

Reacting System of Variable Viscosity on Mhd Mixed Convection over a Stratified Porous Wedge.

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Abstract
The reacting system of variable viscosity on MHD mixed convection over a stratified porous wedge has been studied in the presence of suction or injection. The wall of the wedge is embedded in a uniform Darcian porous medium in order to allow for possible fluid wall suction or injection and has a power-law variation of the wall temperature. An approximate numerical solution for the steady boundary-layer flow over a wall of the wedge has been obtained by solving the governing equation analytically and with the help of a computer program (MAPLE).
The results were illustrated graphically and necessary conclusions were derived.
Keywords: Reacting system, Variable viscosity, Magneto hydrodynamics (MHD), Mixed Convection, Porous Wedge, suction, injection, power-law, boundary layer flow.

1. Introduction

Mathematics is a body of knowledge that deals with concepts such as quantity, structure, space, change and also the academic discipline that studies them. It is as well referred to as “the science that draws necessary conclusions”. The viscosity of a magneto hydrodynamic fluid is influenced or altered by changing the values of some crucial parameters. Many Practitioners of mathematics have made their contributions towards this area of fluid dynamics. Mixed convections induced by the simultaneous action of buoyancy forces resulting from thermal diffusion is of considerable interest in nature and in many industrial applications such as geophysics, oceanography, drying processes, and chemical engineering. Ariel (1994) has considered the stagnation-point flow of electrically-conducting fluids in the presence of large transverse magnetic field strengths. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. In recent investigations, Chambre and Acrivos (1956) analyzed catalytic surface reactions in hydrodynamic flows. Falkner-Skan transformation is used to reduce boundary-layer equations into ordinary differential equations for similar flows. The solutions of the Falkner-Skan equations are sometimes referred to as wedge-flow solutions with only two of the wedge flows being common in practice. Recently, MHD laminar boundary layer flow over a wedge with suction or injection had been discussed by Kafoussias and Nanousis (1997) and Kumari (1998) discussed the effect of large blowing rates on the steady laminar incompressible electrically conducting fluid over an infinite wedge with a magnetic field applied parallel to the wedge. Here in this research work, we are interested in checking for the effect of altering the reacting term Q, and the Prandtl number Pr, on the temperature of the fluid.

2. Mathematical Formulation

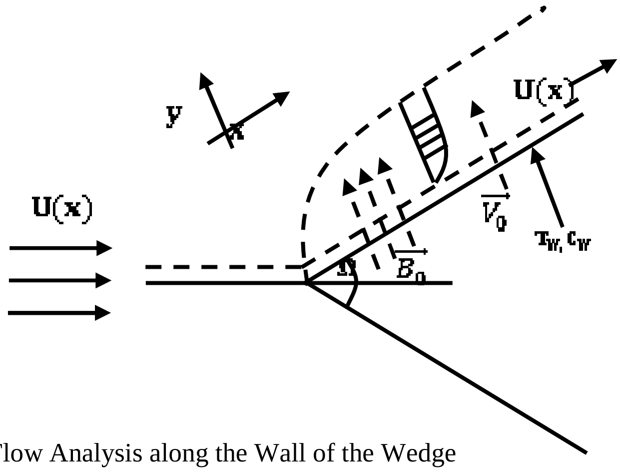

Figure 1: Flow Analysis along the Wall of the Wedge

Two dimensional MHD laminar boundary layer flow of an incompressible, viscous, electrically conducting double diffusive and Boussinesq fluid over a wall of the wedge with suction or injection is considered. Let the x-axis be taken parallel to the wedge and y-axis be taken normal to it as cited in figure 1. A uniform transverse magnetic field of strength B_0 is applied parallel to the y-axis. The fluid is assumed to be Newtonian; electrically conducting; non-linear variable viscosity and its property variations due to temperature are limited to density and viscosity. The density variation and the effects of the buoyancy are taken into account in the momentum equation. Since the magnetic Reynolds number is very small for most used in industrial applications, we assume that the induced magnetic field is negligible.

Mathematical representations of the problem are:

$$\frac{\delta u}{\delta x}+\frac{\delta v}{\delta y}=0 \quad (1)$$

$$u\frac{\delta T}{\delta x}+v\frac{\delta T}{\delta y}=\alpha\frac{\delta^2 T}{\delta y^2}+\frac{\sigma B_o{}^2}{\rho C_p}u^2+Q(T-T_o) \quad (2)$$

The boundary conditions are:

$$u=0,\ v=v_0, T=T_w\ \ at\ y=0 \quad (3)$$

$$u=U(x), T=T_\infty(x)=(1-n)T_0+nT_w(x)\ \ at\ y\to\infty \quad (4)$$

where n is a constant which is the thermal stratification parameter and is such that 0 ≤ n < 1. The *n* defined above as thermal stratification parameter, is equal to $m_1/(1+m_1)$ of Nakayama and Koyama (1989), Anjali Devi and Kandasamy (2001), where m_1 is a constant. T_0 is constant reference temperature say, $T_\infty(0)$. The suffixes w and ∞ denote surface and ambient conditions. Following the lines of Yih and Bansal (1998), the following change of variables are introduced

$$\psi(x,y)=\sqrt{\frac{2U\alpha_g x}{1+m}}f(x,\eta) \quad (5)$$

$$\eta(x,y)=y\sqrt{\frac{(1+m)U}{2\alpha_g x}} \quad (6)$$

where α_g is the effective thermal diffusivity of the porous medium $\left(\alpha_g=\frac{k_e}{\rho c_p}\right)$, where k_e is the porous medium effective thermal conductivity. The variations of viscosity are written in the form of $:\frac{\mu}{\mu_0}=e^{-\alpha\theta}$, where μ_0 is the viscosity at temperature T_w and α is the viscosity parameter.

Under this consideration, the potential flow velocity can be written as

$$U(x)=Ax^m\beta_1=\frac{2m}{1+m} \quad (7)$$

where A is a constant and β_1 is the Hartree pressure gradient parameter that corresponds to $\beta_1 = \frac{\Omega}{\pi}$ for a total angle Ω of the wedge. The wall temperature is assumed to have power-law variation forms as shown by the following equations:

$T_w = T_\infty + c_1 x^n$

Where c_1 is constant and n is the power of index of the wall parameter. The wall temperature is assumed to have the power index n. The continuity equation (1) is satisfied by the stream function $\psi(x,y)$ defined by

$$u = \frac{\delta\psi}{\delta y}\quad v = -\frac{\delta\psi}{\delta x} \tag{8}$$

3 Method of Solution

To transform (2) and (3) into a set of ordinary differential equations, the following dimensionless variables are introduced

$$\theta = \frac{T-T_\infty}{T_w-T_\infty}\,(temperature\ variable) \tag{9}$$

$$\Pr = \frac{\kappa_e}{\alpha} = \frac{\mu c_p}{\kappa}\,(Prandtl number) \tag{10}$$

$$M^2 = \frac{\sigma B_0^2}{\rho c k^2}\,(magnetic parameter) \tag{11}$$

$$Ec = \frac{c^2}{c_p(T_w-T_\infty)(k^2)^{\frac{2m}{1-m}}}\,(Eckert number) \tag{12}$$

$$\lambda = \frac{\kappa_e}{KA}\,(Dimensionless\ porous\ medium\ parameter) \tag{13}$$

$$\frac{\delta^2\theta}{\delta\eta^2} = \frac{2x}{(1+m)}P_r\left(\frac{\delta f}{\delta\eta}\cdot\frac{\delta\theta}{\delta x} - \frac{\delta f}{\delta x}\cdot\frac{\delta\theta}{\delta\eta}\right) - \frac{2x}{(1+m)}P_r\frac{f}{2x}\cdot\frac{\delta\theta}{\delta\eta} + \frac{2x}{(1+m)}P_r\frac{\theta n}{x}\cdot\frac{\delta f}{\delta\eta} - \frac{2x}{(1+m)}P_r\frac{\sigma B_0{}^2}{\rho U}\cdot\frac{U^2}{C_p(T_w-T_\infty)}\left(\frac{\delta f}{\delta\eta}\right)^2 - \frac{2x}{(1+m)}P_r\frac{Q(T-T_0)}{U(T_w-T_\infty)} \tag{14}$$

The boundary conditions are:

$$\eta = 0:\ \frac{\delta f}{\delta\eta} = 0,\ \frac{f}{2}\left(\frac{x}{U}\frac{dU}{dx}+1\right) + x\frac{\delta f}{\delta x} = -v_0\sqrt{\frac{(1+m)x}{2vU}}\quad,\theta = 1$$

$$\eta \to \infty:\ \frac{\delta f}{\delta\eta} = 1,\theta = 0 \tag{15}$$

$$\frac{\delta^2\theta}{\delta\eta^2} = \frac{(1-m)}{(1+m)}P_r\ \xi\left(\frac{\delta\theta}{\delta\xi}\cdot\frac{\delta f}{\delta\eta} - \frac{\delta f}{\delta\xi}\cdot\frac{\delta\theta}{\delta\eta}\right) - \frac{f}{1+m}P_r\cdot\frac{\delta\theta}{\delta\eta} + \frac{2\theta n}{(1+m)}P_r\cdot\frac{\delta f}{\delta\eta} - \frac{2}{(1+m)}\xi^{\frac{2}{1-m}}\cdot P_r\frac{Ax^m}{c_p(T_w-T_\infty)k^{\frac{2}{1-m}}}\cdot\frac{\sigma B_0{}^2}{\rho}\left(\frac{\delta f}{\delta\eta}\right)^2 - \frac{2}{(1+m)}\xi^{\frac{2}{1-m}}P_r\cdot\frac{Q(T-T_0)}{Ax^m(T_w-T_\infty)k^{\frac{2}{1-m}}} \tag{16}$$

$$x^m = (\xi k^{-1})^{\frac{2}{1-m}m} = \frac{\xi^{\frac{2m}{1-m}}}{k^{\frac{2m}{1-m}}}$$

We define $\xi = kx^{\frac{1-m}{2}}$ as the dimensionless distance along the wedge ($\xi > 0$).in these system of equations $f(\xi,\eta)$is the dimensionless stream function;$\theta(\xi,\eta)$ be the dimensionless temperature; Pr, the Prandtl number etc. which are defined above. The parameter indicatesthe indicates the dimensionless distance along the wedge is ($\xi > 0$). It is obvious that to retain the ξ-derivative terms, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. In addition, owing to the coupling between adjacent stream wide location through the ξ-derivatives, a locally autonomous aolution, at any given stream wise location cannot be obtained. In such a case, an implicit marching numerical solution scheme is usually applied proceeding the solution in the ξ-direction, i.e., calculating unknown profiles at ξ_{i+1} when the same profiles at ξ_i are known. The process starts at $\xi = 0$ and the solution proceeds from ξ_i to ξ_{i+1} but such procedure is time consuming.

However, when the terms involving $\frac{\delta f}{\delta\xi}$and $\frac{\delta\theta}{\delta\xi}$and their η derivatives are deleted, the resulting system of equations resembles, in effect, a system of ordinary differential equations for the functions f and θ with ξ as a parameter and the computational task is simplified. Thus, we have

$$\frac{\delta^2\theta}{\delta\eta^2} = -\frac{f}{1+m}P_r\cdot\frac{\delta\theta}{\delta\eta} + \frac{2\theta n}{(1+m)}P_r\cdot\frac{\delta f}{\delta\eta} - \frac{2}{(1+m)}\xi^{2\frac{(1+m)}{1-m}}P_r\frac{\sigma B_0^2}{\rho C k^2}\cdot\frac{C^2}{C_p(T_w-T_\infty)(k^2)^{\frac{2m}{1-m}}}\cdot\left(\frac{\delta f}{\delta\eta}\right)^2 - \frac{2}{(1+m)}\xi^{2\frac{(1+m)}{1-m}}P_r\frac{Q(T-T_0)}{C(T_w-T_\infty)(k^2)^{\frac{2+2m}{1-m}}} \quad (17)$$

thus, we arrive at

$$\theta'' = -\frac{f}{1+m}P_r\theta' + \frac{2\theta n}{(1+m)}P_r\cdot f' - \frac{2}{(1+m)}P_rM^2\xi^{2\frac{(1+m)}{1-m}}E_c(f')^2 - \frac{2}{(1+m)}\xi^{2\frac{(1+m)}{1-m}}P_r\frac{Q(T-T_0)}{c(T_w-T_\infty)(k^2)^{\left(\frac{1+m}{1-m}\right)}} \quad (18)$$

$$\theta'' = -\frac{f}{1+m}P_r\theta' + \frac{2\theta n}{(1+m)}P_r\cdot f' - \frac{2}{(1+m)}P_rM^2\xi^{2\frac{(1+m)}{1-m}}E_c(f')^2 - \frac{2}{(1+m)}\xi^{2\frac{(1+m)}{1-m}}P_r\frac{Q\theta}{c(k^2)^{\left(\frac{1+m}{1-m}\right)}} \quad (19)$$

the boundary conditions are

$$\eta = 0, \quad f(0) = \frac{2}{1+m}S, \quad \theta'(0) = 0, \quad \theta(0) = 1,$$
$$\eta \to \infty; \quad f'(\infty) = 1, \ \theta(\infty) = 0 \quad (20)$$

we now substitute the values for the constants

E_c = 0.001, m = 0.0909, M^2 =1.0, n = 0.5, $P_r = 0.71$, $\xi = 0.1$, $k = 0.5$, $Q = 1$, $c = 0.1$

$$\theta'' = -0.650838757f\theta' + 0.650838757f'\theta - 0.03981475009f'^2 - 0.273705129\theta$$

If we assume the value of 1.2 to *f*, which implies that f' also becomes zero

4. Results and Discussion

Figure 2 and Figure 3 illustrate the relationship between θ and η as we vary the two parameters Q (reacting term) and Pr (Prandtl number).

Figure 2: depicts that increase in the reacting term causes a corresponding decrease effect on the temperature.

Figure 3: shows that increase in the Prandtl number implies a corresponding decrease in the temperature.

5. Conclusion and Recommendation

5.1 Conclusion

In this paper work, we have studied the effect of the reacting term of variable viscosity on MHD mixed convection over a stratified porous wedge plate. The governing equation was approximated to an ordinary differential equation by some basic transformations. Numerical calculations were carried out for two dimensionless parameters of the problem. The results are presented graphically and the conclusion is hereby drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters.

We therefore conclude that, for a magneto-hydro-dynamic viscous fluid:

(i) The temperature varies with respect to the varying reacting term (increase in the reacting term causes a corresponding decrease effect on the temperature).
(ii) Increase in the Prandtl number implies a corresponding decrease in the temperature.

Thus, the reacting term and the Prandtl number can effect a change in the temperature of a magneto hydrodynamic viscous fluid.

5.2 Recommendation

We therefore recommend that to effect a change in the temperature of a viscous fluid, the Prandtl number and a given reacting term could serve as essential factors to be considered for increase or decrease depending on the choice of the individual performing the experiment.

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Nomenclature

B_0^2 is the strength of the magnetic effect.
u,v are the velocity components in x and y direction respectively.
U is the flow velocity of the fluid away from the wedge
G is the acceleration due to gravity.
β is the coefficient of volume expansion.
K is the permeability of the porous medium
T is the temperature of the fluid
T_w is the temperature of the wall.
T_∞ is the temperature of the fluid far away from the wall, ρ is the density of the fluid.
σ is the electric conductivity of the fluid, α is the thermal diffusivity, Q is the reacting term

Table 1: Relationship between θ(temperature variable) and η(position) where Q (Reacting term) is varied at constant Prandtl number (0.71)

	θ	θ	θ
$\Delta\eta$	Q = 1.0	Q = 0.5	Q = 0.25
0.000	1.0000000000	1.000000000	1.0000000000
0.001	0.9999998632	0.999999932	0.9999999660
0.002	0.9999994529	0.999999726	0.9999998635
0.003	0.9999987692	0.999999384	0.9999996928
0.004	0.9999978126	0.999998907	0.9999994539
0.005	0.9999965831	0.999998291	0.9999991460
0.006	0.9999950810	0.999997540	0.9999987711
0.007	0.9999933064	0.999996653	0.9999983272
0.008	0.9999912597	0.999995629	0.9999978154
0.009	0.9999889408	0.999994470	0.9999972357
0.010	0.9999863503	0.999993175	0.9999965882
0.011	0.9999834882	0.999991744	0.9999958720
0.012	0.9999803547	0.999990177	0.9999950891
0.013	0.9999769502	0.999988475	0.9999942375
0.014	0.9999732745	0.999986638	0.9999933184
0.015	0.9999693282	0.999984664	0.9999923327
0.016	0.9999651114	0.999982555	0.9999912786
0.017	0.9999606244	0.999980312	0.9999901560
0.018	0.9999558672	0.999977934	0.9999889671
0.019	0.9999508401	0.999975420	0.9999877098
0.020	0.9999455433	0.999972771	0.9999863863

Table 2: Relationship between θ(temperature variable) and η(position) where Prandtl number is varied at constant Q (Reacting term = 1.0)

	θ	θ	θ
$\Delta\eta$	Pr = 0.61	Pr = 0.71	Pr = 0.95
0.000	1.0000000000	1.0000000000	1.0000000000
0.001	0.9999998824	0.9999998632	0.9999998169
0.002	0.9999995299	0.9999994529	0.9999992681
0.003	0.9999989426	0.9999987692	0.9999983537
0.004	0.9999981205	0.9999978126	0.9999970743
0.005	0.9999970639	0.9999965831	0.9999954301
0.006	0.9999957728	0.9999950810	0.9999934217
0.007	0.9999942476	0.9999933064	0.9999910492
0.008	0.9999924884	0.9999912597	0.9999883134
0.009	0.9999904954	0.9999889408	0.9999852142
0.010	0.9999882686	0.9999863503	0.9999817525
0.011	0.9999858080	0.9999834882	0.9999779281
0.012	0.9999831143	0.9999803547	0.9999737417
0.013	0.9999801871	0.9999769502	0.9999691938
0.014	0.9999770269	0.9999732745	0.9999642846
0.015	0.9999736337	0.9999693282	0.9999590144
0.016	0.9999700077	0.9999651114	0.9999533837
0.017	0.9999661491	0.9999606244	0.9999473930
0.018	0.9999620581	0.9999558672	0.9999410423
0.019	0.9999577345	0.9999508401	0.9999343325
0.020	0.9999531790	0.9999455433	0.9999272635

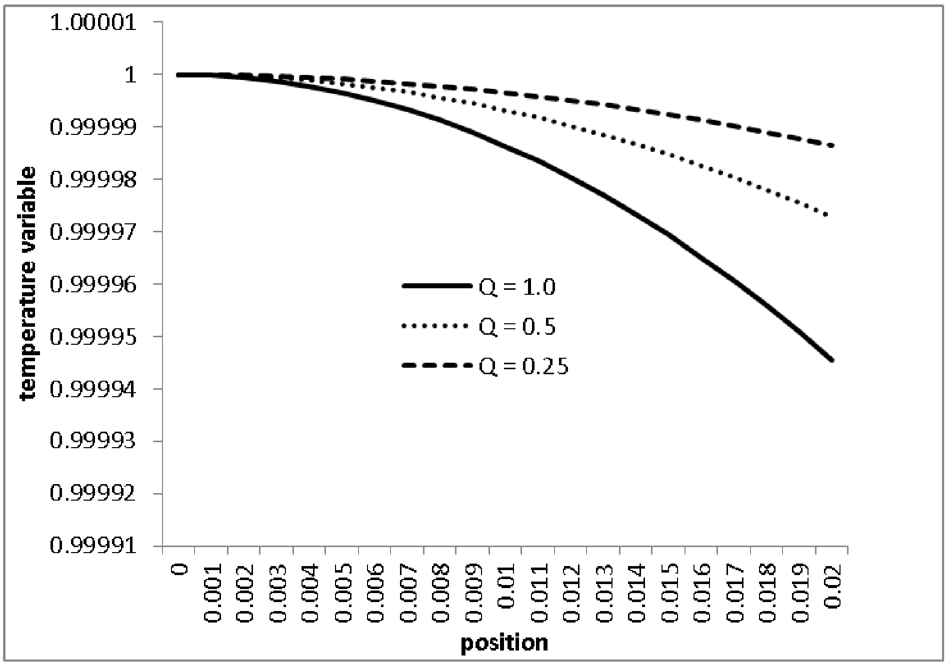

Figure 2: The graph of θ(temperature variable) against η(position) at varying Q (reacting term).

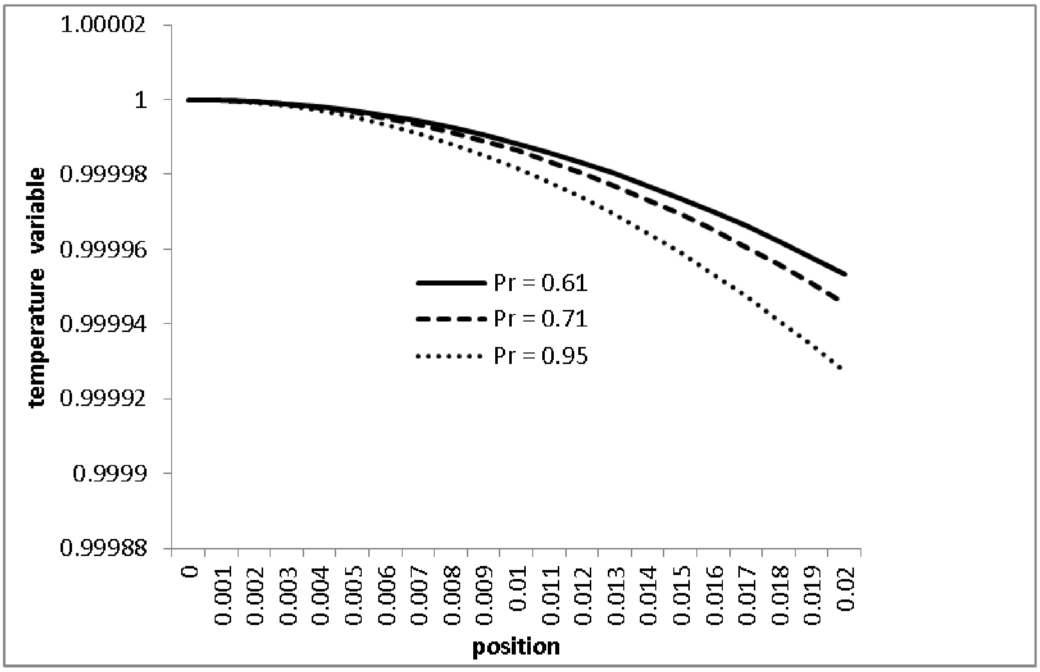

Figure3: The graph of θ(temperature variable) against η(position) at varying Pr (Prandtl number).