support curvatures ,$(R_{ad}\ w_{max})/2$ ,must be applied, reversed, as in Fig.1-c for completion. The effect of clamping makes the effects of any input support-moments negligible; for example zero central plate displacements.

$[(\chi_x)_{0,1/2}]_{mn,2,2;4,2;2,6;6,2} .=(\pi^2)[-2(4/3462.76)+(-2(16/24370)\ )+(-2(40/105968))]=-0.0506$ .

Taking more terms the exact maximum clamping moment of '-0.0513' (1) will be reached .

$[(\chi_x)_{1/2,1/2}]_{mn,2,2;4,2;2;6;6,2}.=(\pi^2)[2(4/3462.76)+(-2(16/24370))+(2(40/105960))]=0.01727$

Allowing for poisson's ratio of '0.3' the central X-moment =0.01727(1+0.3) =0.0225 $qa^2$. More terms may be tried here to move the '0.0225' towards the exact result of '0.0231'(0.0229-Taylor,etal,2002).

### 3. Application to Plate Buckling

Plate buckling may be studied by the differential equation,

$$D(w_{,xxxx} + 2w_{,xx,yy} + w_{,yyyy} . ) + N_{x..}\partial^2 w/\partial x^2 = 0 \qquad (3.1)$$

$$L_{4w} + N_x L_{2w} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.2)$$

Where $N_x$. is the axial compressive force in the X-direction only

Examine a simply supported plate; by Eq.2.9,

$$L_{4w} = A_{mn} (\pi^6 /16)(mn(m^2/a^2+n^2/b^2)^2 .) \qquad \ldots\ldots\ldots \qquad (3.3)$$

We can find $(L_{2w})$ as

$$L_{2w} = J_{2x}. (\partial^2 w /\partial x^2) / w \qquad \ldots\ldots\ldots \qquad (3.4)$$

Multiply through by 'w' and integrate over plate and we have,

$$J_{2x} = A_{mn}. (m\, n\, \pi^2/16) \qquad \ldots\ldots\ldots\ldots. \qquad (3.5)$$

and,

$$N_x. L_{2w} . = N_x .(J_{2x})(m^2.\pi^2/a^2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.6)$$

$$= (N_x)(A_{mn} (m\, n\, \pi^2 /16) (m^2 \pi^2/a^2) \;.. \qquad (3.7)$$

From Eqs. 3.3-3.7

$$N_x = (D\pi^2/b^2 ) (mb/a + n^2 a/m b)^2. = (C) (f(m,n,b,a))$$

For minimum -$N_x$- keep n=1

a=b , N= 4C ; a=2 b , N= 4C for m=2 ; and so on. These are exact, Timoshenko&Kreiger(1)

### 4. Application to Stiffness Matrix Analysis of Buckling of Bars/Frames.

The main objective of the present transform method was to develop a constant stiffness analysis of buckling, and this is now briefly explained.

The basic beam stiffness can be stated as: $\{F_{local} .\}= [K_{local} .]\{Vector\}_{local} .$; (a*=EA/L ;r=EI/L)

$$\left|\begin{array}{c} N \\ V1 \\ M1 \\ V2 \\ M2 \end{array}\right| = \left|\begin{array}{cccccc} -a^* & 0 & 0 & a^* & 0 & 0 \\ 0 & -12r/L^2. & -6r/L & 0 & 12r/L^2. & -6r/L \\ 0 & 6r/L & 4r & 0 & -6r/L & 2r \\ 0 & 12r/L^2. & 6r/L & 0 & -12r/L^2. & 6r/L \\ 0 & 6r/L & 2r & 0 & -6r/L & 4r \end{array}\right| \begin{array}{c} u1 \\ v1 \\ \theta 1 \\ v2 \\ \theta 2 \end{array} \quad .. \ (4.1)$$

So, find for (N=axial force, M=moment,V=shear-force; member from point-1 to 2

$$[\ K_{global}\ .\ ] = [\ T^T\ ]\ [K_{local}\ .]\ [T\ .] \quad ,\ldots\ldots\ldots\ldots\ldots\ldots \quad . \quad (4.2)$$

The acceleration-displacement ratio means relative-bending-moment – displacement ratio at nodal points. When the ratio becomes point-wise constant then a solution has been found and that 'ratio' is the buckling load.In final implementation the computed bending moments(accelerations) are interpreted as gravity loads and analysis progresses by successive approximation using only the same initial elastic stiffness. Convergence is found within five iterations. In the plate analysis demonstrated above, the ratio , ($R_{ad}$ ), can be seen to be buckling load factors in the X,Y- directions. Eqs.4.1,4.2 lead to the computer program.The analysis starts by applying a uniform loading but in iterations the bending moments are the nodal gravity loads.

### 4.1 Buckling of column pinned at both ends.

The column (AB),Fig-2 is divided into 20 elements and 21-nodes ; the results are found in Table-1,below after only three iterations. Note that in this case, Table-1 , the relative moments are the actual moments (columns2 and 3 ) ; '$m_i$ is the normal nodal slope-deflection moment . As shown in Table-1 for the pin-pin column the result of '1.002' compares with exact,1.0 ; it must be recalled that the stiffness solution did not assume a sinusoidal variation which, in a manual solution, would have led to the same correct result. The result confirms that a column buckles into a sine-curve.

### 4.2 Buckling of Sway Frame with Fixed Bases / Pinned Bases

This is output in Table-2 ;Fig.3a,b, for a beam-column stiffness ratio of 1.0. Many more results were output in Johnarry(2009). The intention, here, is to demonstrate the power of the present method; errors are very small(less than one-percent).

Table-3 give results for pinned base portal frame with $I_{beam}$./$I_{col}$ .= 100. Any $I_b$./$I_c$.-ratio can be analysed . The high ratio of 100 may be taken as beam of infinite stiffness relative to the adjoining columns ; in the Table,$P_{cr}$/$P_{Euler}$.= 0.249(Exact,0.25 for infinite value of the stiffness ratio). Fig.3c gives the pinned-bases frame-diagram.

**Conclusion**

The acceleration-displacement ratio transforms method, here, has been shown to, more simply, and exactly, solve the thin plate problem in transverse bending for displacements, bending moments and buckling. The plate under central point load is solved more exactly in convergent displacements and

bending moments. All solutions are, in the first instance, with respect to curvatures relative to boundary values; this allows for possible corrections if support tuning moments have any additional effects. The employment of a constant initial elastic stiffness for buckling by stiffness matrix is most desirable and the new method of acceleration- displacement ratio transform has been shown to achieve it; additional applications may be published. Results for bars and portal building frames are demonstrated for buckling tests. Even for the limited number of discrete elements the results are exact to within 1.0-percent. Three iterations are, usually, sufficient in the successive approximation analysis.

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Table-1
pin-pin-col; iteration =3, bars = 20; nodes=21 ;
E=200,I=1E+08,L=10000,$P_E$=1974 ;Fig.2

| Node | mi | rel-mom,m | y; | **moyope**; | weight |
|---|---|---|---|---|---|
| 1 | 0.033 | 0.000 | -0.000 | 0.000 | 0.000 |
| 2 | -3178.815 | -3178.815 | -1.607 | 1.002 | 5.107 |
| 3 | -6279.420 | -6279.420 | -3.173 | 1.002 | 19.927 |
| 4 | -9225.088 | -9225.088 | -4.662 | 1.002 | 43.009 |
| 5 | -11943.527 | -11943.527 | -6.036 | 1.002 | 72.093 |
| 6 | -14366.936 | -14366.936 | -7.261 | 1.002 | 104.323 |
| 7 | -16436.387 | -16436.387 | -8.308 | 1.002 | 136.549 |
| 8 | -18100.791 | -18100.791 | -9.150 | 1.002 | 165.614 |
| 9 | -19318.322 | -19318.322 | -9.766 | 1.002 | 188.662 |
| 10 | -20059.930 | -20059.930 | -10.142 | 1.002 | 203.447 |
| 11 | -20308.916 | -20308.916 | -10.268 | 1.002 | 208.536 |
| 12 | -20057.795 | -20057.795 | -10.142 | 1.002 | 203.418 |
| 13 | -19312.686 | -19312.686 | -9.765 | 1.002 | 188.595 |
| 14 | -18092.527 | -18092.527 | -9.149 | 1.002 | 165.522 |
| 15 | -16427.439 | -16427.439 | -8.307 | 1.002 | 136.458 |
| 16 | -14357.994 | -14357.994 | -7.260 | 1.002 | 104.244 |
| 17 | -11936.252 | -11936.252 | -6.035 | 1.002 | 72.037 |
| 18 | -9219.818 | -9219.818 | -4.661 | 1.002 | 42.977 |
| 19 | -6276.302 | -6276.302 | -3.173 | 1.002 | 19.914 |
| 20 | -3177.169 | -3177.169 | -1.606 | 1.002 | 5.103 |
| 21 | -0.019 | -3177.169 | -0.000 | 1.002 | 5.103 |

ratio,$P_c/P_E$-weighted=1.002017,analysis:K Y.=Mi= gravity force;
$m_i$=slope-defl-mom;moyope=(m/y)/$P_E$=(critical-load)/Euler-load ;Error=0.2% ;weighting factor = $m_{rel}.y_i$.

Table-2 (Fig.3a,b).

Fixed-base portal+mirror;iteration=3,bars=34;$(I_{beam})/(I_{col})=1$

| node | Mi | $m_{relative}$.=m | y; | moyope; | weight |
|---|---|---|---|---|---|
| 1 | 4050.175 | 0.000 | -0.000 | 0.000 | 0.000 |
| 2 | 3197.095 | -853.044 | -0.091 | 0.759 | 0.078 |
| 3 | 1979.468 | -2070.655 | -0.221 | 0.758 | 0.458 |
| 4 | 539.338 | -3510.767 | -0.376 | 0.757 | 1.320 |
| 5 | -957.547 | -5007.634 | -0.537 | 0.756 | 2.689 |
| 6 | -2341.093 | -6391.163 | -0.686 | 0.755 | 4.388 |
| 7 | -3455.925 | -7505.977 | -0.807 | 0.754 | 6.059 |
| 8 | -4178.111 | -8228.146 | -0.886 | 0.753 | 7.287 |
| 9 | -4427.671 | -8477.688 | -0.913 | 0.753 | 7.738 |
| 10 | -4177.179 | -8227.179 | -0.886 | 0.753 | 7.285 |
| 11 | -3454.260 | -7504.242 | -0.807 | 0.754 | 6.057 |
| 12 | -2338.832 | -6388.796 | -0.686 | 0.755 | 4.385 |
| 13 | -955.033 | -5004.980 | -0.537 | 0.756 | 2.687 |
| 14 | -541.753 | -3508.177 | -0.376 | 0.757 | 1.318 |
| 15 | 1981.568 | -2068.344 | -0.221 | 0.758 | 0.457 |
| 16 | 3198.630 | -851.264 | -0.091 | 0.758 | 0.077 |
| 17 | 4050.921 | -851.264 | 0.000 | 0.758 | 0.077 |
| 18 | -4048.568 | -851.264 | 0.000 | 0.758 | 0.077 |
| 19 | 3196.140 | -853.702 | -0.091 | 0.761 | 0.078 |
| 20 | 1979.168 | -2070.656 | -0.221 | 0.759 | 0.458 |
| 21 | 539.675 | -3510.132 | -0.376 | 0.757 | 1.318 |
| 22 | -956.450 | -5006.239 | -0.537 | 0.756 | 2.687 |
| 23 | -2339.314 | -6389.086 | -0.686 | 0.755 | 4.383 |
| 24 | -3453.683 | -7503.438 | -0.807 | 0.754 | 6.053 |
| 25 | -4175.589 | -8225.326 | -0.885 | 0.753 | 7.279 |
| 26 | -4425.183 | -8474.902 | -0.912 | 0.753 | 7.730 |
| 27 | -4175.013 | -8224.715 | -0.885 | 0.753 | 7.278 |
| 28 | -3452.514 | -7502.198 | -0.807 | 0.754 | 6.051 |
| 29 | -2337.696 | -6387.362 | -0.686 | 0.755 | 4.381 |
| 30 | -954.511 | -5004.161 | -0.537 | 0.756 | 2.685 |
| 31 | 541.716 | -3507.916 | -0.375 | 0.757 | 1.317 |
| 32 | 1981.006 | -2068.608 | -0.221 | 0.759 | 0.457 |
| 33 | 3197.648 | -851.948 | -0.091 | 0.760 | 0.077 |
| 34 | 4049.579 | -851.948 | 0.000 | 0.760 | 0.077 |

$P_{cr}/P_{Euler}$ = 0.7541421 analysis: [K]{Y}={Fi}=Mi=gravity-force

Table-3;pinned-base,1-cell-Portal-Fram; direct;iteration=**20** (3,will do);bars=34;$I_{beam}/I_{col.}$=100

| node | mi | $m_{relative}$=m | y; | moyope; | weight; |
|---|---|---|---|---|---|
| 1 | -0.202 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 2628.370 | 2628.370 | 0.864 | 0.247 | 0.009 |
| 3 | 5233.774 | 5233.774 | 1.720 | 0.247 | 0.037 |
| 4 | 7790.714 | 7790.714 | 2.560 | 0.247 | 0.082 |
| 5 | 10278.283 | 10278.283 | 3.375 | 0.247 | 0.142 |
| 6 | 12670.287 | 12670.287 | 4.158 | 0.247 | 0.216 |
| 7 | 14947.424 | 14947.424 | 4.902 | 0.247 | 0.300 |
| 8 | 17089.115 | 17089.115 | 5.599 | 0.247 | 0.392 |
| 9 | 19073.607 | 19073.607 | 6.243 | 0.248 | 0.487 |
| 10 | 20885.162 | 20885.162 | 6.827 | 0.248 | 0.583 |
| 11 | 22507.680 | 22507.680 | 7.346 | 0.248 | 0.677 |
| 12 | 23932.301 | 23932.301 | 7.795 | 0.249 | 0.763 |
| 13 | 25135.898 | 25135.898 | 8.169 | 0.249 | 0.840 |
| 14 | 26112.773 | 26112.773 | 8.465 | 0.250 | 0.904 |
| 15 | 26850.174 | 26850.174 | 8.679 | 0.251 | 0.954 |
| 16 | 27350.355 | 27350.355 | 8.809 | 0.252 | 0.986 |
| 17 | 27601.090 | 27601.090 | 8.854 | 0.253 | 1.000 |
| 18 | -0.208 | 27601.090 | 0.000 | 0.253 | 0.000 |
| 19 | 2628.365 | 2628.365 | 0.864 | 0.247 | 0.009 |
| 20 | 5233.539 | 5233.539 | 1.720 | 0.247 | 0.037 |
| 21 | 7790.900 | 7790.900 | 2.560 | 0.247 | 0.082 |
| 22 | 10278.357 | 10278.357 | 3.375 | 0.247 | 0.142 |
| 23 | 12671.202 | 12671.202 | 4.158 | 0.240 | 0.216 |
| 24 | 14946.470 | 14946.470 | 4.902 | 0.247 | 0.300 |
| 25 | 17089.078 | 17089.078 | 5.599 | 0.247 | 0.392 |
| 26 | 19073.756 | 19073.756 | 6.243 | 0.248 | 0.487 |
| 27 | 20884.490 | 20884.490 | 6.827 | 0.248 | 0.583 |
| 28 | 22507.979 | 22507.979 | 7.346 | 0.248 | 0.677 |
| 29 | 23932.600 | 23932.600 | 7.795 | 0.249 | 0.763 |
| 30 | 25136.121 | 25136.121 | 8.169 | 0.249 | 0.840 |
| 31 | 26112.969 | 26112.969 | 8.465 | 0.250 | 0.904 |
| 32 | 26850.254 | 26850.254 | 8.679 | 0.251 | 0.954 |
| 33 | 27352.088 | 27352.088 | 8.809 | 0.252 | 0.986 |
| 34 | 27351.789 | 27351.789 | 8.854 | 0.250 | 0.991 |

$P_{cr}/P_{Euler}$.,weighted=0.2494865;analysis:Fi=Mi=gravity-force;Three iterations give almost same results(not shown here)

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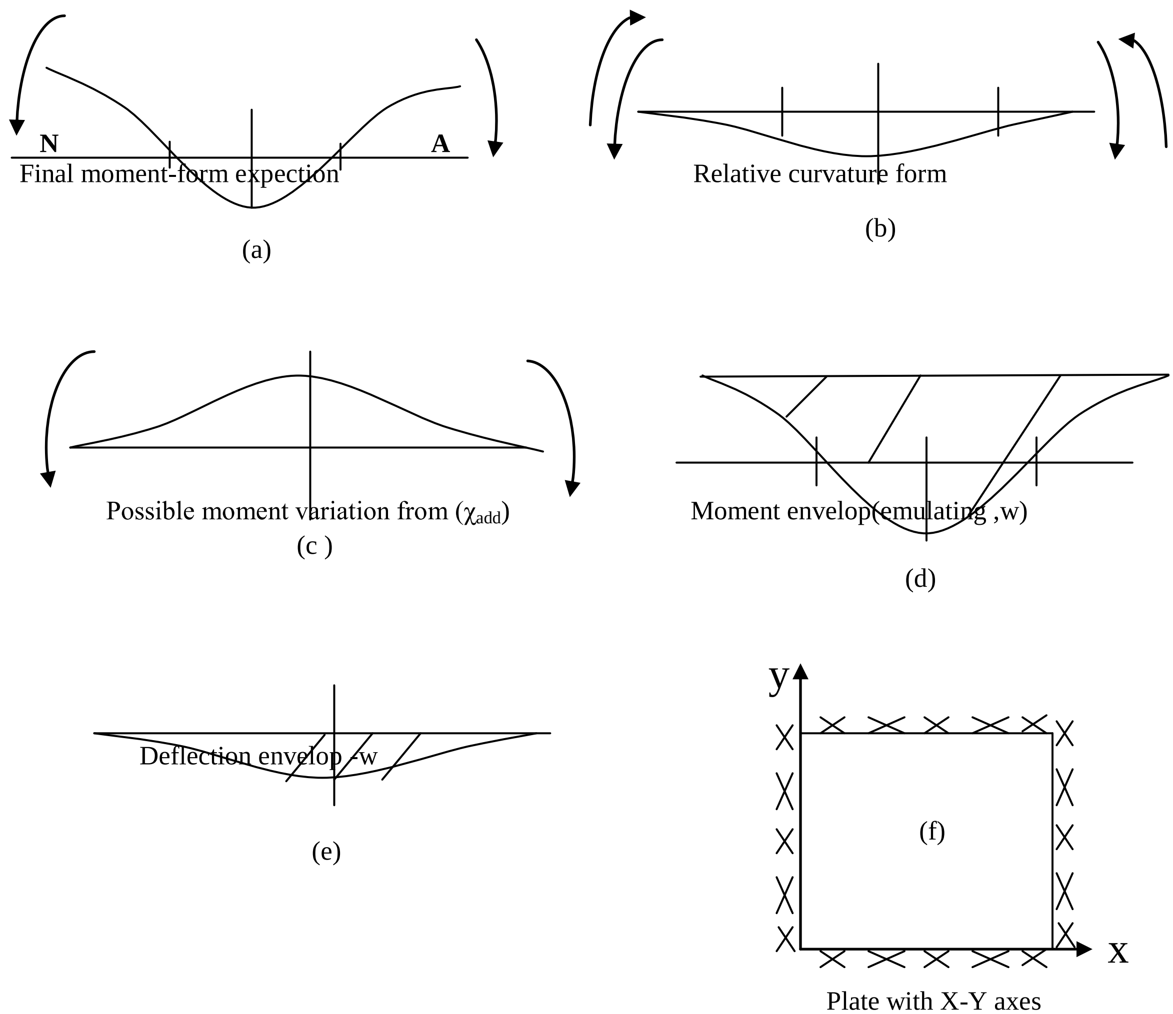


**Fig.1:Clamped rectangular plate solution.**

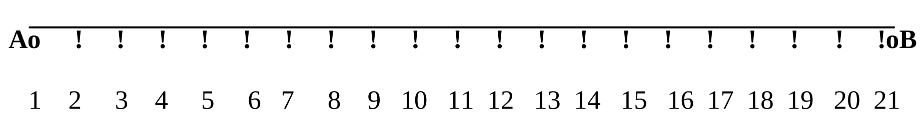


**Fig.-2 Pin-ended Column divided into 20-elements,21-nodes**

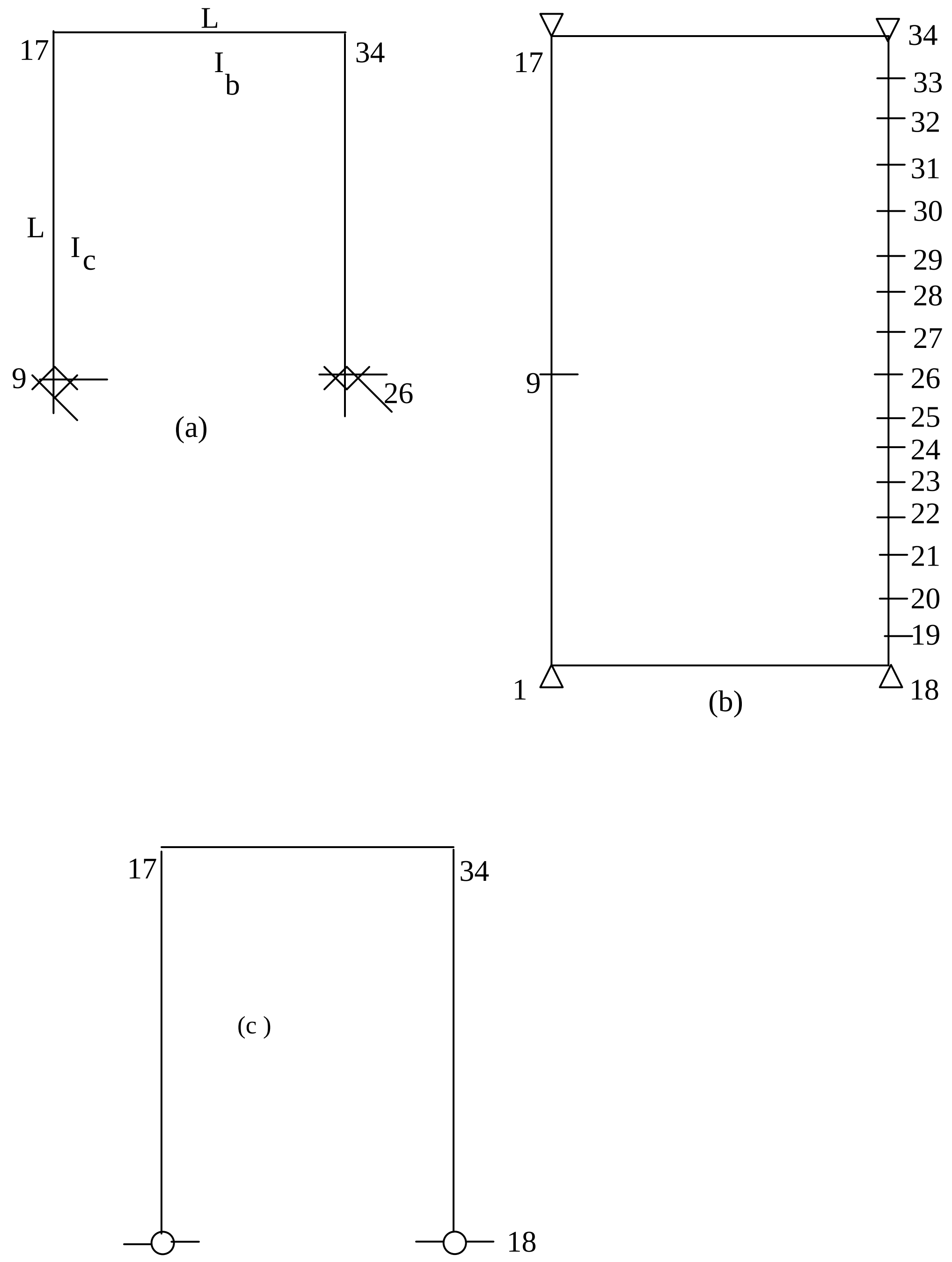


**Fig.3, Portal Frame (a) Fixed bases; (b) Analyse (a) with its mirror; (c)Pinned Base;Analyse direct.**