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Modeling the Grade Point Average (GPA) System in Nigerian Universities

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Abstract

We proposed a mathematical model that captures students' academic progress via the GPA evaluation system. Our purpose is to understand the effect of failing a course (carryovers), low grade point and probation on the overall academic progress (CGPA) of students. The model is analyzed for the existence and stability of the student progress free equilibrium (SPFE) state. Stability analysis revealed that the model is locally asymptotically stable under certain conditions on the model parameters. A quantitative analysis using numerical experiments with the Maple software was also carried out. From the result of this work, there is a direct relationship between high grade point and high GPA and low grade point and low GPA. Students with high CGPA on a consistent manner tend to graduate in the fourth year while students with low grade and failing courses graduate in the fifth or sixth year. The GPA system goes hand in hand with the semester and course credit system, hence the need for students to understand the functionalities of the impact of the credit system on his overall academic progress.

Keywords: Mathematical Model, Course credit system, Semester system, Grade Point Average (GPA) system, Student Academic Progress

1. Introduction

The Nigerian educational system at tertiary levels such as colleges of education, polytechnics and universities has been reshaped with a new system called the 'semester system' and the 'course credit system'. These systems began in 1988 with the first set of graduating students in 1990 (NUC, 1990). The principal reason concerned the inherent advantages of the system and its positive contributions on the future of the students and the development of the country's educational system. The semester system and the course credit system now replaced the traditional three term system (called 'almighty June'). In the traditional three terms or annual evaluation system, one final examination is conducted at the end of an academic session, usually in June of every year. Under this system of evaluation, teachers have no proper plan and strategy to teach their courses, the failure rate is high as students obtain comparatively low marks due to elaborate and wide areas to concentrate and prepare for examination. Other advantages attached to the semester and course credit system, to mention but a few, are convenience in accumulation and transfer of credits, offering students a wider choice of optional courses (electives), easy division of subjects into manageable units and its tendency to keep students busy throughout the sessions.

There is a considerable little variation in the grading system in the Nigerian Universities and other tertiary institutions. The Colleges of Education in line with the National Commission of College of Education (NCCE), has adopted the grading system involving the use of both letter (A-F) and figures (0-5). This is because it is more consistent with the degree classifications in use in Nigeria universities. Thus, the grading system for the NCE is similar to what is in use in the Universities.

According to the National Universities Commission (NUC, 1990), a University shall run the semester system for all its academic programmes. Each academic session shall comprise two semesters and each semester shall last for a period of eighteen (18) weeks consisting of one (1) week for registration, fifteen (15) weeks for lectures and two (2) weeks for examination. One week of lectures shall consist of 40 hours of classroom and/or laboratory activities. Teaching and laboratory activities shall commence daily and shall start at 8.00 a.m. and end by 5.00 p.m. with a lunch break in the afternoon. Courses can be taken at any level by any student provided there are no (constraints) prerequisites for these courses. For instance, a 100 level student can offer a course at any level provided the student has the prerequisites required for the course, while a 400 level student can still offer a 100 level course if such a student so desires. However it is generally desirable that lower level core courses are taken and passed before proceeding to high level ones. The semester system also allows a student to spread his programme evenly over the semesters provided such a student keeps to the rules and regulations of the system.

In line with the guidelines of the National Universities Commission, the University shall also operate the course credit system. One credit unit is defined as one hour of lecture or tutorial per week per semester and its equivalents shall be: two hours of seminar; three hours of laboratory or field work, clinical practice/practicum or studio practice; six hours of teaching practice; and one week of industrial attachment. There are minimum and maximum numbers of units a student can register per semester. Every semester is as important as the other. A wise student is encouraged to attempt a reasonable number of units he/she can cope with to ensure a qualitative performance.

Apart from the end of semester examination, there are continuous assessments during each semester. These tests at the end of the semester examination make up the set of semester examinations for each course. Appropriate scores to make the final mark of 100% varies from one faculty to another, but continuous assessment scores must be within 30% to 40% of the 100% final mark

2. Literature Review/Empirical studies

Ishitani & Desjardins (2002), in their work found that the higher a student's first year grade point average, the less likely it is he or she would drop out of university. Academic achievement is often measured by the grade point average (GPA) of the student. Pascarella and Terezini (2005) referred to GPA as the "lingua franca" of the academic instructional world. A GPA 'buys' the student's academic standing to job enrolment, employment opportunities and admission to post graduate study. Adelman (1999) found that having first year grades in the top two quartiles increases a student's chance of degree completion two or three times over students with grades in the bottom quartile.

Given the predictive power of academic achievement, knowledge of the environmental variables that enhance student high or low GPA should be a critical aspect of higher education research. Previous work studying the factors that influence high or low GPA include the following Hijazi and Naqvi,(2006), Clavier, C. W. (2013), Hake, (1998), Galiher (2006), Broh, (2000), Stacey, A. P. (2010), Meagher, *e.t al.* (2011), Noble, J. P., & Sawyer, R. L. (2004), Kaylor and Sherman (2009), and Darling (2005), while comparison between the annual grading system and the semester system (Asfandyar, Y. and Muhammad H. (2012) and effect of failed course (carryovers) in the semester system (Nwadiani, M. and Ofoegbu, F. I. (2011). None of these works provides a mathematical explanation of the grade point average (GPA) and how it affects students' academic progress in the long run in term of fail course(s).

In this research work, we model the GPA system of evaluating and assessing undergraduate students in Nigeria universities using differential equations. Our focus is to clearly understand the variables and parameters that influence low and high GPA in term of fail course(s) and low grade.

3. Model Formulation

The model seeks to understand the GPA system of grading and the variables or parameter that influences high and low GPA of students using a system of differential equations. Since the course credit system is employed, six compartments involving cumulative credit units at each level from 100 to 400 level, first spill-over (FSO) and second spill-over (SSO) is considered. We defined $C_i(t)$ as each level, $i = 1, 2, \cdots, 6$ as the cumulative total credit registered. A student will normally graduate at 400 level ($i = 4$) if he or she was able to pass all prescribed course registered for within the first four years. For all students admitted for a 4-year course, a minimum of 120 credit units must be earned before graduation, hence $C_{4,5,6}(t) \geq 120$. Where a student was unable to graduate in the 4th year, he will spill over to the next year, refer to as first spill over (FSO) year and subsequently if he fails to graduates moves to the second or last spill over year.

According to the NUC regulations, a four year course programme will have an additional two year grace period. Hence it is expected that students admitted for a 4-year programme must graduate within the 4th and the 6th year. This allowance will take care of certain factors beyond the students' control, like sickness, accidents etc and

factors within his control like failure in previous year and carelessness. The student progress to the next level at the rate β_i, where β_i is the current cumulative grade point average (CGPA) of the second semester of level i. A student probates if his/her CGPA is less than one. It is assume that a student can only probate at 100 level, 200 level and 300 level at a rate η_i^{*}, where , $i = 1, 2, 3$. The total number of credit units a student fails (also refers to carryovers) in the previous year (first and second semester examination inclusive) is denoted by η_i, where $i = 1, 2, \cdots, 5$. We assume that for student on probation, $\eta_i = \eta_i^{*}$. The probability of a student graduating is denoted by α_i, where $i = 4, 5, 6$. A student withdraws if he could not pass all the prescribed courses at the end of the second spill over year and was unable to earn a minimum of 120 credit units.
All parameters used in the formulation of the model are assumed to be non-negative.

Given the above description, the relationship between students' academic progress and GPA is governed by the following system of ODEs.

$$\frac{dC_1}{dt} = -(\eta_1 + \beta_1)C_1(t) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$
$$\frac{dC_2}{dt} = (\eta_1 + \beta_1)C_1(t) - (\eta_2 + \beta_2)C_2(t) \quad \ldots \quad \ldots \quad \ldots \quad (2)$$
$$\frac{dC_3}{dt} = (\eta_2 + \beta_2)C_2(t) - (\eta_3 + \beta_3)C_3(t) \quad \ldots \quad \ldots \quad \ldots \quad (3)$$
$$\frac{dC_4}{dt} = (\eta_3 + \beta_3)C_3(t) - (\eta_4 + \beta_4 + \alpha_4)C_4(t) \quad \ldots \quad \ldots \quad \ldots \quad (4)$$
$$\frac{dC_5}{dt} = (\eta_4 + \beta_4)C_4(t) - (\eta_5 + \beta_5 + \alpha_5)C_5(t) \quad \ldots \quad \ldots \quad \ldots \quad (5)$$
$$\frac{dC_6}{dt} = (\eta_5 + \beta_5)C_5(t) - (\beta_6 + \alpha_6)C_6(t) \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

In the table below, variables and parameters used in the model are defined.
Table 1 Model variables and parameters

Variable	Description
$C_1(t)$	Total number of credit units a students registered (cumulative) at the end of 100 level
$C_2(t)$	Total number of credit units a students registered (cumulative) at the end of 200 level
$C_3(t)$	Total number of credit units a students registered (cumulative) at the end of 300 level
$C_4(t)$	Total number of credit units a students registered (cumulative) at the end of 400 level
$C_5(t)$	Total number of credit units a students registered (cumulative) at the end of the first spill over year, if the student did not graduate at the official year (400 level)
$C_6(t)$	Total number of credit units a students registered (cumulative) at the end of the second spill over year, if the student did not graduate at the end of the first spill over year.
Parameter	**Description**
$\beta_i(t)$	Denotes cumulative grade point average(CGPA) at the end of each level $(i = 1,2, \ldots ,6)$
η_i	Denotes number of course units a student carry over at previous level ($i = 1,2, \ldots ,5$)
η_i^{*}	Denotes number of course units a student who is on probation at level ($i = 1,2, \ldots ,5$) is expected to registered in the current level $i+1$. (for example $\eta_1 = \eta_1^{*}$ for a student who is on probation after 100 level.
α_i	Denotes the probability that a student graduates at i level $(i = 4,5,6)$

The following diagram describes the dynamics of student academic progress and will be used in the formulation of the model equations

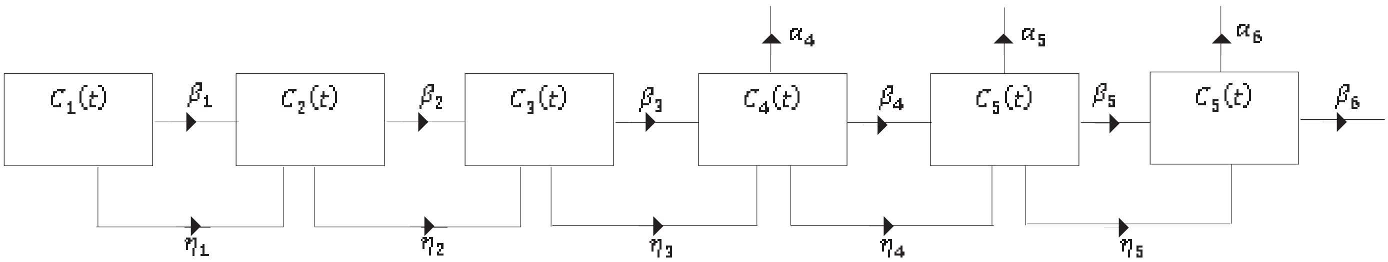

Figure 1: Flow diagram for the dynamics of student academic progress

3.1 Mathematical Analysis

We discuss the existence and uniqueness of the student progress free equilibrium (SPFE) of the model and their stability analysis in this section. The SPFE of the model is obtained by setting the left hand side of equation (1) - (6) to zero and solving the resulting system of equations simultaneously. The system of equation (1) - (6) has a unique student progress free equilibrium $(SPFE_{E_0}) = (C_1, C_2, C_3, C_4, C_5, C_6) = (0,0,0,0,0,0)$

The following lemmas shall be necessary in our analysis;

Lemma 1

Let R be a commutative subring of ${}^nF^n$, where F is a field (or a commutative ring), and $M \in {}^mF^m$, If $M = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}$ where A, B, C, D are $n \times n$ block matrices over F, then

$det_F M = det_F(\mathbf{AD} - \mathbf{BC})$, whenever at least one of the blocks A, B, C, D is an $n \times n$ zero matrix.

Proof

As a first case suppose that $B = C = 0$, be the $n \times n$ zero matrix, so that $M = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix}$

be a block matrix. It is a well-known fact that

$$det_F \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix} = det_F \mathbf{A} \cdot det_F \mathbf{D} \qquad \dots \qquad \dots \qquad \dots \qquad (7)$$

Equation (7) is the result of the Lemma above in the special case when $B = C = 0$.

To prove (7) we use the Laplace extension of $det_F \mathbf{M}$ by the first n rows, which give the result immediately. A more elementary proof is describe thus: Generalized to the case where $\mathbf{A}$ is $r \times r$, but $\mathbf{D}$ is till $n \times n$. The result is now obvious if $r = 1$, by expanding by the first row. Using induction on r and expanding by the first row to perform the inductive steps gives the same result as (7) if we know only that $\mathbf{B} = \mathbf{0}$. We obtain

$$det_F \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = det_F \mathbf{A} \cdot det_F \mathbf{D} \qquad \dots \qquad \dots \qquad \dots \qquad (8)$$

Taking the transpose or by repeating the proof using columns instead of rows, we also obtain the result when $\mathbf{C} = \mathbf{0}$, namely

$$det_F \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{D} \end{pmatrix} = det_F \mathbf{A} \cdot det_F \mathbf{D} \qquad \dots \qquad \dots \qquad \dots \qquad (9)$$

To proof the multiplicative property of equation (7) that is

$$det_F \mathbf{A} \cdot det_F \mathbf{D} = det_F(\mathbf{AD}) \qquad \dots \qquad \dots \qquad \dots \qquad (10)$$

We make the following assumptions

a) Adding a multiple of one row (respectively, column) to another row (respectively, column) of a matrix does not alter the determinant.

b) Multiplying a matrix on the left (respectively, right) by a unitriangular matrix corresponds to performing a number of such operations on the rows (respectively, columns), does not alter the determinant. (A unitriangular matrix is a triangular matrix with all diagonal entries equal to 1).

c) We shall assume that $det_F \mathbf{I}_n = 1$, where $\mathbf{I}_n$ is the $n \times n$ identity matrix.

We observed that

$$\begin{pmatrix} \mathbf{I}_n & -\mathbf{I}_n \\ \mathbf{0} & \mathbf{I}_n \end{pmatrix} \begin{pmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{I}_n & \mathbf{I}_n \end{pmatrix} \begin{pmatrix} \mathbf{I}_n & -\mathbf{I}_n \\ \mathbf{0} & \mathbf{I}_n \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = \begin{pmatrix} -\mathbf{C} & -\mathbf{D} \\ \mathbf{A} & \mathbf{B} \end{pmatrix} \qquad \dots \qquad \dots \qquad \dots \qquad (11)$$

where

$det_F \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = det_F \begin{pmatrix} -\mathbf{C} & -\mathbf{D} \\ \mathbf{A} & \mathbf{B} \end{pmatrix}$, since the first three matrices on the left of (11) are unitriangular.

From (8) and (9), it follows from this that

$$det_F \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{0} \end{pmatrix} = det_F(-\mathbf{C}) \cdot det_F \mathbf{B} = det_F \begin{pmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \qquad \dots \qquad \dots \qquad \dots \qquad (12)$$

Also we have that $\begin{pmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{I}_n & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{I}_n & \mathbf{D} \\ \mathbf{0} & \mathbf{I}_n \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{AD} \\ -\mathbf{I}_n & \mathbf{0} \end{pmatrix}$

Hence the second matrix on the left is unitriangular, so taking determinants and using (8) and the first part of (12) we have

$det_F \mathbf{A} \cdot det_F \mathbf{D} = det_F \mathbf{I}_n \cdot det_F(\mathbf{AD})$;

Since $det_F \mathbf{I}_n = 1$, the multiplicative law (7) for determinants in ${}^nF^n$, follows.

Lemma 2

The eigenvalues λ_i of the 2×2 matrix A satisfy $\operatorname{Re} \lambda_i < 0$ if and only if $\lambda_1 \cdot \lambda_2 = \mathbf{Det}\, A > 0$ and $\lambda_1 + \lambda_2 = \mathbf{Trace}\, A < 0$. They are pure imaginary if and only if $\mathbf{Trace}\, A = 0$. Moreover $\lambda_1 < 0 < \lambda_2$ $(\lambda_2 < 0 < \lambda_1)$ if and only if $\mathbf{Det}\, A < 0$

Proof

Consider a linear system $x' = Ax$ in two dimensions, where the entries of matrix
$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ are real or complex numbers

The Characteristic polynomial of A is given by the relation $\rho(\lambda) = A - \lambda I$

$$\rho(\lambda) = \lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) \qquad \ldots \quad \ldots \quad \ldots \quad (13)$$

If $\lambda_1$and λ_2 are eigenvalues of A (not necessarily distinct), then we have that

$$\rho(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) = \lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1\lambda_2 \qquad \ldots \quad \ldots \quad \ldots \quad (14)$$

Comparing (13) and (14), we have the following identity

$\lambda_1\lambda_2 = \mathbf{Det}\, A$ and $\lambda_1 + \lambda_2 = \mathbf{Trace}\, A$

Since the characteristic polynomial is quadratic the eigenvalues is given by

$$\lambda_1, \lambda_2 = \frac{Trace\,(A) \pm \sqrt{(Trace\,(A))^2 - 4Det\,(A)}}{2} \qquad \ldots \quad \ldots \quad \ldots \quad (15)$$

Let $\Delta = Trace\,(A)^2 - 4Det\,(A)$ be the discriminant, then the nature of the root is determined by the sign of the discriminant, which invariably satisfy the result of the lemma.

Therefore $\lambda_1\lambda_2 = \mathbf{Det}\, A > 0$ and $\lambda_1 + \lambda_2 = \mathbf{Trace}\, A < 0$ for real $\lambda_1\lambda_2$ and

$\lambda_1\lambda_2 = \mathbf{Det}\, A < 0$ and $\lambda_1 + \lambda_2 = \mathbf{Trace}\, A > 0$, for imaginary λ_i

Hence the proof

3.2 Stability Analysis of Student Progress Free Equilibrium (SPFE) state

We now proceed to show the stability of the equilibrium states. To study the behaviour of the system of ODEs (1) - (6) around the student progress free equilibrium states, we applied the linearized stability principles.

Let

$$f_1 = -(\eta_1 + \beta_1)C_1(t) \qquad \ldots \quad \ldots \quad \ldots \quad (16)$$
$$f_2 = (\eta_1 + \beta_1)C_1(t) - (\eta_2 + \beta_2)C_2(t) \qquad \ldots \quad \ldots \quad \ldots \quad (17)$$
$$f_3 = (\eta_2 + \beta_2)C_2(t) - (\eta_3 + \beta_3)C_3(t) \qquad \ldots \quad \ldots \quad \ldots \quad (18)$$
$$f_4 = (\eta_3 + \beta_3)C_3(t) - (\eta_4 + \beta_4 + \alpha_4)C_4(t) \qquad \ldots \quad \ldots \quad \ldots \quad (19)$$
$$f_5 = (\eta_4 + \beta_4)C_4(t) - (\eta_5 + \beta_5 + \alpha_5)C_5(t) \qquad \ldots\ldots \quad \ldots \quad (20)$$
$$f_6 = (\eta_5 + \beta_5)C_5(t) - (\beta_6 + \alpha_6)C_6(t) \qquad \ldots \quad \ldots \quad \ldots \quad (21)$$

At the student progress free equilibrium state ($SPFE_{E_0}$), we evaluate the partial derivative of the system (16) - (21) to get the Jacobian matrix below:

$$J_{SPFE(E_0)} = \begin{bmatrix} -(\eta_1+\beta_1) & 0 & 0 & 0 & 0 & 0 \\ (\eta_1+\beta_1) & -(\eta_2+\beta_2) & 0 & 0 & 0 & 0 \\ 0 & (\eta_2+\beta_2) & -(\eta_3+\beta_3) & 0 & 0 & 0 \\ 0 & 0 & (\eta_3+\beta_3) & -(\eta_4+\beta_4+\alpha_4) & 0 & 0 \\ 0 & 0 & 0 & (\eta_4+\beta_4) & -(\eta_5+\beta_5+\alpha_5) & 0 \\ 0 & 0 & 0 & 0 & (\eta_5+\beta_5) & -(\beta_6+\alpha_6) \end{bmatrix} \ldots \quad \ldots \quad (22)$$

Using the result in Lemma 1 and 2, we partition the matrix (22) as follows:

$$J_{SPFE} = \left(\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array}\right)$$

where A, B, C, and D are block matrices defined as follows

$$\mathrm{A} = \begin{bmatrix} -(\eta_1+\beta_1) & 0 & 0 \\ (\eta_1+\beta_1) & -(\eta_2+\beta_2) & 0 \\ 0 & (\eta_2+\beta_2) & -(\eta_3+\beta_3) \end{bmatrix}, \quad \mathrm{D} = \begin{bmatrix} -(\eta_4+\beta_4+\alpha_4) & 0 & 0 \\ (\eta_4+\beta_4) & -(\eta_5+\beta_5+\alpha_5) & 0 \\ 0 & (\eta_5+\beta_5) & -(\beta_6+\alpha_6) \end{bmatrix}$$

$$B = \begin{bmatrix} \mathbf{0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 0 & (\eta_3 + \beta_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We have that

$\mathbf{Det}\ (J_{SPFE}) = \mathbf{Det}\,(A).\mathbf{Det}\ (D)$, since $\mathbf{Det}\,(B) = 0$

Hence $\mathbf{Det}\ (J_{SPFE}) = (\eta_1 + \beta_1)(\eta_2 + \beta_2)(\eta_3 + \beta_3)(\eta_4 + \beta_4 + \alpha_4)(\eta_5 + \beta_5 + \alpha_5)(\beta_6 + \alpha_6)$

Similarly the $\mathbf{Trace}\ (J_{SPFE}) = \mathbf{Trace}\ (A) + \mathbf{Trace}\ (D)$

$\mathbf{Trace}\ (J_{SPFE}) = -(\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \alpha_4 + \alpha_5 + \alpha_6)$

Theorem 1

The student progress free equilibrium (SPFE) of equation (1)-(6) is stable if $\beta_i > 0, \alpha_i > 0$ and $\eta_i > 0$

Proof:

Consider the Jacobian matrix (22), the required criteria for stable equilibrium (that is, of having the trace of the Jacobian matrix negative and the determinant positive) give the conditions for stability.

From the Jacobian matrix (22), we have that

$\mathbf{Det}\ (J_{SPFE}) = (\eta_1 + \beta_1)(\eta_2 + \beta_2)(\eta_3 + \beta_3)(\eta_4 + \beta_4 + \alpha_4)(\eta_5 + \beta_5 + \alpha_5)(\beta_6 + \alpha_6) > 0$ and

$\mathbf{Trace}\ (J_{SPFE}) = -(\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \alpha_4 + \alpha_5 + \alpha_6) < 0$

provided that $\beta_i > 0,\ \eta_i > 0$ and $\alpha_i > 0$

3.3 Numerical Experiments

The model equations (1) - (6) were all solved numerically using Maple 15 (Maplesoft, Waterloo Maple 2012). The parameter chosen were in consonance with the threshold values obtained in the stability analysis of the student progress free equilibrium states of the models. The data use were from four students from the Nasarawa State University, Keffi, Nigeria, whose result from year one to the year of their graduation were gotten from the examination officer of the Mathematical Sciences Department of the University. The following experiments were carried out.

Experiment 1: Measuring student progress (Low grade, probation and spill over)

Experiment 2: Measuring student progress (Low grade, No probation and spill over)

Experiment 3: Measuring student progress (Low grade, Few carry overs and no spill over)

Experiment 4: Measuring student progress (High grade, High GPA, No probation, No spill over)

Table 2: Summary of student result use for the Numerical Experiment

	Experiment 1				**Experiment 2**				**Experiment 3**				**Experiment 4**			
Level	***CTCR**	***CTCE**	***CTGP**	***CGPA**	**CTCR**	**CTCE**	**CTGP**	**CGPA**	**CTCR**	**CTCE**	**CTGP**	**CGPA**	**CTCR**	**CTCE**	**CTGP**	**CGPA**
100	39	30	73	1.87	36	33	87	2.42	36	36	74	2.06	36	36	155	4.31
200	84	50*	99	1.18	82	77	182	2.22	78	69	157	2.01	78	78	340	4.36
300	118	68	145	1.23	117	101	233	1.99	114	99	242	2.12	114	114	499	4.38
400	166	96	211	1.27	150	125	297	1.98	160	148	375	2.34	148	148	660	4.46
***FSO**	208	121	245	1.18	171	134	306	1.79	0	0	0	0	0	0	0	0
***SSO**	228	228	274	1.20	177	134	306	1.73	0	0	0	0	0	0	0	0

***CTCR :** Cumulative Total Credit Registered per session ***CTCE –** Cumulative Total Credit Earned per session
***FSO:** First Spill Over ***SSO –** Second Spill Over ***CTGP –** Cumulative Total Grade Point ***CGPA –** Cumulative Grade Point Average

Table 3: Estimated parameter values use in the model

	Experiment 1	**Experiment 2**	**Experiment 3**	**Experiment 4**
Parameters	**Values**	**Values**	**Values**	**Values**
$C_1(t)$	39	36	36	36
$C_2(t)$	84	82	78	78
$C_3(t)$	118	117	114	114
$C_4(t)$	166	150	160	148
$C_5(t)$	208	171	0	0
$C_6(t)$	228	177	0	0
β_1	1.87	2.42	2.06	4.31
β_2	1.18	2.22	2.01	4.36
β_3	1.23	1.99	2.12	4.38
β_4	1.27	1.98	2.34	4.46
β_5	1.18	1.79	0.00	0.00
β_6	1.23	1.70	0.00	0.00
η_1	9	3	0	0.00
η_2	25	2	9	0.00
η_3	16	11	0	0.00
η_4	21	9	0	0.00
η_5	0	5	0	0.00
α_4	0.01	0.00	0.70	0.90
α_5	0.20	0.00	0.00	0.00
α_6	0.87	0.40	0.00	0.00

4. Results and Discussion

Table 2 present a summary of four students' computed result used for the numerical experiments, while Table 3 shows the parameter values used. These values were found to meet the criteria for stability of the student progress free equilibrium (SPFE) states of the model. The grade point average (GPA) system allows courses registered to be weighted by a combination of grade (A, B, C, D, E, F) and grade points (5, 4. 3, 2, 1, 0) to the number of credits given for a particular course. The Cumulative Grade Point Average (CGPA) is a number that represent the average of a student's grade during his/her time in the university.

We present the following results from the numerical experiments carried out.

Experiment 1: Low Grade point, Probation, Spill-over

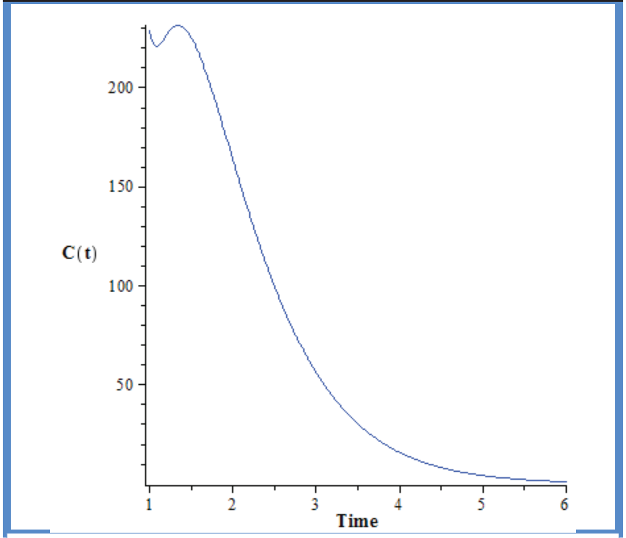

Figure 2: Effect of Low Grade Point with many carryover courses (71 credit units) on students' official year of graduation (See parameter values in Table 3 above).

Experiment 2: Low Grade, No Probation, Spill-over

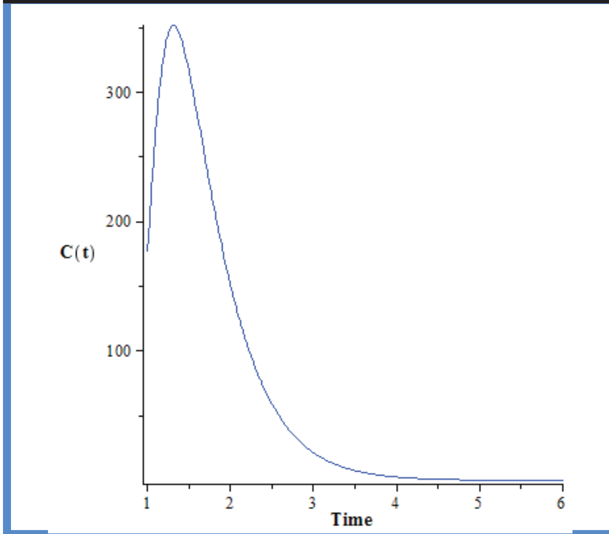

Figure 3: Effect of Low Grade Point with many carryover courses (31 credit units) on students' official year of graduation (See parameter values in Table 3 above).

Experiment 3: Low Grade point, Few carry overs No spill-over

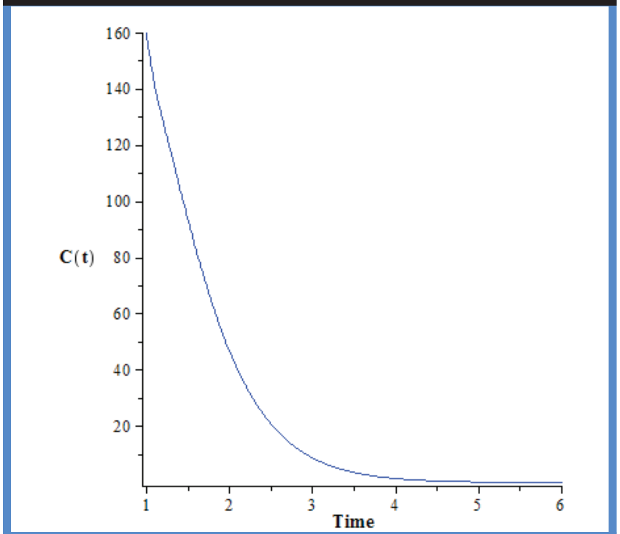

Figure 4: Effect of Low Grade Point with 9 credit unit carryover courses on students' official year of graduation (See parameter values in Table 3 above).

Experiment 4: High Grade, No Probation No spill-over

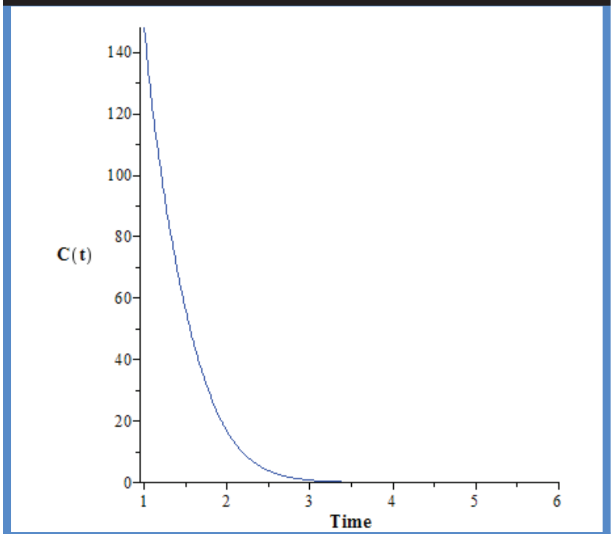

Figure 5: Effect of High Grade point with no carryover courses on students' official year of graduation (See parameter values in Table 3

From figure 2 above, we observed that the students has low grade point resulting from low grade score like (D = 2, E = 1, F = 0). Failing a course implies that the grade point is zero and when this course is registered in the next year, it increases the cumulative credit registered C(t) as compare with students in the same level who passed the course with high grade in the previous year. Since the GPA system account for every semester in a consistent manner through cumulative calculation, it then has a way of increasing a student stay in the university. In Figure 2, the student graduate after six years due to probation $(GPA < 1.00)$ in year two and many carry overs. From the result in figure, we conclude that having repeatedly failing one or more courses in the semester system has a way of increasing one stay in the university.

From figure 3, we observed that the total cumulative units registered increases for student with consistent carryovers (in this case the total credit for all repeat courses was 31). This is very high. When a carryover course is a pre-requisite course to one or more courses, the student will not be able to register such courses until he has re-written and passed the carryover courses in the current year. This student also was not able to graduate at the 400 level official year of graduation; rather he graduated at the 6th year.

The student result progress shown in Figure 4 indicates that the student had carryover once at 200 level (a total of 9 credit units), but had a recurrent mid-grade (like C = 3, D = 2) on a consistent basis. The student graduated at the fourth year though the probability of not graduating could still be high.

From figure 5, we observed that the student had high grade point in a consistent manner leading to a consistent high cumulative grade point average (CGPA). The student had no carryover and as such graduated at 400 level which is the official graduating year. The level where a student had carryover, no matter the total number of credits involves (small or high), the probability of not graduating at the official year could still be high.

The GPA system and the course credit system must be clearly understood by undergraduate students, so that they can set realistic goals and developed good study habits. Student must understand the negative effect of failing any course irrespective of the level. The GPA system gives the students the great opportunity to decide which class of degree he/she will come out with even at the first examination in the university, so students must take the 100 level very serious as they would do when in their final year.

5. Conclusion

We proposed a mathematical model that captures students' academic progress via the GPA evaluation system. Our purpose is to understand the effect of failing a course (carryovers), low grade point and probation on the overall academic progress (CGPA) of students. From the result of this work, there is a direct relationship between high GPA and high grade point, and low GPA and low grade point. Students with high CGPA on a consistent manner tend to graduate in the fourth year while students with low grade and failing courses graduate in the fifth or sixth year. The GPA system goes hand in hand with the semester and course credit system, hence the need for students to understand the functionalities of the impact of the credit system on his overall academic progress. The GPA is of great importance to the student present and future aspiration, hence the need to find suitable and specific strategies to help improve once current GPA and subsequent CGPA. The probability of graduation at the 4th, 5th or 6th year is directly related to the student academic performance, mostly in having good grade on a continuous manner and not failing any course. Student must developed good study pattern, control environmental factors within his learning situation and external factors from family or society, if he is to have high CGPA and graduate at the official graduation year.

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