Social-economic advantages of c-centres and m-centres with weight in Kosovo

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Abstract

This Graph theory has an enormous number of applications because through them, we may model different complex problems such as road and crossroad placement, electricity network and computer network placement, objects with public preeminence, etc. Deriving from the chaos that appears in the capital city, particularly in front of Clinic University or preeminent public objects, we decided to give a mathematical solution to these problems. Oftentimes we encounter on the problem for finding the most suitable place for building an object that may serve for different social reasons in peripheral areas. This paper gives a special significance to object placement for social-economic reasons in Kosovo and aims to give a solution for this problem.

Additional to the theoretical elaboration on the graph concepts theory, Floyd's algorithm for minimum distances, and concepts for c and m centers with weight we will model the graph of the most important roads in Kosovo, whereas vertices we have used cities and crossroads of Kosovo. Therefore, we will find c and m centers with and without weight in Kosovo, and we will give some reasons for their social-economic advantages

Keywords: Graph algorithm, c-center, m-center, Kosovo, economic advantages

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1. Introduction

The object placement problem is a classic problem, which has been studied since 1909 by the Weber Location Problem. Resembling problems require a solitary attention, hence for a group of objects the best locations should be positioned in those places that encounter most of the service requirements for a group of customers. In a boarder meaning we will use "facility". This signifies the intention for factories, hospitals, electronic communications centres and energy warning sirens, customs inclusion. Inclusiveness of placement determination has led to a special interest for model analysis, designing different algorithms for optimal solutions, in order to make decision for facilities placement, along with the determination on how to assign the requirements for the placed facilities, so that resources will be used more efficiently (Hamdy A. Taha,2017)..

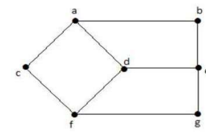
2. GRAPH THEORY FUNDAMENTAL CONCEPTS

2.1 Graph definition. Example

Let V be a nonempty finite set: V={v_1,v_2,v_3..v_n }.A particular set can be described along with diagrams, where its elements are marked as nodes. Every corresponding node is called a vertex. If two of its elements v_i and v_j not necessarily different, are considered then the connection between their points in a diagram with straight or arc segments, where one end point is vi and the other v_j is called edge, and is denoted like v_i,v_j. Two particular points can be connected by more than one edge. For ends with the same point the edge is represented as loop. Let E be a set of edges with edges at the vertices of the set V.

Definition 2.1.1 Considering the above conditions the pair G=(V,E), where V is a nonempty finite set of vertices, while E is a finite set of edges is called graph.

Example 2.1.1. Given the diagram for the graph G=(V,E) (figure 2.1.1) find V and E



Solution. V={a, b, c, d, e, f, g}, E={ab, ac, ad, be, ed, eg, df, fg, fc}.

Definition 2.1.2

- Vertices of an edge are called its ends
- Two vertices are called neighbours, if there is an edge connecting them
- Two edges are called incidents, if they have common vertex
- Graph is called simple, if there are no loops
- An empty graph is the one that has vertices, but no edges

Definition 2.1.3 Graph H=(V_H, E_H) is called subgraph of graph G=(V_G, E_G), if $V_H \subseteq V_G$ and $E_H \subseteq E_G$. Denoted as $H \leq G$.

Definition 2.1.4 A directed graph is called an ordered pair G=(V,E), where V is a nonempty finite set, while E is the subset of cartesian production $V \times V$. If the element (x_i,x_j) appears p-times in E, then the graph is called p-graph. Consequently, if each pair (x_i,x_j) appears only once in E, the graph is called 1-graph [4].

Definition 2.1.5 An ordered system of vertices (x_0, x_1, x_2,..., x_r), $r \geq 1$ of graph G=(V, E) such that each of the edges x_i,x_{i+1}, for i=0, 1, ..., r-1, belong to graph G, is called path in G. The vertex x_0 is called the beginning of the path while x_r is called the end of the path.

Definition 2.1.6 Let the graphs G_1=(V_1, E_1) and G_2=(V_2, E_2), where $V1 \cap V2 = \varnothing$ (disjoint) be given. Graph $G=(V,E)=G_1 \cup G_2$, where $V=V_1 \cup V_2$ and $E=E_1 \cup E_2$ is called the union of graphs G_1 and G_2.

Definition 2.1.7 A graph is called connected, if it is a union of two graphs. On the contrary, we say that the graph is disconnected.

2.2 Weighted graphs. The minimum distance- shortest path problem

Let G=(V, E) be a directed graph, with respect to which a function (reflection) W:E→ R_0^+ is given, with each edge of $e \in E$ associated with only one non-negative real number W(e), which is called weight of edge e or length with weight (abbreviated length) of edge e or distance with weight (abbreviated distance) between the edges of edge *e*. If the edge is *e*=(v_i, v_j), then we note the length with its weight

$$W_{i,j}: W(v_i, y_j)= W_{i,j} .$$

Definition 2.2.1. A graph G equipped with a function W of weights of its edges is called a weighted graph and is denoted by G^W.

For the subgraph H of the G^W graph, the number $W(H) = \sum_{e \in E(H)} w(e)$ is called the total weight of the subgraph H.

In particular, if H is a path P=(v_{i_1},v_{i_2},..., v_{i_p}) from G^W, then the sum $W(P) = \sum_{k=1}^{p-1} W_{i_k, i_{k+1}}$ is called the weighted length (abbreviated length) of the path P. We consider each distinct vertex of the graph G^W as a path of length 0.

In a directed graph there may be several paths starting at vi and ending at v_j . Of interest is knowing those paths that have the smallest length, otherwise we say the search for the shortest path starting at a vertex v_i and ending at a vertex v_j. The length of the shortest path starting at vi and ending at v_j is called the minimum distance between vertices vi and vj and is denoted by d(v_i, v_j),denoted.

$$d(v_i, v_j) = min\{W(P) | P \text{ is path starting at } v_i \text{ and ending at } v_j\}$$

It is cleary that shortest paths are required between elementary paths. Since the number of graph vertices is finite, the number of different elementary paths between two vertices is also finite. This makes it possible to find the shortest paths, but aim for the best way, which is given by certain algorithms.

Note: When the graph is undirected, as for example in the graph of the mathematical model of building a freeway at minimum cost between two cities (see [4], example on page 270), taking cost as a weight, and considering that the freeway works in both directions, a such graph is considered as a directed graph with bidirectional edges [8].

2.3 Floyd's algorithm

This algorithm solves the problem of minimum distances and shortest paths between any two vertices of the 1-graph with discrete distance matrix A=(l_{ij})$_{nxm}$.

Initial step:We take matrices A_0=($l_{ij}^{(0)}$) $_{nxm}$ and S_0=($s_{ij}^{(0)}$) $_{nxm}$ such that $l_{ij}^{(0)}$= l_{ij} and $s_{ij}^{(0)}$=j for each i,j=1,2,…,n. We take k=l and go to the general step.

General step (k):We find the matrices A_k=($l_{ij}^{(k)}$)$_{nxm}$ and S_k=($s_{ij}^{(k)}$)$_{nxm}$ where:

$$l_{ij}^{(k)} = \begin{cases} \min(l_{ij}^{(k-1)}, l_{ik}^{(k-1)} + l_{kj}^{(k-1)}) & for\ i \neq k, j \neq k \\ l_{ij}^{(k-1)} & for\ i = k\ and|or\ j = k \end{cases}$$

$$s_{ij}^{(k)} = \begin{cases} k\ for\ i,j \neq k\ such\ that\ l_{ik}^{(k-1)} + l_{kj}^{(k-1)} < l_{ij}^{(k-1)} \\ s_{ij}^{(k)} \quad \text{on contrary} \end{cases}$$

We make k equal to (k+1) and repeat the general step until k takes the value n. The matrix An found at the end of the n-steps is the matrix of the minimum distances between the vertices of the graph, so An=A*.[2],[3]

2.4 Centres concept in Graph

In practice, it is often required to find the "most suitable" place to build a facility that serves several peripheral points which may be residential centers or different points that require service. Example, in a given region where it is better to place a health, social-cultural facility, or a shopping center, serving the residential centers, for a telegraphic network where it is better to place the information processing center (central). Depending on the optimality criterion for positioning in this object, we consider two types of problems:

- The first includes those problems where the most suitable place for the facility is considered to be the one from which the minimum distance from the facility to the peripheral points is the smallest possible. This type includes the problem of placing the social-cultural or health facility.
- In problems of the second type, the most suitable place for the object is the one from which the sum of the lengths of all the shortest paths connecting the object with peripheral points is the smallest possible. It is more economical that the total length of the conductors connecting the points of the telegraph network to the switchboard to be minimal, therefore this problem belongs to the problems of the second type. [2]

2.4.1 Graph Centres

If we take a graph which is connected G=(V,E) with n vertices, we put a non-negative number in correspondence with each edge $u \in E$ l(u). We denote by l*(x_i,x_j) the minimum distance from x_i to x_j.

- The vertex $x_c \in V$ such that for each i=1,2,..,n satisfies the inequality:

$max_{1 \leq j \leq n} l^*(x_c, x_j) \leq max_{1 \leq j \leq n} l^*(x_i, x_j)$ it is called the c-center of graphite.

In other words, c-centre is the peak where it is reached

The vertex $x_m \in V$ such that for each i=1,2,...,n satisfies the inequality:

$\sum_{j=1}^{n} l^*(x_m, x_j) \leq \sum_{j=1}^{n} l^*(x_i, x_j)$, is called the m-centre of the graph.

In other words, the m-center is the vertex where it is reached

We assume that for each vertex $x_j \in V$ we have given a positive number w_j that we call its weight. In different problems, w_j it has different meanings, for example, in the placement of a service center in an area, w_j it can also indicate the number of residents of the i-th settlement.

- c-center with weighted graph G is any vertex $x_c \in V$ such that:

$max_{1 \leq j \leq n} \{w_j\ l^*(x_c, x_j)\} \leq max_{1 \leq j \leq n} \{w_j\ l^*(x_i, x_j)$

for each i=1,2,..,n

- m-center with weighted graph G is any vertex $x_m \in V$ such that:

$\sum_{j=1}^{n} w_j\ l^*(x_m, x_j) \leq \sum_{j=1}^{n} w_j\ l^*(x_i, x_j)$ for each i=1,2,..,n

In the above definitions, the center exists only when the left sides of the inequalities are finite numbers. On the contrary, it is said that the graph has no center, namely c-center or m-center. Clearly, G will be centered if and only if there is at least one vertex from which every other vertex can be traversed by paths in G.

In the above definitions, vertices are considered the same "rights". In practice this consideration is not always appropriate. For example, in the establishment of a health or social-cultural facility that serves several residential centers, the number of residents of each residential center should also be taken into account, which means; that the service facility should be as close as possible to centers with more inhabitants. [2],[3]

2.4.2 An approach for finding centres

All four types of centers defined above are very easily found through the matrix A* which we find with the help of Floyd's Algorithm.

i. For simple centers, we add to the matrix A* a column where we put the largest element of each row and a column where we put the sum of each row. The vertex, which belongs to the smallest element of the first added columnin represents the c-center, while the vertex belonging to the smallest element in the second added column represents the m-center of the graph.
ii. To get the weighted c-center and m-weighted center, first the columns of the matrix A* are multiplied by the weights wj of the vertices, respectively, and then it is done in the same way as in point i).

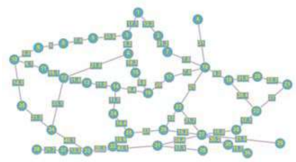


Figure. 3.1.1

3. C-CENTER AND M-CENTER IN KOSOVO (RESULTS)

3.1 Simple c-center and m-center

At the beginning, we give the graph of the most important roads in Kosovo (fig. 3.1.1)

Where the circles with numbers represent the cities or crossroads, while the rectangles represent the distances or weights of the edges.

In table 3.1.1 we have presented the data on the sequence numbers of the vertexes and the number of inhabitants for each settlement.

No.	Country	Number of Inhabitants
1.	Mitrovica	71.636
2.	Vushtrri	68.793
3.	Kline e Eperme	1.719
4.	Podujeva	86.836
5.	Gjyrakoc	2.209
6.	Skenderaj	51.553
7.	Obiliq	21.056
8.	Banje e Pejes	1.300
9.	Vitomeric	5.409
10.	Peje	97.776
11.	Zahac	1.120
12.	Kline	39.527
13.	Kjeve	1.279
14.	Orlat	3.134
15.	Gllogovc	60.111
16.	Komoran	4.393
17.	Fushe Kosove	37.735
18.	Prishtina	207.062
19.	Graqanice	11.720
20.	Novoberda	6.953
21.	Kamenice/Artane	33.599
22.	Gjilan	87.385
23.	Lipjan	58.373
24.	Malisheve	56.889
25.	Bllace	1.445
26	Shtimje	27.818
27.	Ferizaj	107.985
28.	Pozhoran	4.247
29.	Vitia	47.434
30.	Kacanik	33.784
31.	Shterpce	6.906
32.	Suhareke	60.869
33.	Prizren	184.586
34.	Gjakove	95.576
35.	Decan	40.847
36.	Hani i Elezit	9.759
37.	Zhur	5.909
38.	Dragash	34.241
39.	Doganaj	957

Table. 3.1.1

We use the Floyd-id Algorithm for finding A* and we act based on the clarifications that we have given in 2.4.1. and 2.4.2.i), so we obtain the table 3.1.2

Table. 3.1.2

1	0	13	17	73	51	22	34	58	64	70	63	44	55	57	41	47	50	42	51	83	106	90	63	69	83	76	85	100	106	115	109	94	115	75	84	127	128	150	126	150
2	13	0	30	60	64	36	21	72	78	83	76	57	68	66	54	57	37	29	38	70	93	77	50	78	86	63	72	87	93	102	96	97	123	88	97	114	136	158	113	158
3	17	30	0	89	34	5.2	52	41	47	53	46	27	38	40	24	30	50	58	67	99	121	106	79	51	66	89	101	116	122	131	125	77	98	58	67	143	111	133	142	143
4	73	60	89	0	123	84	39	130	124	119	110	91	80	68	65	59	39	31	40	72	95	79	52	80	88	65	74	89	95	104	98	99	125	122	133	116	138	160	115	160
5	51	64	34	123	0	39	85	7.4	13	19	27	46	58	69	57	64	84	92	101	133	155	139	113	81	96	119	134	149	155	164	138	106	94	54	33	176	108	129	155	176
6	22	36	5.2	84	39	0	57	46	52	49	41	22	33	35	18	25	45	53	62	94	116	100	74	46	61	84	95	111	117	126	120	72	93	53	63	137	106	128	137	137
7	34	21	52	39	85	57	0	93	99	95	87	68	57	45	42	35	15	7.8	17	49	71	55	29	57	65	42	51	66	72	81	75	76	102	99	110	92	115	137	92	137
8	58	72	41	130	7.4	46	93	0	6	12	20	39	50	62	65	71	91	99	108	140	163	147	120	74	88	111	126	142	148	157	130	99	87	47	26	168	100	122	147	168
9	64	78	47	124	13	52	99	6	0	5.5	14	33	44	56	71	65	85	93	102	134	157	141	114	68	82	105	120	136	142	151	124	93	81	41	20	162	94	116	141	162
10	70	83	53	119	19	49	95	12	5.5	0	8.1	27	39	51	67	60	80	88	97	128	151	135	109	62	77	100	115	130	136	145	119	88	76	35	14	157	89	111	136	157
11	63	76	46	110	27	41	87	20	14	8.1	0	19	31	42	59	52	72	79	89	120	143	127	100	54	69	92	107	122	128	137	127	79	84	44	22	149	97	119	144	149
12	44	57	27	91	46	22	68	39	33	27	19	0	11	23	40	33	53	60	70	101	124	108	81	35	49	72	88	103	109	118	112	60	71	31	41	130	85	106	129	130
13	55	68	38	80	58	33	57	50	44	39	31	11	0	12	28	21	41	49	58	90	112	97	70	24	38	61	76	91	98	106	101	49	75	43	53	118	88	110	118	118
14	57	66	40	68	69	35	45	62	56	51	42	23	12	0	16	9.4	29	37	46	78	101	85	58	12	26	49	64	80	86	95	89	37	63	54	65	106	76	98	106	106
15	41	54	24	65	57	18	42	65	71	67	59	40	28	16	0	6.8	27	34	44	75	98	82	55	28	42	65	77	92	99	107	102	53	79	71	81	119	93	114	119	119
16	47	57	30	59	64	25	35	71	65	60	52	33	21	9.4	6.8	0	20	28	37	68	91	75	49	21	36	59	70	86	92	101	95	46	73	64	74	112	86	108	112	112
17	50	37	50	39	84	45	15	91	85	80	72	53	41	29	27	20	0	7.6	17	48	71	55	29	41	56	42	50	66	72	81	75	66	93	84	94	92	106	128	92	128
18	42	29	58	31	92	53	7.8	99	93	88	79	60	49	37	34	28	7.6	0	9.2	41	64	48	21	49	57	34	43	58	64	73	67	68	94	91	102	85	107	129	84	129
19	51	38	67	40	101	62	17	108	102	97	89	70	58	46	44	37	17	9.2	0	32	54	38	30	58	66	43	52	56	73	82	76	77	103	101	111	94	116	138	93	138
20	83	70	99	72	133	94	49	140	134	128	120	101	90	78	75	68	48	41	32	0	23	50	62	90	98	75	82	67	104	112	106	109	135	132	143	124	148	170	124	170
21	106	93	121	95	155	116	71	163	157	151	143	124	112	101	98	91	71	64	54	23	0	27	81	112	98	75	59	44	81	90	84	108	127	155	165	101	140	162	101	165
22	90	77	106	79	139	100	55	147	141	135	127	108	97	85	82	75	55	48	38	50	27	0	54	85	71	48	32	17	54	63	57	81	100	139	149	74	113	135	74	149
23	63	50	79	52	113	74	29	120	114	109	100	81	70	58	55	49	29	21	30	62	81	54	0	50	36	13	22	37	43	52	46	47	73	112	123	64	86	108	63	123
24	69	78	51	80	81	46	57	74	68	62	54	35	24	12	28	21	41	49	58	90	112	85	50	0	14	37	53	68	74	83	77	25	51	66	76	95	65	86	94	112
25	83	86	66	88	96	61	65	88	82	77	69	49	38	26	42	36	56	57	66	98	98	71	36	14	0	23	38	53	60	68	63	11	37	77	91	80	50	72	80	98
26	76	63	89	65	119	84	42	111	105	100	92	72	61	49	65	59	42	34	43	75	75	48	13	37	23	0	15	30	37	45	40	34	60	100	114	57	73	95	57	119
27	85	72	101	74	134	95	51	126	120	115	107	88	76	64	77	70	50	43	52	82	59	32	22	53	38	15	0	15	22	30	24	49	68	108	129	42	81	103	42	134
28	100	87	116	89	149	111	66	142	136	130	122	103	91	80	92	86	66	58	56	67	44	17	37	68	53	30	15	0	37	45	40	64	83	123	144	57	96	118	57	149
29	106	93	122	95	155	117	72	148	142	136	128	109	98	86	99	92	72	64	73	104	81	54	43	74	60	37	22	37	0	52	46	71	89	129	150	63	102	124	63	155
30	115	102	131	104	164	126	81	157	151	145	137	118	106	95	107	101	81	73	82	112	90	63	52	83	68	45	30	45	52	0	55	79	98	138	159	12	111	133	72	164
31	109	96	125	98	138	120	75	130	124	119	127	112	101	89	102	95	75	67	76	106	84	57	46	77	63	40	24	40	46	55	0	69	43	83	104	66	56	78	17	138
32	94	97	77	99	106	72	76	99	93	88	79	60	49	37	53	46	66	68	77	109	108	81	47	25	11	34	49	64	71	79	69	0	26	66	87	91	39	61	86	109
33	115	123	98	125	94	93	102	87	81	76	84	71	75	63	79	73	93	94	103	135	127	100	73	51	37	60	68	83	89	98	43	26	0	40	61	109	13	35	60	135
34	75	88	58	122	54	53	99	47	41	35	44	31	43	54	71	64	84	91	101	132	155	139	112	66	77	100	108	123	129	138	83	66	40	0	21	150	53	75	100	155
35	84	97	67	133	33	63	110	26	20	14	22	41	53	65	81	74	94	102	111	143	165	149	123	76	91	114	129	144	150	159	104	87	61	21	0	171	75	96	122	171
36	127	114	143	116	176	137	92	168	162	157	149	130	118	106	119	112	92	85	94	124	101	74	64	95	80	57	42	57	63	12	66	91	109	150	171	0	123	144	83	176
37	128	136	111	138	108	106	115	100	94	89	97	85	88	76	93	86	106	107	116	148	140	113	86	65	50	73	81	96	102	111	56	39	13	53	75	123	0	22	74	148
38	150	158	133	160	129	128	137	122	116	111	119	106	110	98	114	108	128	129	138	170	162	135	108	86	72	95	103	118	124	133	78	61	35	75	96	144	22	0	95	170
39	126	113	142	115	155	137	92	147	141	136	144	129	118	106	119	112	92	84	93	124	101	74	63	94	80	57	42	57	63	72	17	86	60	100	122	83	74	95	0	155
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According to table 3.1.2 we can see that the c-centre is vertex 25, which belongs to the settlement (crossroads) Bllacë, while the m-centre, which is vertex 14 that belongs to the settlement (crossroads) Orllatë.

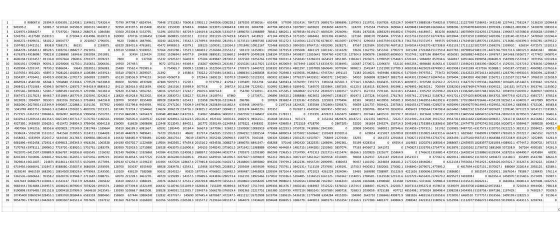

Table. 3.1.3

According to table 3.1.3 we observed that:

- The weighted c-centre is the vertex 24 belonging to the city of Malishevë, while
- The weighted m-centre is the vertex 26 belonging to the city of Shtime.

If we analyse further the appendix 2, we notice that:

- The second c-centre with weight is the peak 26 - the city of Shtime
- The third c-centre with weight comes out the peak 14 -the settlement Orllate, while
- The m-centre with weight comes out the peak 27- the city of Ferizaj
- The third m-centre with weight comes out 25 - the settlement Bllacë.

Notice. In the above graph, we have not considered the permitted speed of vehicle movements on the road. If the speed is also considered, then the c,m-centres may change.

4. SOCIO-ECONOMIC ADVANTAGES OF WEIGHTED C-CENTRES AND WEIGHTED M-CENTERS

According to the results that we obtained for weighted c-centres and m-centres and considering the theoretical aspect for them, we are listing some economic and social favors for c-centres with weight and m-centres with weight:

4.1 Socio-economic advantages of c weighted centers

The object of public importance in Kosovo of the first type (hospital, university, training facility, etc.) should be placed in peak 24 (or as an alternative 26.14), i.e. in the city of Malisheva (Shtimes Orllati), for the reason that:

- The smallest average distance for each end of Kosovo turns out to be Malisheva (the second weighted c-center is Shtimja and the third weighted c-center is Orllati)
- Not far from Malisheva (Shtime, Orllati) are the big cities of Kosovo.
- On average, the cost of transporting goods and services in Kosovo to come for such activities in the capital city decreases
- The capital is relieved from heavy traffic
- Malisheva (Shtimja, Orllati), will develop more economically, for the reason that small commercial services can be opened around these facilities
- The weight of the capital is distributed
- New job opportunities open up for c-centers with weight
- The internal displacement of the population to the Capital is stopped.

4.2 Social-economic advantages of m weighted centers

The object of public importance in Kosovo of the second type (information processing centers, distribution centers should be located at peak 26 (or as an alternative 27,25), i.e. in the city of Shtime (Ferizaj, Bllacë), for the reason that:

- The distribution of networks throughout Kosovo will have a lower cost, if this distribution has its center in Shtime.
- Not far from Shtime are the big cities of Kosovo.
- On average, the cost of building such networks decreases
- The capital is relieved from heavy traffic
- Shtimja is developed more economically, for the reason that small commercial services can be opened around these facilities
- Large interventions in the Capital are avoided, which also lead to road blockages
- New employment opportunities are opening for weighted m-centers

5. CONCLUSION

After a theoretical support that we gave to our aim, we successfully reached some significant results:

- The weighted c centre in Kosovo is Malisheva
- The weighted m centre in Kosovo is Shtimja
- Some socio-economic arguments were given for these centres

We hope that with this paper we have managed to solve some issues on the placement of public facilities in Kosovo.

Moreover, as we know the time complexity of the shortest paths algorithms depends on the number of the vertices taken, number of the edges etc in future work we are planning to generate a software with the optimized time complexity for the problems of the same nature as discussed in this paper.

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