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Non-Compartmental S-I-S Modeling Of Hiv Prevalence In 7 Countries Of The World

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Abstract
Two non-compartmental SIS models were developed, one for infection (I or H) and the other for susceptible (S or SH). HIV percentage prevalence data were obtained via the internet. Data for susceptible percentage to HIV were computed from HIV data, knowing that $\beta + \gamma = 1$. The data were used to validate the models. The coefficients of correlation for different countries tested with the models were found to be reasonably very high, showing that the models fitted the data almost perfectly. The ultimate prevalence, intersection time for HIV and susceptible to HIV prevalence, the peak time and its prevalence, and the exhaustion time of HIV prevalence for different countries were computed. These non-compartmental SIS models could be used for different countries in making valuable predictions.
Keywords: Prevalence, non-compartmental, modelling, Infected, Susceptible, HIV.

1. Introduction

Human immunodeficiency virus infection/acquired immunodeficiency syndrome (HIV/AIDS) is a disease of the human immune system caused by the human immunodeficiency virus (HIV). During the initial infection a person may experience a brief period of influenza-like illness (Chretien, 1990). This is typically followed by a prolonged period without symptoms. As the illness progresses it interferes more and more with the immune system, making people much more likely to get infections, including opportunistic infections, and tumors that do not usually affect people with working immune systems (Sepkowitz,2001, Evian, 2006).
Throughout history, infectious diseases have had a lead impact on human population. Although infectious diseases are present in human populations at all times to some degrees, the effects of epidemics are the most noticeable and spectacular (Leitch *et al*., 1995). In 14th century Europe, some 25 million people out of a total population of about 100 million died from the Black Death (major epidemic impact) (Collins *et al*., 1997). The European conquest of the new world was not caused by guns, swords or barbaric type behavior but by the invisible danger-germs. Infectious diseases have played a major role in shaping the conquest of the new world. It is often said that in the centuries after Columbus landed in the new world on 12 October 1492,more native Americans died each year from infectious diseases brought by the European settlers than were born (Corbett *et a*l.,2003). Disease was the principal reason for the demise of the Indians. The main disease was smallpox, measles, influenza, and typhus. Others are whooping cough, the nymphs and diphtheria. Apart from HIV/AIDS and Tuberculosis for two decades, from 1934 to 1954, tropical Africans suffered a nearly 40% childhood mortality rate from malaria and malaria related diseases (Reviglione et al, 1995, Santiago *et al*., 2005).
In Nigeria, an estimated 3.6 percent of the population is living with HIV and AIDS. Although HIV prevalence (3.1%) is much lower in Nigeria than in other African countries such as south Africa and Zambia , the size of Nigeria's population (about 149 million) meant that by an end of 2009, there were about 3 million people living with HIV. Approximately 192,000 persons died from AIDS in 2009. In addition, Nigeria has one of the largest tuberculosis burdens in the world (311 per 100,000), resulting in the largest burden in Africa. The largest rates of TB/HIV co-infection result in a significant health challenge in the HIV/AIDS response (UNAIDS, 1998).
From Dietz (1988), Reeves *et al*., (2002) and Pedian *et al*., (1991) analysis of smallpox, mathematical modelling of epidemic of infectious diseases (i.e. diseases with causative agent whether as virus, bacterium or protozoa) have been extended greatly in recent years. He was, by a long way the first to express the proportion of susceptible individual of an endemic infection in terms of force of infection (the annual rate of acquiring an infection) and life expectancy. Hundreds of mathematical models have proven particularly powerful in the study of the effects of bacterial, parasitic and viral pathogens (Vynnycky, 1996, Kaslow, 1989).
Mathematical models and computer simulation have become useful in analyzing the spread and control of infectious diseases (Herbert, 1992, Castillo, 1989).
HIV which lead to AIDS and which has defied all medical solution is spreading fast in every country of the world non-stop. Serious study is going on to understand the complicated mechanism of the propagation all over

the world. It looks as if the medication of this disease will not be by administering particular drug to cure the disease straight but by administering different drugs at different stages of its recovery until it disappears completely from the patients Alland *et al*., 1994). To do this the complicated mechanism of its propagation must be understood. Although some team of U.S. scientists have announced their understanding of the complicated propagation mechanism, the HIV/AIDS complicated medication is yet to be unravelled.

It is important to have mathematical knowledge of the variation of the HIV prevalence with time. These maybe connected to the complicated mechanism of its propagation with time. Next will be to model the variation of each stage development with time and when all these models are put together it is possible to arrived at a complicated model that rightly described the complicated mechanisms with time.

The objective of this study is to develop a simple non-compartmental SIS model for HIV prevalence against time for different country. This will help to tell a countries HIV prevalence variation at any time.

The scope of this study is to simply model a non-compartmental SIS prevalence against time only. It does not include SIR, SIRS SEIR, and so on.

2. Model Development

Consider a community of P people with outbreak of HIV. Suppose there are H affected and N unaffected people in the community, so that P = H+N. If β and γ are the proportions (or prevalence) of HIV affected and unaffected (susceptible) people respectively, then β + γ=1 i.e. H = βP and N = γP. If P is sufficiently large (like a country) β and γ can be taken as continuous variables. The rate at which the disease spreads is dβ/dt. Also, dγ/dt is the rate at which the unaffected (susceptible) people diminish. Suppose the spread is by contact i.e. βγ, between the HIV affected and the unaffected people, then the rate of spread of the disease is proportional to the contact. Also, the rate of the unaffected diminishing is proportion to the contact i.e.

$$\frac{d\beta}{dt} \propto \beta\gamma \quad \text{or} \quad \frac{d\beta}{dt} = K_H\,\beta\gamma \tag{1}$$

$$\frac{d\gamma}{dt} \propto \beta\gamma \quad \text{or} \quad \frac{d\gamma}{dt} = -K_N\,\beta\gamma \tag{2}$$

From β + γ =1 (3)

$$\frac{d\beta}{dt} = -\frac{d\gamma}{dt}$$

Introducing equations (1) and (2), KH = K_N

Dividing equation (1) by (2) we have:

$$\frac{d\beta}{dt} = -\frac{d\gamma}{dt} \quad \text{or} \quad K_N d\beta = -K_H d\gamma \tag{4}$$

Substituting (3) into (1) yields

$$\frac{d\beta}{dt} = K_H\,\beta(1-\beta) \tag{5a}$$

Similarly the diminishing rate of unaffected people from equation (2) and (3) will be:

$$\frac{d\gamma}{dt} = -K_N\,\gamma(1-\gamma) \tag{6a}$$

the solutions of equation (5a), and (6a) are:

$$\beta = \frac{Ae^{K_H t}}{1+Ae^{K_H t}} \quad (5b)$$

$$\gamma = \frac{Be^{-K_N t}}{1+Be^{-K_N t}} \quad (6b)$$

The rate of change of prevalence (Dβ) for HIV prevalence is expressed as:

$$D\beta = \frac{d\beta}{dt} = \frac{AK_H e^{K_H t}}{(1+Ae^{K_H t})^2} \quad (7)$$

Equation (5b) and (6b) are the model equations for HIV prevalence Infected and Susceptible for different countries respectively, while equation (7) is the model equation for the annual rate of HIV prevalence infected of different countries.

Note: Percentage prevalence for the susceptible is obtained by subtracting the prevalence of the HIV infected from 100, since $\beta + \gamma = 1$.

2.1 Data Collection

HIV prevalence data with time were obtained via the internet (AFR-country, 2003).

2.2 Curve Fitting

The HIV and susceptible (computed) prevalence data are plotted in a scatter diagram. The developed models are superimposed on the scatter diagram and its fitness is declared as R^2 (coefficient of correlation) by the MATLAB 7.0 toolbox. The R^2 shows the degree of scattered points falling into the profile of the models.

3. Results

The results of the computation are shown in figure 1a and 1b, 2a and 2b------- 7a and 7b, and, tables 2 to 8 respectively

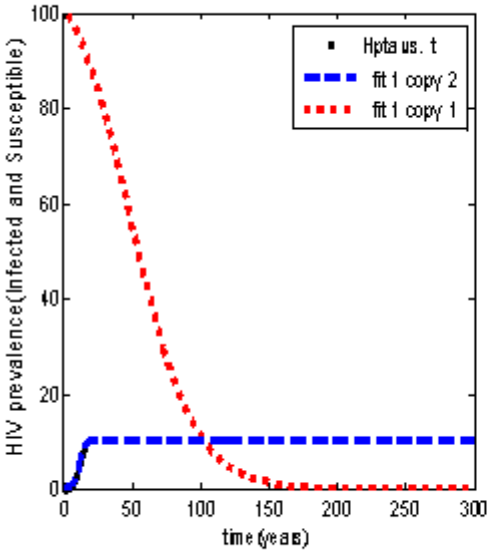

Fig1 a: HIV prevalence (Infected and Susceptible) versus time - Tanzania

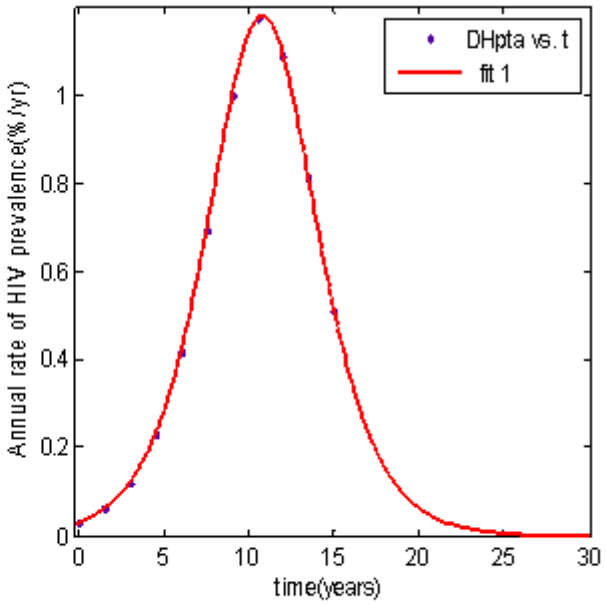

Fig. 1b: Annual rate of HIV prevalence versus time – Tanzania

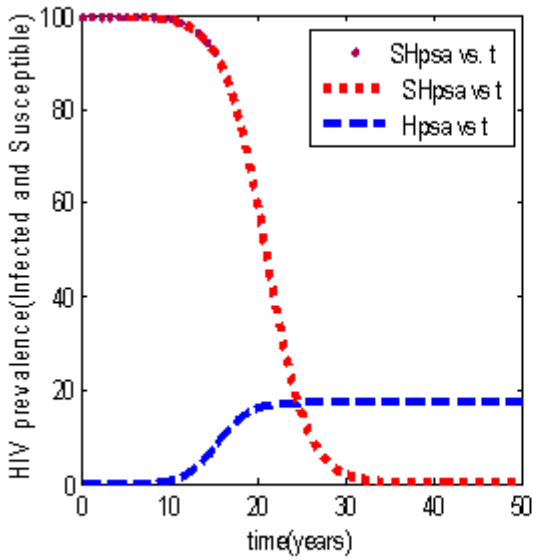

Fig. 2a: HIV prevalence (Infected & Susceptible) versus time – South Africa

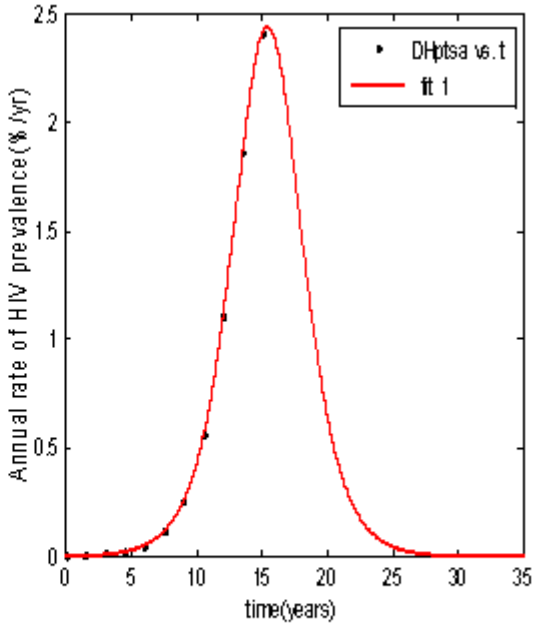

Fig. 2b: Annual rate of HIV prevalence versus time – South Africa

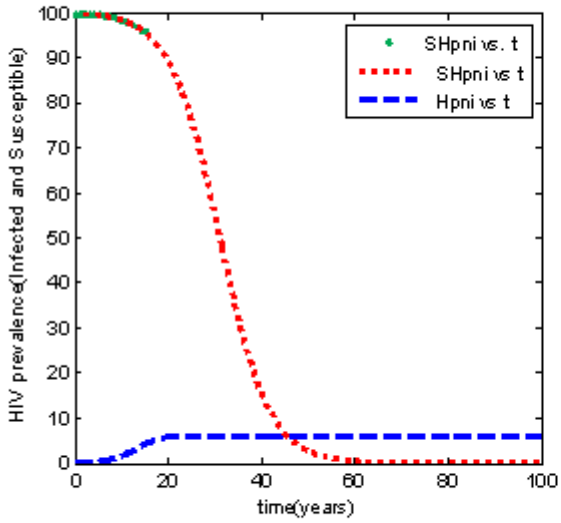

Fig3a: HIV prevalence (Infected and Susceptible) versus time - Nigeria

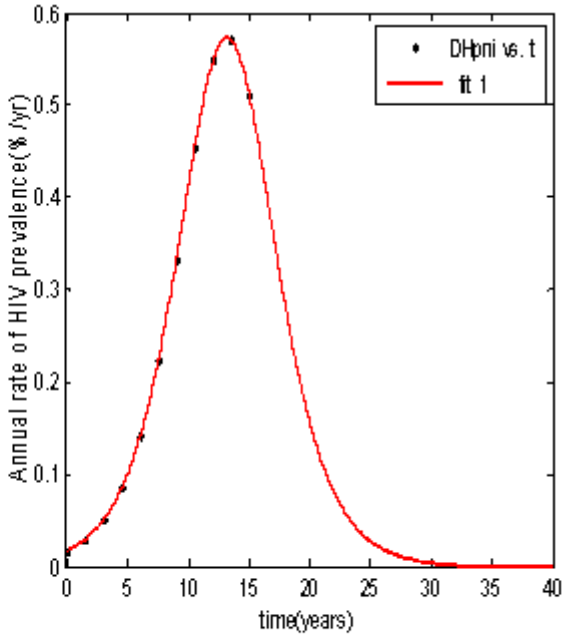

Fig. 3b: Annual rate of HIV prevalence versus time – Nigeria

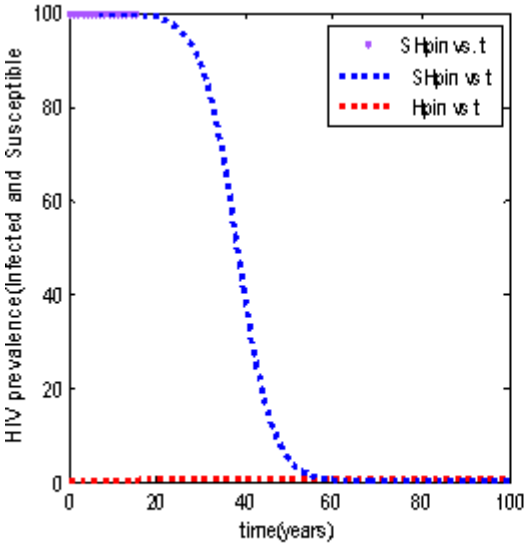

Fig. 4a: HIV prevalence (Infected and Susceptible) Versus time - India

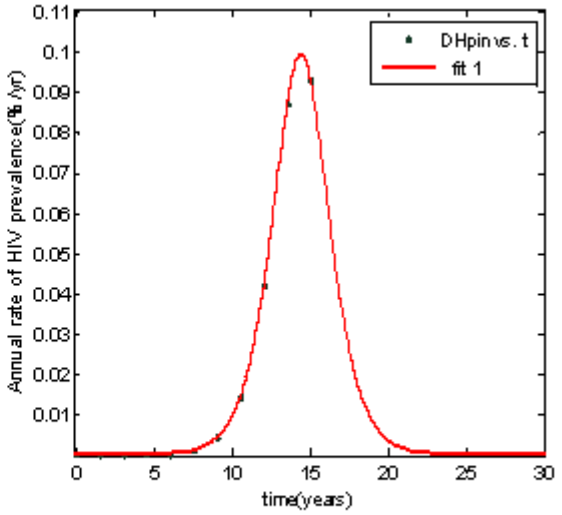

Fig. 4b: Annual rate of HIV Prevalence Versus time - India

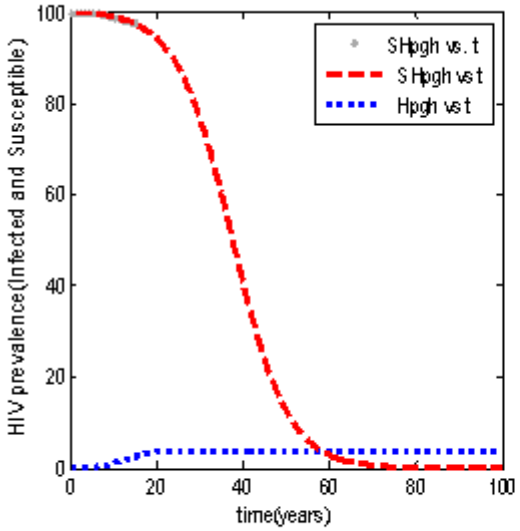

Fig. 5a: HIV prevalence (Infected and Susceptible) Versus time - Ghana

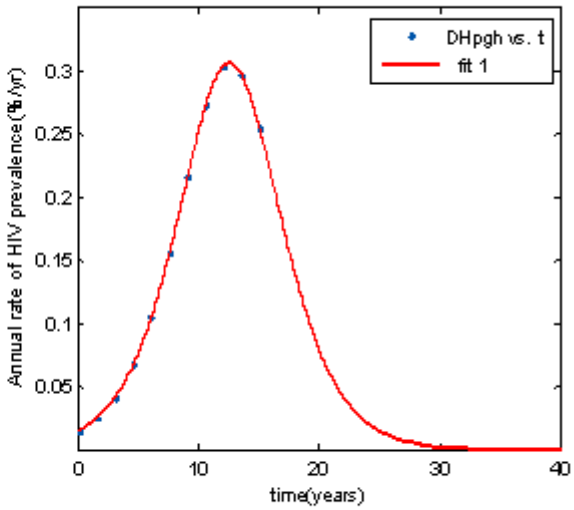

Fig. 5b: Annual rate of HIV Prevalence Versus time - Ghana

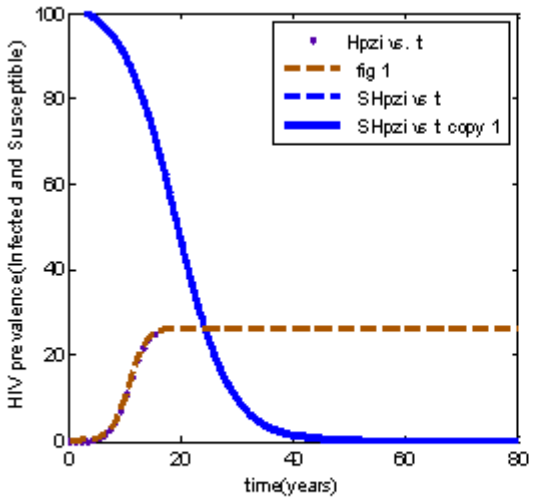

Fig. 6a: HIV prevalence (Infected and Susceptible) Versus time - Zimbabwe

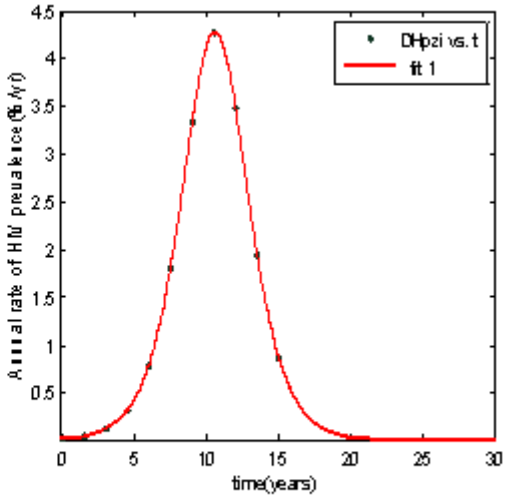

Fig. 6b: Annual rate of HIV prevalence Versus time - Zimbabwe

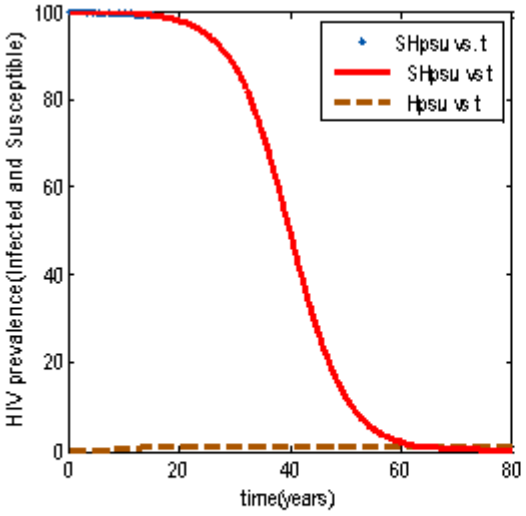

Fig. 7a: HIV prevalence (Infected and Susceptible) Versus time - Sudan

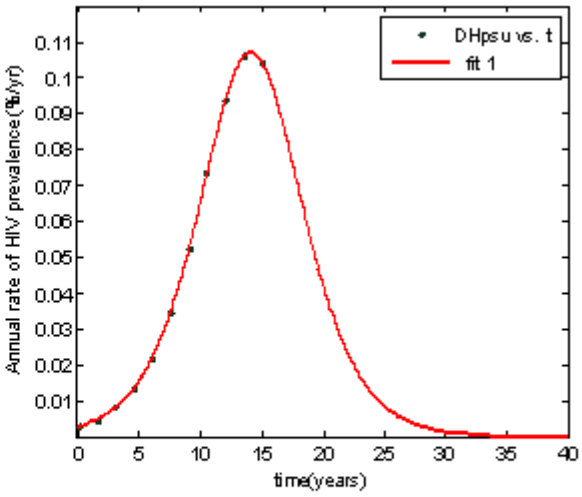

Fig. 7b: Annual rate of HIV prevalence Versus time - Sudan

Table 1: Coefficients and goodness of fit of model on Tanzania.

A= 0.0006758	B= 8.225
F= 10.16	G= 113.2
Kh= 0.4638	Ks= 0.0434
SSE= 0.01621	SSE= 10.33
R^2= 0.9999	R^2= 0.9329
R^2Adj= 0.9999	R^2Adj= 0.933
RMSE= 0.03531	RMSE= 0.8914

t_{PK}: f(10.8)=1.178%/yr
t_{EX}: f(28.9)=0.00105387%/yr
Int:f(101.7)= 10.2748%

Table 2: Coefficients and goodness of fit of model on South Africa

A=0.0001822	B=6387
F=17.46	G=100.1
Kh=0.5593	Ks=0.4215
SSE=0.06915	SSE=0.1604
R^2=0.9991	R^2=0.998
R^2Adj=0.999	R^2Adj=0.9977
RSME=0.07294	RSME=0.1111

t_{PK}: f(15.4)=2.44075%/yr
t_{EX}:f(31.8)=0.0010122%/yr
Int:f(24.4)=17.9245%

Table 3: Coefficients and goodness of fit of model on Nigeria

A=0.007616	B=347.1
F=6.18	G=100.4
Kh=0.3708	Ks=0.1886
SSE=0.01343	SSE=0.12
R^2=0.9995	R^2=0.9957
R^2Adj=0.9994	R^2Adj=0.9977
RSME=0.0321	RSME=0.09607

t_{PK}: f(13.1)=0.573019%/yr
t_{EX}:f(33.9)=0.00103751%/yr
Int:f(45.2)=6.4816%

Table 4: Coefficients and goodness of fit of model on India

A=0.0000006405	B=0.00001502
F=0.4772	G=100
Kh=0.8315	Ks=0.2521
SSE=0.002914	SSE=0.01508
R^2=0.9756	R^2=0.8737
R^2Adj=0.9718	R^2Adj=0.8543
RSME=0.01497	RSME=0.0955

t_{PK}: f(14.3)=0.0991268%/yr
t_{EX}: f(21.5)=0.00106678%/
Int: f(59)=0.517893%

Table 5: Coefficients and goodness of fit of model on Ghana

A=0.01307	B=345.8
F=3.472	G=100.3
Kh=0.3472	Ks=0.1561
SSE=0.01421	SSE=0.06714
R^2=0.9986	R^2=0.9934
R^2Adj=0.9984	R^2Adj=0.9924
RSME=0.03306	RSME=0.07186

t_{PK}: f(12.5)=0.305775%/yr
t_{EX}:f(32.8)=0.00105846%/yr
Int:f(58.8)=3.44978%

Table 6: Coefficients and goodness of fit of model on Zimbabwe

A=0.001063	B=50.74
F=26.46	G=103.7
Kh=0.6471	Ks=0.2075
SSE=0.1594	SSE=42.08
R^2=0.9999	R^2=0.9694
R^2Adj=0.9999	R^2Adj=0.9694
RSME=0.1107	RSME=1.799

t_{PK}: f(10.6)=4.28057%/yr
t_{EX}:f(25.6)=0.00102959%/yr
Int:f(23.9)=26.459%

Table 7: Coefficients and goodness of fit of model on Sudan

A=0.006954	B=2364
F=1.214	G=100.1
Kh=0.3528	Ks=0.195
SSE=0.008832	SSE=0.009894
R^2=0.9888	R^2=0.9875
R^2Adj=0.	R^2Adj=0.9855
RSME=0.02606	RSME=0.02759

t_{PK}: f(14.1)=0.107067%/yr
t_{EX}:f(31.1)=0.0010564%/yr
Int:f(61.5)=1.44055%

Table 8: Comprehensive results of the models for all the countries

S/N	Country	HIV or NOT	R^2	RMSE	$f(t_{pk})=P_{max}$ %/yr	$f(t_{ex})$=A %/yr	$f(t_{int})$=U%
1	Tanzania	H	0.9999	0.0353	f(10.8)=1.178	f(28.9)=0.0010	f(101.7)=10.2748
		SH	0.9329	0.8914	_	_	_
2	South Africa	H	0.9991	0.0729	f(15.4)=2.4408	f(31.8)=0.0010	f(24.4)=17.9245
		SH	0.998	0.1111	_	_	_
3	Nigeria	H	0.9995	0.0321	f(13.1)=0.5730	f(33.9)=0.0010	f(45.2)=6.4816
		SH	0.9957	0.0961	_	_	_
4	India	H	0.9756	0.0149	f(14.3)=0.0991	f(21.5)=0.0010	f(59)=0.5179
		SH	0.8737	0.0955	_	_	_
5	Ghana	H	0.9986	0.0331	f(12.5)=0.3058	f(32.8)=0.0010	f(58.8)=3.450
		SH	0.9934	0.0719	_	_	_
6	Zimbabwe	H	0.9999	0.1107	f(10.6)=4.2806	f(25.6)=0.0010	f(23.9)=26.459
		SH	0.9694	1.799	_	_	_
7	Sudan	H	0.9888	0.0261	f(14.1)=0.1071	f(31.1)=0.0010	f(61.5)=1.4406
		SH	0.9875	0.0276	_	_	_

3. Discussion

From figure 1 to 7, a and b, and tables 1 to 7 respectively, it is seen that the models fitted the countries variations of HIV prevalences with time very well.

In table 8, it is now clear that Zimbabwe has the highest (ultimate) prevalence of 26.459% followed by South Africa of 17.9245% and then Tanzania 10.2748% before Nigeria with 6.4816%. The intersections of HIV and Susceptible to HIV prevalence, also are seen in table 8. At this point of time HIV and susceptible to HIV prevalence are equal. The country with the farthest intersection year (time) is Tanzania with 102years, counting from 1980 i.e. 2082AD. It is followed by Sudan 61.5years, India and Ghana having equal interception of 59years. Although Zimbabwe has the highest (ultimate) prevalence, the HIV prevalence will disappear completely in 25 .6years from 1980 before countries like South Africa, Nigeria, Ghana and even Sudan.

The rate of infection of HIV will peak first in Zimbabwe and Tanzania before other countries on the table 8. Also the correlation coefficient (R^2) for HIV prevalence fitted Tanzania, Zimbabwe, Nigeria, South Africa and Ghana

best than other countries. For susceptible to HIV, the model fitted South Africa, Nigeria and Ghana best (see table 8).

4. Conclusion/Recommendation

Two non-compartmental SIS models were developed, one for infection (I or H) and for susceptible (S or SH). HIV percentage prevalence data were obtained via the internet. Data for Susceptible percentage to HIV were computed from HIV data, knowing that $\beta + \gamma = 1$. The data were used to validate the models. The coefficients of correlation for different countries tested with the models were found to be reasonably very high, showing that the model fitted data almost perfectly. The ultimate prevalence, intersection time for HIV and susceptible to HIV prevalence, the peak time and its prevalence, and the exhaustion time of HIV prevalence for different countries were computed.

These non-compartmental SIS models can be used for different countries to predict different values as shown in table 8.

The modelling of non-compartmental SIR formula is recommended so that it is compared with the compartmental SIR model for HIV. Zimbabwe, South Africa, and Tanzania are advice to embark on a serious campaign against activities leading to HIV propagation.

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Appendix Table: HIV Prevalence data as obtained from the internet

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Country	Notification rate	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Tanzania	HIV prevalence %	0	0.1	0.1	0.3	0.4	0.7	1.0	1.5	2.2	3.1	4.2	5.3	6.5	7.5	8.3	8.9
S. Africa	HIV prevalence %	0	0	0	0	0	0	0	0	0.2	0.5	0.8	1.4	2.4	3.6	5.4	7.8
Nigeria	HIV prevalence %	0	0	0.1	0.1	0.2	0.3	0.4	0.6	0.8	1.1	1.5	1.9	2.4	3.0	3.6	4.1
India	HIV prevalence %	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.1	0.2	0.3
Ghana	HIV prevalence %	0	0.1	0.1	0.1	0.2	0.2	0.3	0.5	0.6	0.8	1.1	1.3	1.6	1.9	2.2	2.5
Zimbabwe	HIV prevalence %	0	0	0	0	0.2	0.6	1.2	2.3	4.2	7.1	10.9	15.0	18.8	21.8	23.9	25.1
Sudan	HIV prevalence %	0	0	0	0	0	0.1	0.1	0.1	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7

Source: http//apps.who.int/tb/surveillance workshop/document/AFR-country data.xls.2003

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