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# Development of a Timed Coloured Petri Net Model for Time-of-Day Signal Timing Plan Transitions

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**Abstract**

In many countries, traffic signal control is one of the most cost effective means of improving urban mobility. Nevertheless, the signal control can be grouped into two principal classes, namely traffic-response and fixed-time. Precisely, a traffic response signal controller changes timing plan in real time according to traffic conditions while a fixed-time signal controller deploys multiple signal timing plans to cater for traffic demand changes during a day. To handle different traffic scenarios via fixed-time signal controls, traffic engineers determine such time-of-day intervals manually using one or two days worth of traffic data. That is, owing to significant variation in traffic volumes, the efficient use of fixed-time signal controllers depends primarily on selecting a number of signal timing plans within a day. In this paper, a Timed Coloured Petri Net (TCPN) formalism was explored to model transition between four signal timing plans of a traffic light control system such that a morning peak signal timing plan handles traffic demand between the hours of 6:00 am and 8:30 am, followed by afternoon I and afternoon II signal timing plans which handle traffic demands from 8:30 am to 3:00 pm and from 3:00 pm to 7:00 pm respectively, while the off peak plan handles traffic demands from 7:00 pm to 9:00 pm. Other hours of the day are ignored since they are characterized by low traffic demands.

**Keywords:** Signal timing plan, Petri nets, Time-of-day, Model, Traffic, Fixed-time.

**1. Introduction**

Traffic controls are mainly based on the estimation of the flow rate of vehicle arrivals. This is the case of both major kinds of techniques of the traffic light control, fixed cycle time and adaptive signal control (Wu, *et al*., 2007). In the fixed signal control (i.e. offline technology), historical data is used to set up control strategies. The fixed signal control is simple and does not require sensors to obtain information about real-time traffic condition. It is just based on the average flow rate of roads. The drawback of the fixed signal control is that it cannot respond to any change in traffic condition. In order to improve such a control, adaptive control strategies (i.e. online technologies) are proposed. They receive real-time data through sensors and create an optimal timing plan. In adaptive control strategy, detectors located on the intersection approaches monitor traffic conditions and feed information on the actual system state to the real-time controller. Moreover, the controller selects the duration of the green phases in the signal-timing plan in order to optimize a performance index (Patel and Ranganathan, 2001; Wey, 2000; Dotoli *et al*., 2003). In both control strategies, the traffic network has to be appropriately modelled either for simulation purposes or in order to determine on line some states of the transportation network that are not available due to detector absence or failures (Gabard, 1991).

Using a fixed-time control strategy, as traffic demand changes over time, especially by time-of-day, traffic engineers develop multiple signal timing plans to accommodate these changes over time in an urban signalized intersection. For example, signal timing plan deployed in the morning peak would be different from that of midday. This is called time-of-day (TOD) mode control. It is the most common traffic control approach for non-adaptive signals in urban signalized intersections. In addition, traffic signals are coordinated in order to provide better progression along major arterials. Timing plans under TOD intervals are different in nature such that when a new timing plan is implemented over the previous timing plan, the progression bandwidth along an arterial could be damaged, which is known as a transition cost. Thus, the number and the selection of the TOD intervals are as important as finding the optimal signal timing plan for each of the intervals. In practice, traffic engineers manually collect traffic count data for one or two days, plot the aggregated volumes and determine the TOD intervals based on engineering judgment. This approach may not be efficient since it cannot keep up with the rapid changes in daily traffic. Thus, an adaptive and automated tool that utilizes a large set of archived traffic data and produces an optimal TOD plan could be very useful (Byungkyu *et al.*, 2003). This would eliminate the shortcomings of the short-term manual counts and the reliance of the expert judgment. A recent study proposed the use of statistical clustering algorithms to determine such TOD intervals (Smith *et al.*, 2002).

In furtherance, Petri nets have been proven to be a powerful modeling tool for various kinds of discrete event systems (Murata, 1989; Peterson, 1981), and its formalism provides a clear means for presenting simulation and control logic. Hence, the Petri nets are applied in traffic control. Coloured Petri nets (CPN) is a graphical oriented language for modeling and validation of systems in which communication, synchronization and resource sharing play an important role. It is an example of high-level Petri nets which combines the strength of Petri nets with the strength of programming languages. That is, Petri nets provide the primitives for describing synchronization of concurrent processes, while programming languages provide the primitives for definition of data types and manipulation of their data values. The inclusion of time concepts into a Coloured Petri Net model results in a model called Timed Coloured Petri Net (TCPN) model (Ganiyu *et al*., 2011b). Thus, with a Timed Coloured Petri Nets, it would be possible to calculate performance measures, such as the speed by which a system operates, mean waiting time and throughput. The objective of this paper is to develop a Timed Coloured Petri Net model for Time-of-Day signal timing plans with emphasis on modelling transition between four signal timing plans of a traffic light control system such that a morning peak signal timing plan handles traffic demand between the hours of 6:00 am and 8:30 am, followed by afternoon I and afternoon II signal timing plans which handle traffic demands from 8:30 am to 3:00 pm and from 3:00 pm to 7:00 pm respectively, followed by the off peak plan handles traffic demands from 7:00 pm to 9:00 pm while other hours of the day are ignored.

## 2. Methodology

### *2.1 Basic Concept of Timed Coloured Petri Nets*

Colored Petri Nets (CPNs) provide a modeling framework suitable for simulating distributed and concurrent processes with both synchronous and asynchronous communication. They are useful in modeling both non-deterministic and stochastic processes as well. Simulation is experimentation with a model of a system (White and Ingalls 2009). A CPN model is an executable representation of a system consisting of the states of the system and the events or transitions that cause the system to change its state. Through simulations of a CPN model, it is possible to examine and explore various scenarios and behaviors of a system. The relatively small basic vocabulary of CPNs allows for great flexibility in modeling a wide variety of application domains, including communication protocols, data networks, distributed algorithms, and embedded systems (Peterson 1981, Jensen and Kristensen 2009). CPNs combine the graphical components of ordinary Petri Nets with the strengths of a high level programming language, making them suitable for modeling complex systems (Jensen, Kristensen, and Wells 2007). Petri Nets provide the foundation for modeling concurrency, communication, and synchronization, while a high level programming language provides the foundation for the definition of data types and the manipulations of data values. The CPN language allows the model to be represented as a set of modules, allowing complex nets (and systems) to be represented in a hierarchical manner. CPNs allow for the creation of both timed and untimed models. Simulations of untimed models are usually used to validate the logical correctness of a system, while simulations of timed models are used to evaluate the performance of a system. Time plays an important role in the performance analysis of concurrent systems.

CPN models can be constructed using CPN Tools, a graphical software tool used to create, edit, simulate, and analyze models. CPN Tools has a graphical editor that allows the user to create and arrange the various Petri Net components. One of the key features of CPN Tools is that it visually divides the hierarchical components of a CPN, enhancing its readability without affecting the execution of the model. CPN Tools also provides a monitoring facility to conduct performance analysis of a system. In addition, unlike traditional discrete event systems, CPNs allow for state space based exploration and analysis, which is complementary to pure simulation based analysis.

In a formal way, a Coloured Petri Nets is a tuple CPN = ( $\Sigma$ , P, T, A, N, C, G, E, I) where:

(i) $\Sigma$ is a finite set of non-empty types, also called colour sets.

(ii) P is a finite set of places.

(iii) T is a finite set of transitions.

(iv) A is a finite set of arcs such that:

$P \cap T = P \cap A = T \cap A = \emptyset$.

(v) N is a node function. It is defined from A into $PxT \cup TxP$.

(vi) C is a colour function. It is defined from P into $\Sigma$

(vii) G is a guard function. It is defined from T into expressions such that:

$\forall t \in T :$[Type(G(t)) =B $\wedge$ Type(Var(G(t))) $\subseteq \Sigma$ ].

(viii) E is an arc expression function. It is defined from A into expressions such that:

$\forall a \in A:$ [Type(E(a)) = $C(p)_{MS}$ $\wedge$ Type(Var(E(a))) $\subseteq \Sigma$ ] where p is the place of N(a).

(ix) I is an initialization function. It is defined from P into closed expressions such that:
$\forall p \in P:$ [Type(I(p)) = $C(p)_{MS}$].

(Jensen, 1994)

The set of *types* determines the data values and the operations and functions that can be used in the net expressions (i.e., arc expressions, guards and initialization expressions). If desired, the types (and the corresponding operations and functions) can be defined by means of a many-sorted sigma algebra (as in the theory of abstract data types). The *places*, *transitions* and *arcs* are described by three sets P, T and A which are required to be finite and pairwise disjoint. By requiring the sets of places, transitions and arcs to be finite, we avoid a number of technical problems such as the possibility of having an infinite number of arcs between two nodes. The *node* function maps each arc into a pair where the first element is the source node and the second the destination node. The two nodes have to be of different kind (i.e., one must be a place while the other is a transition). Multiple arcs is a modelling convenience. For theory, they do not add or change anything. The *colour* function C maps each place, p, to a type C(p). Intuitively, this means that each token on p must have a data value that belongs to C(p). The *guard* function G maps each transition, t, into a boolean expression where all variables have types that belong to Σ. When we draw a CP-net we omit guard expressions which always evaluate to true. The *arc expression* function E maps each arc, a, into an expression of type $C(p)_{MS}$. This means that each arc expression must evaluate to multi-sets over the type of the adjacent place, p. The *initialization* function I maps each place, p, into a closed expression which must be of type $C(p)_{MS}$ (Jensen, 1994).

A timed non-hierarchical Coloured Petri Nets is a tuple TCPN = (CPN, R, $r_o$) such that:

(i) CPN satisfying the above definition.

(ii) R is a set of time values, also called *time stamps.* It is closed under + and including 0.

(iii) $r_o$ is an element of R called the *start time*

(Huang and Chung, 2008).

*2.2 The Case Study*

Controlling traffic signal timing at an optimal condition is undoubtedly one of the most cost effective means of improving mobility of urban traffic system. In the light of this, majority of studies related to the signal control have focused on developing better traffic signal timing plans by developing computerized programs including TRANSYT-7F (Wallace *et al.*, 1998), PASSER-II (Messer *et al.*, 1974) or better optimization techniques. Using a fixed-time control strategy, as traffic demand changes over time, especially by time-of-day, traffic engineers develop multiple signal timing plans to accommodate these changes over time in an urban signalized intersection. However, the number of signal timing plans should be minimum as possible. The greater the number of plans the more maintenance would eventually be required to upkeep them.

In order to develop the proposed Timed Coloured Petri Net model, a study case is considered in this section. Figure 1.1 depicts the layout of a signalized intersection under consideration. Traffic is currently ruled in the intersection by a fixed time control strategy with associated signal-timing plans. The layout consists of two roads named ARTERIAL S1 and ARTERIAL S3. With respect to the four input links of the intersection, the direction from ARTERIAL S1 is identified as North while its opposite direction identified as South. Similarly, the direction from ARTERIAL S3 is identified as East while its opposite direction is identified as West. To efficiently control significant variation in traffic volumes at the considered intersection, there exists a morning peak plan that operates from 06:00 am – 8:30 am, an afternoon I plan that operates from 8:30 am – 3:00 pm, an afternoon II plan that operates from 3:00 pm – 7:00 pm and an off-peak plan that operates from 7:00 pm – 9:00 pm. Tables 1.1, 1.2, 1.3 and 1.4 report the four aforementioned signal-timing plans. Vehicle streams are represented with letters from A to H while pedestrian streams are represented with letters from I to L. Also, yellow (i.e. amber) phases are taken into account such that a morning peak, afternoon I, afternoon II and off peak signal timing plans of the considered intersection consist of 25, 27, 26 and 25 phases respectively, with phases 10, 11 and 12 of each the morning peak, afternoon I and afternoon II signal timing plans representing amber phases.

2.2.1 Developing the TCPN Model of Time-of-Day Signal Timing Plan Transitions

The proposed Timed Coloured Petri Net (TCPN) model is developed consisting of two parts, namely the *signal timing plan sub-model* and the *traffic light sub-models*. Based on the Timed Coloured Petri Net formalism, the tokens required for all the two parts mentioned above comprise four elements. The token elements (i.e. *i*, *ct*, *p* and *n*) and their interpretations are as enumerated:

- The element *i* denotes the traffic light of each stream (i.e. stream $A_T$, $B_T$, $C_L$, $D_L$, $E_T$, $F_T$, $G_L$, $H_L$, $M_R$, $N_R$, $O_R$ and $P_R$ traffic lights).
- The element *ct* represents the cycle time (i.e. *ct1* for morning peak, *ct2* for afternoon I, *ct3* for afternoon II and *ct4* for off peak period).
- The element *p* represents the period for the giving time-of-day (i.e. *p1* for morning peak period, *p2* for afternoon I period, *p3* for afternoon II period and *p4* for off peak period).
- The element *n* counts the numbers of repetition cycles starting from the initial period.

However, the parameters necessary to describe the traffic behaviours in the considered intersection are as follows:

- The phase durations (in seconds) of green, red and yellow signal lights of each stream as reported in signal timing plans shown in Tables 1.1, 1.2, 1.3 and 1.4.
- The numbers of repetition cycles $n_i$ given by equation (1.1).
- $$n_i = \sum_{i=1}^{n} r_i, n \leq 4 \qquad (1.1)$$
  where $r_i$ is the numbers of repetition for the period $p_i$. All the numbers of repetition cycles (i.e. $r_i$) and the corresponding values of $n_i$ are depicted in Table 1.5.

2.2.1.1 The Signal Timing Plan Sub-model

The operation of a signal control system on an arterial corridor requires a timing plan for each signal in the corridor. A coordination timing plan consists of three main elements: cycle length, splits, and offsets. Moreover, the signal timing sub-model should be able to assign the cycle times to the different periods. In this sub-model, there are no durations and time delay. Indeed, it is used to estimate the cycle time of a period. Likewise, the number of the repetition cycle has to be counted. Once the number of repetition cycles equal to $n_i$, the current period $p_i$ goes to the next period $p_{i+1}$. However, the Signal Sub-model requires four places (i.e. p1, p2, p3 and DB1) and three transitions (i.e. t2, t3 and t4). The function of the place DB1 is to keep the cycle time and the current period together at the same time. Finally, there are four arcs between transition t2 and the place p2. The arc expressions determine when the current period goes to next period.

2.2.1.2 The Traffic Light Sub-models

To correctly control an intersection via traffic light signal indications, each traffic signal must follow a defined sequence of active colour lights, normally from green to yellow and red, and then backing to green. As a result, the traffic light part of the proposed TCPN model modelled the changing rule of traffic lights according to the four signal-timing plans shown in Tables 1.1, 1.2, 1.3 and 1.4. In particular, vehicles are allowed to pass through an intersection when green lights are turned on. On the other hand, vehicles are inhibited to pass through an intersection during red and yellow signal indications as these, in Nigerian context, correspond to *stop* and *stop at the stop line*, respectively (Ganiyu *et al.*, 2011a).

By considering the intersection shown in Figure 1.1, there are twelve vehicle streams identified, namely C-left, D-left, G-left, H-left, M-right, N-right, O-right, P-right, A-through, B-through, E-through and F-through denoted by $C_L$, $D_L$, $G_L$, $H_L$, $M_R$, $N_R$, $O_R$, $P_R$, $A_T$, $B_T$, $E_T$ and $F_T$, respectively. As a result, the traffic light part of the TCPN modelling the intersection would be divided into twelve sub-models. These are called streams $C_L$, $D_L$, $G_L$, $H_L$, $M_R$, $N_R$, $O_R$, $P_R$, $A_T$, $B_T$, $E_T$ and $F_T$ sub-models. Each of the first four streams (i.e. $C_L$, $D_L$, $G_L$ and $H_L$) is controlled by one set of traffic light (i.e. Red, Yellow and Green signal lights) while each of the next four streams (i.e. $M_R$, $N_R$, $O_R$ and $P_R$) is controlled only by a Right turn green arrow light. However, the last four streams (i.e. $A_T$, $B_T$, $E_T$ and $F_T$) are individually controlled by two sets of traffic lights. To be precise, each of the last four streams is allocated two lanes and controlled correspondingly by two sets of traffic lights placed on a long arm cantilever. As an example, two sets of traffic lights represented by A.1 and A.2 in Figure 1.1 concurrently control the Stream $A_T$ vehicles. As reflected by signal heads depicted in Tables 1.1, 1.2, 1.3 and 1.4, in modelling stream $A_T$ traffic lights, the two sets of traffic lights would be merged and represented as follows:

- Place $A_G$ models the state of green light controlling the Stream $A_T$ vehicles
- Place $A_Y$ models the state of yellow light controlling the Stream $A_T$ vehicles
- Place $A_R$ models the state of red light controlling the Stream $A_T$ vehicles

This is also applicable to the two sets of traffic lights ruling each of the streams $B_T$, $E_T$ and $F_T$. That is, the two sets of traffic lights controlling each of these streams would be merged and modelled as stated in Table 1.6. Besides, the model representations of the other traffic lights (i.e. streams $C_L$, $D_L$, $G_L$, $H_L$, $M_R$, $N_R$, $O_R$ and $P_R$ traffic lights) are also explicated in Table 1.6. Moreover, taking model representation of the stream $A_T$ traffic lights as an example, the presence of token in each of the places $A_G$, $A_Y$ and $A_R$ means green, yellow and red signal lights turn on respectively, and turn off otherwise. This is also applicable for each of the streams $C_L$, $D_L$, $G_L$, $H_L$, $M_R$, $N_R$, $O_R$, $P_R$, $B_T$, $E_T$ and $F_T$ traffic lights.

Furthermore, for each of the twelve traffic light sub-models, the transitions required and their functions are enumerated in Tables 1.7(a) and 1.7(b). Based on the rules of Timed Colored Petri Nets, all the transitions would be drawn as rectangles. These represent individual events taking place in the traffic lights of the intersection. As with places, the names of the transitions are written inside the rectangles (e.g. t1, T8, t3, etc). A number of directed *arcs* connecting places and transitions are associated with appropriate arc expressions which consist of one or two of the following decision variables:

- Element *i* denotes the traffic light of each stream (i.e. stream $A_T$, $B_T$, $C_L$, $D_L$, $E_T$, $F_T$, $G_L$, $H_L$, $M_R$, $N_R$, $O_R$, $P_R$ traffic lights would be denoted by *ic*, *id*, *ig*, *ih*, *im*, *inn*, *io*, *ip*, *ia*, *ib*, *ie* and *iff* respectively).
- Time stamp derived from durations of green, red and yellow signal lights of each stream shown in Tables 1.1, 1.2, 1.3 and 1.4. This would be introduced using the symbol @. It should be noted that the time stamp could be defined as seconds, microseconds, milliseconds, etc, depending on the choice of modeller. Here, one time stamp unit is assumed to represent one millisecond in the developed TCPN model.

2.2.2 Discussion of Result

Figure 1.2 shows the developed Timed Coloured Petri Net (TCPN) model of Time-of-Day signal timing plan transitions of the considered intersection. The developed TCPN model is characterized by the following:

- Eight sets of traffic lights controlling the stream $A_T$, $B_T$, $C_L$, $D_L$, $E_T$, $F_T$, $G_L$ and $H_L$ vehicles and four right-turn green arrow lights controlling the stream $M_R$, $N_R$, $O_R$ and $P_R$ vehicles.
- One signal timing plan sub-model and twelve traffic light sub-models (i.e. streams $A_T$, $B_T$, $C_L$, $D_L$, $E_T$, $F_T$, $G_L$, $H_L$, $M_R$, $N_R$, $O_R$ and $P_R$ sub-models) that allow easy model modification or development for other periodic interval in a day in the considered intersection.
- The modelling of signal timing transitions of different periods that varies with the traffic flow in a day.
- The presence of tokens in the places *AR, BR, CR, DR, ER, FR, GR, HR, P8, P10, P14, P21, P23, P27, P33, P37, P40, P44, P48, P51*, and *DB1* which constitutes the initial marking or lost time phase of the model.

- The model execution is assumed to begin at the signal timing plan sub-model

**3. Conclusion and Future Work**

In this medium, we have been able to develop a Timed Coloured Petri Net model for Time-of-Day signal timing-plans selection of a named signalized intersection. Precisely, a Timed Coloured Petri Net formalism was explored with emphasis on the application of the methodology in modelling transition between four signal timing plans of a traffic light control system such that a morning peak signal timing plan handles traffic demand between the hours of 6:00 am and 8:30 am, followed by afternoon I and afternoon II signal timing plans which handle traffic demands from 8:30 am to 3:00 pm and from 3:00 pm to 7:00 pm respectively, followed by an off peak plan that handles traffic demands from 6:00 pm to 9:00 pm while other hours of the day are ignored. Furthermore, future and further research may be geared towards developing a Timed Coloured Petri Net model for Time-of-Day signal timing-plans selection of arterial traffic characterized by coordinated actuated controls. Also, the occurrence graph, as one of the analysis methods of Timed Coloured Petri Net models, could be used to verify the safeness and liveness properties associated with the developed Timed Coloured Petri Net model.

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Table 1.1: Signal timing plan of a morning peak period

| Streams | Signal Heads | Phases | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| $A_T$ | A.1 A.2 | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | G | G | G |
| $B_T$ | B.1 B.2 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | Y | Y | R | R | R |
| $C_L$ | C.1 | G | G | G | G | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G |
| $D_L$ | D.1 | R | R | R | G | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $E_T$ | E.1 E.2 | R | R | R | R | R | R | R | R | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $F_T$ | F.1 F.2 | R | R | R | R | R | R | R | R | G | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $G_L$ | G.1 | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | R | R | R | R | R | R | R | R |
| $H_L$ | H.1 | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | R | R | R | R | R | R | R | R |
| I | I.1I.2I.3 I.4 | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | G | G | G | G |
| J | J.1J.2J.3J.4 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | Y | R | R |
| K | K.1K.2K.3K.4 | R | R | R | R | R | R | R | G | G | G | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R |
| L | L.1L.2L.3L.4 | R | R | R | R | R | R | R | G | G | G | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R |
| $M_R$ | M.1 | | | | | | | | | | | | | | | G | G | G | G | G | | | | | | |
| $N_R$ | N.1 | | | | | | | | | | | | | | | | G | G | G | | | | | | | |
| $O_R$ | O.1 | | | | | G | G | G | G | | | | | | | | | | | | | | | | | |
| $P_R$ | P.1 | G | G | G | G | G | G | G | G | | | | | | | | | | | | | | | | | G |
| Phase Duration (sec) | | 15 | 3 | 2 | 4 | 3 | 3 | 2 | 1 | 6 | 1 | 2 | 1 | 2 | 2 | 1 | 9 | 3 | 2 | 1 | 12 | 1 | 2 | 3 | 8 | 1 |
| Cycle Duration $Ct_1$ (sec) | | 90 | | | | | | | | | | | | | | | | | | | | | | | | |

*Legend* G: Green signal Y: Yellow signal R: Red signal

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Table 1.2: Signal timing plan of an afternoon I period

| Streams | Signal Heads | Phases | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| $A_T$ | A.1 A.2 | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | G | G | G |
| $B_T$ | B.1 B.2 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | Y | Y | R | R | R |
| $C_L$ | C.1 | G | G | G | G | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G |
| $D_L$ | D.1 | R | R | R | G | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $E_T$ | E.1 E.2 | R | R | R | R | R | R | R | R | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $F_T$ | F.1 F.2 | R | R | R | R | R | R | R | R | G | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $G_L$ | G.1 | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | R | R | R | R | R | R | R | R |
| $H_L$ | H.1 | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | R | R | R | R | R | R | R | R |
| I | I.1I.2I.3 I.4 | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | G | G | G | G |
| J | J.1J.2J.3J.4 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | Y | R | R |
| K | K.1K.2K.3K.4 | R | R | R | R | R | R | R | G | G | G | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R |
| L | L.1L.2L.3L.4 | R | R | R | R | R | R | R | G | G | G | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R |
| $M_R$ | M.1 | | | | | | | | | | | | | | | G | G | G | G | G | | | | | | |
| $N_R$ | N.1 | | | | | | | | | | | | | | | | G | G | G | | | | | | | |
| $O_R$ | O.1 | | | | | G | G | G | G | | | | | | | | | | | | | | | | | |
| $P_R$ | P.1 | G | G | G | G | G | G | G | G | | | | | | | | | | | | | | | | | G |
| Phase Duration (sec) | | 15 | 3 | 2 | 4 | 3 | 3 | 2 | 1 | 6 | 1 | 2 | 1 | 2 | 2 | 1 | 9 | 3 | 2 | 1 | 12 | 1 | 2 | 3 | 8 | 1 |
| Cycle Duration $Ct_1$ (sec) | | 90 | | | | | | | | | | | | | | | | | | | | | | | | |

*Legend* G: Green signal Y: Yellow signal R: Red signal

Table 1.3: Signal timing plan of an afternoon II period

| Streams | Signal Heads | Phases | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| $A_T$ | A.1 A.2 | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | G | G | G |
| $B_T$ | B.1 B.2 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | Y | Y | R | R | R |
| $C_L$ | C.1 | G | G | G | G | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G |
| $D_L$ | D.1 | R | R | R | G | G | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $E_T$ | E.1 E.2 | R | R | R | R | R | R | R | R | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $F_T$ | F.1 F.2 | R | R | R | R | R | R | R | R | G | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $G_L$ | G.1 | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | R | R | R | R | R | R | R | R |
| $H_L$ | H.1 | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | R | R | R | R | R | R | R | R |
| I | I.1I.2I.3 I.4 | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | G | G | G | G |
| J | J.1J.2J.3J.4 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | Y | R | R |
| K | K.1K.2K.3K.4 | R | R | R | R | R | R | R | G | G | G | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R |
| L | L.1L.2L.3L.4 | R | R | R | R | R | R | R | G | G | G | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R |
| $M_R$ | M.1 | | | | | | | | | | | | | | | G | G | G | G | G | | | | | | |
| $N_R$ | N.1 | | | | | | | | | | | | | | | | G | G | G | | | | | | | |
| $O_R$ | O.1 | | | | | G | G | G | G | | | | | | | | | | | | | | | | | |
| $P_R$ | P.1 | G | G | G | G | G | G | G | G | | | | | | | | | | | | | | | | | G |
| Phase Duration (sec) | | 14 | 3 | 2 | 4 | 3 | 3 | 2 | 1 | 6 | 1 | 2 | 1 | 2 | 2 | 1 | 9 | 3 | 2 | 1 | 12 | 1 | 2 | 3 | 8 | 2 |
| Cycle Duration $Ct_3$ (sec) | | 90 | | | | | | | | | | | | | | | | | | | | | | | | |

*Legend* G: Green signal Y: Yellow signal R: Red signal

Table 1.4: Signal timing plan of an off peak period

| Streams | Signal Heads | Phase | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| $A_T$ | A.1 A.2 | G | G | G | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G |
| $B_T$ | B.1 B.2 | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | Y | R | R | R | R | R | R | R | R | R | R |
| $C_L$ | C.1 | G | G | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G |
| $D_L$ | D.1 | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | Y | R | R | R | R | R | R | R | R | R | R |
| $E_T$ | E.1 E.2 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | G | Y | Y | R | R | R |
| $F_T$ | F.1 F.2 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | Y | Y | R | R | R | R |
| $G_L$ | G.1 | R | R | R | R | R | R | R | R | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| $H_L$ | H.1 | R | R | R | R | R | R | R | R | G | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| I | I.1I.2I.3 I.4 | G | Y | Y | Y | Y | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| J | J.1J.2J.3J.4 | R | R | R | R | R | R | R | R | R | R | R | R | G | G | Y | R | R | R | R | R | R | R | R | R | R | R |
| K | K.1K.2K.3K.4 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | G | G | G | Y | Y | Y | R | R |
| L | L.1L.2L.3L.4 | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | G | Y | Y | Y | R | R | R |
| $M_R$ | M.1 | | | | G | G | G | G | G | G | G | | | | | | | | | | | | | | | | |
| $N_R$ | N.1 | | | | | | | | G | G | G | G | G | | | | | | | | | | | | | | |
| $O_R$ | O.1 | | | | | | | | | | | | | | G | G | G | G | | | | | | | | | |
| $P_R$ | P.1 | G | G | G | G | | | | | | | | | | | | | | | | | G | G | G | G | G | G |
| Phase Duration (sec) | | 1 | 2 | 1 | 1 | 1 | 1 | 2 | 1 | 11 | 1 | 2 | 2 | 1 | 7 | 5 | 3 | 2 | 3 | 2 | 7 | 2 | 1 | 2 | 2 | 1 | 1 |
| Cycle Duration $Ct_4$ (sec) | | 75 | | | | | | | | | | | | | | | | | | | | | | | | | |

*Legend* G: Green signal Y: Yellow signal R: Red signal

Table 1.5. Numbers of repetition, execution time and cycle time for the periods

| **Period** | **Index (*i*)** | **Period ($p_i$) Execution Time (Hrs)** | **Period ($p_i$) Cycle Time (Secs)** | **Repetition Cycle ($r_i$)** | **Total number of Repetition Cycle ($n_i$)** |
|---|---|---|---|---|---|
| Morning Peak | 1 | 2.5 | 90 | 100 | 100 |
| Afternoon I | 2 | 6.5 | 120 | 195 | 295 |
| Afternoon II | 3 | 4 | 90 | 160 | 455 |
| Off Peak | 4 | 2 | 75 | 96 | 551 |

Table 1.6: Model Representation of Major Traffic Light States of the Considered Intersection

| **Sub-models** | **Places** | **Model Representations** |
|---|---|---|
| Stream $B_T$ | $B_G$ | Models the state of green light controlling the Stream $B_T$ vehicles. |
| | $B_Y$ | Models the state of yellow light controlling the Stream $B_T$ vehicles. |
| | $B_R$ | Models the state of red light controlling the Stream $B_T$ vehicles. |

| | | |
|---|---|---|
| Stream $E_T$ | $E_G$ | Models the state of green light controlling the Stream $E_T$ vehicles. |
| | $E_Y$ | Models the state of yellow light controlling the Stream $E_T$ vehicles. |
| | $E_R$ | Models the state of red light controlling the Stream $E_T$ vehicles. |
| Stream $F_T$ | $F_G$ | Models the state of green light controlling the Stream $F_T$ vehicles. |
| | $F_Y$ | Models the state of yellow light controlling the Stream $F_T$ vehicles. |
| | $F_R$ | Models the state of red light controlling the Stream $F_T$ vehicles. |
| Stream $C_L$ | $C_G$ | Models the state of green light controlling the Stream $C_L$ vehicles. |
| | $C_Y$ | Models the state of yellow light controlling the Stream $C_L$ vehicles. |
| | $C_R$ | Models the state of red light controlling the Stream $C_L$ vehicles. |
| Stream $D_L$ | $D_G$ | Models the state of green light controlling the Stream $D_L$ vehicles. |
| | $D_Y$ | Models the state of yellow light controlling the Stream $D_L$ vehicles. |
| | $D_R$ | Models the state of red light controlling the Stream $D_L$ vehicles. |
| Stream $G_L$ | $G_G$ | Models the state of green light controlling the Stream $G_L$ vehicles. |
| | $G_Y$ | Models the state of yellow light controlling the Stream $G_L$ vehicles. |
| | $G_R$ | Models the state of red light controlling the Stream $G_L$ vehicles. |
| Stream $H_L$ | $H_G$ | Models the state of green light controlling the Stream $H_L$ vehicles. |
| | $H_Y$ | Models the state of yellow light controlling the Stream $H_L$ vehicles. |
| | $H_R$ | Models the state of red light controlling the Stream $H_L$ vehicles. |
| Stream $M_R$ | $M_G$ | Models the state of a Right turn green arrow light controlling the stream $M_R$ vehicles. |
| Stream $N_R$ | $N_G$ | Models the state of a Right turn green arrow light controlling the stream $N_R$ vehicles. |
| Stream $O_R$ | $O_G$ | Models the state of a Right turn green arrow light controlling the stream $O_R$ vehicles. |
| Stream $P_R$ | $P_G$ | Models the state of a Right turn green arrow light controlling the stream $P_R$ vehicles. |

Table 1.7(a): Major Transitions of the Twelve Traffic Light Sub-models

| Sub-models | Transitions | Actions |
|---|---|---|
| Stream $A_T$ | $t_1$ and $T_5$ | Concurrently turns off and on red and green signal lights respectively for Stream $A_T$ vehicles in the first instance. |
| | $T_9$ | Concurrently turns off and on red and green signal lights respectively for Stream $A_T$ vehicles in the second instance. |
| | $T_6$ | Concurrently turns off and on green and yellow signal lights respectively for Stream $A_T$ vehicles. |
| | $T_7$ | Concurrently turns off and on yellow and red signal lights respectively for Stream $A_T$ vehicles. |
| Stream $B_T$ | $t_{12}$ and $T_{13}$ | Concurrently turns off and on red and green signal lights respectively for Stream $B_T$ vehicles. |
| | $T_{14}$ | Concurrently turns off and on green and yellow signal lights respectively for Stream $B_T$ vehicles. |

| | | |
|---|---|---|
| | $T_{15}$ | Concurrently turns off and on yellow and red signal lights respectively for Stream $B_T$ vehicles. |
| Stream $C_L$ | $t_{20}$ and $T_{21}$ | Concurrently turns off and on red and green signal lights respectively for Stream $C_L$ vehicles in the first instance. |
| | $T_{25}$ | Concurrently turns off and on red and green signal lights respectively for Stream $C_L$ vehicles in the second instance. |
| | $T_{22}$ | Concurrently turns off and on green and yellow signal lights respectively for Stream $C_L$ vehicles. |
| | $T_{23}$ | Concurrently turns off and on yellow and red signal lights respectively for Stream $C_L$ vehicles. |
| Stream $D_L$ | $t_{44}$ and $T_{45}$ | Concurrently turns off and on red and green signal lights respectively for Stream $D_L$ vehicles. |
| | $T_{46}$ | Concurrently turns off and on green and yellow signal lights respectively for Stream $D_L$ vehicles. |
| | $T_{47}$ | Concurrently turns off and on yellow and red signal lights respectively for Stream $D_L$ vehicles. |
| Stream $E_T$ | $t_{49}$ and $T_{50}$ | Concurrently turns off and on red and green signal lights respectively for Stream $E_T$ vehicles. |
| | $T_{51}$ | Concurrently turns off and on green and yellow signal lights respectively for Stream $E_T$ vehicles. |
| | $T_{52}$ | Concurrently turns off and on yellow and red signal lights respectively for Stream $E_T$ vehicles. |
| Stream $F_T$ | $t_{26}$ and $T_{27}$ | Concurrently turns off and on red and green signal lights respectively for Stream $F_T$ vehicles in the first instance. |
| | $T_{28}$ | Concurrently turns off and on green and yellow signal lights respectively for Stream $F_T$ vehicles. |
| | $T_{29}$ | Concurrently turns off and on yellow and red signal lights respectively for Stream $F_T$ vehicles. |
| Stream $G_L$ | $t_{57}$ and $T_{58}$ | Concurrently turns off and on red and green signal lights respectively for Stream $G_L$ vehicles. |
| | $T_{59}$ | Concurrently turns off and on green and yellow signal lights respectively for Stream $G_L$ vehicles. |
| | $T_{60}$ | Concurrently turns off and on yellow and red signal lights respectively for Stream $G_L$ vehicles. |

Table 1.7(b): Major Transitions of the Twelve Traffic Light Sub-models

| **Sub-models** | **Transitions** | **Actions** |
|---|---|---|
| Stream $H_L$ | $t_{36}$ and $T_{37}$ | Concurrently turns off and on red and green signal lights respectively for Stream $H_L$ vehicles. |
| | $T_{38}$ | Concurrently turns off and on green and yellow signal lights respectively for Stream $H_L$ vehicles. |
| | $T_{39}$ | Concurrently turns off and on yellow and red signal lights respectively for Stream $H_L$ vehicles. |
| Stream $M_R$ | $T_{18}$ | Turns on a Right turn green arrow light for Stream $M_R$ vehicles. |
| | $T_{19}$ | Turns off a Right turn green arrow light for Stream $M_R$ vehicles |

| | | |
|---|---|---|
| Stream $N_R$ | $T_{42}$ | Turns on a Right turn green arrow light for Stream $N_R$ vehicles. |
| | $T_{43}$ | Turns off a Right turn green arrow light for Stream $N_R$ vehicles |
| Stream $O_R$ | $T_{55}$ | Turns on a Right turn green arrow light for Stream $O_R$ vehicles. |
| | $T_{56}$ | Turns off a Right turn green arrow light for Stream $O_R$ vehicles |
| Stream $P_R$ | $T_{32}$ | Turns on a Right turn green arrow light for Stream $P_R$ vehicles |
| | $T_{33}$ | Turns off a Right turn green arrow light for Stream $P_R$ vehicles |
| | $T_{34}$ | Turns on a Right turn green arrow light for Stream $P_R$ vehicles in the second instance |

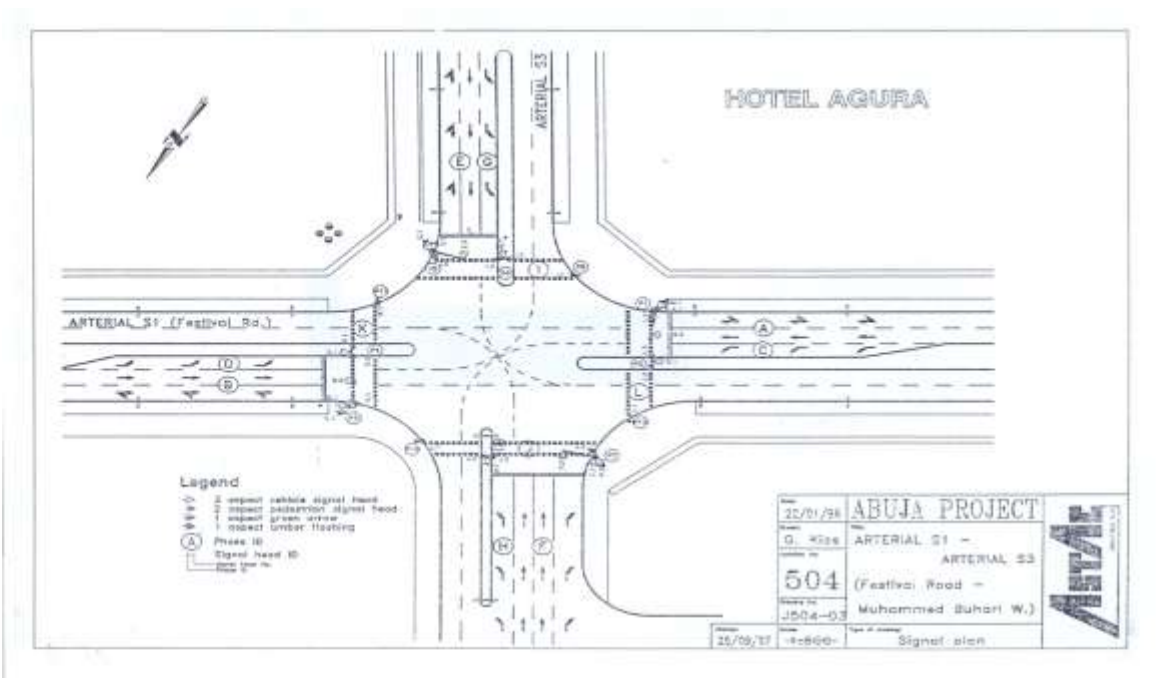


Figure 1.1: Layout of the considered intersection

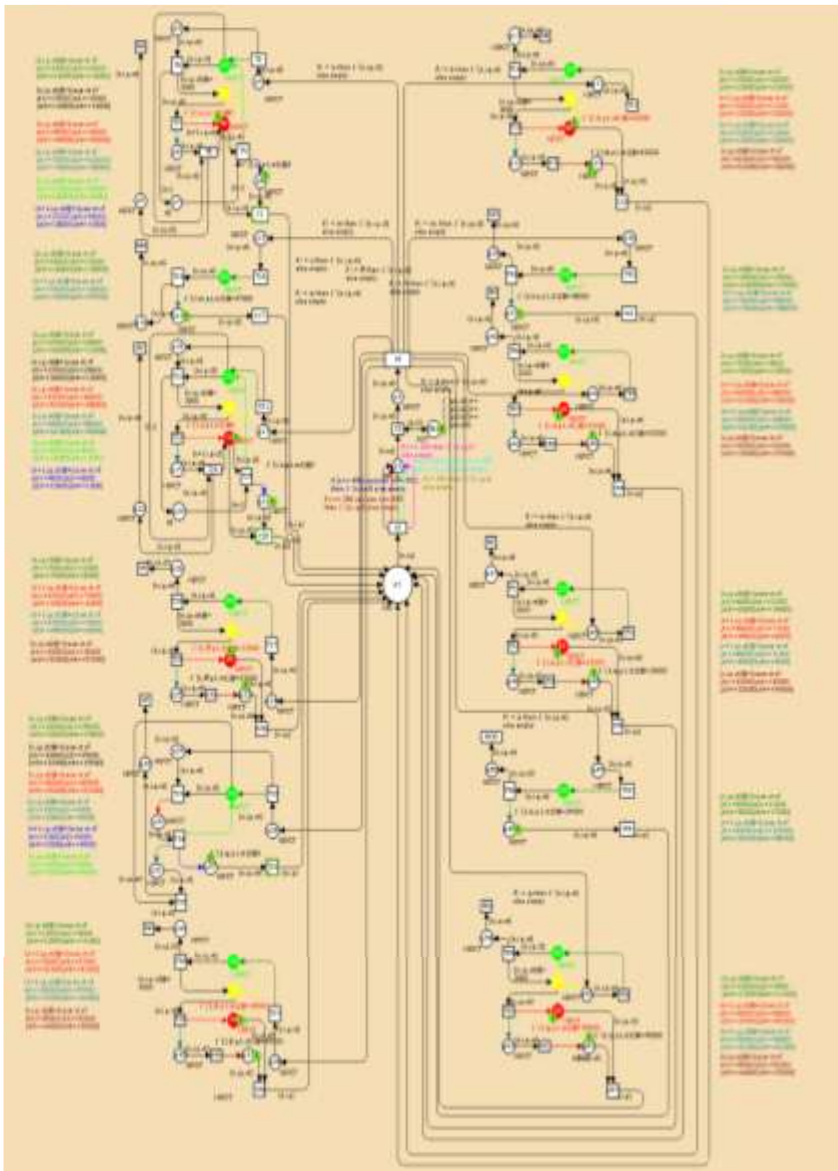

Figure 1.2: The developed TCPN model of Time-of-Day signal timing plan transitions of the considered intersection