# Modeling Inflation Rates using Periodogram and Fourier Series Analysis Methods: The Nigerian Case

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**Abstract**
This work considers the application of Periodogram and Fourier Series Analysis to model all-items monthly inflation rates in Nigeria from 2003 to 2011. The main objectives are to identify inflation cycles, fit a suitable model to the data and make forecasts of future values. To achieve these objectives, monthly all-items inflation rates for the period were obtained from the Central Bank of Nigeria (CBN) website. Periodogram and Fourier Series methods of analysis are used to analyze the data. Based on the analysis, it was found out that inflation cycle within the period was fifty one (51) months, which relates to the two government administrations within the period. Further, appropriate significant Fourier series model comprising the trend, seasonal and error components is fitted to the data and this model is further used to make a forecast of the inflation rates for thirteen months and this forecast compares favourably with the actual values for the thirteen months.
**Keywords:** Fourier Series analysis, periodogram, frequency domain, time series, forecasting.
**JEL Subject Classification:** E31, E37.

## 1.0 INTRODUCTION

Inflation is referred to as the persistent and appreciable rise in the general level of prices (Thinngan, 2002). It is not every rise in the price level that is termed inflation. Therefore, for a rise in the general price level to be considered inflation, such a rise must be enduring, constant and sustained. This rise should affect all commodities and should not be temporal.

Inflation rate has been regarded as one of the major economic indicators in any country. According to Ayinde et al (2010), inflation is undeniably one of the most leading and dynamic micro-economic issues confronting almost all economies of the world. Its dynamism has made it an imperative issue to be considered.

In view of its importance to the economic growth of a country, many researchers and economists have applied various time series models to forecast or model inflation rates of countries. These models include Autoregressive Integrated Moving Average (ARIMA) Models and Seasonal Autoregressive Integrated Moving Average (SARIMA) Models all in the time domain, but not much have been done using the Frequency domain models. Recently, the Central Bank of Nigeria Governor, Sanusi Lamido Sanusi, called on researchers to come up with a model that could be adopted to forecast inflation, in view of the recent economic meltdown all over the world; as this would further help in understanding more the current behaviour of inflation rates. Consequently, the researchers explore the frequency domain approach to model inflation rates as a time series, using Periodogram and Fourier Series Analysis methods, because of their simple way of modeling seasonality and eliminating significant peaks without re-estimating the model. We also seek to use this model to make forcast of future values.

Some econometric models have been used to model inflation rates, but they are restrictive in their theoretical formulations and often lack to incorporate the dynamic structure of the data and have tendencies to inflict improper restrictions and specifications on the structural variables (Saz, 2011). Odusanya and Atanda (2010) determined the dynamic and simultaneous interrelationship between inflation and its determinants – growth rate of Gross Domestic Product (GDP), growth rate of money supply ($M_2$), fiscal deficit, exchange rate (U.S dollar to Naira), importance and interest rates, using econometric time series model. Ayinde et al (2010) examined the factors affecting inflation in Nigeria using cointegration and descriptive statistics. They observed that there were variations in the trend pattern of inflation rates and some variables considered were significant in determining inflation in Nigeria. These variables include annual total import, annual consumer price index for food, annual agricultural output, interest rate, annual government expenditure, exchange rate and annual crude oil export. Stockton and Glassman (1987) conducted a comparative study on three different inflation processes namely rational expectations model, monetarist model and the expectation augmented Philips curve that are based on economic theory relationships that explain and form inflation. These processes were compared to one another, utilizing their out-of-sample forecast performance on an eight-quarter horizon and in addition a simple Autoregressive Integrated Moving Average (ARIMA) model was used as a bench mark to substantiate the theoretical validity of the econometric models. Their findings showed that the ARIMA model outperformed both the rational expectations model and the monetarist model and was found to perform just as good as the Philips curve in all specifications. They included that upon theoretical effort put to explain causes of inflation, a simple ARIMA model of inflation turned in such a respectable forecast performance relative to the theoretically based specifications.

A reason why simple time series models tend to outperform their theoretical counterparts lies in the restrictive nature of econometric models with their improper restriction and specifications on structural variables. The absence of restriction in the ARIMA model gives it the necessary flexibility to capture dynamic properties and thus significant advantage in short-run forecasting (Saz, 2011). Encouraged by these empirical results on the superiority of ARIMA models, Saz (2011) applied Seasonal Autoregressive Integrated Moving Average (SARIMA) model to forecast the Turkish inflation.

Having known the advantages of using Fourier Series Analysis to model periodic data, Ekpenyong and Omekara (2007) modeled the mean monthly temperature of Uyo Metropolis using Fourier Series Method. Also Ekpenyong (2008) modeled rainfall data using Pseudo-additive Fourier series model, which he modified from Fourier Series Model. In the same vein, we seek to use Fourier Series Analysis and Periodogram analysis to investigate the properties of inflation rates in Nigeria and also model the rates so as to make good forecast of it.

## 2.2 METHOD OF ANALYSIS

The main methods of analysis of this work are Periodogram and Fourier Series Analysis. If the series is largely influenced by seasonality or periodicity, one can immediately inspect and guess the period or the frequency of the series; but if the period of the series cannot be guessed accurately, it calls for the construction of a Periodogram to determine the period or frequency of the series.

In this work, since inflation is affected by other factors, Periodogram analysis becomes necessary to determine the inflation cycle of the series for the period under study.

In constructing the Periodogram, time series is viewed in the frequency-domain view point as one that consists of sine and cosine waves at different frequencies.

$$X_t = T_t + \sum_{i=1}^{N} \left[a_i Cos\,\omega_i t \; b_i Sin\,\omega_i t\right] + Z_t \quad --- (1)$$

Where : $T_t$ is the trend equation
$X_t$ is the series
$\omega_i$ = the angular frequency measured in radians
$Z_t$ is the error term
$a_i$, $b_i$ are the coefficients

in short, the series consists of the trend, seasonal and error components. The series is first detrended and the ordinary least squares estimates of the parameters obtained on the detrended series as:

$$\hat{a}_i = \frac{2}{N}\sum_{i=1}^{N} \Delta X_t Cos\omega_i t \quad ---(2)$$

$$\hat{b}_i = \frac{2}{N}\sum_{i=1}^{N} \Delta X_t Sin\omega_i t \quad ---(3)$$

$$Since \sum_{i=1}^{N} Cos\omega_i t \;\; Cos\omega_j t = \begin{cases} 0, i \neq j \\ N/2, i = j \end{cases} \quad --- (4)$$

$$\sum_{i=1}^{N} Sin\omega_i t \;\; Cos\omega_j t = 0 \;\; for\,all\,i\,and\,j \quad --- (5)$$

$$Also \sum_{i=1}^{N} Sin\omega_i t \; Sin\omega_j t = \begin{cases} n/2, i = j \\ 0, otherwise \end{cases} \quad ---(6)$$

$$And \sum_{i=1}^{N} Sin\omega_i t = 0 \quad --- (7)$$

$$\sum_{i=1}^{N} Cos\omega_i t = 0 \quad --- (8)$$

for $1 < i < N/2$, $where\;\omega_i = 2\pi f_i$

The above results are obtained from complex variables as orthogonality and independent properties of Fourier Series or sinusoidal models. In case of time series with even number of observations N = 2q, q = N/2, the same definitions are applicable except for

$$\hat{a}_q = \frac{1}{N}\sum_{i=0}^{N} (-1)\,\Delta X_t$$

$$\hat{b}_q = 0 \qquad \text{- - - (9)}$$

Therefore the Periodogram for a time series $\Delta X_t$ with N = 2q + 1 (odd number of observations) is defined as the function of intensities I ($f_i$) at frequency $f_i$ as:

$$\mathrm{I}\,(f_i) = \frac{N}{2}\left[a_i^2 + b_i^2\right],\ i = 1, 2, ..., q \qquad \text{- - - (10)}$$

where $f_i$ is the ith harmonic of frequency $\frac{1}{N}$ and $0 \leq f_i \leq 0.5$

for even number of observations,

$$\mathrm{I}\,(f_{0.5}) = Na_q^2 \qquad \text{- - - (11)}$$

The plot of the intensities against the frequencies($f_1$) or periods $\left(\frac{1}{f_i}\right)$ is the Periodogram.

The Periodogram measures the amplitude of a time series for all possible frequencies and wavelengths. It can be interpreted as the amount of the total series sums of squares, that is explained by specific frequencies. The Period or frequency of the series is identified as that with the largest intensity, I ($f_i$). The frequency would then be used as the Fourier frequency $f_i$ to obtain the parameter estimates of the model.

The combination of methods of estimating the components of time series gives a general model of Fourier Series Analysis used in forecasting time series.

The general Fourier Series model is given by:

$$X_t = T_t + \alpha_t\, Cos\omega t + \beta_i Sin\omega t + \alpha_2\, Cos2\omega t + \beta_2 Sin2\omega t + ... + \alpha_k Cosk\omega t + \beta_k Sink\omega t + Z_t \qquad --- (12)$$

$$= T_t + \sum_{i=1}^{k}\left[\alpha_i Cosi\omega t + \beta_i Sini\omega t\right] + Z_t \qquad ---(13)$$

The estimated model for forecasting time series is given by:

$$\hat{X}_t = T_t + \sum_{i=1}^{k}\left[\hat{\alpha}_i Cosi\omega t + \hat{\beta}_i Sini\omega t\right] + \hat{Z}_t \qquad ---(14)$$

where $\hat{X}_t$ = estimated values

$\hat{T}_t$ = estimated trend equation

$\hat{\alpha}_i,\ \hat{\beta}_i\ (i = 1, ..., k)$ = parameter estimates

$\omega$ = $2\pi f_i$

k = highest harmonic of $\omega$

$\hat{Z}_t$ = estimate of the random error.

The highest harmonic, k in Fourier series analysis model is the number of observations per season divided by two (2) for an even number of observations and (n – 1)/2 for an odd number of observations (Priestly, 1981).

The trend is first isolated or removed by fitting the linear trend model or quadratic trend or even the overall mean of the data, using the method of least squares. The trend equation is fitted based on its significance in the model. Thus the trend equation includes:

$$\left.\begin{aligned} X_t &= a_o + b_o t &&\text{- Linear trend}\\ X_t &= a_o + b_o t + C_o t^2 &&\text{- Quadratic trend}\\ \mu &= \frac{\sum X_t}{N} && \end{aligned}\right\} \qquad \text{- - - (15)}$$

The trend, after estimating, is now removed from series and the detrend series used to estimate seasonal variation.

The sine and cosine functions in equation (2.14) give the estimated model for the seasonality of the model. It is given by:

$$\Delta\hat{X}_t = \sum_{i=1}^{k}\hat{\alpha}_i Cosi\omega t + \hat{\beta}_i Sini\omega t \qquad ---(16)$$

The above equation is cast as a Multiple Linear Regression to obtain the estimates of $\alpha_i$ and $\beta_i$.

The original value of $Z_t$ is obtained by subtracting the trend and seasonal components from the original value ($X_t$). This gives the random error. To estimate $Z_t$, we first of all determine if the residual values are random. This can be done by assessing the autocorrelation function of the residual. If the residual or error component is not random, a first order autoregressive model can be fitted to the error values as:

$$Z_t = \phi Z_{t-1} + \mu_t \qquad --- (17)$$

where $Z_t$ = error term

$\phi$ = the autoregressive coefficient

$\mu_t$ = the purely random process (white noise)

The estimated equation is given as:

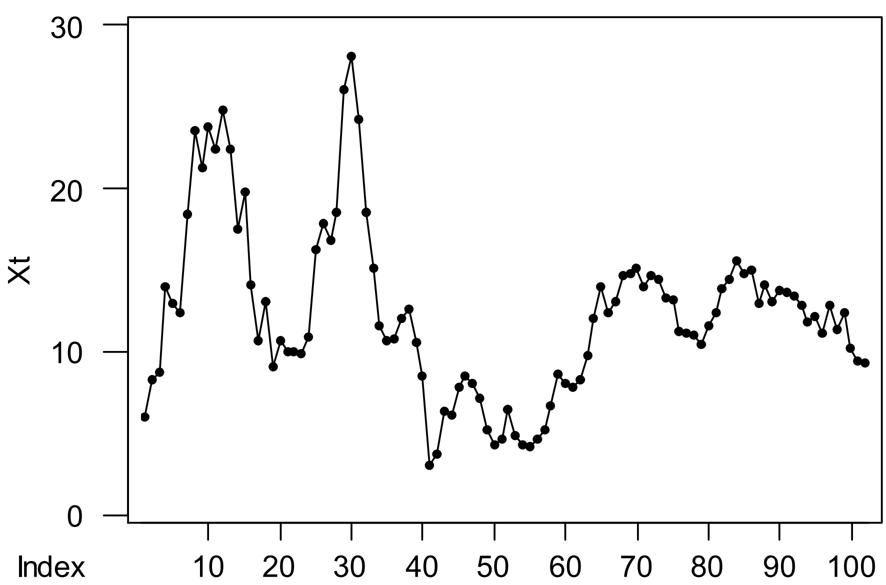
$$\hat{Z}_t = \hat{\phi} Z_{t-1} \qquad --- (18)$$

These estimations are done using statistical packages such as MINITAB and SPSS/EXCEL.

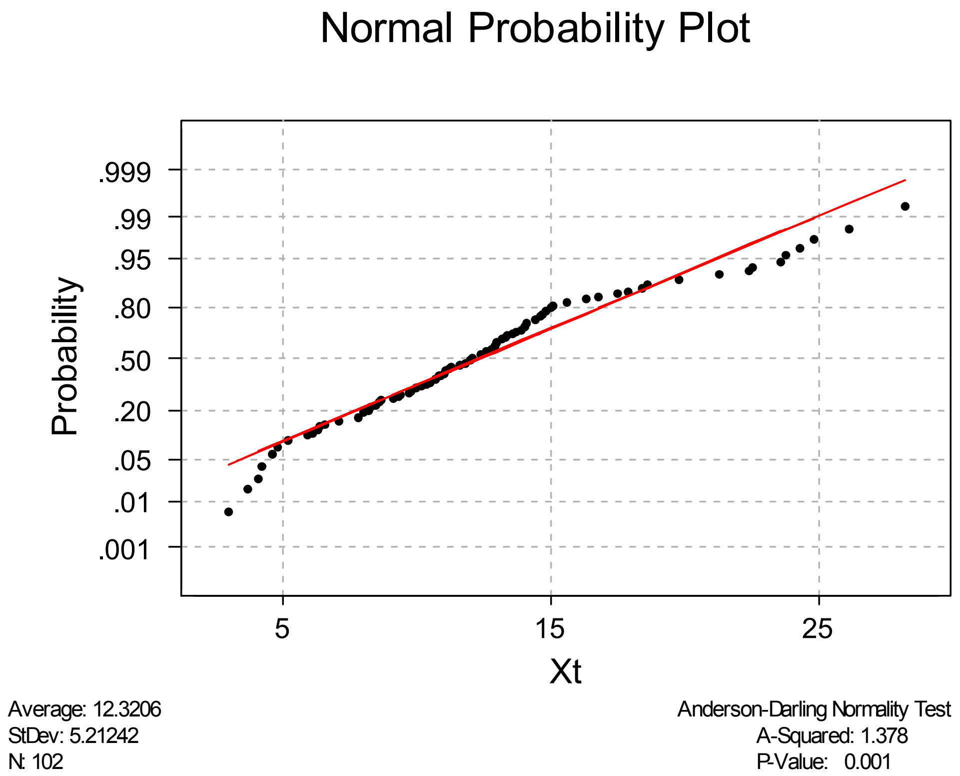
## 3.0 DATA ANALYSIS

A clear assessment of the graphical representation of original series shows that there is an existence of trend cyclical and seasonal variations, but the cycle or period of the series cannot be exactly ascertained (see figure I). In addition the normal probability plot of the series indicates that the series is not normal or stable. It is therefore important to transform the data for stability and normality. A square root transformation was therefore appropriate as shown in figure III below.

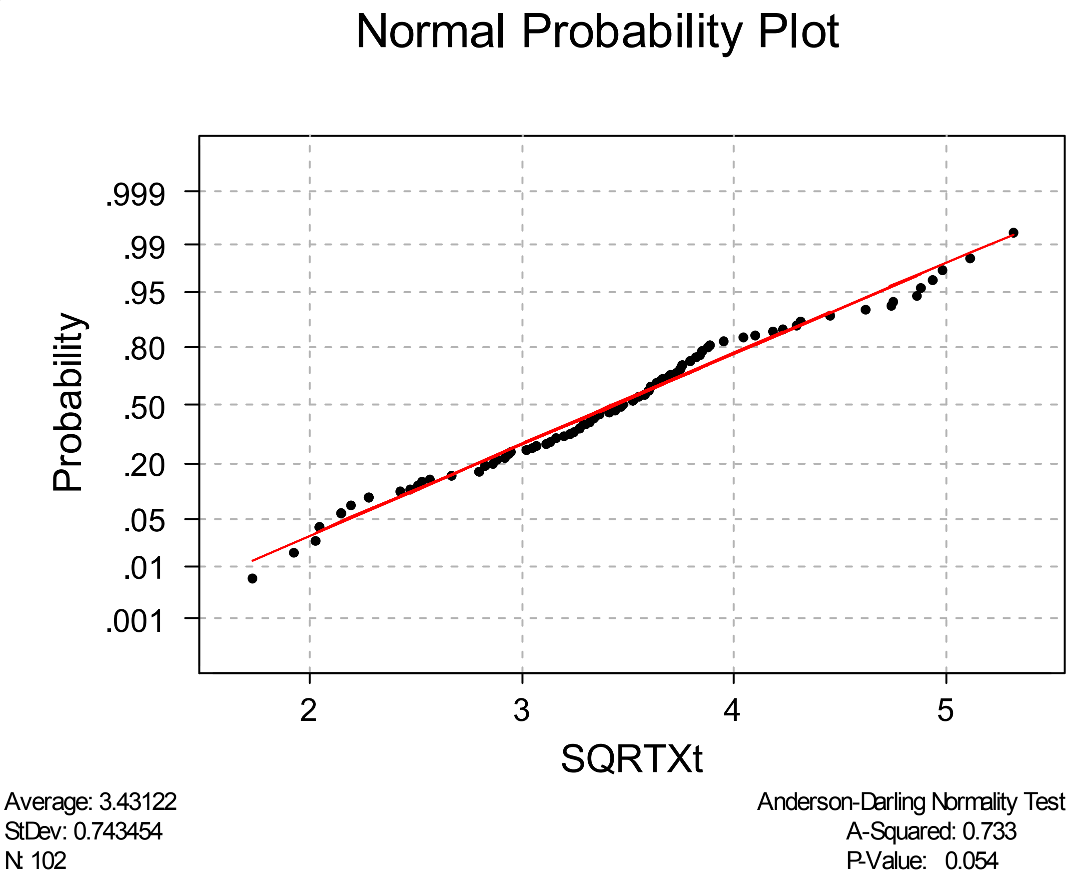
**Figure I: Plot of Original Series**



**FigureII: Normal P-P Plot of the Series**




**FigureIII: Normal P-P Plot of the transformed Series**




## 3.2 PERIODOGRAM ANALYSIS

The intensities $I(f_i)$ at various frequencies are obtained using equations (2), (3) and (10) as shown in

Table 6. The plots of these intensities against the frequencies and periods are given in the figures below.

**FIGURE IV: Plot of Intensities against Frequency**

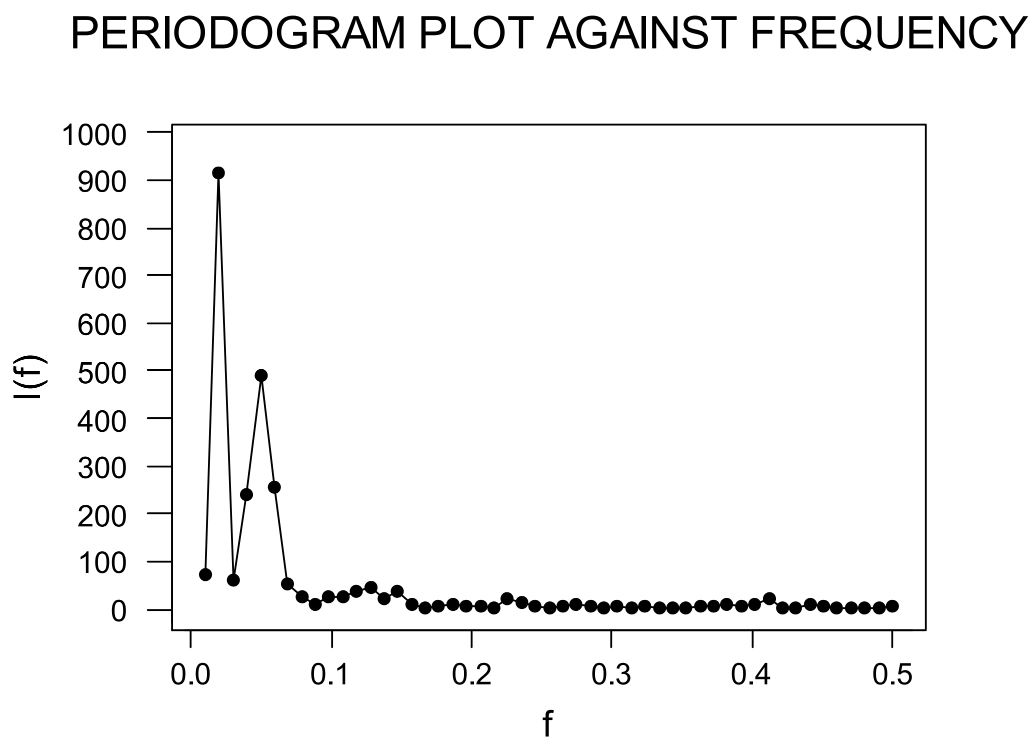
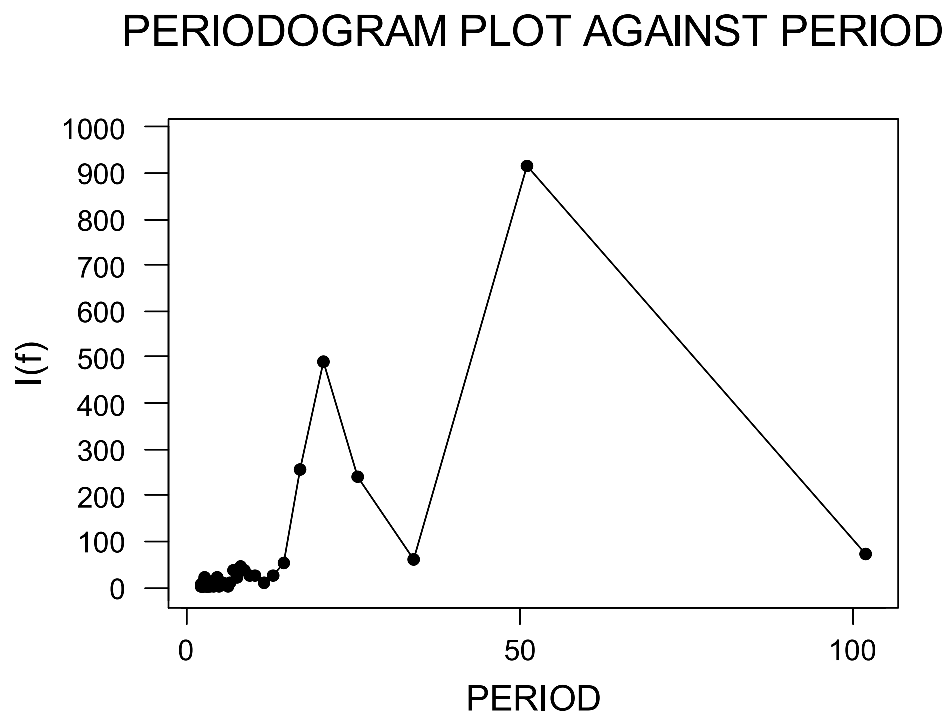



**FIGURE V: Plot of Intensities against Periods**




From figures I, II and III, it is observed that the frequency and period with the largest intensity are 0.0196 and 51 months respectively. Hence the period or cycle for the data is 51 months. These are now used to fit the Fourier series model.

### 3.3 FITTING THE GENERAL FOURIER SERIES MODEL

The trend ($T_t$) is estimated as:

$T_t = a_o + b_o t$ and its analysis is given in table 1.

The regression equation is

$$T_t = 3.6529 + 0.0043t$$

From table 1, it is seen that the parameter estimate $a_o$ is significant in the trend equation at 5% level of significance. Therefore $T_t = 3.6529$.

The seasonal component is then estimated from the detrended series as:

$$k = \frac{(n-1)}{2} = \frac{51-1}{2} = {}^{50}\!/_{2} = 25$$

$$and \ \omega = 2 x \pi \ x \ 0.019608$$

$$\therefore X_t = \sum_{i=1}^{25} \left[\alpha_i Cos i\omega t + \beta_i Sin i\omega t\right]$$

The parameters $\alpha_i^{'s}$ and $\beta_i^{'s}$ are obtained by method of least squares as shown in the table 2 .

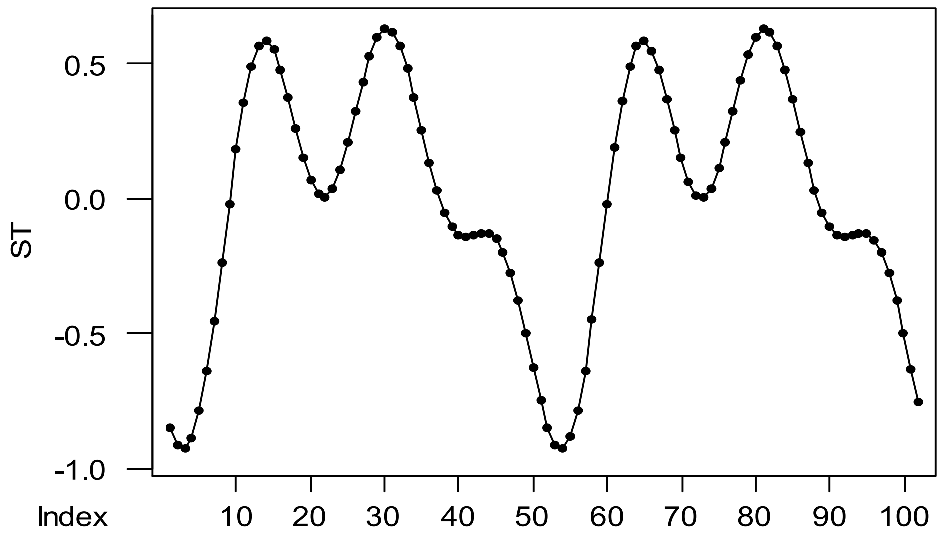
From table 2, it is observed that the parameter estimates that are significant in the model are: $\hat{\alpha}_1, \ \hat{\alpha}_2 \ and \ \hat{\beta}_3.$

Therefore, the estimated seasonal model is

$$SQRT\Delta\hat{X}_t = \hat{\alpha}_1 Cos\omega t + \alpha_2 Cos2\omega t + \hat{\beta}_3 Sin3\omega t$$

$$SQRT\Delta\hat{X}_t = -0.5071 Cos\omega t - 0.2432 Cos2\omega t - 0.3088 Sin3\omega t \qquad ...(20)$$

**Figure VI: Plot of Estimated Seasonal Component**




In assessing the autocorrelation and partial autocorrelation function of the error component, it was found out that the error was not random. The behaviour of the autocorrelation and partial autocorrelation function suggest an autoregressive model of order one. Table 3 shows the test for significance of the model.

From table 3, the parameter estimate in the error component is significant in the model.

Hence $\hat{Z}_t = 0.9030 Z_{t-1}$ ...(21)

The general model for the series, which consists of the estimated trend, seasonal and error component is given as:

$SQRT\hat{X}_t$ = 3.6529 – 0.5071Cos $\omega$ t – 0.2432Cos $\omega$ t – 0.3088Sin3 $\omega$ t + 0.903$Z_{t-1}$ ....(22)

The model is now used to estimate inflation rates. (See Appendix I).

The plots of the original estimated series of both the actual and transformed values are shown in figures VII and VIII below:

**Figure VII: Plot of Actuals and Estimates of the Transformed Series**

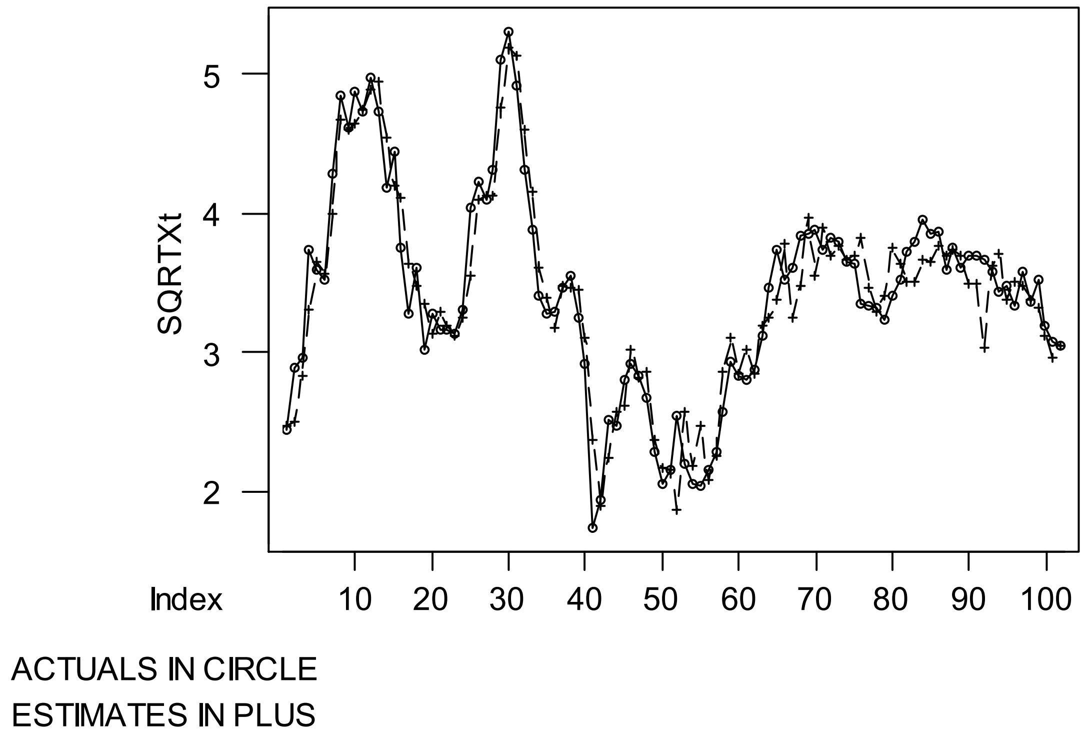
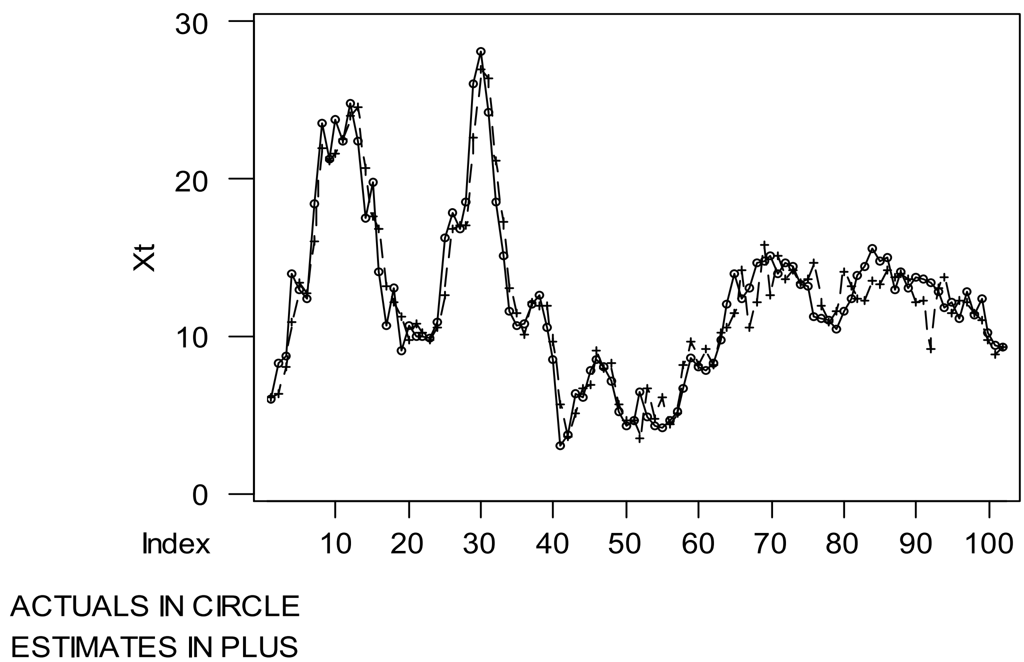



**Figure VIII: Plot of Actuals and Estimates of the Original Series**




The above plots show that the model fit well to the data.

### 3.4 FORECAST FOR THE FUTURE RATES

A forecast from September 2011 to September 2012 was made using the estimated general model and the forecast is shown in the table below:

### 4.0 DISCUSSION

The periodogram analysis reveals that there exist both short term and long term cycles within the period under study. The long term cycle is 51 months while the short term cycle is approximately 20 months. This is known by checking the second largest intensity in Appendix III. This goes a long way to buttress the fact that inflation is influenced by cyclical or periodic variation. It can also be deduced from the periodogram the relationship between the inflation cycle and the various government administrations within the period. Under this period of study, it is known that two administrations existed, namely Obasanjo's and Yaradua/Jonathan's administrations. The inflation cycle relates to these two administrations. The first fifty one months represents the administration of President Olusegun Obasanjo, while the second cycle represents Yaradua/Jonathan's administration.

From the foregoing, it can be deduced that economic policies, activities, implementation of policies, budgets, etc of government administration influence inflation rates in Nigeria. Most economists have confirmed that an expansionary budgetary provision among other factors helps to increase inflation rates. A clear look at the original plot of the series shows that during Obasanjo's regime, there were higher inflation rates than the Yaradua/Jonathan's administration. This may be as a result of incessant supplementary budgets raised and the inability of the administration to follow budgetary provisions in the implementation of the budget. But the latter administration records lower inflation rates, which may be as a result of reduced passage of supplementary budgets and implementation of some reforms in the economic sectors.

## 5.0 CONCLUSION

The periodogram analysis has identified a major inflation rate cycle for the period under study to be fifty one (51) months and the former series analysis has established a former series model for the all-items inflation rates to be

$$SQRT\hat{X}_t = 3.6529 - 0.5071\mathrm{Cos}\,\omega t - 0.2432\mathrm{Cos}\,\omega t - 0.3088\mathrm{Sin}3\omega t + 0.903Z_{t-1}$$

This model is used to make good forecast of inflation rates. Therefore Fourier series models can also be used to model inflation rates because of its advantage of identifying inflation cycles in addition to establishing a suitable model for the series.

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**Table 1: Test for Significance of the Parameter Estimates in the Trend Model**

| Predictor | Coef | StDev | T | P |
|---|---|---|---|---|
| Constant | 3.6529 | 0.1469 | 24.87 | 0.000 |
| t | -0.004304 | 0.002475 | -1.74 | 0.085 |

S = 0.7361   R-Sq = 2.9%   R-Sq(adj) = 2.0%

**Table 2: Test for Significance of the Parameter Estimates in the Seasonal Component**

| Predictor | Coef | StDev | T | P |
|---|---|---|---|---|
| Noconstant | | | | |
| coswt | -0.5071 | 0.1041 | -4.87 | 0.000 |
| sinwt | 0.0387 | 0.1041 | 0.37 | 0.712 |
| cos2wt | -0.2432 | 0.1041 | -2.34 | 0.023 |
| sin2wt | 0.1524 | 0.1041 | 1.46 | 0.149 |
| cos3wt | -0.0323 | 0.1041 | -0.31 | 0.758 |
| sin3wt | -0.3088 | 0.1041 | -2.97 | 0.005 |
| cos4wt | -0.0326 | 0.1041 | -0.31 | 0.756 |
| sin4wt | -0.0813 | 0.1041 | -0.78 | 0.438 |
| cos5wt | 0.0120 | 0.1041 | 0.12 | 0.909 |
| sin5wt | -0.0951 | 0.1041 | -0.91 | 0.365 |
| cos6wt | -0.1057 | 0.1041 | -1.01 | 0.315 |
| sin6wt | -0.0322 | 0.1041 | -0.31 | 0.759 |
| cos7wt | 0.0163 | 0.1041 | 0.16 | 0.876 |
| sin7wt | 0.0834 | 0.1041 | 0.80 | 0.427 |
| cos8wt | 0.0563 | 0.1041 | 0.54 | 0.591 |
| sin8wt | 0.0132 | 0.1041 | 0.13 | 0.900 |
| cos9wt | -0.0449 | 0.1041 | -0.43 | 0.668 |
| sin9wt | 0.0050 | 0.1041 | 0.05 | 0.962 |
| cos10wt | -0.0113 | 0.1041 | -0.11 | 0.914 |
| sin10wt | -0.0420 | 0.1041 | -0.40 | 0.688 |
| cos11wt | -0.0230 | 0.1041 | -0.22 | 0.826 |
| sin11wt | -0.0119 | 0.1041 | -0.11 | 0.910 |
| cos12wt | 0.0714 | 0.1041 | 0.69 | 0.496 |
| sin12wt | 0.0119 | 0.1041 | 0.11 | 0.910 |
| cos13wt | 0.0230 | 0.1041 | 0.22 | 0.826 |
| sin13wt | -0.0025 | 0.1041 | -0.02 | 0.981 |
| cos14wt | 0.0529 | 0.1041 | 0.51 | 0.613 |
| sin14wt | 0.0183 | 0.1041 | 0.18 | 0.861 |
| cos15wt | -0.0212 | 0.1041 | -0.20 | 0.839 |
| sin15wt | 0.0095 | 0.1041 | 0.09 | 0.928 |
| cos16wt | -0.0235 | 0.1041 | -0.23 | 0.822 |
| sin16wt | 0.0108 | 0.1041 | 0.10 | 0.918 |
| cos17wt | 0.0021 | 0.1041 | 0.02 | 0.984 |
| sin17wt | 0.0165 | 0.1041 | 0.16 | 0.875 |
| cos18wt | 0.0004 | 0.1041 | 0.00 | 0.997 |
| sin18wt | 0.0164 | 0.1041 | 0.16 | 0.875 |
| cos19wt | 0.0192 | 0.1041 | 0.18 | 0.855 |
| sin19wt | -0.0428 | 0.1041 | -0.41 | 0.683 |
| cos20wt | 0.0309 | 0.1041 | 0.30 | 0.768 |
| sin20wt | -0.0322 | 0.1041 | -0.31 | 0.759 |
| cos21wt | 0.0041 | 0.1042 | 0.04 | 0.969 |
| sin21wt | 0.0088 | 0.1041 | 0.08 | 0.933 |
| cos22wt | -0.0011 | 0.1042 | -0.01 | 0.992 |
| sin22wt | -0.0039 | 0.1041 | -0.04 | 0.970 |
| cos23wt | 0.0248 | 0.1042 | 0.24 | 0.813 |
| sin23wt | -0.0205 | 0.1041 | -0.20 | 0.845 |
| cos24wt | -0.0086 | 0.1042 | -0.08 | 0.935 |
| sin24wt | -0.0243 | 0.1040 | -0.23 | 0.816 |
| cos25wt | 0.0184 | 0.1045 | 0.18 | 0.861 |
| sin25wt | -0.0052 | 0.1038 | -0.05 | 0.961 |

S = 0.7436

**Table 3: Test for Significance of Parameter Estimate I the Error Component**

Final Estimates of Parameters

| Type | Coef | StDev | T |
|---|---|---|---|
| AR 1 | 0.9030 | 0.0441 | 20.49 |

Number of observations: 102

**Table 4: Forecasts for Future Periods**

| FORCAST | ACTUALS | t |
|---|---|---|
| 9.2881 | 10.3 | 103 |
| 9.1516 | 10.5 | 104 |
| 9.2528 | 10.5 | 105 |
| 9.3567 | 10.3 | 106 |
| 9.436 | 12.6 | 107 |
| 9.4507 | 11.9 | 108 |
| 10.5905 | 12.1 | 109 |
| 11.6472 | 12.9 | 110 |
| 12.1333 | 12.7 | 111 |
| 12.7911 | 12.9 | 112 |
| 13.3627 | 12.8 | 113 |
| 12.3368 | 11.7 | 114 |
| 10.7698 | 11.3 | 115 |

**Table 5: Actuals and Estimates of Nigeria All-items Inflation Rates(2003-20011)**

| t | ESTSQRTXt | ESTXt | Xt | SQRTXt |
|---|---|---|---|---|
| 1 | 2.45894 | 6.0464 | 5.9 | 2.42899 |
| 2 | 2.49951 | 6.2476 | 8.3 | 2.88097 |
| 3 | 2.82162 | 7.9615 | 8.7 | 2.94958 |
| 4 | 3.29974 | 10.8883 | 14 | 3.74166 |
| 5 | 3.6537 | 13.3495 | 12.9 | 3.59166 |
| 6 | 3.56475 | 12.7074 | 12.4 | 3.52136 |
| 7 | 3.99915 | 15.9932 | 18.4 | 4.28952 |
| 8 | 4.68373 | 21.9373 | 23.6 | 4.85798 |
| 9 | 4.60276 | 21.1854 | 21.3 | 4.61519 |
| 10 | 4.65238 | 21.6446 | 23.8 | 4.87852 |
| 11 | 4.75027 | 22.5651 | 22.4 | 4.73286 |
| 12 | 4.90114 | 24.0212 | 24.8 | 4.97996 |
| 13 | 4.95641 | 24.566 | 22.5 | 4.74342 |
| 14 | 4.55181 | 20.719 | 17.5 | 4.1833 |
| 15 | 4.1966 | 17.6114 | 19.8 | 4.44972 |
| 16 | 4.108 | 16.8757 | 14.1 | 3.755 |
| 17 | 3.63175 | 13.1896 | 10.7 | 3.27109 |
| 18 | 3.48247 | 12.1276 | 13 | 3.60555 |
| 19 | 3.3535 | 11.246 | 9.1 | 3.01662 |
| 20 | 3.12435 | 9.7616 | 10.7 | 3.27109 |
| 21 | 3.28923 | 10.819 | 10 | 3.16228 |
| 22 | 3.19264 | 10.193 | 10 | 3.16228 |
| 23 | 3.11341 | 9.6933 | 9.8 | 3.1305 |
| 24 | 3.24506 | 10.5304 | 10.9 | 3.30151 |
| 25 | 3.55351 | 12.6274 | 16.3 | 4.03733 |
| 26 | 4.1022 | 16.828 | 17.9 | 4.23084 |
| 27 | 4.12568 | 17.0213 | 16.8 | 4.09878 |

| | | | | |
|---|---|---|---|---|
| 28 | 4.13136 | 17.0681 | 18.6 | 4.31277 |
| 29 | 4.75936 | 22.6515 | 26.1 | 5.10882 |
| 30 | 5.19483 | 26.9863 | 28.2 | 5.31037 |
| 31 | 5.14782 | 26.5001 | 24.3 | 4.9295 |
| 32 | 4.6063 | 21.218 | 18.6 | 4.31277 |
| 33 | 4.16304 | 17.3309 | 15.1 | 3.88587 |
| 34 | 3.60814 | 13.0187 | 11.6 | 3.40588 |
| 35 | 3.38407 | 11.4519 | 10.7 | 3.27109 |
| 36 | 3.17771 | 10.0978 | 10.8 | 3.28634 |
| 37 | 3.48145 | 12.1205 | 12 | 3.4641 |
| 38 | 3.45813 | 11.9586 | 12.6 | 3.54965 |
| 39 | 3.45463 | 11.9345 | 10.5 | 3.24037 |
| 40 | 3.10574 | 9.6456 | 8.5 | 2.91548 |
| 41 | 2.357 | 5.5554 | 3 | 1.73205 |
| 42 | 1.8921 | 3.5801 | 3.7 | 1.92354 |
| 43 | 2.2347 | 4.9939 | 6.3 | 2.50998 |
| 44 | 2.56834 | 6.5964 | 6.1 | 2.46982 |
| 45 | 2.61611 | 6.844 | 7.8 | 2.79285 |
| 46 | 3.01449 | 9.0871 | 8.5 | 2.91548 |
| 47 | 2.80696 | 7.879 | 8 | 2.82843 |
| 48 | 2.86217 | 8.192 | 7.1 | 2.66458 |
| 49 | 2.36306 | 5.5841 | 5.2 | 2.28035 |
| 50 | 2.15326 | 4.6365 | 4.2 | 2.04939 |
| 51 | 2.15144 | 4.6287 | 4.6 | 2.14476 |
| 52 | 1.86214 | 3.4676 | 6.4 | 2.52982 |
| 53 | 2.56943 | 6.602 | 4.8 | 2.19089 |
| 54 | 2.17886 | 4.7474 | 4.2 | 2.04939 |
| 55 | 2.46632 | 6.0827 | 4.1 | 2.02485 |
| 56 | 2.07906 | 4.3225 | 4.6 | 2.14476 |
| 57 | 2.23993 | 5.0173 | 5.2 | 2.28035 |
| 58 | 2.86022 | 8.1808 | 6.6 | 2.56905 |
| 59 | 3.10472 | 9.6393 | 8.6 | 2.93258 |
| 60 | 2.84175 | 8.0755 | 8 | 2.82843 |
| 61 | 3.01905 | 9.1147 | 7.8 | 2.79285 |
| 62 | 2.84466 | 8.0921 | 8.2 | 2.86356 |
| 63 | 3.19439 | 10.2041 | 9.7 | 3.11448 |
| 64 | 3.2467 | 10.5411 | 12 | 3.4641 |
| 65 | 3.37694 | 11.4037 | 14 | 3.74166 |
| 66 | 3.77535 | 14.2533 | 12.4 | 3.52136 |
| 67 | 3.24661 | 10.5405 | 13 | 3.60555 |
| 68 | 3.47907 | 12.104 | 14.7 | 3.83406 |
| 69 | 3.96803 | 15.7453 | 14.8 | 3.84708 |
| 70 | 3.54911 | 12.5962 | 15.1 | 3.88587 |
| 71 | 3.89074 | 15.1378 | 14 | 3.74166 |
| 72 | 3.69084 | 13.6223 | 14.6 | 3.82099 |
| 73 | 3.76744 | 14.1936 | 14.4 | 3.79473 |

| | | | | |
|---|---|---|---|---|
| 74 | 3.66179 | 13.4087 | 13.3 | 3.64692 |
| 75 | 3.69333 | 13.6407 | 13.2 | 3.63318 |
| 76 | 3.83035 | 14.6716 | 11.2 | 3.34664 |
| 77 | 3.45736 | 11.9534 | 11.1 | 3.33167 |
| 78 | 3.29035 | 10.8264 | 11 | 3.31662 |
| 79 | 3.40812 | 11.6153 | 10.4 | 3.2249 |
| 80 | 3.75526 | 14.102 | 11.6 | 3.40588 |
| 81 | 3.63406 | 13.2064 | 12.4 | 3.52136 |
| 82 | 3.50865 | 12.3106 | 13.9 | 3.72827 |
| 83 | 3.50096 | 12.2567 | 14.4 | 3.79473 |
| 84 | 3.67314 | 13.4919 | 15.6 | 3.94968 |
| 85 | 3.64484 | 13.2849 | 14.8 | 3.84708 |
| 86 | 3.76182 | 14.1513 | 15 | 3.87298 |
| 87 | 3.70015 | 13.6911 | 12.9 | 3.59166 |
| 88 | 3.7377 | 13.9704 | 14.1 | 3.755 |
| 89 | 3.6965 | 13.6641 | 13 | 3.60555 |
| 90 | 3.48558 | 12.1493 | 13.7 | 3.70135 |
| 91 | 3.4968 | 12.2276 | 13.6 | 3.68782 |
| 92 | 3.03385 | 9.2043 | 13.4 | 3.6606 |
| 93 | 3.61712 | 13.0835 | 12.8 | 3.57771 |
| 94 | 3.70553 | 13.7309 | 11.8 | 3.43511 |
| 95 | 3.38216 | 11.439 | 12.1 | 3.47851 |
| 96 | 3.50707 | 12.2995 | 11.1 | 3.33167 |
| 97 | 3.47876 | 12.1018 | 12.8 | 3.57771 |
| 98 | 3.38236 | 11.4403 | 11.3 | 3.36155 |
| 99 | 3.32326 | 11.0441 | 12.4 | 3.52136 |
| 100 | 3.11168 | 9.6825 | 10.2 | 3.19374 |
| 101 | 2.96192 | 8.773 | 9.4 | 3.06594 |
| 102 | 3.04434 | 9.268 | 9.3 | 3.04959 |

**Table 6: ESTIMATES, FREQUENCIES, PERIOD AND INTENSITIES**

| a | b | a**2 | b**2 | I(f) | f | PERIOD |
|---|---|---|---|---|---|---|
| 8.4864 | 0.663 | 72.019 | 0.44 | 72.459 | 0.009804 | 102 |
| -30.243 | 1.6932 | 914.639 | 2.867 | 917.506 | 0.019608 | 51 |
| 7.5888 | 0.0204 | 57.59 | 0 | 57.59 | 0.029412 | 34 |
| -13.5048 | 7.6398 | 182.38 | 58.367 | 240.746 | 0.039216 | 25.5 |
| -21.828 | 3.8556 | 476.462 | 14.866 | 491.327 | 0.04902 | 20.4 |
| -2.142 | -15.8406 | 4.588 | 250.925 | 255.5 | 0.058824 | 17 |
| 3.5598 | -6.0486 | 12.672 | 36.586 | 49.258 | 0.068627 | 14.571 |
| -1.9482 | -4.2228 | 3.795 | 17.832 | 21.628 | 0.078431 | 12.75 |
| -2.652 | -0.0204 | 7.033 | 0 | 7.034 | 0.088235 | 11.333 |
| 0.4284 | -4.9062 | 0.184 | 24.071 | 24.254 | 0.098039 | 10.2 |
| 4.4778 | 1.581 | 20.051 | 2.5 | 22.55 | 0.107843 | 9.273 |
| -5.5182 | -1.6932 | 30.451 | 2.867 | 33.317 | 0.117647 | 8.5 |
| 0.5916 | -6.4056 | 0.35 | 41.032 | 41.382 | 0.127451 | 7.846 |
| 0.7344 | 4.2126 | 0.539 | 17.746 | 18.285 | 0.137255 | 7.286 |

| | | | | | | |
|---|---|---|---|---|---|---|
| -4.4472 | -3.927 | 19.778 | 15.421 | 35.199 | 0.147059 | 6.8 |
| 2.7948 | 0.6324 | 7.811 | 0.4 | 8.211 | 0.156863 | 6.375 |
| -0.9996 | -0.153 | 0.999 | 0.023 | 1.023 | 0.166667 | 6 |
| -2.346 | 0.2142 | 5.504 | 0.046 | 5.55 | 0.176471 | 5.667 |
| 0.459 | -2.8458 | 0.211 | 8.099 | 8.309 | 0.186275 | 5.368 |
| -0.6222 | -2.1828 | 0.387 | 4.765 | 5.152 | 0.196078 | 5.1 |
| 1.8462 | 0.0306 | 3.408 | 0.001 | 3.409 | 0.205882 | 4.857 |
| -1.2138 | -0.6426 | 1.473 | 0.413 | 1.886 | 0.215686 | 4.636 |
| -0.0102 | -4.5186 | 0 | 20.418 | 20.418 | 0.22549 | 4.435 |
| 3.6006 | 0.5712 | 12.964 | 0.326 | 13.291 | 0.235294 | 4.25 |
| 0.6426 | -1.3158 | 0.413 | 1.731 | 2.144 | 0.245098 | 4.08 |
| 1.1424 | -0.1632 | 1.305 | 0.027 | 1.332 | 0.254902 | 3.923 |
| 1.377 | -0.9282 | 1.896 | 0.862 | 2.758 | 0.264706 | 3.778 |
| 2.6724 | 0.8976 | 7.142 | 0.806 | 7.947 | 0.27451 | 3.643 |
| 0.3774 | 2.346 | 0.142 | 5.504 | 5.646 | 0.284314 | 3.517 |
| -1.1016 | 0.4488 | 1.214 | 0.201 | 1.415 | 0.294118 | 3.4 |
| 0.8772 | -1.122 | 0.769 | 1.259 | 2.028 | 0.303922 | 3.29 |
| -1.224 | 0.5202 | 1.498 | 0.271 | 1.769 | 0.313725 | 3.188 |
| 1.224 | -1.0914 | 1.498 | 1.191 | 2.689 | 0.323529 | 3.091 |
| 0.0918 | 0.8058 | 0.008 | 0.649 | 0.658 | 0.333333 | 3 |
| -0.0816 | -0.2958 | 0.007 | 0.087 | 0.094 | 0.343137 | 2.914 |
| 0 | 0.8058 | 0 | 0.649 | 0.649 | 0.352941 | 2.833 |
| -1.9278 | -0.6732 | 3.716 | 0.453 | 4.17 | 0.362745 | 2.757 |
| 0.9588 | -2.2134 | 0.919 | 4.899 | 5.818 | 0.372549 | 2.684 |
| 0.8058 | -2.3052 | 0.649 | 5.314 | 5.963 | 0.382353 | 2.615 |
| 1.5606 | -1.6728 | 2.435 | 2.798 | 5.234 | 0.392157 | 2.55 |
| 2.8764 | 0.9792 | 8.274 | 0.959 | 9.233 | 0.401961 | 2.488 |
| 4.2126 | 0.4182 | 17.746 | 0.175 | 17.921 | 0.411765 | 2.429 |
| 0.6528 | -0.7038 | 0.426 | 0.495 | 0.921 | 0.421569 | 2.372 |
| -0.0612 | -0.2346 | 0.004 | 0.055 | 0.059 | 0.431373 | 2.318 |
| 1.734 | -2.346 | 3.007 | 5.504 | 8.51 | 0.441176 | 2.267 |
| 1.2546 | -1.0812 | 1.574 | 1.169 | 2.743 | 0.45098 | 2.217 |
| 0.2448 | -0.7548 | 0.06 | 0.57 | 0.63 | 0.460784 | 2.17 |
| -0.4488 | -1.275 | 0.201 | 1.626 | 1.827 | 0.470588 | 2.125 |
| 0.7038 | -0.255 | 0.495 | 0.065 | 0.56 | 0.480392 | 2.082 |
| 0.9282 | -0.306 | 0.862 | 0.094 | 0.955 | 0.490196 | 2.04 |
| 1.7442 | -0.0112 | 3.042 | 0 | 3.042 | 0.5 | 2 |