Arbitrage Pricing Model; Determining the Number of Factors and Their Consistency Across Markets

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Abstract
Purpose - The discovery of a true financial equilibrium model that could explain the prices of stocks has long been a sought after challenge and a vital area of research in modern financial theory. The concept is based on the fact that the price of the stock is affected by the present value of the future cash flows from the stock, and anything that will affect the discount rate of these future cash flows. Many brokerage firms, financial institutions and financial consulting firms use multi-index models to aid in the investment process. Thus the APT model is becoming increasingly popular and has been a subject of several empirical studies. These models have been tested on both developed and developing markets. The purpose of this research is to analyze the Arbitrage Pricing Theory (APT) introduced by Ross (1976), which is a more simplified, multifactor model, with fewer relative assumptions to other models, across different representative markets, giving particular attention to the number of factors.
Design/methodology/approach – The research is quantitative in nature and principal component analysis will be used to determine the ideal number of factors that should be included in the model, as well as the identity of these factors.
Findings - Results indicate that the ideal number of factors vary from four to five factors across markets, with their identity differing across markets. Findings provide valuable insights for professionals in the market as well as academics who want to gain further knowledge on the number of factors.
Research limitations/implications – The application of Principal Component Analysis (PCA) is based only on a sample of stocks and not on the whole population in the stock market, and thus there remains a question of how accurate these approximations actually are.
Practical implications – The APT is a popular multi-index model that should be used by financial analysts to allow risk to be more tightly controlled and allow investors to protect against specific type of risk to which he or she is particularly sensitive or to make specific bets on certain types of risks.
Originality/value – No research has yet been carried out across different markets for the same time period as will be carried out in this research, and thus the empirical study in this research aims to add knowledge on whether the number of factors will be consistent across borders or will change from market to market.
Keywords Arbitrage Pricing Theory, Number of factors, Emerging markets
Paper type Research Paper

1. Introduction
One of the most sought after challenges throughout time has been the discovery of a model that can truly explain what determines a stock price. There have been several studies, and researchers have come up with several proposals as to the determinants of stock valuation. We can identify from previous studies a few of the determinants that affect a stock value. A stock value will be determined by the cash flows that investors seek from owning a stock, whether these cash flows are in the form of dividends or capital gains return (Iqbal and Haider, 2005).
Modigliani and Miller (1961) found that investors are keener and primarily focused on the dividends that they will get from a stock, as opposed to capital gains from increase in the price of the stock. Thus anything that is expected to affect the income component as unanticipated changes in rates of inflation, or unanticipated changes in industrial production, would logically affect a stock valuation. Following the same logic any factor that would affect the discount rate and thus the present value of the stock and cash received through dividends, as the changes in structure of interest rates, would also logically affect the value of the stock (Chen et al., 1986).
It seems important to quickly shade some light on the Capital Asset Pricing Model (CAPM) being the primary model to measure the risk return tradeoff developed by Sharpe (1964), Linter (1965). The Capital Asset Pricing Model is represented as:

\[ E(R_i) = R_f + \beta_i (E(R_m) - R_f) \]

Where:
\( E(R_i) \) is the expected return on a given asset
\( R_f \) is the risk free rate of return.
\( \beta_i \) is the sensitivity of the asset’s return to the excess return on the market portfolio.
\( E(R_m) \) is the expected return on the market portfolio, the market portfolio has a beta of 1.

The Capital Asset Pricing Model breaks down risk into two components, systematic and non-systematic.
risk also known as idiosyncratic risk. The model states that investors shouldn’t be rewarded for facing non-systematic risk, as that can be diversified away by holding a large portfolio of stocks.

The CAPM model focuses on studying the systematic or market risk, and proposes a way of measuring the reward an investor should accept for bearing a given level of risk. It is a one factor model, which assumes that there is a market portfolio with a beta of 1, and that any asset’s return can be measured through its sensitivity (correlation) to the market portfolio. However the CAPM was criticized as having many restrictive assumptions. By assuming that the market portfolio has a beta of 1, it ignores the fact that stocks have different sensitivities to different risk factors.

As a result, the Arbitrage Pricing Theory was introduced by Ross (1976), and is a more simplified, multifactor model, with fewer assumptions than that of the CAPM, and thus, in some cases, can be viewed as being superior to the CAPM.

The Arbitrage Pricing Theory can be summarized through the following equation:

\[ R_i = \mu_i + b_{i1}F_1 + b_{i2}F_2 + \cdots + b_{ik}F_k + \varepsilon_i \]

This equation means that the stock return \( R_i \) is composed of the following three components:

- An expected return component \( \mu_i = E(R_i) \),
- An unexpected return component \( b_{i1}F_1 + b_{i2}F_2 + \cdots + b_{ik}F_k \),
- And finally an idiosyncratic or stock specific component \( \varepsilon_i \).

The expected component is a superposition of factors \( F_1, F_2, \cdots, F_k \), representing unanticipated changes in macro-economic components, such as inflation rate, gross domestic product (GDP), interest rate etc., each with an expected value of zero and weighed by the sensitivities \( b_{i1}, b_{i2}, \cdots, b_{ik} \) of the stocks to these unanticipated changes. These sensitivities are known as the factor loads or the beta loadings. Any anticipated change in the factors mentioned above is assumed to be incorporated in the expected return component. Further it is assumed that \( E(\varepsilon_i, \varepsilon_j) = 0 \) for an investor who holds a well-diversified portfolio, as the residual risk will tend to go to zero and only the systematic risk presented by the indexes will matter. Ross (1976) further shows that the expected return of a stock can be approximated by:

\[ E(R_i) = \lambda_0 + \lambda_1b_{i1} + \lambda_2b_{i2} + \cdots + \lambda_kb_{ik} \]

Where \( \lambda_0 \) is the risk free rate of return; this is the lowest rate of return the investor would be willing to accept given they don’t take on any level of risk. \( \lambda_j, j = 1,2,\cdots,k \) is the risk premium (expected return) that an investor expects for bearing additional risk for changes in the movement of the associated macro-economic factors, and \( b_{ij}, j = 1,2,\cdots,k \) is the sensitivity of the stock to the individual factor movements.

The APT asserts that the return on stocks should be described by the linear relationship given above as opposed to, say, a nonlinear relationship. This linearity is one of the most attractive features of the theory. Ross (1976) further states that if this relationship did not hold, and the factors were not able to explain the stock return, then people could enter into arbitrage opportunities by selling short the overpriced stocks, and simultaneously buying the underpriced stocks, thus granting the model its name “The Arbitrage Pricing Theory” (APT). The APT theory is one of the most prominent theories in finance, and has been applied on different markets to see whether the theory holds or not. However, the APT leaves unanswered the following two questions:

1. The number of macro-economic factors to be included in the model (the value of \( k \)).
2. Whether there is only one set of \( k \)-factors or there are different equivalent sets of \( k \)-factors. If the former case holds true: how can we identify these factors? And if the latter case holds true: what are the conditions that determine equivalent factors?

In most of the previous tests of the APT, researchers focused on whether the Arbitrage Pricing Theory does apply in a certain market, and have used ad-hoc factors already determined from existing tests. There hasn’t been much insight or testing into the number or identity of risk factors and their associated risk premiums that should be included in the model. Apparently, this should be crucial to find the factors that should be included, if the model is to be regarded as an accurate and easy to use model by professionals in the stock market. However Berry et al., (1988) gave us a good idea of the characteristics the factors must fulfill to be included in the model, following are the three characteristics:

1. They have to be market wide factors that have a prevalent influence on the stock returns.
2. The factors must also have an effect on the expected return, which can be identified through empirical analysis of statistical stock returns.
3. The risk factor must be unpredictable or unforeseeable by the market, to allow it to become a risk factor.

Thus Berry et al., (1988) gave us a guideline as to the factors that could be included in the model, and which could be used as a standard for later research.

2. Research objectives and Importance

This research generally aims to extend the research made on the APT factors, adding knowledge and providing further insight as what factors should be considered, and the possible identification of these factors. Narrowing it
down the specific objectives will be achieve through:

2.1 The academic perspective
The theory of the APT remains a relatively new and lacking theory, and we stand to gain further knowledge of the theory through more empirical research on the subject. This is what the empirical research aims to achieve, by testing for the number and identification of factors that is vital to render the model useful. Almost no research has been performed across several markets during the same time period. This research perform tests across different continents; thus a representative market will be chosen from Africa, U.S Market, Asia, Europe and Australia, across the same time period. The idea is to empirically determine the number of factors that should be included in the model and if there are universal economic variables that affect the local stock market returns, regardless of where the exchange is taking place, and whether the factors will be consistent across the different markets.

2.2 The practical perspective
The APT remains a valuable risk model used by BIRR, Portfolio Analysis, Inc., a consulting company, founded in the late 1980’s and set up by the APT – theorists, and finance professors as Edwin Burmeister, Roger Ibbotson, Stephen Ross, and Richard Roll(BIRR), who use the APT as their primary risk model. However the evidence on the factors included in the APT remains inconclusive and lacking. Dhrymes et al., (1984) and Shanken (1982) criticize the model for being short of identifying the needed factors, and in this way they state that this renders the model non testable and thus invaluable. The research results of this paper will allow for a conclusion that could be used by portfolio managers and other professionals using the Arbitrage Pricing Theory as their method of asset pricing in the market.

3. Measuring APT
The statistical method employed in this research uses a method known as Principal Component Analysis (PCA) to determine the factors to be included in the APT. It identifies the factors, which are uncorrelated with each other, but can significantly explain the variability of return of the stocks.

Gehr (1978) was the first who applied factor analysis (a method very similar to principal component analysis) to the U.S stock market returns to test the APT for the number of factors. This was extended further by Roll and Ross (1980) who used factor analysis through conducting a three-part examination: the first test was of the ability of the APT to model return, they then examined the correlation between residuals, and finally they considered the difference between factor structures across groups of securities. Roll and Ross (1980) applied factor analysis to 42 groups of 30 securities listed on the New York Stock Exchange from the date 1962 to 1972. The results of their first pass test show that in over 38% of the groups, there was less than a 10% chance that a sixth factor was needed to explain variations in return, and in over three fourths of the groups, there was a 50% chance that five factors were sufficient to explain stock variations. They then tried several second pass regression tests, and their major findings were that there are at least three factors are significant in explaining the equilibrium prices. They proposed the following five-factor structure that could be chosen from a varied set of risk factors:

- change in industrial production (IP),
- change in expected inflation (EI),
- change in unanticipated inflation (UI),
- excess return on long-term corporate bonds over long-term government bonds (CG), and
- Excess return of long-term government bonds over T-bills (GB).

Roll and Ross(1980) acknowledge the weak nature of their tests, even if the data results seemed to support the APT. Though Roll and Ross (1980) accept that their tests may not be very strong, however, they are able to show that at least there are three factors needed to explain stock return, providing support for the multi-factor model as a method of pricing of assets, as opposed to the CAPM.

Cho et al., (1984), went on to develop on these tests and answer the logical question of whether the results achieved by Roll and Ross (1980) would be consistent with the CAPM, or whether there actually were additional factors that were needed to explain the market variation. They repeated the Roll and Ross (1980) test for a later time period, and their results indicate that more factors have significant influences. They then simulated a set of data using the zero- Beta form of CAPM, and they enforced the same means and variances on the returns for each stock that were present in the original data. They did the aforementioned step to allow the rate on the zero Beta portfolio and the Beta on each asset to change over time. When they applied the Roll and Ross (1980) tests to the data, they found that the number of factors is consistent with the zero Beta form of the CAPM.

The fact that more factors were significant when the actual returns were analyzed supports the theory of Roll and Ross (1980) that there are additional factors at play. And though this analysis suggests that there is more than just one factor, providing additional support for the APT, contrary to what was suggested by the CAPM. There still, however, remain questions about the ability to implement the APT using factor analysis, due to the lack of knowledge of what these additional factors maybe.
Trzcinka (1986) performed a statistical test using eigenvalues calculated from sequentially larger covariance matrices. He wanted to test the results already stated by Dhrymes et al. (1984) using a larger number of securities; 865 securities. The results he came up with support Dhrymes et al. (1984) conclusions for very large portfolios, that as we increase the number of securities the number of factors also increases. However, he also found that at least one eigenvalue was increasing without bound, indicating that the appropriate model may be a one factor model. And this maybe one of the few tests made that provided evidence in support of the CAPM vs. the APT.

A plausible explanation for the results achieved by (Dhrymes, and Gultekin, 1986; and Trzcinka, 1986) may be that the assertions of the APT hold firm for infinite numbers of stocks. When any statistical technique is applied to a finite numbers of stocks, the results are only approximations. What seems to be lacking in this direction is a robust and practical estimation of the accuracy of these approximations.

Factor analysis is a statistical testing technique that is used to test for the number of factors to be included in the APT model. One of the primary theories of the APT is that the mean squared error terms should converge to zero. This is because with a well-diversified portfolio one should expect that the industry specific risk component should sum up to zero.

However, Connor and Korajczyzk (1993) argue that given an infinite number of assets it can’t be shown that the mean squared error terms will converge to zero. Thus he argues against the testability of the APT through factor analysis, this argument runs similar to the argument by Roll (1977) against the testability of the CAPM. He further implies that if one of the factors from a sample is altered the eigenvectors and eigenvalues will change, and thus it is impossible to deduce the correct economic factors underlying factor analysis.

Factor analysis is only a technique that is used to uncover the economic factors by examining how assets covary together, and it should be held separate from the development of the APT. Though factor analysis can produce many different factor structures as the portfolios are changed, however the APT maybe designed more in the spirit of macroeconomic variable modeling (Chen, 1983).

4 The Macro-Economic Factor Model

The Macro-Economic Factor model uses exogenous factors such as inflation, interest rates, business cycle, oil prices, exchange rates … etc. as the risk factors that should be included in the APT equation. It allows for the selection of macro factors based on logical reasoning, and then sets out to test whether these factors would be significant or not. This section will allow us to view the macro variables selected by different researchers, and which were found to be significant when tested, this could provide us with insight when we try to identify the factors in the findings and analysis chapter.

Chen (1983) was the first to suggest that statistical factors could be interpreted as economic variables. The basic idea was, as previously identified in the introduction chapter, that investors are concerned with anything that would affect their returns; macro-economic factors affect the discount rate and the expected cash flows which in turn affect present value of returns achieved by investors.

Chen et al., (1986) proposed the use of the most probable macroeconomic variables that could explain the stock returns. They propose that one should use the most probable indexes on the basis of economic theory that should enter the return generating process. By reasoning that stocks should be affected by any influence that affects either future cash flows from holding a security or the value of these cash flows to investors.

Chen et al. (1986) look at pricing relative to a set of observable macroeconomic variables, or factors, they selected the variables primarily based on economic intuition. The strong intuition underlying their choice of factors is derived largely from several previously tested models.

Chen et al. (1986) went about trying to find primarily the relation between non-equity economic variables and stock returns. They have identified seven systematic or market influences including innovations in industrial production, expected inflation, risk premia, term structure of interest rates, selected market indices, changes in real consumption and oil prices. They then develop a linear model to test variables on the return of a chosen set of assets.

They find that industrial production, changes in risk premiums, and changes in the yield curve are significant in explaining the security returns. They also find that the two measures of inflation are significant though at a lower level. They conclude that the return on stocks is shaped in accordance with systematic economic news, and thus the model used above does indicate significant explanatory value that is consistent with the APT.

Thus one can view the factors as a multivariate substitution for the unobservable equilibrium benchmark suggested by the CAPM. They reasoned that these measures affect the cash flows that come from holding a stock or the value of these cash flows if they were discounted. Their results show that the Marco variables have significant explanatory influences on pricing. They also found that when they added the Beta of each portfolio with the market, it did not show up as significant in the second stage (cross-sectional) regression. Chen et al. (1986) could not conclude that they found the correct state variables for asset pricing, however they gave the direction in which further research could be completed. We will now move on to other studies that were made, analyzing the
different factors considered by researchers.

Up to now the macro-economic variable method has proved to be an acceptable method of asset pricing, using an integration of the statistical method and economic variable method allows the researchers to explain an adequate amount of variation in stock return. What could be noticed above is that though researchers considered different sets of factors, however they usually found that five factors would be enough to explain a sufficient amount of variation. Also we could notice that different factors were found to be significant in each study, and this could perhaps be because different markets are influenced by different factors, and the time period considered will also play a role in the significance of the factors.

Chen and Jordan (1993) test for the factors in the APT model using both factor loadings and macroeconomic variable models. They use the macro-economic factors that were stated by Chen et al. (1986) in their second approach. They don’t find any significant difference in the results between the two models. They acknowledge that there remains a lack of formal discipline in the selection of macroeconomic factors that should be included. However, they argue that the APT model does explain the variation in the stock’s expected return well using macroeconomic innovations, which can be used by investors and portfolio managers to provide an effective management for the different types of risk. They realize that greater computer power and software applications, and a reliable database of asset returns and risk factor realizations will render the APT model more favorable to use.

Shanken and Weinstein (2006), examined the relationship between expected return and measures of systematic or market risk, using the five macroeconomic factors that were studied by Chan et al. (1985) and Chen et al. (1986), took yet another approach to the asset pricing empirical work, by looking at pricing relative to a set of observable macroeconomic variables, that were chosen mainly based on intuition, this intuition was derived through studying the work of previous authors and previous models as Brown and Weinstein (1983), Connor and Korajczyk (1988) and Lehman and Modest (1988). They use the following model:

$$R_{pt} = \alpha_p + \beta_{sp}MP_t + \beta_{UPR}UPR_t + \beta_{UTS}UTS_t + \epsilon_{pt}$$

Where $R_{pt}$ stands for the excess return on size portfolio $p$ for month $t$, $MP$ stands for the percentage change in industrial production led by 1 month, $DEI$ stands for the change in expected inflation, $UI$ stands for the contemporaneous unanticipated inflation, $UPR$ stands for the excess return of low grade corporate bonds over long term government bonds, $UTS$ stands for the excess return of long term government bonds over T-bills with 1 month to maturity and finally the $\epsilon_{pt}$ values are assumed to be zero mean disturbances with constant covariance matrix conditional on the factors. The returns and factors are assumed to be independent and identically distributed over time. Using excess return allows the authors to change the zero beta rate and facilitates the imposition of some restrictions considered below, but will have little effect on the results.

Shanken and Weinstein (2006) apply the two pass technique like Chan et al. (1985) and Chen et al. (1986) and they group their securities into portfolios based on annual rankings of the market value of equity. Chan et al. (1985) and Chen et al. (1986) had employed versions of the two pass regression methodology developed by Fama and MacBeth (1973), and their results are impressive, suggesting that several of the factors are priced, that is the betas on these factors explain the cross sectional variation in the asset returns. However contrary to Chan et al. (1985) and Chen et al. (1986) who estimate the betas using backward looking returns relative to the ranking dates, Shanken and Weinstein (2006) use post ranking returns throughout, which offers them different conclusions to those reached by Chan et al. (1985) and Chen et al. (1986).

Their experimental design is comparable to that of Chan et al. (1985) and Chen et al. (1986), except that the industrial production factor is significantly priced in the overall period of 1958-1983. They find that the sample mean of bond return premium, which was a highly significant factor in previous studies, is negative and insignificant for the period they chose.

They also fail to find any evidence of factor pricing in the sub-period of 1968-1977, this is surprising as it is contrary to the t-statistics by Chen et al. (1986) which were greater than 2.5 for all five factors during the same period. They report that though the most positive results they obtained was for industrial production, they had reservations about factors. They argue that factors ought to capture innovations in the relevant state variables.

5. The Egyptian stock market and emerging markets

The Egyptian Stock Exchange, established in 1883, is the oldest stock market in Africa. It was one of the major stock markets until 1952 when some political changes caused the stock market activities to weaken and ultimately to cease. Thus the Egyptian stock market, like many emerging stock markets, remains relatively illiquid and is denominated by a few stocks. Furthermore, the availability of stock data remains to a large extent underdeveloped. In the only research made on the APT in the Egyptian stock market by Omran (2005), he applied the multivariate technique of the principal component analysis to the Egyptian stock market, using weekly volume of trade and number of transactions from the period of March 2001 to October 2001, on 41 of the most actively traded companies in the market. He used the varimax orthogonal rotation technique to identify the three primary components that could explain the variability of the stock returns in the aforementioned period.
In his results, the first factor that seemed to explain the variability was a weighted average of six companies in the telecommunications, media and construction industry, that seem to be the major driving force of the Egyptian stock market. The second factor was the activities in the development and housing sector companies. And finally the third factor was activities in the consumer staples, especially mills. He found that, together, these three factors could explain about 90% of the stock return variability in the market. However, he leaves it open for further research to assure whether these three factors were priced according to the framework of the arbitrage pricing theory.

There is another set of papers written on emerging markets such as the Egyptian market, and although they may not have used the APT model per say, yet these researches were studying the main principle of the APT, that is, they were studying the effect of macroeconomic variables on the stock market.

Most of the results achieved by the studies made on developed economies are consistent and agree that regardless of the techniques being used, and the economies being studied, there does seem to be a real effect of economic variables and in particular factors as interest rates, inflation, money supply, industrial production and exchange rates on the stock market. All of which were discussed above.

However, the results are not consistent when studying merging markets. Several group of studies including Chowdhury and Rahman (2004), Arnold and Vrugt (2006), Beltratti and Mornara (2006), Chowdhury et al. (2006), Corradi et al. (2006), Teresience et al. (2008), focus on how volatility and thus risk of the macro economy affect the stock market volatility.

The results they achieved for the emerging markets are inconsistent and mixed. This could be explained perhaps by considering that emerging markets are inefficient markets, information diffusion in such markets is not readily available, and thus these markets may be driven by different market sentiments and behavioral aspects, and this could show us a need for the APT to include a behavioral factor as well.

Coetzee (2002) performed a study on South Africa using quarterly data from 1991-2001 and found that there is a significantly negative relationship between monetary variables such as inflation, short-term interest rates, the random dollar exchange rate and stock prices both in the short run and the long run.

Moolman and Du Toit (2005) developed on the aforementioned study from the period 1993-2003, by breaking the factors into two groups; they established that discounted future dividends determine the long run behavior of the stock market, while short term interest rates, the Rand dollar exchange rate and the S&P 500 index determine the short run behavior of the market. Durodola (2006) found that both domestic factors as well as foreign GDP influence the long-run behavior of the South African stock market index.

Chowdhury and Rahman (2004) used a Vector Autoregressive and a seasonality adjusted forecasting model for Bangladesh and found that there is a one way influence from macroeconomic volatility to stock market volatility. Chowdhury et al. (2006) argue that the basic idea is that since there is a strong link between the macro-economy and the stock market, any shock in the macroeconomic variables should present a strong source of market risk, which should affect any market portfolio and cannot be diversified away.

Adam and Tweneboah (2008) studied the factors that affect the stock market return in Ghana; they found that contrary to what would be expected there was in fact a positive relationship between inflation and stock returns, and they explained that this was an indication that investors were being compensated for inflationary pressures.

6. Research methodology
This research applies the strategy of applied research. Applied research uses accumulated theories, knowledge methods and techniques used from previous research with the aim of meeting a specific purpose.

The strategy of applied research will be used in this study to nourish the expansion of knowledge, and aims at advancing fundamental knowledge about the real world. That is primarily what we’re dealing with; we try to see how the APT can be used to explain the returns of assets in the stock market. Through this paper we will use applied research with the aim at supporting the Arbitrage Pricing Theory, and enhancing the reader’s understanding of the relationship between the economic factors and the stock market return variation.

The strategy will allow for the generation of new ideas, and improvement on the theory made through research. These ideas and principles may not be put to immediate use, but will likely aid in forming the foundations of modern progress and development in the field of asset pricing. Further these extracted ideas and principles may be used to stimulate the thinking of everyday practitioners, as in our case they could be used by portfolio managers and investors, and will help them to revolutionize and improve how to deal with investment decisions.

Though any research includes an amount of quantitative and qualitative strategies, however this research uses a large amount of statistical techniques to analyze the data and answer the research question and thus it is mainly considered a quantitative research.

6.1 Sampling
The number of stocks listed in the New York Stock Exchange is 2,308, the number of listings in the Hong Kong Stock Exchange is 1,421, those listed in the Frankfurt Stock Exchange are 765, the number of stocks listed in the
Australia Stock Exchange is 2,221, finally the number of stocks listed in the Egyptian Stock Exchange is 212 (the previous data is obtained from Wikipedia). The preceding data gives us the population, out of the mentioned population 30 stocks will be chosen in each market as representatives of the respective market.

The reason for choosing 30 stocks was based primarily on the fact mentioned above that principal component analysis requires a very complex level of math and thus one can only study a limited number of securities at any point in time. There is also an aim at holding the number of stocks constant across the different markets, this is because (Drymes et al. 1984, 1985), found that the number of factors increases with the number of stocks in the sample; suggesting the number of factors to be around 13 or 17 for 90 stocks on basis of chi-square tests. This was in contrast to Brown and Weinstein (1983) who found the number of factors to be insensitive to the size of the sample. So to overcome this dilemma the sample size will be held constant across the markets.

Further the focus of this research is to find out whether the number of factors would be the same in the Egyptian market in correspondence with the international markets, and since the primary area of our focus stemmed from the Egyptian market so the EGX 30 will be taken as a good representative of a well-diversified market portfolio.

6.2 Data Collection
The data collected for this study is the daily closing prices of the most actively traded (as acquired from the volume level) stock across the different markets chosen from the period of January 2002 to December 2007. Brealey et al. (2006) determine that “five years of data is the recommended length of data to use in most financial analysis”, thus the choice for the period under study will be for five years. Choosing the research date to be before 2008 was intended, this is because of the stock market depression that occurred during 2008. Since different markets will be compared, it is important that the data not be distorted or affected by that depression, and rather be functioning in a normal environment, to render this comparison more reliable. Particularly this is also because after the Egyptian Stock market was just starting to recover from the shock of the 2008 depression, the country went through a revolution, bringing the stock market back down again, and rendering this period very unstable for use. The data for the daily stock return for all markets is obtained from Yahoo Finance, except for the Egyptian market, which is not available on Yahoo Finance, and thus was obtained directly from the Egyptian Stock Exchange office.

The stocks will be chosen according to the constituent stocks of the corresponding market index, the reason for that is that the market index by definition includes the most actively traded stocks, and is a representation of the whole market against which investors can benchmark the performance of their portfolios, and thus the constituent stocks should give us a good picture of the market taken as a whole (Christopher et al., 2010). However since some stocks within these indexes are lacking in data within the chosen period, these stocks will have to be replaced with other stocks outside of the index having high volumes of trade, and thus are also actively traded stock.

Thus the data collected in this research is the daily return for a five year period, on thirty stocks from the respective market, the stocks will be chosen mainly as stocks that are constituents of market indexes. The market index used for the Frankfurt Stock Exchange is DAX30, the index used for the New York Stock Exchange is the Dow Jones index, the index used for the Hong Kong Stock Exchange is Hang Seng Corporate Sustainability Index, the index used for the Australia Stock Exchange is ASX 20 and ASX 50, the index used for the Egyptian Stock Exchange is EGX30. Data on the constituents of each of the above mentioned portfolios are included in the Appendix.

6.3 Research Question and Hypothesis
What is the number and identity of factors that should be included in the Arbitrage Pricing Theory? And are they consistent from market to market?

In order to answer this question the following two hypothesis are formulated as follows:

$H_0 = \text{The ideal number of factors is 5 factor.}$

We have seen in several previous studies that the ideal number of factors found to be significant were five factors. Roll and Ross (1980) state that there was a 50% chance that five factors were sufficient to explain stock variations. Further in the economic variable model section several researchers including Brown and Weinstein (1983), Burmeister et al. (1986, 1987, 1988), and Shanken and Weinstein (2006) presumed a five factor model to study whether certain supposed macro factors were significant or not. Thus in the next section we will study the five markets to see whether the number of factors will come out to be five factors for all the markets.

$H_a = \text{The identity of factors are inconsistent from market to market.}$

The second hypothesis aims to test whether the factors will be found to be consistent across markets. Previous studies (Chamberlain and Rothschild 1982, and Chamberlain 1983) emphasize the factors are interchangeable as $B_1B_2' = B_2B_1'$, and as we can relate the loadings, thus it doesn’t matter what the choice of factors will be. In the following section we will test the markets and give the most probable identity of the factors, aiming to conclude whether the identity of factors will remain consistent or will change from market to market.

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7. Research analysis, findings and discussion

The Statistical Package for the Social Sciences (SPSS) will be used for the testing and analysis of the markets in this research. PCA will be applied to each portfolio, and the prominent risk factors that affect the level of return will be analyzed. In deciding on the number of factors to be retained in the analysis of each market 2 main criteria will be used. The first criterion, proposed by (Kaiser, 1960), is the greater than one eigenvalue criterion, and it is the default criteria used by statistical software SPSS. The Kaiser rule is described as follows. In a correlation matrix, since the variables are standardized, the mean eigenvalue is 1. Thus, any variable that displays an eigenvalue greater than 1.00 accounts for a greater amount of variance. This means that these components are accounting for a meaningful amount of variance and should, therefore, be retained. The Kaiser rule has the positive feature of being simple to use, as well as eliminating any subjective decisions. On the other hand, this criterion has also been criticized for often including too many factors than necessary.

This brings us to the second criterion empowered by the logic of analysis. A component is considered important if it has at least 3 variables with significant loadings on it. Further, as a rule of thumb, it was considered that any loading factor of value greater than 0.3 is important, though this maybe be arbitrary. Raubenheimer (2004) reflects that researchers particularly in the field of exploratory analysis will usually use a level of 0.4 for the central factor, and 0.25 for other factors. Thus the component loadings matrix, obtained through the extracted components chosen by the Kaiser rule will be viewed, and the factors eliminated until the remaining factors have at least three factors with corresponding loading factors greater than 0.3.

After deciding on the number of factors to retain in each market an analysis of the communalities table will be made. The communalities table shows the proportion of each variable’s variance that could be explained through the retained principal components. It is computed as the sum of the squares of the factor loadings for the given variable. It could also be defined as the squared factor loading and is denoted by \( h^2 \). The communality will always be 1.0 for a full orthogonal PCA, as all the variances are explained by all the variables. The extracted communality however -the column we care about for analysis- is the percentage of variance of a given variable that is explained through the retained factors. It thus always results in a coefficient less than 1.0.

Therefore the variables that are well represented through the chosen components or factors are those variables with high communality values. Usually a communality level of greater than 0.75 is considered high, while that lower than 0.25 is considered low. However some variables have high communalities but may be meaningless unless the factor on which the variable is loaded is interpretable. And thus what is more critical at this point is not the communality coefficient, but the extent to which this variable plays a role in the interpretation of the factor, and it is usual to find that the variables with higher communalities will often play greater roles in interpretation.

Following that the rotated factors will be analyzed to gain more insight into the analysis. Rotation serves a very important role in analysis because it makes the output more understandable and facilitates the interpretation of factors. Orthogonal rotation is used to preserve no correlation of the factors. In practice we use a method known as “Varimax Rotation”.

The Varimax rotation method maximizes the dispersion of loadings within factors, and thus it attempts to load a smaller number of variables into each factor so that factor clusters become easily interpretable. Varimax is considered the most common rotation option because it yields results that make it as easy as possible to identify each variable with a single factor, by investigating how groupings of items measure the same concept (Zhang et al., 2008; Ozbay et al., 2011). Usually the variables that load on a given factor share the same conceptual meaning, and are measuring a certain construct. The factor loadings from the Varimax rotation signify the contribution of each variable in a particular principal component (Ozbay et al., 2011).

Thus for the purposes of this research we have to figure out which variables load highly on each factor, and find out what these variables have in common, and what might be the logical explanation of that factor. The rotated solution matrix shows the factor loadings, or component loadings, and these are the correlation coefficients between the variables and factors. In the analysis the loadings will be broken down into 3 categories: any variable with a loading above 0.7 loads highly on the factor, any variable with loading of 0.6-0.7 loads highly moderate on the factor and finally any variable with a loading of 0.3-0.6 is considered to have a moderate loading on the factor.

Further the industry classification will be based on the Industry Classification Benchmark (ICB). This industry classification taxonomy was developed by Dow Jones and Financial Times & London Stock Exchange (FTSE), and is a definitive system used to classify over 70,000 companies and 75,000 securities to different sectors within the economy. Based on the ICB all companies are classified within a system of 10 industries. The system provides a supreme data source for global sector analysis, and is maintained by FTSE International limited.

Finally the extracted factors from principal component analysis will be used to form a regression line that could best explain the portfolio return this is known as principal component regression. Principal component regression (PCR) is used to establish a relationship between the output variable (\( y \)) which in our case is the return on the portfolio, and the extracted principal components of the input factors (\( P_1 \)), (Saufie et al., 2011).

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7.1 Frankfurt Stock Exchange:
Applying principal component analysis to the 30 stocks, and using the extraction method suggested by Kaiser, yielded an extraction of 6 components. However 2 of the 6 components had variables with very low loading on them. Thus these components were successively eliminated and 4 factors retained in accordance with the second criterion. Kan and Zhou (1999) used a method known as the "generalized method of moments", and found that the US stock market has a 4-factor structure, thus supporting the four factor structure found for this market. The results are also supported by the work of (Roll and Ross, 1980), who find that at the very least, three factors are needed to explain the portfolio return.

The 4 components in Figure 4.1 measure a cumulative variance of 47.701%, which is almost half of the total variance. The highest eigenvalue is reported by component 1 at 9.198, and subsequently it measures the largest portion of the variance at 30.661%, demonstrating that component 1 is greatly significant. This is followed by a drop to an eigenvalue of 2.646 for component 2, and it respectively explains 8.821% of the variance. Components 3 and 4 are almost equal; each explaining almost 4% of the remaining variance.

Next the communalities provides that the following stocks showed high communality values: Henkel, Fresenius Medical Care, Infineon and DT. Their communality values range approximately from 90 to 95% and that is the proportion of their variance being explained by the chosen components. This is considered quiet high. On the other hand, several stocks such as Deutsche Telekom, Aareal Bank, ADIDAS, Beiersdorf, and Fresenius have very low communality values of less than 20%, i.e. less than 20% of their variance could be explained through the chosen components, signifying that these stocks are less affected by the common factors and instead their variance is greatly derived from endogenous, firm specific factors.

The rotated component matrix shows that when the stocks of Factor 1 are broken down into industries we find the following:

• First, there are eight stocks that load highly on Factor 1. Four of these stocks are from the financial sector, two are from the consumer goods sector, although they fall under the super sector of the automotive industry, one stock is from the technology sector, one stock is from the conglomerate sector, and one stock is from the utilities sector.

• Second, there are seven stocks that load highly moderate on the same factor. From these stocks two are from the Industrial sector, two from the consumer services sector, one from the technology sector, one from the utilities sector and one from the conglomerate sector.

• Finally, we have two stocks that load only moderately on Factor 1 and both are from the industrial sector.

First and foremost we will analyze all of the above industries giving particular interest to those with high loadings. Claessens (2003), states that the most possible factors affecting the financial sector are economic growth rate as measured by GDP, and inflation. While (Cheng et al., 2011) note that the most probable factors affecting the electronics industry is industrial production as measured by GDP, money supply and exchange rate. In a recent research issued by Rosenthal (2011), it was also noted that the major factor affecting the energy industry is general economic growth rate. The same results were found for the automotive industry. If we look closely at what all these industries have in common is the economic growth rate, this is generally one of the most prominent macro-
factor affecting all sector performance (Lipsey and Chrystal, 2004). It is also one of the most reoccurring factors found significant in the literature review. Thus we are likely to believe that the first macro factor common to them would be “economic growth rate” as measured by the change in GDP. It is important here to note that economic growth rate and industrial production are essentially the same, as economic growth rate is often measured through increase in the industrial production rate (Saloman et al, 1989).

For Factor 2 we find that:

- Two stocks load highly on Factor 2; one being from the health care sector and the other being from the consumer goods sector selling personal care and home care products.
- Two stocks also load moderately though negatively on Factor 2, one being from the industrial sector and the other being from the technology sector.

If we analyze the stocks that load highly positively on Factor 2, they both come from industries that are more or less stable industries with lower levels of risk. On the other hand if we look at the two stocks that load negatively, we see that one comes from the technology and the other from the industrial sectors. Both of these are highly changing industries, and are prone to higher levels of risk (Drefyus, 2011 and Icon, 2010). Thus, maybe Factor 2 could involve the risk preference of investors, or it is a measure of the “risk premia”. This would make sense as the risk premium was found one of the significant variables, when tested through the economic variable model by several researchers discussed in the literature (Chen et al., 1986; Brown and Weinstein, 1983).

For Factor 3 we find that:

- Two stocks load highly on Factor 3, one form the industrial and one from the technology sector.
- Two stocks load moderately, though again negatively, and are from the financial sector.

We could consider Factor 3 as the level of “Exchange rate”. This is because depreciation of the local currency vs. appreciation for a market like the German market, which exports more than imports (Deutsche Bundersbank, 2010), would mean the German products would become cheaper and this would positively affect the industrial and technology, as it increases their level of trade. On the other hand depreciation in the currency poses currency risk that would negatively affect the financial sector (Tambakis, 2003). Thus my choice for the third factor would be the “exchange rate”.

Finally for Factor 4 we find that:

- Seven stocks load only moderately on Factor 4 and are from several industries. Two are conglomerate companies; one stock is from the telecommunications, one stock is from the financial sector, one from the industrial sector, one from the consumer goods sector, and one from the basic materials sector.
- There is only one stock that loads moderately but negatively on Factor 4 and it comes from the consumer goods industry.

Factor 4 has to be a factor that positively affects a variety of industries at the same time. Thus it is likely that Factor 4 could be the “per capita income”, an increase in the level of income would logically positively affect all industries, as it means an increase in the ability to invest in these industries. On the other hand, an increase in the level of income would negatively affect the inferior goods, and this may explain the negative effect we find on one of the stocks from the consumer goods industry (Lipsey and Chrystal, 2004).

The following model is the most probable model achieved and could be used to explain the return on the portfolio:

\[ R_p = 0.001 + 0.765(F_1) + 0.39(F_2) + 0.264(F_3) + 0.217(F_4) \]

Where in accordance with the arbitrage pricing theory and as previously identified;

- 0.001 would be the expected return given that all other betas are zero.
- Factor 1 represents changes in Economic Growth Rate and its associated beta is 0.765.
- Factor 2 represents changes in the Risk Premia and has an associated beta of 0.39.
- Factor 3 represents the changes in the Exchange Rate with an associated beta of 0.264.
- Finally Factor 4 represents the change in the Per Capita Income and has an associated beta of 0.217.

### 7.2 New York Stock Exchange

If we look at the New York stock market, the number of components that should be retained in accordance with the Kaiser rule are 5 components. The 5 components had more than 3 variables that were highly loaded on them and thus none of the factors needed to be eliminated.
Figure 2
Extraction of the Sum of Squared Loadings for the New York Stock Exchange

The five factors together explain a total amount of variance of 54%. Again logically the first component explains the largest amount of variance, this time explaining approximately 38% of the variance, which is a pretty significant amount, and it has a corresponding eigenvalue of 11.338. After that we notice a significant drop with component 2 and 3 explaining approximately 5% of variance and having an approximate eigenvalue of 1.5. This is closely followed by components 4 and 5, which explain an average approximate variance of 3.5%, with respective eigenvalues of approximately 1.0. If we recall from the literature review that in one of the earliest tests performed by (Roll and Ross, 1980) on the New York Stock Exchange, they also found that there was a 50% chance that five factors were sufficient to explain stock variations, supporting the results achieved in this research.

The communalities table provides that the highest communalities are given by Chevron Corp., Exxon Mobil, AT&T, and Verizon Communication, where the variance explained by the 5 components varies from approximately 75 to 85%. None of the stocks show very low communality values, perhaps the lowest is that achieved by Kraft Foods and McDonalds Corp. at 22.6% and 28.7% respectively. The rest of the stocks having communality values higher than 30%.

The rotated component matrix of Factor 1 provides that:
- Two stocks load highly moderate on Factor 1. One stock is from the financial sector and the other stock is from the technology sectors.
- On the other hand we have 18 stocks loading moderately on Factor 1 and come from various industries. 5 are from the consumer services industry, 3 from the financial industry, 3 from the consumer goods industry, 2 from the technology industry, 2 from the basic materials industry, 2 from the conglomerate industry and 1 stock being from the industrial sector.

The first factor here has loadings from all of the sectors in the economy, and though the majority of the stocks are from the consumer service industry, however we still cannot say that this is the sector that dominates the loadings on factor one. So factor one has to be the most prominent factor that affects all of the economy, as discussed in the Frankfurt Stock Exchange this factor would be the change in “economic growth rate” (Lipsey and Chrystal, 2004). This is in addition that if we go back to the literature review and look at the most reoccurring factor, industrial production used to measure economic growth rate was found to be significant in the majority of the research (Chen et al. 1986 and Roll and Ross 1980).

For Factor 2 we find that:
- Two stocks load highly on Factor 2 and are both from the technology sector.
- Three stocks load highly moderate and are again all 3 from the technology sector.
- Four stocks load moderately. 2 from the consumer services sector, 1 from the basic materials, and 1 stock is a conglomerate.

Factor 2 on the other hand is dominated by the technological sector, with five factors loading highly and highly moderately from the technology sector, while the other four factors from various other sectors only load moderately. Logically and from previous research it is known that the largest factor affecting the technology industry is the amount of Research and Development (R&D) (Albert et al. 2004, Hung-Yi Wu et al. 2011). This factor would also be applicable to the other stocks, as one of the stocks is involved in the health care sector.
under consumer services super industry, and of course R&D is a necessity in the health care industry. The other stock is Walt Disney, which is an entertainment conglomerate and constantly needs R&D to develop itself. The stock from the conglomerate industry is General Electric which is a company specialized in energy, technology, infrastructure, capital finance and consumer & industrial products, and thus R&D here would also play a dominant role. And thus the level of “R&D (Research and Development)” would be the choice for Factor 2.

For Factor 3 we find that:
- One factor loads highly on Factor 3 and is from the healthcare sector.
- Two factors load highly moderate on the 3rd factor and are again from the healthcare sector.
- Two factors load moderately, and are both from the consumer goods sector.

The most prominent factor affecting the healthcare sector as well as the consumer goods sector would be the “Per Capita Income”, since one of the most important factors affecting the healthcare industry is the amount of financing available that affects the ability of the people to achieve healthcare and to buy health insurance (Ensor and Cooper, 2004). The level of income available would also logically affect the ability of people to consume the goods (Lipsey and Chrystal, 2004), and thus would be the choice for factor three.

For Factor 4 we find that:
- Two stocks load highly on the 3rd factor and are both from the Oil and Gas industry.
- Two stocks load moderately on the same factor one being from the basic materials industry and the other being a conglomerate.

Factor 4 seems to be dominated by the oil and gas industry, and also the basic materials industry which is a closely linked industry. Whether the basic materials sector or the oil and gas sector are considered, it is noticeable, that they are both sectors that are involved in the discovery, development and processing of raw materials. Since these sectors supply materials necessary for construction, it is greatly sensitive to change in the business cycle, as it depends on a strong economy (Investopedia, 2012), so factor four would likely be considered as changes in the “business cycle”. Salmon et al. (1989), include the business cycle as one of the significant factors in explaining the variation in their asset pricing model.

Finally for Factor 5 we find that:
- Two factors load highly on the factor and are both from the telecommunications industry.
- Four factors load moderately and are all from the financial sector.

Factor 5 is denominated primarily by the telecommunication and financial industry. Since the factors that load highly are from the telecommunications sector, thus the analysis of this factor will be based primarily on the telecommunications sector. Weingarten and Benito-Martín (1994), in a study they made on the telecommunications sector in the U.S market argue that the rate of technological change should greatly affect this sector as it deals with the methodology that deals with new services like wireless, Internet and video telephony, and that this would be tremendously important to future industry structural evolution. Thus Factor 5 would likely be the rate of “technological development”.

Following we will look at the regression model that best explains the portfolio returns for the New York Stock Market from the extracted five factors:

\[ R_p = 0.001 + 0.654(F_1) + 0.499(F_2) + 0.350(F_3) + 0.324(F_4) + 0.305(F_5) \]

Where, as previously identified and in accordance with the arbitrage pricing theory;
- 0.001 would be the expected return given that all other betas are zero.
- Factor 1 stands for changes in Economic Growth Rate and its associated beta is 0.654.
- Factor 2 stands for changes in the Research and Development and has an associated beta of 0.4999.
- Factor 3 is the changes in the Per Capita Income with an associated beta of 0.35.
- Factor 4 standings for the changes in the Business Cycle and has associated beta of 0.324.
- Finally Factor 5 is the change in the levels of Technological development and has an associated beta of 0.305.

7.3 Australian Securities Exchange

For the Australian Securities Exchange market the number of components that are extracted in accordance with the Kaiser rule are 5 components, all of the 5 components have more than 3 variables with high loadings on them and thus are all retained. This result is the same as the result achieved for the New York Stock Exchange and supporting literature discussed above (Roll and Ross, 1980 and Brown and Weinstein, 1983).
Here the 5 components together explain a lower cumulative percentage than the above markets, explaining only 40.544% of the variance. This drop in percentage seems to be due to a decrease in the percentage of variance explained by the first component, as it explains a total variance of 23.592% and has a respective eigenvalue of 7.077, which is the lowest in comparison to the above analyzed markets. Component 2 has an eigenvalue of approximately 1.7 and explains a portion of the variance equivalent to 5.786%. Finally Components 3, 4 and 5 all have eigenvalues of approximately 1.1 and explain from 3.5% to 4% of the variance.

As is expected, since the 5 components explain a lower cumulative percentage of variance, thus it would logically mean that the communality values would be lower as well. The highest communality value is given by BHP Biliton and RIO TINTO Ltd. both having approximately 70 to 75% of their variance being explained by the 5 components. At the other hand even though AMP Ltd. has the lowest communality value at 19.1%, we have 5 other stocks whose communality values range within the 20’s.

The rotated component matrix for Factor 1 provides the following results:

- Two stocks load highly on Factor 1 and are both from the basic materials sector.
- Three stocks load highly moderate, 2 again from the basic materials and the other from the oil and gas industry, which is a closely related industry.
- Four stocks load moderately on the 1st Factor. 2 are again from the basic materials sector, 1 is from the oil and gas sector as well, and finally 1 stock is from the financial sector.

The most prominent sector that is affected by Factor 1 is the basic materials sector along with other sectors. As mentioned above in the New York Stock Exchange market, when Factor 4 was being analyzed, it was argued that this sector is greatly sensitive to change in the business cycle, as it depends on a strong economy. This factor would also logically highly affect all sectors in the economy (Lipsey and Chrystal, 2004). So I’m likely to consider Factor 1 for the Australian market to be the “business cycle”.

For Factor 2 we find that:

- Two stocks load highly on the 2nd factor and are both from the financial sector.
- One stocks loads highly moderate and is again from the financial sector.
- Seven load moderately. The majority again at 4 stocks coming from the financial sector, 2 from the consumer goods sector, 1 from the basic materials sector, and 1 from the retail sector.

We find that the majority of the stocks that load on Factor 2 whether highly, highly moderate, or moderately, are dominated by the financial sector. These stocks are mainly involved in providing banking services within the financial sector. This sector is highly impacted by the level of interest rates (Gert, 2010). Where he argues that “while the effects of changes in interest rates and their structure on financial institutions differ, recent changes in asset and funding structures of banks make them generally more vulnerable to a changing interest rate environment”

Thus changes in the level of “interest rates” would be the choice for Factor 2.

For Factor 3 we find that:

- Only eight stocks load moderately on the 3rd factor. 3 are from the industrial sector, 2 are from the consumer goods sector, 1 from the basic materials sector, 1 from the healthcare sector and finally the last
being from the financial sector.

The identification of Factor 3 will be made alongside Factor 4 for the similarity of their analysis.

For Factor 4 we find that:

- Again only nine stocks load moderately on Factor 4. 4 of these stocks come from the financial sector, 2 from the oil and gas sector, 1 from the telecommunications sector, 1 from the healthcare sector and 1 from the consumer goods sectors.

For Factors 3 and 4 we can see that all the stocks that load on both factors have only moderate loadings, and are stocks that come from various industries. Thus unlike the other factors they don’t seem to be dominated by a single industry, and rather are factors that are common to the economy as a whole. When deciding on these factors, it would be logical to go back to the literature study, and find out what the predominant factors are, and thus we’d get a labeling for Factors 3 and 4.

From the literature review the results show that the most prominent factor affecting any economy is the industrial production rate, used to measure economic growth rate, and was found to highly significant in several studies (Roll and Ross, 1980; Chen et al., 1986; Shanken and Weinstein, 2006), and thus we are likely to conclude that “economic growth rate” this as our label for Factor 3. This is followed by changes in the “inflation” and thus would be the labeling for both Factor 3 and 4. Inflation is also one of the most studied macro-economic factors and though the sign was not always consistent, majority of the cases found that there was a positive relationship between inflation and stock returns (Salmon et al, 1989; and Adam and Tweneboah, 2008).

Finally for Factor 5 we notice that:

- Two stocks load moderately but negatively on the 5th factor, one being from the healthcare industry and the other stock from the industrial sector.
- On the other hand one stock loads moderately but positively on the 5th factor and is from the consumer services industry.

Since Factor 5 has a negative influence on two sectors, the likely identity of the factor would be “unanticipated inflation”. It would be necessary here to differentiate between the inflation mentioned in Factor 4 and the one mentioned for this factor. There are two measures that could be used for inflation, as performed by (Chen et al., 1986). In Factor 4 we are measuring changes in expected inflation, as put by Chen et al. (1986) that there is always a chance that future inflation predictions could be influenced by other economic forces, and it would make sense that investors would expect to earn more return given a higher level of expected inflation. However, for the unanticipated inflation mentioned here we find that this negatively impacts both the level of the discount rate and the size of future cash flows, and since it was unanticipated investors would not demand a higher level of return. Chen et al. (1986) states:

“Since changes in inflation have the general effect of shifting wealth among investors, there is no strong a priori presumption that would sign the risk premia for UI or DEI, but the negative signs on the premia for these variables probably mean that stock market assets are generally perceived to be hedges against the adverse influence on other assets that are, presumably, relatively more fixed in nominal terms”

Further if we look at the regression model presented below, we will find that Factor 5 has an associate beta of -0.096, that is an increase in Factor 5 would decrease the return on the portfolio and this result would be similar to the model tested by (Burmeister et al., 1994).

The regression model that best describes the market is as follows:

$$ R_p = 0.001 + 0.554(F_1) + 0.465(F_2) + 0.554(F_3) + 0.3361(F_4) - 0.096(F_5) $$

Where 0.001 is the constant return one would get assuming all the other factors are 0.

Factor 1 stands for changes in the phase of the business cycle and has an associated beta of 0.554.

Factor 2 stands for changes in the level of interest rates and has an associated beta of 0.465.

Factor 3 is the changes in the economic growth rate with an associated beta of 0.554.

Factor 4 is the change in the expected inflation rate and has an associated beta of 0.3361.

Factor 5 is the change in the rate of unanticipated inflation with a negatively associated beta where each unit increase in inflation should decrease the return on the portfolio by 0.096.

7.4 Hong Kong Stock Exchange

Now we will have a look at the results achieved when we apply the calculations to Hong Kong market.
As with the Frankfurt Stock Exchange Market, we find that the number of components that could be considered significant is four factors. The four factors explain a total variance of approximately 51%, and again as with the Frankfurt Stock Exchange market, the first component compromises most of the variance at 36.873% and has a corresponding eigenvalue of 11.062. This is followed by component 2 which explains a lower amount of variance at 6.1% with a corresponding eigenvalue of 1.831. Component 3 and 4 also explain approximately the same amount of variance at almost 4%, and have eigenvalues of 1.285 and 1.123 respectively.

The communalities table shows that the amount of variance that are explained by the 4 components for each stock are lower than that achieved for the Frankfurt Stock Exchange market. The highest communality values are achieved by the Cheung Kong Holdings Ltd., Henderson Land Development Co, and Sun Hung Kai Properties where they explain about 70 – 76% of the variance. On the other hand only Galaxy Entertainment Group has a low communality value bellow than 20%. Thus most of the stock’s variance is explained by the common space at an average level.

The rotated component matrix for 1 provides the following:

- Four variables load highly on Factor 1 and are all from the financial sector.
- Four variables loads highly moderate on Factor 1 and are all conglomerates.
- Finally twelve variables load moderately on the factor. 6 are from the financial sector, 2 are conglomerates, 2 are from the consumer services sector, 1 is from the utilities sector, and 1 from the consumer goods

What is noticeable is that the 4 stocks that load highly on Factor 1 are involved in property development, which is part of the real estate financial sector. Again the 4 stocks that load highly moderate on Factor 1 are conglomerates that are all involved in property development as well as having business in other sectors. We find a variety of sectors that load moderately, but the majority of the stocks come from the financial sector as well. Thus the conclusions are likely to be based on the factor that it is one that has the largest impact on the financial sector and in particular to the property development sector as well as an impact on other sectors as well. Bo (2010) made a study on the Hong Kong market and found that the interest rates had a significant effect on the property development market.

The first factor could most probably be “interest rates”. This is because if we look closely at the conglomerates we will find that other than providing financial services, they are mainly involved also in infrastructure development, telecommunications and the utilities sector. Increased interest rates would affect all of the above. We have previously discussed the impact that changes in the level of interest rates have on the financial sector, when analyzing Factor 2 in the Australian market, referring in our analysis to (Gert, 2010).

For Factor 2 we find that:

- Six stocks load highly moderate on Factor 2. 3 being from the financial sector, 1 from the industrial sector, 1 from the oil and gas sector and 1 company being a conglomerate.
- Eight stocks load moderately on Factor 2. 4 are from the financial sector, 2 are from the consumer services sector, 2 are conglomerate companies, and 1 stock from the consumer goods sector.

Again for Factor 2 we find that the majority of the stocks whether loading highly moderate, or moderately come from the financial sector. If we look at the other sectors involved which are the oil and gas sector or the consumer
services or the consumer goods sector, we would notice that all of these sectors are greatly impacted by any increase in the money supply (Antoniou et al., 1998). And thus we are likely to believe that Factor 2 would be “money supply”.

For Factor 3 we find that:
- Two stocks load highly on the 3rd factor both being from the utilities sector.
- One stock loads highly moderate and is again from the utilities sector.
- Two stocks load moderately. 1 from the financial and the other being a conglomerate.

Here we find that Factor 3 could be explained through the utilities sector. The utility industry across various countries is usually a public industry that is controlled mainly by the government, (Christopher, 2007). Thus we should logically find that the largest factor that could impact the utilities sector is the government expenditure as could be measured through the fiscal policy of the government. The choice for Factor 3 would be “government expenditure” as it addresses the use of government spending and taxation to influence the economy (Lipsey and Chrystal, 2004).

Finally for Factor 4 we find that:
- One stock loads highly on the 4th factor and is from the financial sector.
- Two stocks load highly moderate. 1 is from the financial sector and the other is a conglomerate.
- Two stocks load moderately. 1 stock again from the financial sector and the other stock from the consumer services sector.

The financial sector and property development seem to be playing a great role across the different factors for the Hong Kong market. Thus following the same logic used by (Omran, 2005), in his research, where he was able to explain the factors in terms of performance of certain industries, building on that logic and considering that the market index is considered as one of the significant factors by (Salomon et al, 1989), we could possibly form an index of the financial sector stocks within the Hong Kong Stock Exchange to represent Factor 4, “financial index”.

Following we will look at the regression model that best explains the portfolio returns from the extracted factors:

\[ R_p = 0.002 + 0.67(F_1) + 0.584(F_2) + 0.319(F_3) + 0.316(F_4) \]

Where 0.002 is the constant return one would get assuming all the other factors are 0.

Factor 1 represents changes in the level of interest rates and its associated beta is 0.67.

Factor 2 represents changes in the level of money supply and has an associated beta of 0.584.

Factor 3 is the changes in the government expenditure as measured through the change in the government’s fiscal policy, with an associated beta of 0.319.

Factor 4 is the change in the phase of the financial index and has an associated beta of 0.316.

7.5 Egyptian Stock Exchange

What we notice under the Egyptian market is that when the Kaiser rule is applied we find that the number of factors to be retained is 8 factors, and this by far was the largest number of factors throughout the markets. Using the second rule, discussed earlier, of having more than 3 variables load highly on each component forces us to rule out 2 components, and thus the remaining number of significant components is 6.
Figure 5
Extraction of the Sum of Squared Loadings for the Egyptian Stock Exchange:

The retained 6 components explain a cumulative variance of 46.967%, and this is acceptable as we find that the number of retained components throughout the chosen markets explain from 40% to 50% of the total variance. The first component here is a very significant component as it explains approximately 25.7% of the variance, and has an eigenvalue of 7.71. This is followed by a significant drop in eigenvalues and variance explained. Component 2 has an eigenvalue of 1.511 and explains 5.038%, component 3, 4, 5, and 6 are roughly similar with eigenvalues of 1.323, 1.260, 1.193 and 1.090 respectively all explaining almost 4% of the variance. However the individual t-test results for all factors are rather large, except for factor 6, the t-test result for factor 6 comes up to be 2.51, and although this is still higher than boundary given by $-2.364 < t < 2.364$ however it is still comparatively very small, which could show us that factor 6 is a trivial factor and perhaps should not be retained, leaving us with five factors retained.

The communality levels provide that the variables that are greatly affected by the extracted factors are El Qahera Housing and United Housing and Development, both with communality levels of almost 70%, it is interesting to note that both of these variables come from the same sector. The rest of the communality levels are moderate with the lowest being Orascom Construction Industries and Extracted Oil, at almost 25-27%.

The rotated component matrix of Factor 1 when analyzed provides:
- There are no stocks that load highly on Factor 1, and only one stock that loads moderately high and that is from the consumer goods industry.
- There are twelve stocks that load moderately on the same factor. The industry divisions of those twelve stocks areas follows: 4 from the financial sector, 4 from the basic materials sector, 2 from the industrial sector, 1 from the technological sector and 1 from the telecommunications sector.

The identification of Factor 1 and Factor 2 will be made simultaneously, thus we must first look at the decomposition of Factor 2:
- As with Factor 1 there are no factors that load highly on Factor 2 as well. And again there is just one stock that loads moderately high on factor 2 and it comes from the consumer goods sector.
- However, here we have thirteen stocks that load moderately on Factor 2. 4 from the financial sector, 4 from the basic materials sector, 2 from the industrial sector, 2 from the consumer goods sector, and 1 from the technological sector.

What we could notice is that Factor 1 and 2 are very similar to Factors 3 and 4 in the Australian stock market. Both the Factors 1 and 2 have all the stocks loading highly moderate or moderately on them, and are stocks that come from various industries, and thus again here the component identification is not dominated by a single industry, and rather they both must be factors that are common to the economy as a whole. Consequently, to identify these factors the identification made in the Australian market will be used, and which stemmed from previous literature and conclude that the identity of factor 1 is "economic growth rate", while the identity of component 2 is changes in the "expected inflation".

As for the breakdown of Factor 3:
- We have two stocks that load moderately high on Factor 3 and both are from the financial sector.
- We have seven stocks loading moderately on Factor 3, 6 load positively and 1 loads negatively and is...
Again what is noticeable in this case and as with the previous markets, the identity of Factor 3 is not denominated by a single sector, and it is rather widely stemmed from all sectors. However since one of the stocks is negatively affected by this factor and using the same logic argued by in the Frankfurt stock exchange when analyzing Factor 4, we are compelled to assume that the most probable identity of this factor is the change in “Per Capita Income” (Lipsey and Chrystal, 2004).

Next we will move on to analyzing Factor 4:
- We have one stock loading highly on Factor 4 from the telecommunication sector.
- Another stock loading moderately high on Factor 4 and is again from the telecommunications sector.
- And finally one stock loading moderately on Factor 4 and is from the financial sector.

Since Factor 4 seems to be dominated largely by the telecommunications sector, thus the identity of Factor 4 should stem from the factor that most affects the telecommunications sector. This factor would be the rate of “technological development”. The argument for this factor is presented in Factor 5 in the U.S market, and supported by (Weingarten and Benito-Martin, 1994).

Subsequently looking at Factor 5 we find the following:
- One stock loads highly on Factor 5 and is from the financial sector.
- There are no stocks loading highly moderate on Factor 5, and 3 stocks that load moderately on the aforementioned factor. 1 from the financial sector, 1 from the basic materials sector, and 1 from the consumer goods sector.

Since the variable that loads highly on Factor 5 is from the financial sector and following the same logic used above in the Australian Stock exchange referring to (Gert, 2010), we could assume that the identity of Factor 5 would be the change in the levels of “interest rate”.

Following is the regression model that best explains the return on the stock market portfolio:

\[ R_p = 0.002 + 0.525(F_1) + 0.549(F_2) + 0.404(F_3) + 0.310(F_4) + 0.367(F_5) \]

Where 0.002 is the constant return one would get assuming all the other factors are 0.

Factor 1 is changes in the level of economic growth rate and its associated beta is 0.525.

Factor 2 is changes in the level of expected inflation and has an associated beta of 0.549.

Factor 3 represents the changes in the per capita income with an associated beta of 0.404.

Factor 4 represents unexpected changes in the technological development and has an associated beta of 0.310.

Factor 5 is the change in the changes in interest rate and has an associated beta of 0.367.

In the above section the various markets were analyzed to identify the number of components significant in each market and try to figure out their identity. Five markets were studied each as a representative of the different continents. Three of the five markets indicate the significant number of factors to be five, while the other two markets indicated the significant number of factors to be four. Thus we are likely to base our conclusion that though the number of factors may vary slightly, the larger probability shows the ideal number of factors to be five, and this would be in line with majority of the results found in the literature review (Roll and Ross, 1980; Burmeister et al., 1986, 1988). The identification of the factors differed from market to market, in accordance with the stocks that were loaded on each component. However, the most reoccurring factor identified for all the market was the “economic growth rate” and the phase of the “business cycle”, which is also in line with the previous literature (Saloman et al., 1989; Lipsey and Chrystal, 2004). Finally the most probable APT model that could be considered, as presented through principal component regression were provided in the final portion of each subsection, with the most significant APT model being presented in the New York Stock Exchange.

8. Conclusion, limitations and recommendations
The Arbitrage Pricing Theory, one of the most popular asset pricing models, is a multi-linear model that tries to explain asset return in terms of a set of exogenous factors. However, the model comes with a paramount limitation; the inability of the model to identify the factors that should be included. Without knowledge of those factors, several researchers have criticized that this renders the model impractical for use.

The specific objectives intended in this research were approached in two main directions, an empirical direction, and a theoretical direction; aiming to detect the number of factors (H1) and discover whether the number of the factors themselves would be consistent from market to market or would change, aiming also at figuring out the identity of risk factors that should be included in the Arbitrage Pricing Theory, and whether the identification would be distinctive or not (H2).

The methodology of “principal component analysis” was applied to 30 of the most actively traded stocks across several representative markets during the years 2003-2007. Further “Varimax orthogonal rotation technique” was used to uncover the most probable identity of these factors. Finally, principal component regression was used
to present us with the model in accordance with the extracted and identified factors.

Though most of the previous researches conducted aren’t all settled on the same result, however, the majority of the research presented in the literature had pointed towards the probability that the correct number of factors that should be included in the Arbitrage Pricing Theory is five factors (Roll and Ross, 1980). Addressing also that the number of factors is not necessarily consistent with different sample size, and their identity would unlikely to be consistent across different time periods (Shanken, 1982, 1985).

The objectives of this research were made under an empirical direction: a test was performed on several representative markets, across different continents. Regarding the number of factors the results achieved in this research show that the number of these components varied from four to five components across the different markets. This particular result was one of the primary analysis points in this research and since three of five markets gave us a requirement of five components, thus the results achieved are in line with most of the results achieved from the literature. Several empirical studies have been conducted on the number of factors that should be included (Roll and Ross, 1980; Chen et al., 1986; Berry et al., 1988), concluding mainly that five variables are sufficient to describe the security returns. The four factor structure was also supported by (Kan and Zhou, 1999). The results also showed that the market portfolio that is best explained by the extracted factors is the New York Stock Exchange, as revealed by the regression results.

The research’s results lead us to conclude that portfolio managers and other professionals using the Arbitrage Pricing Theory as their method of asset pricing should possibly consider a four to five factor model. This partially supports the first hypothesis, concluding the five factors would be sufficient to explain a large amount of the variation in stock return.

In regards to the second hypothesis, it is also supported, with the identity of the factors being inconsistent as shown in the table below.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Frankfurt Stock Exchange</th>
<th>New York Stock Exchange</th>
<th>Australian Securities Exchange</th>
<th>Hong Kong Stock Exchange</th>
<th>Egyptian Stock Exchange</th>
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</thead>
<tbody>
<tr>
<td>Business Cycle</td>
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<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economic Growth Rate</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange Rate</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected Inflation</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Index</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
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</tr>
<tr>
<td>Government Expenditure</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Per Capita Income</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest Rates</td>
<td></td>
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<tr>
<td>Money Supply</td>
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<td>X</td>
<td></td>
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<tr>
<td>Research and Development</td>
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<td>X</td>
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<td>Risk Premia</td>
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<td>X</td>
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<tr>
<td>Technological Development</td>
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<tr>
<td>Unanticipated Inflation</td>
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<td>X</td>
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</tr>
</tbody>
</table>

The identity of the factors would have to depend on an exhaustive and in depth study of the market, and the economic conditions that are prevalent during that time. Perhaps professionals could use Varimax orthogonal rotation to help them identify the factors, as proposed in this research, then use the identified factors as the pre-specified factors in an economic factor model and test for their significance, swapping factors as they find appropriate.

The econometric techniques needed to identify the exact factor structure in security returns remains insufficiently developed, and this is particularly with respect to incorporating conditioning information. The identity of the exact factors that could be related to sources of economic risk are not yet clearly understood. There remains a requirement of additional research in the asset pricing theory, macroeconomics, and econometrics to understand the relationship between return factors and economic risk. Ross’s (1976) addition created a fundamental building block in the asset pricing theory, however the model needs to further evolve to become
useful. Recommendation for further research could involve increase in the time period, or increasing the sample size, to see whether the number of factors would remain consistent for each market, or would increase with increase in either cases. Another recommendation would be a check of robustness, studying the period after 2008, to analyze what would happen to the factors and whether they would remain consistent across the time period, it would be interesting to note the effect of the depression and revolution on the factors in the APT. Particularly due to globalization the depression had an effect on several markets in a similar way, thus we may want to see whether these factors become universal. Finally a study could be made of comparison of several markets across the MENA region, especially considering that most of the stock exchanges in the MENA region are emerging, which could give us greater insight on the emerging markets.

References


