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The existence of common fixed point theorems of generalized contractive mappings in cone metric spaces

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Abstract
The purpose of this paper is to the study of the existence of common fixed point theorem for a sequence of self maps satisfying generalized contractive condition for a cone metrice space and obtains some new results in it. Also the paper contains generalized fixed point theorems of [10, 13, 19] and many others from the current literature.

1. Introduction and preliminaries

The well known Banach contraction principal and its several generalizations in the setting of metric spaces play a central role for solving many problems of non linear analysis. For example, see[2,5,6,15,16,17]
Huang and Zang[8] generalized the concept of the metric spaces by introducing cone metric spaces and proved some fixed point theorems for mappings satisfying some contractive conditions, subsequentialy, several other authors [1,9,16,19,21] studied the existence of fixed points and common fixed points of mappings satisfying contractive type condition on a normal cone metric space.

Recently Razapour and Hamlbarani[19] omitted the assumption of normality in cone metric space, which is milestone in developing fixed point theory in cone metric space. In[11] the authors introduced the concept of a compatible pair of self maps in a cone metric space and established a basic result for a non normal cone metric space with an example which [12] weakly compatible maps have been studied. In this paper we prove a common fixed point theorem for a sequence of self maps satisfying a generalized contractive condition for a non normal cone metric space.

Definition:-1.1 [sec [8]] :- Let E be a real Banach space a sub set of p of E is called a cone whenever the following condition holds.

(c_1) P is closed, nonempty and $P \neq \{0\}$
(c_2) $a,b \in R, a,b \geq 0$ and $x, y \in P$ imply $ax+by \in P$,
(c_3) $P \cap (-P) = \{0\}$

Given a cone $P \subset E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y-x \in P$. We shall write $x<y$ to indicate that $x \leq y$ but $x \neq y$ while $x \ll y$ will stand for $y-x \in p^0$ where P^0 stands for the interior of P. If $p^0 \neq \emptyset$.then P is called a solid cone(see[20]).

There exist two kinds of cones-normal (with the normal constant k) and non-normal cone [6]. Let E be a real Banach space, $P \subset E$ a cone and $\leq$ partial ordering defined by P. Then P is called normal it there is a number $k>0$ such that for all $x,y \in P$.

$0 \leq x \leq y$ implies $\|x\| \leq k\|y\|$
(1.1)
or equivalently if $(\forall\, n)\ x_n \leq y_n \leq z_n$
and $\lim_{n\to\infty} x_n = \lim_{n\to\infty} z_n = x$ imply $\lim_{n\to\infty} y_n = x$
(1.2)

The least positive number K satisfying (1.1) is called the normal constant of P.

Example 1.2 (see[20]) let $E = C^1_{\mathbb{R}}[0,1]$ with $\|x\| = \|x\|_\infty + \|x^1\|_\infty$ on $p = \{x \in E : x(t) \geq 0\}$.This cone is not normal. Consider for example, $x_n(t) = t^n/n$ and $y(t) = 1/n$, then $0 \leq x_n \leq y_n$ and $\lim_{n\to\infty} y_n = 0$ but $\|x_n\| = \max_{t\in[0,1]} \left|\frac{t^n}{n}\right| + \max_{t\in[0,1]} \left|t^{n-1}\right| = \frac{1}{n} + 1 > 1$;hence x_n does not converge to zero. It follows by (1.2) that P is a non-normal cone.

<u>**Definition 1.3**</u> (see [13]): Let P be a cone in a real Banach space E. If for $a \in P$ and $a \leq ka$ for same $K \in [0,1]$,then $a=0$.

Definition1.4 (see [10]): Let P be a cone in a real Banach space E with non-empty interior. If for a∈E and a≪c for all c∈p0,then a=0.

Remarks 1.5(see [19]) $\lambda p^0 \subseteq p^0$ for $\lambda>0$ and $p^0 + p^0 \subseteq p^0$

Definition 1.6(see [8.22]) Let X be a nonempty set suppose that the mapping
$d : X\times X\rightarrow E$ satisfies.
(d_1) $0\leq d(x,y)$ for all x,y∈X and d(x,y)=0 if amd only if x=y.
(d_2) d(x,y)=d(y,x) for all x,y∈X
(d_3) $d(x, y)\leq d(x,z)+d(z,y)$, x,y,z∈X ……..
Then d is called a cone metric [8] or K_metric [22] on X and (x,d) is called a cone metric[8] or k-metric space[22] (we shall use the first term).The concept of a cone metric space is more general than that of a metric space, because each metric space is a cone metric space where $E=\mathbb{R}$ and $P=[0,+\infty]$.

Example 1.7(see[8])Let $E\neq\mathbb{R}^2$ $p = \{(x,y) \in\mathbb{R}^2 : x\geq0,y\geq0\}$, $X=\mathbb{R}$ and $d : X\times X\rightarrow E$ defined by $d(x,y)=(|x-y|, a|x-y|)$,where $a\geq0$ is a constant. Then(x,d) is a cone metric space with normal cone P where K=1

Example 1.8(see[18] Let $E=l^2$,$P=\{x_n\}_{n\geq1}\in E:x_n\geq0$, for all n, (X, ρ) a metric space ,and $d:X\times X\rightarrow E$ defined, by $d(x,y)=\{\rho(x,y/2^n\}_{n\geq1}$. Then (X,d) is a cone metric space.

Clearly, the above examples show that class of cone metric space contains the class of metric spaces.

Definition 1.9 (sec[8]) let (X,d) be cone metric space. We say that $\{x_n\}$ is

(i) a Cauchy sequence if for every ε in E with 0≪ε, then there is an N such that for all n,m>N,$d(x_n,x_m) \ll \varepsilon$.

(ii) a convergent sequence if for every ε in E with 0≪ε, then there is an N such that for all n>N, $d(x_n,x) \ll \varepsilon$ for some fixed x in X.

A cone metric space X is said to be complete if every Cauchy sequence in X is convergent in X.

In the following (X,d) will stands for a cone metric space with respect to a cone P with $P^0\neq\emptyset$ in a real Banach space E and ≤ is partial ordering in E with respect to P.

Remarks 1.10 It follows from above definition that if $\{x_{2n}\}$ is a subspace of a Cauchy sequence $\{x_n\}$ in a cone metric space (x,d) and $x_{2n}\rightarrow u$ as $n\rightarrow\infty$ then $x_n\rightarrow u$ as $n\rightarrow\infty$.

Definition 1.11 (see[13]) Let (X,d) be a cone metric Space and P be a cone in a real Banach space E. If u≤v,v≪w, then u≪w.

Lemma 1.12 (see[13]) Let(X,d) be a cone metric space and P be a cone in a real Banach space E and $l,l_1,l_2 >0$ are some fixed point real number. If $x_n\rightarrow x$, $y_n\rightarrow y$ in X and for some a∈p
$l_a\leq l_1 d(x_n, x)+l_2 d(y_n, y)$
for all n>N, for some integer N then a=0

2. Generalized contraction mapping

Let X be a cone metric space and T:X→X be a mapping then T is called generalized contractive mapping if it satisfies the following condition:

$$d(T_x,T_y) \leq \alpha\, d(x,y) +\beta\, d(x,T_x)+\gamma(y, T_y)+\delta[d(x,T_x)+(y, T_y)] +\eta[d(x,T_y)+(y, T_y)+\mu[d(x,T_y)+(x, T_x)] \qquad (2.1)$$

For all x, y ϵ X and α, β, γ, δ, η, μ ϵ[0,1] are constants such that

$$\alpha+\beta+\gamma+2\delta+2\eta+3\mu<1$$

Remarks (2.1):

(i) If (i)δ=η=μ=0 and α, β, γ ∈[0,1], then (2.1) reduce to contraction mapping defined by Banach[3]

(ii) α=β=γ=μ=0 and δ, η∈[0,1/2] then (2.1) reduce to contraction mapping defined by Kannan[14]

(iii) α=β=γ=δ=η=0 and μ∈[0,1/3] then (2.1) reduce to contraction mapping following the condition hold.

3. Main Results

In this section we shall prove some fixed point theorems of generalized contractive mapping.

Theorem 3.1: let (X,d) be a complete cone metric space with respect to a cone *p* contained in real Banach space E. let $\{T_n\}$be a sequence of self maps on x satisfying generalized contractive condition (2.1) with for some α, β, γ, δ, η,μ ϵ[0,1] for x_0 ∈X, let $x_n =T_n x_{n-1}$ for all n . then the sequence $\{x_n\}$ converges in X and its limit v is a common fixed point of all the maps of the sequence $\{T_n\}$. This common fixed point is unique if α+2η+μ<1

Proof:- taking $x=x_{n-1}$, $y= x_n$ $T= T_n$ and $T= T_n+1$ in (2.1) we have

$d(T_n x_{n-1}, T_{n+1}x_n) \le \alpha d(x_{n-1}, x_n)+\beta d(x_{n-1}, T_n x_{n-1})$
$+\gamma d(x_n, T_{n+1}x_n)+\delta[d(x_{n-1}, T_n x_{n-1})+d(x_n, T_{n+1}x_n)]$
$+\eta[d(x_{n-1}, T_{n+1}x_n)+d(x_n, T_n x_{n-1})]$
$+\mu[d(x_{n-1}, T_{n+1}x_n)+d(x_{n-1}, T_n x_n)]$

As $x_n= T_n x_{n-1}$, we have

$d(x_n, x_{n+1}) \le \alpha d(x_{n-1}, x_n)+\beta d(x_{n-1}, x_n)$
$+\gamma d(x_n, x_{n+1})+\delta[d(x_{n-1}, x_n)+d(x_n, x_{n+1})]$
$+\eta[d(x_{n-1}, x_{n+1})+d(x_n, x_n)]+\mu[d(x_{n-1}, x_{n+1})+d(x_{n-1}, x_n)]$
$\le \alpha d(x_{n-1}, x_n)+\beta d(x_{n-1}, x_n)+\gamma d(x_n, x_{n+1})$
$+\delta[d(x_{n-1}, x_n)+d(x_n, x_{n+1})]+\eta[d(x_{n-1}, x_n)+d(x_n, x_{n+1})]$
$+\mu[d(x_{n-1}, x_n)+d(x_n, x_{n+1})+d(x_{n-1}, x_n)]$

Writing $d(x_n, x_{n+1})=\rho_n$ we have

$\rho_n \le (\alpha+\beta+\delta+\eta+2\mu)\rho_{n-1}+(\gamma+\delta+\eta+\mu)\rho_n$

$(1-\gamma-\delta-\eta-\mu)\rho_n \le (\alpha+\beta+\delta+\eta+2\mu)\rho_{n-1}$

This implies that

$\rho_n \le t\rho_{n-1}$ Where

$t=\dfrac{\alpha+\beta+\delta+\eta+2\mu}{1-\gamma-\delta-\eta-\mu}$

As $(\alpha+\beta+2\delta+\gamma+2\eta+3\mu<1)$, we obtain that t<1

Now $\rho_n \le t\rho_{n-1} \le t^2\rho_{n-2} \le .... \le t^n\rho_0$

Where $\rho_0 = d(x_0, x_1)$ also for n>m we have

$d(x_n, x_m) \le d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) + ... + d(x_{m+1}, x_m)$

$$\le (t^{n-1}+t^{n-2}+........+t^m)d(x_1,x_0) \le \frac{t^m}{1-t}d(x_1,x_0)$$

$$= \frac{t^m}{1-t}\rho_0$$

As t<1 and p is closed, thus we obtain that

$$d(x_n,x_m) \le \frac{t^m}{1-t}\rho_0 \quad (3.2)$$

Now for $\varepsilon \epsilon p^0$,there exists r>0 such that ε-yϵ p^0, if ‖y‖<r. choose a positive integer N_ε such that for all $n \ge N_\varepsilon$ ‖ $\frac{t^m}{1-t}\rho_0$ y‖<r which implies $\mathcal{E} - \frac{t^m}{1-t}\rho_0 \in p^0$ and $\frac{t^m}{1-t}\rho_0 - d(x_n,x_m) \in p$ by using (3.2).

So we have $\mathcal{E}-d(x_n,x_m)\in p^0$ for all $n>N_\varepsilon$, and for all m by definition(1.11). this implies $d(x_n,x_m)<<\mathcal{E}$. for all $n>N_\varepsilon$ and for all m hence $\{x_n\}$ is a Cauchy sequence in X. by the completeness of X, there exists, Z∈x such that $x_n \to z$ as $n\to\infty$. for an arbitrary fixed m we show that $T_m z=z$. now

$d(T_m z,z) \le d(T_m z,T_n x_{n-1})+d(T_n x_{n-1},z)$
$=d(x_n,z)+ d(T_m z,T_n x_{n-1})$

Using (2.1) we have

$d(T_m z,z) \le d(T_m z,T_n x_{n-1})+ d(T_n,x_{n-1} z)$

$= d(x_n, z) + d(T_m z, T_n x_{n-1})$
$\leq d(x_n, z) + \alpha\, d(z, x_{n-1}) + \beta\, d(z, T_m z) + \gamma\, d(x_{n-1}, T_n x_{n-1})$
$+ \delta[d(z, T_m z) + d(x_{n-1}, T_n x_{n-1})] + \eta[d(z, T_n x_{n-1}) + d(x_{n-1}, T_m z)]$
$+\mu[d(z, T_n x_{n-1}) + d(z, T_m z)]$
$= d(x_n, z) + \alpha d(z, x_{n-1}) + \beta d(z, T_m z) + \gamma d(x_{n-1}, x_n) + \delta[d(z, T_m z) + d(x_{n-1}, x_n)]$
$+\eta[d(z, x_n) + d(x_{n-1}, T_m z)] + \mu[d(z, x_n) + d(z, T_m z)]$
$\leq d(x_n, z) + \alpha d(z, x_{n-1}) + \beta d(z, T_m z) + \gamma d(x_{n-1}, z)$
$+\delta[d(z, T_m z) + d(x_{n-1}, z) + d(z, x_n)] + \eta[d(z, x_n) + d(x_{n-1}, z) + d(z, T_m z)]$
$+\mu[d(z, x_n) + d(x_{n-1}, z) + d(z, T_m z)]$
$= (1+\delta+\eta+\mu) d(x_n, z) + (\alpha+\gamma+\delta+\eta+\mu)\, d(z, x_{n-1}) + (\beta+\delta+\mu+\eta)\, d(T_m z, z)$

So we have

$(1-\beta-\delta-\mu-\eta) d(T_m z, z) \leq (1+\delta+\eta+\mu)\, d(x_n, z) + (\alpha+\gamma+\delta+\mu+\eta) d(z, x_{n-1})$

As $x_n \to z$, $x_{n-1} \to z (n\to\infty)$ and $(1-\beta-\delta-\eta-\mu)>0$, Using lemma 1.12 we have $d(T_m z, z)=0$ and we get $T_m z=z$, thus z is a common fixed point of all the maps of sequence $\{T_n\}$.

Uniqueness:-

Let $T_n v=v$ for all n be another common fixed point of all the maps of the sequence $\{T_n\}$. Now $d(v,z)= d(T_n v, T_n z)$

$$\leq \alpha d(v,z) + \beta d(v, T_n v) + \gamma\, d(z, T_n z) + \delta[d(v, T_n v) + d(z, T_n z)] + \eta[d(v, T_n z) + d(z, T_n v)] + \mu[d(v, T_n z) + d(v, T_n v)]$$

Which gives

$d(v,z) \leq (\alpha+2\eta+\mu) d(v,z)$

as $\alpha+2\eta+\mu<1$ using definition 1.3 we have $d(v,z)=0$, i.e. v=z. thus v is the unique common fixed point of all the maps of the sequence $\{T_n\}$.

Theorem 3.2:- let (X,d) be a compact cone metric space with respect to a cone p contained in a real Banach space E. Let $\{S_n\}$be a sequence of self maps in X satisfying for some $\alpha_n, \beta_n, \gamma_n\, \delta_n, \eta_n, \mu_n \in[0,1]$ with $\alpha_n+\beta_n+\gamma_n+2\delta_n+2\eta_n+3\mu_n<1$ and $\alpha_n+2\eta_n+\mu_n<1$ there exists positive integer m_i for each *i* such that for all x,y∈X.

$$\begin{aligned} d(s_i^{m_i}x, s_j^{m_j}y) \leq{}& \alpha_n d(x,y) + \beta_n d(x, s_i^{m_i}x) + \gamma_n d(y, s_i^{m_i}y) \\ &+ \delta_n[d(x, s_i^{m_i}x) + d(y, s_j^{m_j}y)] + \eta_n[d(x, s_j^{m_j}y) + d(x, s_i^{m_i}x)] \quad (3.3) \\ &+ \mu_n[d(x, s_j^{m_j}y) + d(x, s_i^{m_i}x)] \end{aligned}$$

Then all the maps of the sequence $\{s_n\}$ have a unique common fixed point in X.

Proof: - from theorem 3.1 all the maps of the sequence { $S_i^{m_i}$ },have a unique common fixed point , say z. hence S_i^{mi} z=z

For all i. now $S_i^{m_i}$ z=z implies $S_i^{m_i}$ $s_1z=s_1z$. taking $x=s_1z$, y=z, i=1 and j=2 in (3.3), we have $s_1z = z$. continuing in similar way it follows that $s_iz = z$ for all i . thus z is a common fixed point of all the maps of the sequence $\{s_i\}$. Its uniqueness follows from the fact that $s_iz = z$ implies $S_i^{m_i}$ z=z for all i.

In theorem 3.1 taking $T_1= T_2= T_3=$……………………..$= T_n=$………$= T$, we get the following general form of Banach contraction principle in a cone metric space which is not necessarily normal.

Theorem 3.3:- let(X,d) be a complete cone metric space with respect to a cone p contained in real Banach space E. Let T be a self map in X satisfying generalized contractive condition (2.1) with $\alpha+\beta+\gamma+2\delta+2\eta+3\mu<1$ and for some $\alpha, \beta, \gamma, \delta, \eta, \mu\in[0,1]$then for each x∈X sequence $\{T_x^n\}$ converges in X and its limit *u* is a fixed point T. This fixed point is unique if $\alpha+2\eta+\mu<1$.

Theorem 3.4:- let (X,d)be a complete cone metric space with respect to a cone p contained in a real Banach space E. suppose the mapping T:X→X satisfying for some positive integer n

$$d(T^n x,T^n y)\le \alpha_n d(x,y)+\beta_n d(x,T^n x)+\gamma_n d(y,T^n y)$$
$$+\delta_n[d(x,T^n x)+d(y,T^n y)]+\eta_n[d(x,T^n y)+d(y,T^n x)]$$
$$+\mu_n[d(x,T^n y)+d(x,T^n x)]$$

For all x,y ∈ X and α_n, β_n, γ_n, δ_n, η_n, μ_n ∈[0,1] are constants such that $\alpha_n+\beta_n+\gamma_n+2\delta_n+2\eta_n+3\mu_n<1$ then T has a unique fixed point in X.

Proof :- from theorem 3.3 T^n has a unique fixed point u. but $T^n(Tu)=T(T^n u)=Tu$, so Tu is also a fixed point of T^n hence Tu=u, u is a fixed of T. since the fixed point of T is unique.

Corollary 3.5:- Let (X,d) be a complete cone metric space with respect to a cone p contained in real Banach space E . Suppose the mapping T:X→X satisfies for some positive integer m, n.

$$d(T_x^m x,T_y^n y)\le \alpha_n d(x,y)+\beta_n d(x,T^m x)+\gamma_n d(y,T^n y)$$
$$+\delta_n[d(x,T^m x)+d(y,T^n y)]+\eta_n[d(x,T^n y)+d(y,T^m x)]$$
$$+\mu_n[d(x,T^n y)+d(x,T^m y)]$$

For all x,y ∈ X and α_n, β_n, γ_n, δ_n, η_n, μ_n ∈[0,1] are constants such that $\alpha_n+\beta_n+\gamma_n+2\delta_n+2\eta_n+3\mu_n<1$ and $\delta_n=\eta_n$ then T has a unique fixed point in X.

Proof:- by theorem 3.4 we get x ∈ X such that $T^m x=T^n y=x$. the result then follows from the fact that

$$d(T\ x,x)=d(TT^m x,T^n y)=d(T^m Tx,T^n x)$$
$$\le \alpha_n d(Tx,x)+\beta_n d(Tx,T^m Tx)+\gamma_n d(x,T^n x)$$
$$+\delta_n[d(Tx,T^m Tx)+d(x,T^n x)]+\eta_n[d(Tx,T^n x)+d(x,T^m Tx)]$$
$$+\mu_n[d(Tx,T^n x)+d(x,T^m Tx)]$$
$$\le \alpha_n d(Tx,x)+\beta_n d(Tx,Tx)+\gamma_n d(x,x)$$
$$+\delta_n[d(Tx,Tx)+d(x,x)]+\eta_n[d(Tx,x)+d(x,Tx)]$$
$$+\mu_n[d(Tx,x)+d(Tx,Tx)]$$
$$=(\alpha_n+2\eta_n+\mu_n)d(Tx,x)$$

Which implies Tx=x.

Theorem 1[8] and theorem 2.3 [20]:-. Let (X,d) be a complete cone metric space. Suppose the mapping T: X→X satisfies the contractive condition

$d(T_x,T_y)<=kd(x,y)$

For all x,y ϵ X where k∈[0,1], is a constant. Then T has a unique fixed point in X. And for any x∈X, the iterative sequence $\{T^n x\}$ converges to the fixed point.

Theorem 3[8] and theorem 2.6[20]:- Let (X,d) be a complete cone metric space. Suppose the mapping T:X→X satisfies the contractive condition

$d(T_x,T_y)<=k[d(x,T_x)+d(y,T_y)]$

For all x,y e X where k∈[0,1/2], is a constant. Then T has a unique fixed point in X. And for any x∈X, the iterative sequence $\{T^n x\}$ converges to the fixed point.

Theorem 4[8] and Theorem 2.7[20]:- Theorem 1[8] and theorem 2.3 [20]. Let (X,d) be a complete cone metric space. Suppose the mapping T:X→X satisfies the contractive condition

$d(T_x,T_y)<=k[d(y,T_x)+d(x,T_y)]$

For all x,y e X where k∈[0,1], is a constant. Then T has a unique fixed point in X. And for any x∈X, the iterative sequence $\{T^n x\}$ converges to the fixed point.

Remark 3.8:- Above theorems os [8] and [20] follows Theorem 3.3 of this paper by taking

(i) $\beta,\gamma,\delta,\eta,\mu$ and α=k
(ii) $\alpha,\gamma,\delta,\eta,\mu$ and β=k
(iii) α ,β, , δ,η,μ and γ =k
(iv) α, β ,γ,η,μ and δ =k
(v) α, β, γ, δ ,μ and η = k

(vi) α ,β ,γ ,δ ,η and μ=k

Precisely, Theorem 3.3 synthesize and generalizes all the results of [9] and [20] for a non normal cone metric space. Theorem 3.2 is a generalized form of Banach contraction principle in a complete cone metric space which is not necessarily normal

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