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Numerical Analysis of Dynamic Viscosity Effect Associated with a Continuously Moving Heated Horizontal plate

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Abstract

This work presents numerical studies of the effects of varying dynamic viscosity coupled with viscous-energy dissipation function on the convective heat transfer in a fluid-filled rectangular cavity. The cavity was filled with fluids as quenching media.

The flow governing equations including the momentum and energy equations were solved using the finite difference method. The study was carried out for different fluids such as oil with Prandtl number, Pr = 10, air with Prandtl number, Pr = 0.7 and liquid metal with Prandtl number, Pr = 0.01, for various dynamic viscosity parameters in the range $5*10^{-1} \leq \mu \leq 9*10^{-1}$, and heat capacity in the range $1 \leq Cp \leq 10$ in order to characterize the nature of the flow patterns and energy distribution.

The results revealed that the dynamic viscosity has significant influence on the velocity and temperature profiles for a particular specific heat capacity and Prandtl number higher than unity at fixed viscous dissipation. Further results show that an increase in the dynamic viscosity for a Prandtl number greater than unity leads to a significant decrease in the maximum velocity attainable in the cavity. It was concluded that the dynamic viscosity and specific heat capacity have significance influence on energy distribution and the rate of heat transfer in the enclosure. The results would be useful as baseline design data for manufacturing and material processing industries involved with wire drawing, continuous rolling.

Keywords: Mixed Convection, Heat transfer, Dynamic Viscosity, Isotherms, Finite difference scheme

1. Introduction

Heat transfer in the boundary layer adjacent to continuous moving surfaces has many important applications in many manufacturing processes including the cooling and drying of paper and textiles, wire drawing, continuous casting, metal extrusion, glass fiber production and hot rolling (Sami *et al.*, 2003; Chen, 2000; Ali and Al-Yousef, 1998). The flow over a material moving continuously through a fluid is induced by the movement of the solid materials and by thermal buoyancy. Hence surface motion and buoyancy effect will determine the momentum and thermal transport processes. Thermal buoyancy effect due to the heating or cooling of a continuously moving surface, under some circumstances may alter significantly the flow pattern, thermal field and heat transfer behaviour in the manufacturing process (Chen, 2000).

Many researchers have investigated the effects of buoyancy force caused by continuously moving surfaces on quiescent fluid for different orientations. The numerical simulation of thermal transport associated with a continuously moving flat sheet in materials processing was carried out by Karwe and Jaluria (1991; 1998). Ali and Al-Yousef (1998) examined this effect on vertical surfaces and Chen (2000) examined its effect on vertical and inclined surfaces. They concluded that the buoyancy force has significance on the velocity and temperature distribution and hence on the heat transfer rate from the surface. Wong (2007) investigated the effect of the combined buoyancy- and lid-driven convection in a square cavity in which the influence of pressure on the flow was studied. Al-Sanea and Ali (2000) investigated the effect of buoyancy parameter on moving plate in rolling and extrusion processes. The laminar mixed convection adjacent to a vertical, continuously stretching sheet was studied numerically by Chen (1998). Fan et al., (1997) carried out the numerical investigation of the mixed convective heat and mass transfer over a horizontal plate. A numerical study of the flow and heat transfer characteristics associated with a heated continuously stretching surface being cooled by a mixed convection flow was carried out by Chen (2000). They concluded that the buoyancy force has pronounced effects on the flow field, the local Nusselt number and friction coefficient.

In all the above cited works, the effect of viscous-energy dissipation function on heat transfer at high temperature was not considered. However, high speed and temperature have appreciable effects on the energy distributions and flow field (Oosthuizen and Naylor, 1999). From the findings, it is clear that the effect of varying dynamic viscosity coupled with viscous-energy dissipation function on temperature profile, heat transfer rate and flow field in a rectangular enclosure bounded by a continuously moving heated horizontal plate have not been considered. This fact motivates the present study.

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The aim of this work is thus to present the effect of varying dynamic viscosity coupled with viscous-energy dissipation function on the flow patterns, energy distribution and heat transfer rate within rectangular cavity.

2. The Physical and the Mathematical Models

Figure 1 shows a continuously moving horizontal plate emerging from a slot at a velocity U_w and temperature T_w into an otherwise quiescent fluid. The plate forms the upper wall of the rectangular enclosure under consideration. The enclosure is also bounded by a fixed horizontal isothermal wall on the lower part, a fixed isothermal vertical wall bordering the extrusion die surface on the left and an adiabatic vertical wall on the right. The temperature T_w of the upper horizontal wall is higher than that of the lower horizontal wall (i.e. $T_w > T_\infty$) as a result of which free convective motion ensued in the enclosure.

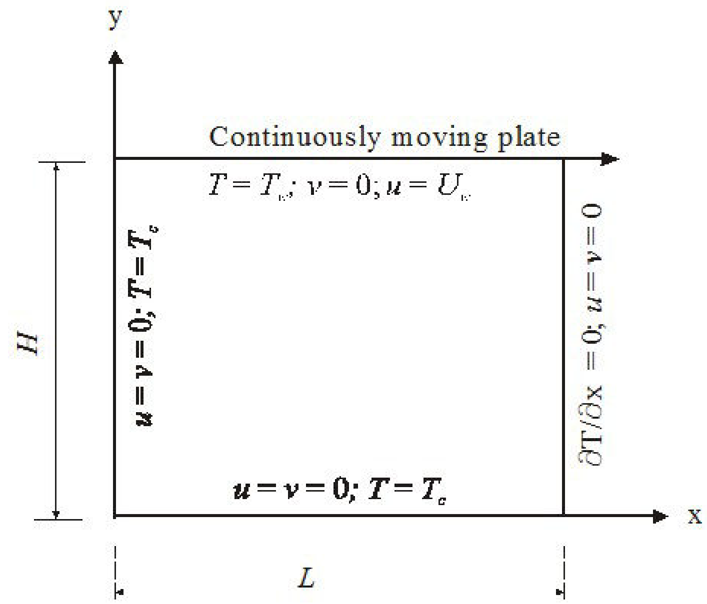

Figure 1. Schematic representation of the physical model with the boundary constraints and the coordinate axes

The flow is assumed steady, incompressible, laminar, and two-dimensional and the fluid is Newtonian. The heat transfer by radiation and the internal heat generation are assumed negligible, while viscous-energy dissipation function effect is considered. The fluid properties are assumed independent of temperature except for the buoyancy term in the momentum equation for which the Boussinesq approximation is used. The extrusion die wall is stationary and impermeable for which the non-slip boundary conditions applied.

The flow governing equations at every point of the continuum comprise the expressions for the conservation of mass, momentum and energy with viscous dissipation term included. These equations for a two-dimensional rectangular domain are (Ozisik, 1985) :

2.1 Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

2.2 the Navier-Stokes equations in the x- and y-directions:

x- direction:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

Y - directions:

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \beta g(T - T_\infty) \tag{3}$$

where $\beta g(T - T_\infty)$ is the body force per unit volume in *y* direction.

2.3 The thermal energy transport equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\phi \qquad (4)$$

where ϕ is the viscous-energy-dissipation function, defined as:

$$\phi=2\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right)+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2 \qquad (5)$$

The consideration of this function becomes important if either the fluid viscosity or the flow velocities are high [10].

The prescribed boundary conditions for the velocities and the temperature are:

$$\begin{aligned}&u=U_w,\ v=0,\ T=T_w\ at\ y=H,\ 0\le x\le L;\\&u=0,\ v=0,\ T=T_\infty\ at\ y=0,\ 0\le x\le L;\\&u=0,\ v=0,\ T=0\ at\ x=0,\ 0\le y\le H;\\&\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}=\frac{\partial T}{\partial x}=0\ at\ x=L,\ 0\le y\le H.\end{aligned} \qquad (6)$$

3. Method of Analysis and the Solution Techniques

The Navier-Stokes equations are class of partial differential equations that could be classified as elliptic, parabolic or hyperbolic depending on the problem under consideration. These equations in their incompressible form can be solved by using either the vorticity-stream function approach or in their primitive-variable form. In this work, the former approach is adopted and so equations (2) and (3) are reduced to vorticity transport equation by eliminating the pressure gradient terms between them using the continuity equation (1), and the expression for scalar value of the vorticity, ω, in the two-dimensional Cartesian coordinate system defined as

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}. \qquad (7)$$

The resulting expression is the dimensional vorticity transport equation:

$$-\beta g\frac{\partial T}{\partial x}+\upsilon\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right) \qquad (8)$$

The velocity components in a two-dimensional Cartesian coordinates are defined as the derivatives of the stream-function, ψ, as follows:

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \qquad (9)$$

which on substitution in equation (7) gives the Poisson equation for the stream function

$$\omega=-\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right) \qquad (10)$$

The derived and energy equations and the prescribed boundary conditions were cast in non-dimensional form so that the results obtained could be generalized for a wide range of physical situations using L, $(T_w - T_\infty)$, U_W, $U_W L$ and U_W/L respectively for length, temperature, velocity, stream function and vorticity following (Sami *et al.,*2003):

$$\begin{aligned}&X=\frac{x}{L},\quad Y=\frac{y}{L},\quad \theta=\frac{(T-T_\infty)}{(T_w-T_\infty)},\quad \Psi=\frac{\psi}{U_wL},\\&U=\frac{u}{U_w},\quad V=\frac{v}{U_w},\ \Omega=\frac{\omega}{U_w/L},\end{aligned} \qquad (11)$$

The normalized form of the X- and Y-velocity components, stream function, vorticity and energy transport equations are:

$$U = \frac{\partial \Psi}{\partial Y}, \quad V = -\frac{\partial \Psi}{\partial X} \tag{12}$$

$$-\Omega = \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} \tag{13}$$

$$U\frac{\partial \Omega}{\partial X} + V\frac{\partial \Omega}{\partial Y} = \frac{1}{Re}\left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2}\right) - \frac{Gr}{Re^2}\frac{\partial \theta}{\partial X} \tag{14}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{\Pr Re}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) + \left(\frac{Ec}{Re}\right)\left[2\left(\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right) + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right)^2\right] \tag{15}$$

In the above equations, *Ec* stands for Eckert number, *Re* Reynolds number, *Gr* Grashof number, and *Pr*, Prandtl number. The Eckert number relates the flow viscous-dissipation term to the energy distributions. The number is a criterion for deciding whether the viscous-energy dissipation effect should be considered in the heat transfer analysis or not. The Prandtl number relates the rates of diffusion of heat and momentum and it is function specific heat capacity, Cp, dynamic viscosity, μ, and thermal conductivity, k. The Grashof number is a dimensionless parameter representing the ratio of the buoyancy force to the viscous force in the free-convection flow problem. It indicates whether the flow is laminar or turbulent and the dynamic process that is dominant.
The boundary conditions in non-dimensional form are:

$$\begin{aligned}
&\Omega \neq 0;\ \Psi \neq 0;\ V = 0;\\
&U = \theta = 1 \text{ at } Y = 1;\ 0 \leq X \leq 1;\\
&\Omega \neq 0;\ \Psi = U = V = \theta = 0\\
&\text{at } Y = 0;\ 0 \leq X \leq 1;\\
&\Omega \neq 0;\ \Psi = U = V = \theta = 0\\
&\text{at } X = 0;\ 0 \leq Y \leq 1;\\
&\Omega \neq 0;\ \Psi = \frac{\partial U}{\partial X} = \frac{\partial V}{\partial X} = \frac{\partial \theta}{\partial X} = 0 .\\
&\text{at } X = 1;\ 0 \leq Y \leq 1
\end{aligned} \tag{16}$$

The vorticity and energy transport equations (14) and (15) are non-linear in character. There are no known general analytical solutions for these coupled equations. One of the most fruitful approaches to the solution of equations (12) – (15) is the finite difference method, which involves the approximation of each term of the differential equations by their corresponding differential quotient. The resulting linear equations were solved simultaneously by adopting the relaxation technique.
The convective heat transfer into the enclosure is computed in terms of the Nusselt number, which is a dimensionless number that describes the ratio of the heat transfer by convection and conduction across the fluid layer. The temperature gradient that would result from the heat exchange process between the fluid and the wall can be related to the local Nusselt number, Nu_x, through the following expression:

$$Nu_x = \frac{h_x x}{k} = -\left(\frac{\partial \theta}{\partial Y}\right)_{Y=1} \tag{17}$$

The average Nusselt number is obtained by the integration of the local Nusselt number over the entire length of the heated wall:

$$\overline{Nu} = \frac{\dot{Q}_{conv}}{\dot{Q}_{cond}} = -\int_0^1 \left.\frac{\partial \theta}{\partial Y}\right|_{Y=0 \text{ or } 1} dX \tag{18}$$

The flow steady state was determined by monitoring the convergence of the temperature and vortex field, using the following criterion:

$$\frac{\sum_{i=2}^{N}\sum_{j=2}^{M}\left|\phi_{ij}^{n+1} - \phi_{ij}^{n}\right|}{\sum_{i=2}^{N}\sum_{j=2}^{M}\left|\phi_{ij}^{n+1}\right|} < \delta \qquad (19)$$

The parameter ϕ stands for Ψ, θ or Ω and n denotes the number of iterations before convergence of the results. The value of δ used in different literatures varies between 10^{-3} and 10^{-8} (Chung, 2002)

4. Discussion of Numerically Generated Results

The effect of the convergence criterion on the numerical results was studied by computing the average Nusselt number at different value of convergence parameter, δ, between 10^{-1} and 10^{-8}. The results which are presented show that a value of $\delta = 10^{-4}$ was adequate for convergence. The results of the grid independence tests show that a 41 by 41 grid system is sufficient for good numerical stability, field resolution and accurate results as reported in a similar work carried out by Waheed (2009).

In order to ascertain the validity of the code used in this work and the accuracy of the present simulation, the Nusselt number for a convective flow of a classical problem was computed for a Prandtl number, Pr = 0.7 and Rayleigh number, Ra = 1000. The Nusselt number computed with the help of the program used in this work is Nu = 1.1210 which compared very well with Nu = 1.132 computed by Waheed (2006) for the same Rayleigh and Prandtl numbers with about 2% discrepancy. Further validation of the results was done as follow: for $Ra = 10^5$ and Pr = 0.7, the computed Nusselt number by Hong (1992) is Nu = 4.5885 and the one computed by Waheed (2006) is 4.6201. The computed Nusselt number from the present simulation is 4.7438 which is in good agreement with the above two results.

Figures 2, 3 and 4 show the temperature profiles respectively for Pr = 0.01, Pr = 0.7 and Pr = 10.0 at Eckert number, Ec = 1.0 and Richardson number, Gr/Re^2 = 1.0 for various values of dynamic viscosity, μ. It is evident from figures 3 and 4 that an increase in the dynamic viscosity results in negligible effect on the wall temperature gradient close to the moving plate for liquid metal and gases, and hence produces an insignificant effect in the surface heat transfer rate. Also Figure 5 reveals that the effects of dynamic viscosity are found to be more pronounced for a fluid with a high Prandtl number, Pr = 10.

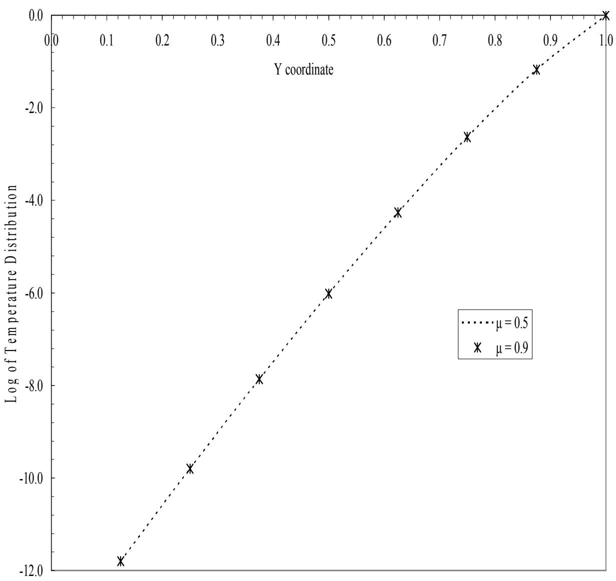

Figure 2: Plot of Temperature, θ versus vertical coordinate, Y for viscosities; $\mu=0.5$, $\mu=0.9$ *and* Pr = 0.01

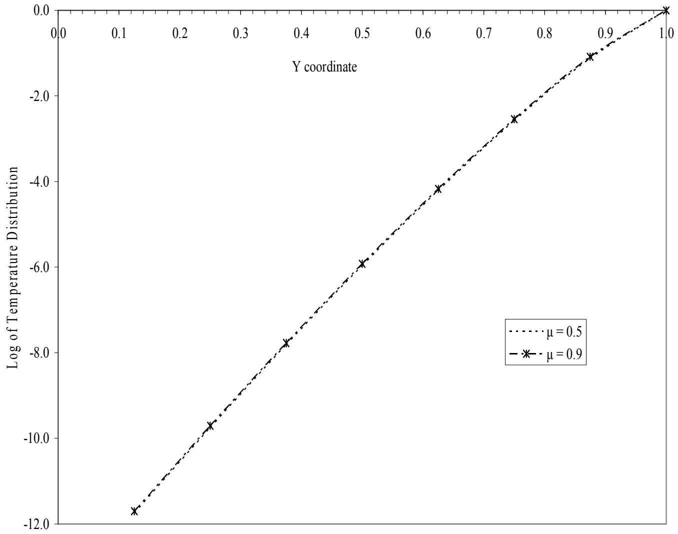

Figure 3: Plot of Temperature, θ versus vertical coordinate, Y for viscosities; μ*=0.5,* μ*=0.9 and* Pr = 1.0

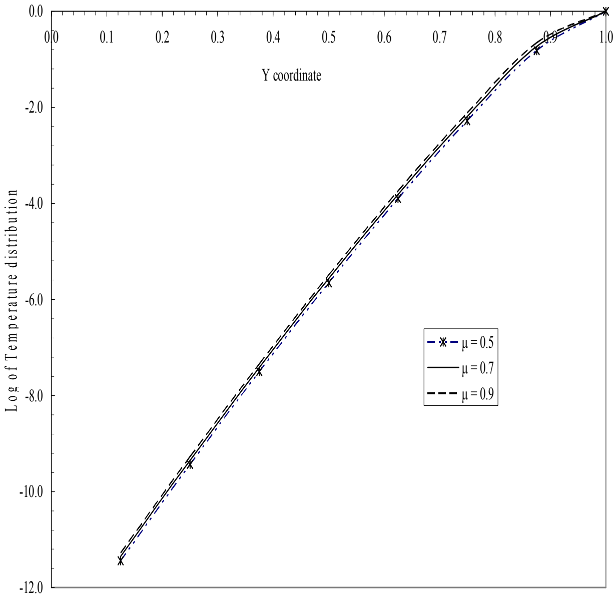

Figure 4: Plot of Temperature, θ versus vertical coordinate, Y for various viscosity, μ, Pr = 10.0

Figure 5 shows the variations of Nusselt number with the dynamic viscosity. It is observed that for a particular value of *Pr*, Pr >>1 the Nusselt number increases with the dynamic viscosity. Also an increase in the Prandtl number results in an increase in the Nusselt number. Figure 7 shows that an increase in dynamic viscosity results in significant enhancement in the Nusselt number at high value of specific heat capacity, Cp but it has negligible effect at low values.

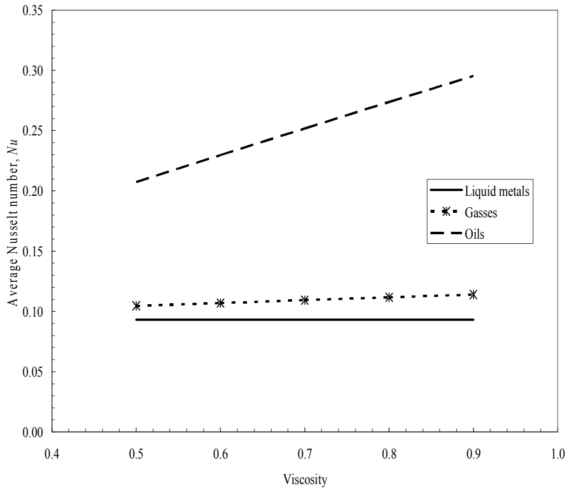

Figure 5: Plot of Average Nusselt number, *Nu* versus viscosities, μ, for different fluids

It is clear from Figure 6 that an increase in dynamic viscosity at high Prandtl number produces a decrease in the velocity gradient that deteriorates the momentum transport, which in turn decreases the flow rate. For liquid metal, i.e. fluid with $Pr << 1$, the effect of convection on the fields is very weak. Small *Pr* could result from very small momentum diffusivity (i.e. very weak convection) or very high thermal diffusivity. These diffusion rates are precisely the quantities that determine the thickness of the boundary layers for a given external flow field. Large momentum or thermal diffusivity means that the viscous or temperature influence is felt farther out in the flow field. It implies that positive increase in the dynamic viscosity decelerates the flow at a particular Prandtl number.

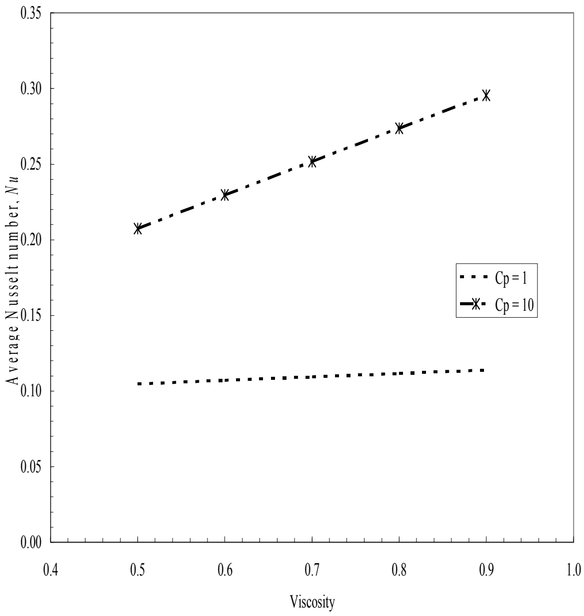

Figure 6: Plot of Average Nusselt number, *Nu* versus viscosities, μ, for heat capacities; *Cp=1.0 and Cp=10.0*

5. Conclusion

The flow and thermal fields along a continuously moving horizontal sheet of extruded material were studied for different quenching media, i.e. oil, air and liquid metal with Pr = 10, 0.7 and 0.01 respectively for varying dynamic viscosity, in order to characterize the flow patterns and energy distribution with viscous-energy dissipation function. The numerical model based on finite difference procedure was validated with published results. It can be concluded that the dynamic viscosity and specific heat capacity have significance influence on energy distribution and the rate heat transfer in the enclosure. The dynamic viscosity has significance effect on energy distribution and heat transfer rate from the surface for particular Prandtl number and specific heat capacity.

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Nomenclature

B_f - Buoyancy force parameter $\left(= \mathrm{Gr_x}/\mathrm{Re_x^3} = g\beta(\mathrm{T_W} - T_\infty)\nu/\mathrm{u_W^3} \right)$

c_p - Specific heat capacity of fluid, J/kgK

Ec - Eckert number $\left(= U_w^2/\Delta T\, c_p\right)$

g - Acceleration due to gravity, m/s^2

Gr - Grashof number, (= $g\beta(T_w - T_c)L^3/\nu^2$)

Pr – Prandtl number, $C_p\mu/k$

H - Height of the enclosure

h - Heat transfer coefficient

k - Fluid thermal conductivity, W/mK

L - Length of the enclosure

M - Number of horizontal grid lines

n - Number of iterations

N - Number of vertical grid lines

Nu - Average Nusselt number,($= \dot{Q}/k\Delta T = hL/k$)

Nu_x- Local Nusselt number $\left(= hx/k\right)$

p - Pressure, N/m^2

Pr- Prandtl number (= ν/α)

Ra - Rayleigh number (= $GrPr = g\,\beta(T_w - T_\infty)L^3/\alpha\nu$)

Re - Reynolds number (= $U_w L/\nu$)

Ri - Richardson number, $= Gr/\mathrm{Re}^2 = g\beta(T_w - T_\infty)L/U_w^2$
T - Temperature, K
u - Horizontal velocity component, m/s
U - Dimensionless horizontal velocity component $(= u/U_w)$
v - Vertical velocity component, m/s
V - Dimensionless vertical velocity component $(= v/U_w)$
x - Horizontal coordinate, m
X - Dimensionless coordinate $(= x/L)$
y - Vertical coordinate, m
Y - Dimensionless coordinate $(= y/L)$

Greek Alphabet

α - Thermal diffusivity $(= k/\rho c_p)$
β - Volumetric thermal expansion coefficient, 1/K
δ - Convergence parameter
θ - Dimensionless temperature = $(T - T_\infty)/(T_w - T_\infty)$
μ - Dynamic viscosity, Pas
ν - Kinematic viscosity, m^2/s
ρ - Density, kg/m^3
ϕ - Viscous-energy-dissipation function
ψ - Stream function, m^2/s
Ψ - Dimensionless Stream-function $(= \psi/(U_W L))$
ω - Vorticity, 1/s
Ω - Dimensionless vorticity $(= \omega \mathrm{L}/U_W)$

Sub- and Superscripts

w - Condition at surface
∞ - Condition at ambient medium