Optimal Triangular Partially Balanced Incomplete Block Designs for Complete Diallel Cross

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Abstract

It is well known that complete diallel cross system IV and triangular partially balanced incomplete block designs have one to one correspondence. In the present paper, we have identified some triangular partially balanced incomplete block designs which generates two types of optimal incomplete block designs for complete diallel cross system IV.(i)The first type is with minimal number of experimental units in proper and non-proper setting for estimating the general combining ability (gca) effects of lines (ii)the second types is with minimum number of experimental units in proper setting for estimating general combining ability (gca)) effects of lines in presences of specific combining ability (sca)

Keywords: Triangular partially balanced incomplete block design, complete diallel cross, General combining ability, of Specific combining ability, optimality

1.   INTRODUCTION

Diallel crossing is a very useful method for conducting plant and animal breeding experiments, especially for estimating combining ability effects of lines. Suppose there are p inbred lines and it is desired to perform a diallel crossing experiments involving p(p-1)/2 crosses of the type (ixj) = (jxi) for i ≠ j = 1, 2, … , p. This is a complete diallel cross system IV mating design of Griffing (1956), who studied the detailed analysis of such mating designs laid out in a randomized complete block design. Henceforth, we will call it Complete Diallel Crosses (CDC).

Optimal block designs for CDC have been considered by Gupta and Kageyama (1994), Dey and Midha (1996) , Das et.al (1998), Parsadet al(1999) , Sharma (2004), Parsadet al(2005) and Sharma and Fanta (2010) by using nested incomplete (NBIB) designs, triangular PBIB design nested balanced block(NBB) designs circular designs and PBIB designs for the cas where the model includes general combining ability(gca) effects apart from block effects but no specific combing ability (sca) effects.

Chai and Mukerjee(1999) considered the case when sca effects are also present in the model and showed that the triangular designs thar=t are proved to be optimal for gcs comparison by Dey and Midha(1996) and Das.et.al(1998) remain optimal for gca comparison even sca effects are presents in the model. Das.et.al (1998) stated a condition in theorem 4.1 on page 334 for a block designs for CDC derived from a triangular designs to be universally optimal. Chai.et al.(2002) proved that the optimal designs for CDC given by Guota and Kegeyama(1994) andDas et.al(1998) , among the others for gca comparisons are also optimal in presence of sca effects even though that are not triangular designs.

The objective of the present paper is to identify triangular designs which can be used to construct optimal incomplete block designs for CDC with Minimum number of experimental units   from catalogue of Clatworthy (1973) and not listed by Dey and Midha (1996), Parsad and Gupta (2004), Das.et al (1998) and Sharma and Fanta (2010). In this investigation we have identified some TPBIB design which can generate optimal incomplete block designs for CDC experiments in proper and non proper settings with minimal number of experimental units for estimating the general combining ability (gca) effects of lines and also can generate optimal incomplete block designs for CDC experiment in proper settings for estimating the gca of lines in presence of specific combing ability. Those designs do not obey the condition given by by Das et. al(1998). The block design in proper and non-proper setting are found universally optimal in sense Kiefer (1975) for estimating the gca effects of lines for CDC experiments. In addition, to the block effects and gca, the model also include sca.

2.   METHOD OF CONSTRUCTION

Our method of construction of IBD for CDC experiment is essentially the same as that of Agarwal and Das (1987) and Dey and Midha (1996) but with some difference. That is those authors put crosses in original blocks of the TPBIB designs but we have taken the crosses in block as block of the mating designs.

Assume that there are p inbred lines and it is desired to find an IBD design for a mating design involving p(p-1)/2 crosses. Consider a two associate triangular PBIB design d_1 as auxiliary design with parameters v = p(p-1)/2, b, r, k = b-1, λ_1, λ_2, n_1, n_2, p^i_{jk} for i, j, k = 1,2. The treatments of the design d_1 are indexed by a pair (ixj) for i < j, i, j, = 1,2,…, p.

Two treatments, say (α, β) and (γ, δ), are first associates if ether α =γ, β≠δ, or α≠δ, β =γ; they are second

associates or $\alpha \neq \delta, \beta = \gamma$; they are second associates otherwise. from the design d_1 , we derived a block design, say d_2 by replacing a treatment in d_1 by cross(ixj) for I <j , I,j = 1,2,…,p. Clearly d_2 involves p (p-1)/2 crosses, b blocks each of size k-1. Each cross is replicated r times. Now in d_2 we interchange the position of the blocks and block size, that is, now we have a design d_3 with parameters v = p (p-1)/2, b′ = k-1, k′ = b, r. When p is odd, design d_3 can be divided into two same IBD d with parameters v = p (p-1)/2, b′′ = (k-1)/2, k′′ = b, r = 1. Similarly when p is even, design d_3 can be divided into two same IBD design d* with parameters v = p (p-1)/2, b′′′ = (k-1)/2, **k**′′′ = (b, b, …, b/2), r = 1 The method of construction is illustrated below for both cases when p is odd (even).

Example 2.1: Let p = 5, we take the following ordered 10 crosses indexed as given below.

1	2	3	4	5	6	7	8	9	10
1x2	1x3	1x4	1x5	2x3	2x4	2x5	3x4	3x5	4x5

Now we consider a triangular PBIB design d_1 (T28, Clatworthy, 1973) with parameters v = 10, b =5, k =4, r =2, λ_1 =1, λ_2 = 0. From design d_1, a mating design d_2 for diallel crossing can be obtained by replacing each treatment with the cross corresponding to i^{th} treatment, we get the required mating design for CDC.

```
          d1                                  d2
B1   1    2    3    4          B1   1x2  1x3  1x4   1x5
B2   5    6    7    1          B2   2x3  2x4  2x5   1x2
B3   8    9    2    5          B3   3x4  3x5  1x3   2x3
B4   10   3    6    8          B4   4x5  1x4  2x4   3x4
B5   4    7    9    10         B5   1x5  2x5  3x5   4x5
```

Now consider columns of d_2 as blocks, we get design d_3 for CDC experiment system

```
        d3                              d              d
B1    B2    B3    B4          B1    B2    B3    B4
1x2   1x3   1x4   1x5         1x2   1x3   1x4   1x5
2x3   2x4   2x5   1x2         2x3   2x4   2x5   1x2
3x4   3x5   1x3   2x3         3x4   3x5   1x3   2x3
4x5   1x4   2x4   3x4         4x5   1x4   2x4   3x4
1x5   2x5   3x5   4x5         1x5   2x5   3x5   4x5
```

Thus we get IBD for CDC experiment with parameters v =10, b′ =4, k′ =5, r =2. By observing the above design we find that it can be divided into two same IBD designs d where blocks have been designated in italic and bold letters, for CDC experiment having parameters v = 10, b′′ = 2, k ′′ = 5, and r =1.

Example 2.2: Let p = 6, we take the following ordered 15 crosses

1x2	1x3	1x4	1x5	1x6	2x3	2x4	2x5	2x6	3x4	3x5	3x6	4x5	4x6	5x6
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Now we take triangular design d_1 (T48, Clatworthy, 1973) with parameters v =15, b =6, r=2, k =5, λ_1 =1, λ_2 = 0. From this design, a mating design d_2 for CDC can be obtained by replacing each ith treatment corresponding i^{th} cross. We get the required mating design for CDC. Now consider columns of mating design as blocks, we get design d_3 for CDC experiment.

```
             d1                              d2
B1   1    2    3    4    5     B1   1x2  1x3  1x4   1x5   1x6
B2   6    7    8    9    1     B2   2x3  2x4  2x5   2x6   1x2
B3   10   11   12   2    6     B3   3x4  3x5  3x6   1x3   2x3
B4   13   14   3    7    10    B4   4x5  4x6  1x4   2x4   3x4
B5   15   4    8    11   13    B5   5x6  1x5  2x5   3x5   4x5
B6   5    9    12   14   15    B6   1x6  2x6  3x6   4x6   5x6
```

Now consider columns of design d_2 as blocks, we get design d_3 for CDC experiment.

```
         d3
 B1    B2    B3    B4    B5
1x2    1x3    1x4    1x5    1x6
2x3    2x4   2x5    2x6    1x2
3x4    3x5   3x6    1x3    2x3
4x5    4x6   1x4    2x4    3x4
1x6    2x6   3x6    4x6    5x6
5x6    1x5   2x5    3x5    4x5
```

Thus we get IBD for CDC experiment with parameters v =15, b′ =6, k′ =5, r =2. By observing the above design we find that it can be divided into two same IBD designs in non proper setting for CDC experiment having parameters v = 15, b′′′ =3, **k**′′′ =(6,6,3), and r =1 as given below. We will denote both designs by d*.

d*.				d*.		
B_1	B_2	B_3		B_1	B_2	B_3
1x2	1x3	1x4		1x4	1x5	1x6
2x3	2x4	2x5		2x5	2x6	1x2
3x4	3x5	3x6		3x6	1x3	2x3
4x5	4x6				2x4	3x4
5x6	1x5				3x5	4x5
1x6	2x6				4x6	5x6

So we get three types of design for CDC experiments (i) with equal block size and having 2 replications for each cross.(ii) with equal block size and having one replication for each cross.(iii) with unequal block size and having one replication for each cross. The first two are proper setting and the third is non proper setting.

Now we give below the theorem of Choi et.al (2002) which establishes that the design d3 and d are optimal for the estimation of gca comparison in presence of sca effects in the model. It is important to note that the design d is balanced for gca comparisons but not balanced for sca comparison because each cross is replicated only once in each design d that is r_c =1. It means that all sca comparisons are not steamtable.

Theorem 2.1: considering a binary diallel cross design D with parameters v =p(p-1)/2 , r_c ,b, k such that it is orthogonally blocked with respect to lines, that is each lines occirs in every block r/b times, that is r = rc(p-1) is the constant number of replication of each the lines. Theb=n D is optimal for the estimation of gca comparisons even in the presence of sca effects in the model. According to the above theorem, the design d3 and d are orthogonally blocked with respect to the lines, hence optimal for gca comparison in the presence of sca effects in the model. Now we state the following theorem.

Theorem 2.2: for a positive prime p>3, if there exists a two associates triangular PBIB design with parameters v= p(p-1)/2, b, r, k = b-1, λ_1, λ_2, n_1, n_2 for i j, = 12, then there is always exist binary CDC design d3 with the parameters v = p(p-1)/2, b- p-1, k = p and r= 2. Then d3 is universally optimal for the estimation of gca comparisons even in the presence of sca effects in the model.

Note: Sharma and Fanta (2010) envisaged 85 PBIB designs (Clatworthy ,1973) to construct optimal block designs for CDC experiments. Out of which 20 and 29 PBIB design gave optimal block design for CDC experiments in which cross is replicated once and twice, respectively. The 29 blocks designs are optimal for gca comparison in the presence of sca effects according to theorem 2.1

In net section we give the method of analysis of the design d_3

3. ANALYSIS OF DESIGN D₃

For the analysis of data obtained from d_3 , we will follow Sing and(Hinklemann's(1998) two stage procedure for estimating gca and sca effects. The first stage is to consider the proposed design for estimation of the effects say = (τ, …. , τp (p-1)/2)' for design d_3 by the following model

$$Y = \mu\, 1_n + X\tau + D\beta + e \qquad (3.1)$$

Where **Y** is the nx1 vector observations, **1** is the nx1 vectors of ones, **X** is the nxv design matrix for treatments and **D** is an nxb design matrix for blocks, that is, the(h, u)[th] ((h,i)[th] elements of **X**(respectively, of **D**) is 1 if the hth observations pertains to the uth cross (to i[th] block) ,and is zero otherwise(h= 1,…,n; u = 1, ….,v; and 1,…,b), **μ** is a general mean, **τ** is a v × 1 vector of treatment parameters, **β** is a b × 1 vector of block parameters and **e** is an n × 1 vector of residuals. It is assumed that vector **β** is fixed and **e** is normally distributed with E (**e**) = 0, V (**e**) = δ²**I**, where **I** is the identity matrix of conformable order.

Following Tocher (1952), Raghava Rao(1971), Dey(1986), the least square method for the analysis of a proposed design leads to the following reduced normal equations for the crosses for model (3.1).

$$C_d\,\boldsymbol{\tau} = Q_d \qquad (3.2)$$

Where $C_d = R - NK^{-1}N'$ and $Q_d = (Q_{1d}, \ldots, Q_{vd}) = T - NK^{-1}B$

In the above expressions above, **R** and **K** are diagonal matrices of order v × v and b × b with common diagonal elements 2 and p in the diagonal, respectively. $N = X'D$ is

the v × b incidence matrix of the designs d; $T = X'\,Y$ and $B = D'Y$ are vector of cross totals and block totals of order v × 1 and b × 1 respectively for design d. `

Hence a solution to (3.2) is given by $\hat{\tau} = C_d^- Q_d \qquad (3.3)$

Where C_d^- is the generalized inverse of C_d with the property that $CC^{-1}C = C.$ The sum of squares due to crosses are $Q_d' C_d^- Q_d$ with degree of freedom (d.f) = rank (C_d) for design d with expectation and variance of Q_d being

$$E(Q_d) = C_d\tau \quad Var(Q_d) = \sigma^2 C_d \qquad (3.4)$$

Now, we will utilize the above equation to estimate the genetics parameters in the proposed design. The second stage is to utilize the fact that the cross effects can be expressed in terms of gca and sca. So we can write effects as follow.

$$\boldsymbol{\tau_{ij}} = g_i + g_j + s_{ij} \qquad (3.5)$$

Where $g_i(g_j)$ is the gca for the $i^{th}(j^{th})$ parent , s_{ij} ($s_{ij} = s_{ji}$) is the sca for cross between the ith and the jth parent ($i < j = 0,1,….,p-1$) and we also assume that, $\sum_i g_i = 0$, and $\sum_j s_{ij} = o$. for all i.

In matrix notation equation (3.5) can be written as $= \boldsymbol{\tau = Zg + s}$ (3.6)

Where Z $(z_{ui})(u = 1,2,….,v: i = 0.1,….,p-1)$ is the cross and gca relation matrix

Z_{ui} = 2 if the uth cross has both parents

    = 1 if the uth cross has only on parent

    = 0, otherwise

Following the approached used in Kepthorne and Curnow (1961) ,equation (3.2)can be written as $\mathbf{C_d \tau = C_d Zg\ C_d s}$

That is, $E\mathbf{(Q_d) = C_d\,Zg\ C_d s}$ (3.7)

Since the matrix **C** is singular, we use unified theory of least square due to Rao(1973, page ,300). So estimator of **g** is

$\hat{\mathbf{g}} = (\,\boldsymbol{Z'C_d\ C_d^- C_d\ Z})^- \boldsymbol{Z'\ Q_d} = (\,\boldsymbol{Z'C_d Z})^- \boldsymbol{Z'\ Q_d}$ (3.8)

Here the matrix $(\,\boldsymbol{Z'C_d Z}) = \frac{1}{2}(p-2)[\ \mathbf{I_p} - \frac{1p1'p}{P}\ ]$

Hence trace $(\,\boldsymbol{Z'C_d Z})$ = 2(p-1)(p-2) (3.9)

So $\hat{\boldsymbol{g}} = \frac{1}{2}(p-2)\boldsymbol{Z'\ C_d \tau}$

Hence $\hat{\boldsymbol{g}} = \mathbf{H_1 \tau}$, where $\mathbf{H_1} = \frac{1}{2}(p-2)\boldsymbol{Z'\ C_d}$ (3.10)

Now, $var(\hat{\boldsymbol{g}}) = \mathbf{H_1 C^- H_1}\sigma^2 = \frac{1}{2}(p-2)\mathbf{I_p}\,\sigma^2$ (3.11)

Since the covariance matrix of $\hat{\mathbf{g}}$ is a constant times the identity matrix, the design d is variance balanced for the general combining ability. We thus have the following result.

Theorem 3.1: for positive prime p> 3m if there exists a two associates triangular PBIB with parameters v = p(p-1)/2,b r,k =b-1, λ_1, λ_2, n_1, n_2,P_{jk}^i for k =1, 2, then there always exists variance-balanced CDC design with the parameters, v = p(p-1)/2, b=p-1 , k = p ,and r =2 for the estimation of gca comparisons.

Now, substituting the estimate of **g** in equation (3.6), we obtain the estimator of **s,** namely insertion of solution of equation (3.10**).**

$\hat{\boldsymbol{s}} = (\boldsymbol{C_d^- - 1/2(p-2)ZZ'})\mathbf{Q_d} = (\boldsymbol{C_d^- - 1/2(p-2)ZZ'})\mathbf{C_d\tau = H_2\tau}$ (3.12)

Where $\mathbf{H2} = (\boldsymbol{C_d^- - 1/2(p-2)ZZ'})\mathbf{C_d}$

$Var\,(\hat{\boldsymbol{s}}) = \acute{H}_2 \boldsymbol{C^-} H_2\,\delta^2$ (3.13)

Since $\mathbf{H_1 1_v = 0}$, $\mathbf{H_2 1_v = 0}$, $\mathbf{H_1 \acute{H}_2 = 0}$,we have rank $(\mathbf{H_1})$ = p-1 and rank $(\mathbf{H_2})$ = v-p.

It follows that **g** and **s** are represented by treatment contrasts which carry p-1 and v-p degree of freedom, respectively and the contrasts representing **g** are orthogonal to those representing **s**. This means that the proposed design d_3 allows for gca and sca effects to be estimated independently. The sum of squares due to gca and sca for design d are given by

$SS(gca) = \boldsymbol{Q'\ Z(Z'C_d Z)^- Z'\ Q_d}$

(3.14)

$SS(sca) = \boldsymbol{\acute{Q}_d\,(C_d^- - 1/(p-2)ZZ'Q_d}$ (3.15)

ANOVA is then given in Table 1

Table 1: Analysis of variance for design d

Source of variation	Degrees of Freedom	Sum of squares
Block	p-2	$\boldsymbol{B'B/(p-1) - G^2/p(p-1)}$
Crosses (adjusted for blocks)	Rank($\mathbf{C_d}$)	$\boldsymbol{\acute{Q}_d C_d^- Q_d}$
gca	rank $(\mathbf{H_1})$	$\boldsymbol{Q'_d\ Z(Z'CdZ)^- Z'\ Q_d}$
sca	rank $(\mathbf{H_2})$	$\boldsymbol{Q'\ Z(Z'C_d Z)^- Z'\ Q_d}$
Residual	by subtraction	by subtraction
Total	n-1	$\boldsymbol{yy'} - G^2/p(p-1)$

Where G = grand total of all n observations

Now we discuss about analysis and optimality of design d*

4.   ANALYSIS AND OPTIMALITY OF DESIGN d*

The data obtained from design d*, we consider the same model as given in equation (3.1)

$\qquad\qquad$ **Y = μ 1_n + Xτ+Dβ+ e** (4.1)

Where **Y** is the nx1 vector observations, **1** is the nx1 vectors of ones, **X** is the nxv design matrix for treatments and **D** is an nxb design matrix for blocks, that is, the(h, u)th ((h,I)th elements of **X**(respectively, of **D**) is 1 if the hth observations pertains to the uth cross (to ith block) ,and is zero otherwise(h= 1,…,n; u = 1, ….,v; and 1,…,b), **μ is** a general mean, **τ** is a v × 1 vector of treatment parameters, **β** is a b × 1 vector of block parameters and **e** is an n

× 1 vector of residuals. It is assumed that vector $\boldsymbol{\beta}$ is fixed and $\mathbf{e}$ is normally distributed with E ($\mathbf{e}$) = 0, V ($\mathbf{e}$) = $\delta^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix of conformable order.

5. Efficiency Factor

Now we will show that the design d and d* are optimal in the sense of Kempthorne (1956). If instead of the designs d and d* , one adopts a randomized complete block design with r blocks, each block having all the p (p-1)/2 crosses, the **C** matrix of the randomized block design i.e. can easily be shown to be , see Dey and Midha(1996).

$$\mathbf{C}_R = r\ (p-2)\ (\ \mathbf{I}_p - p^{-1}\ \mathbf{1}p\ \mathbf{1}'p) \qquad\qquad (4.2)$$

Hence the variance of the best linear unbiased estimator of any elementary contrast among gca effects in the case of randomized block experiment is $2\sigma^2/$ r (p-2), σ^2 is the per observation variance. Thus the efficiency factors e_1 and e_2, respectively of the design d and d*, relative to randomized block designs under the assumption of equal intrablock variances are given by

$$e_1 = \theta\ /\ r(p-2) = r(p-2)/\ r\ (p-2) = 1, \qquad\qquad (4.3)$$

Similarly $e_2 = \theta\ /\ r(p-2) = r(p-2)/\ r\ (p-2) = 1 \qquad\qquad (4.4)$

where $\theta = (\ p-2)$ and r =1

Furthermore, on the lines of Sharma and Fanta (2010), it can be easily proved that the design d* are optimal.

Now, we are giving the list of practicable useful designs for 5 $\leq P \leq$ 11 in Table 2

Table 2

No.	Ref. No of lines	Number of experimental units	Efficiency factor	Optimality status
1	5	10	1	Universally Optimal
2	6	15	1	Universally Optimal
3	7	21	1	Universally Optimal
4	8	28	1	Universally Optimal
5	9	36	1	Universally Optimal
6	10	45	1	Universally Optimal
7	11	55	1	Universally Optimal

Remarks 1: Designs T33, T34, T77, T84, T94 and T95 were reported by Dey and Midha(1996) as efficient designs. These designs are also optimal in sense of Kempthorne(1956) by our method.

Remarks 2: The design reported in Table 2 belong to series v =p (p-1)/2, b = p, r=2, k = p-1, , $\lambda_1 = 1$, $\lambda_2 = 0$, Shrikhande(1952) and Bose and Shimamoto(1952). Their duals are reduced BIB designs with parameters v* = p, b* = p(p-1)/2, r* = p-1, k* = 2, $\lambda_1^* = 1$. These designs are easy to construct Das and Giri(1986). Hence further optimal design for CDC experiment for 12, 13, 14 and 15 inbred lines can be obtained by the duals design.

6. Conclusion

We have searched some more triangular designs to obtain optimal designs for CDC experiment for comparisons of gca in the presence of sca effects in the model and also in the absence of sca.

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