

Using Holt Winter's Multiplicative Model to Forecast Assisted Childbirths at the Teaching Hospital in Ashanti Region, Ghana

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Abstract
The use of maternal healthcare facilities is an important indicator of the impact of the free maternal healthcare policy aimed at improving health status of pregnant women in Ghana. This study investigated the pattern of quarterly assisted deliveries at Komfo Anokye Teaching Hospital (KATH), Kumasi, Ghana from 2000 to 2011. The Holt Winters multiplicative and additive forecasting models were considered. The Multiplicative model reported a Root Mean Square Error of Prediction (RMSEP) of 31.10, Root Mean Square Error (RMSE) of 188.080, Mean Absolute Percentage Error (MAPE) of 6.2951 and Mean Absolute Scaled Error (MASE) of 0.7086 while the additive model reported RMSEP, RMSE, MAPE and MASE of 49.59 201.83, 6.3098 and 0.7106 respectively .The multiplicative model further passed the Shapiro-Wilks test (p-value 0.07358). Results identified the second and fourth quarters as peak seasons and the first quarters as deep seasons for assisted childbirths in the hospital. The negative binomial regression confirmed this by identifying April, May, October and November as peaks months with May being the most significant month.
Keywords: Holt Winter's multiplicative and addictive models, Forecasting, Seasonal, Negative binomial model.

1. Introduction

Pregnant women are often faced with limited options for childbirth and postnatal care as a result of financial constraints. The choice of health care facility is a vital health issue with regards to the well-being and survival of both mother and child. More than 600,000 women die due to childbirth or pregnancy related complications around the world annually (Zozulya, 2010). The lifetime risk of dying due to maternal health causes is about one in six in the poorest countries, compared with about one in 30,000 in the Western World (Fiagbe, et al. 2012). The World Health Organisation (WHO, 2005) states that midwives are the most appropriate primary carers for women during pregnancy and childbirth as these are normal biological processes where most women will achieve successful outcomes if given support and patience. Governments have implemented programmes to bridge equity gaps in access to health care and also ensure sustainable financing arrangements that protect the poor for the free delivery policy in the country. The MDGs aimed at ending extreme poverty worldwide by the year 2015 has MDG5 focused specially on improving maternal health with set target of reducing maternal mortality to less than 185 per 100,000 by 2015 through improved health assistance to pregnant women and removal of inhibiting barriers to health care services.

The Government of Ghana introduced the Exemption Fee Delivery in 2005 and replaced it with the Free Maternal Health Care Policy in 2008 in an effort to increase facility assisted delivery and maternal healthcare. Adjepong .et. al. (2012), reported that the free maternal healthcare policy has resulted in a significant additional enrolment of 90 women at the Mampong hospital while Dzakpasu et. al. (2012) shows increase in facility delivery by 2.3% and 7.5% for the 2005 and 2008 policies respectively in Brong Ahafo region. Reports from Central Region showed that increase in facility deliveries occurred mostly at health centres with rate of 13.7% to 22.3% (Penfold et. al., 2007). KATH in Kumasi is the second-largest hospital in the country and the only tertiary health institution in the Ashanti Region. It is the main referral hospital for the Ashanti, Brong Ahafo, Northern, Upper East and Upper West Regions. Management of KATH cites congestion as one of the serious challenges facing maternal healthcare as health policies have increased recorded deliveries in the Hospital. Most emergency cases within the Kumasi metropolis are referred to KATH. Questions as to what statistical model would be reliable for forecasting the incidence of assisted delivery in the facility as well as indicate months with significant deliveries remained unanswered. It is against this background that this study was undertaken to assess the pattern of assisted delivery as well as predict future assisted delivery at KATH.

2. Theoretical Model

To assess and predict the future incidence of assisted childbirths, monthly recorded data on the number of assisted childbirths at KATH was obtained from the Bio-Statistics Department of the Obstetrics & Gynaecology directorate of the Hospital for the period 2000-2011. The Holt Winters exponential smoothing model was used to forecast the expected deliveries. To describe the number of deliveries, the Negative binomial and Poisson models were fitted to the data.

2.1 Holt Winters' Predictive Model

Winters (1960), generalised Holt's (1957) linear method by adding a seasonal equation to Holt's equations. It is either additive or multiplicative based on the pattern shown by the time plot. It is advisable to fit both additive and multiplicative models and then select the best model based on the model adequacy checks. The model assigns more weight to more recent values and lesser weight assigned to values from the distant past.

The additive model is generally stated as:

$$L_t = \alpha(y_t - S_{t-p}) + (1-\alpha)(L_{t-1} + b_{t-1}) \quad (1)$$

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \quad (2)$$

$$S_t = \gamma(y_t - L_t) + (1-\gamma)S_{t-p} \quad (3)$$

The multiplicative model is generally stated as:

$$\mathrm{a}_t = \alpha\frac{Y_t}{\mathrm{s}_{t-p}} + (1-\alpha)(\mathrm{a}_{t-1} + \mathrm{b}_{t-1}) \quad (4)$$

$$\mathrm{b}_t = \beta(\mathrm{a}_t - \mathrm{a}_{t-1}) + (1-\beta)\mathrm{b}_{t-1} \quad (5)$$

$$\mathrm{s}_t = \gamma\frac{Y_t}{\mathrm{a}_t} + (1-\gamma)\mathrm{s}_{t-p} \quad (6)$$

Where $0 \le \alpha \le 1,\ \ 0 \le \beta \le 1,\ \ 0 \le \gamma \le 1$: α, β and γ are the smoothing parameters.

L_t is the smoothed level at time t, b_t is the change in the trend at time t, S_t is the seasonal smooth at time t, p is the number of seasons per year. The Holt-Winters algorithm requires starting value for each of the component (or initialising) values. Most commonly:

$$\mathrm{a}_p = \frac{1}{p}(Y_1 + Y_2 + \ldots + Y_p) \quad (7)$$

$$\mathrm{b}_p = \frac{1}{p}\left[\frac{Y_{p+1} - Y_1}{p} + \frac{Y_{p+2} - Y_2}{p} + \ldots + \frac{Y_{p+p} - Y_p}{p}\right] \quad (8)$$

The formulas for linear and trend component are the same for both additive and multiplicative models

$$\mathrm{s}_1 = \frac{Y_1}{\mathrm{a}_p},\quad s_2 = \frac{Y_2}{\mathrm{a}_p},\quad \ldots,\quad s_p = \frac{Y_p}{\mathrm{a}_p}, \left(\begin{array}{l}\text{Seasonal component for}\\ \text{multiplicative. model.}\end{array}\right)$$

$$\mathrm{s}_1 = Y_1 - \mathrm{a}_p,\quad s_2 = Y_2 - \mathrm{a}_p,\quad \ldots,\quad s_p = Y_p - \mathrm{a}_p \left(\begin{array}{l}\text{Seasonal component for}\\ \text{additive. model.}\end{array}\right)$$

So we have our forecast for time period $p+t$ to be given as: $F_{t+p} = L_t + pb_t + S_{t+p-1}$ (9)

Where: L_t is the smoothed estimate of the level at time t

b_t is the smoothed estimate of the change in the trend value at time t

S_{t+p-1} is the smoothed estimate of the appropriate seasonal component at t

2.2 Assessing Model Adequacy

Model adequacy check is by error term analysis. The error term is expected to be uncorrelated and normally distributed. If the predictive model cannot be improved upon, there should be no correlations between forecast errors for successive predictions. Ljung-Box test of goodness of fit and the Shapiro-Wilks test for normality were used for model adequacy tests.

2.3 Assessing Prediction Adequacy

The test of any prediction model is how well it predicts when compared to actual data values. The **RMSE, RMSEP, MASE and MAPE** are used for comparing prediction models:

$$RMSEP = \sqrt{\frac{1}{\mathrm{P}}\sum_{t=1}^{\tau}(y_t - \hat{y}_t)^2} \quad (10)$$

Where P is the number of predictions

$$RMSE_i = \sqrt{\frac{1}{n}\sum_{t=1}^{T}\left(y_t - {}_{t-i}y_t^f\right)^2} \quad (11)$$

Where ${}_{t-i}y_t^f$ is the forecast at i periods before t, n is the number of observations and T is the number of prediction made.

$$MASE = \frac{\sum_{t=1}^{n}|e_t|}{\frac{n}{n-1}\sum_{i=2}^{n}|Y_i - Y_{i-1}|} \quad (12)$$

where e_t is the forecast error for a given period t, (Y_i), the actual value and $F_t = Y_{i-1}$ which is the actual value from the prior period to the forecast.

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \quad (13)$$

where A_t is the actual value, F_t is the forecast value and n is the number of fitted points.
The Models with smaller values are always deemed appropriate.

2.4 Modelling the Frequency of Deliveries.

The Generalized Linear Model is an extension of the Linear Model to include response variables that follow any probability distribution in the exponential family of distributions. Many commonly used distributions in the exponential family are the normal, binomial, Poisson, exponential, gamma and inverse Gaussian distributions. Generalized linear models are applicable when we have a single response variable Y and associated explanatory variables $x_1, x_2 \ldots x_p$ where typically $x_1 \equiv 1$ to include the usual constant term in the model.

2.4.1 Negative Binomial

Negative binomial distribution is of the form

$$P(Y = y_i / \beta, x_i) = \frac{\Gamma(\theta + y_i)}{\Gamma(y_i + 1)\Gamma(\theta)}\left(\frac{\lambda_i}{\lambda_i + \theta}\right)^{y_i}\left(1 - \frac{\lambda_i}{\lambda_i + \theta}\right)^{\theta} \quad (14)$$

The conditional mean $E[y_i / x_i] = \lambda_i = e^{x_i\beta}$ and variance $\lambda_i(1 + \eta^2\lambda_i)$
$Var[y_i / x_i] = e^{x_i\beta}(1 + \eta^2 e^{x_i\beta})$, $\eta^2 = \frac{1}{\theta}$ (η^2 with is not allowed to vary over the observations).

2.4.2 Poisson Regression

The poisson regression is of the form $f(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$ (15)

Where e is the base of the natural logarithm $(e = 2.71828\ldots)$, k is the number of occurrences of an event, λ is a positive real number, equal to the expected number of occurrences that occur during the given interval. The log transformed model is $\log(u_i = x_i'\beta)$. The regression parameter β , and also the extra parameter η are estimated by maximum likelihood method. The parameter η measures the degree of over (or under) dispersion. $Exp(\beta_j)$ represents a multiplicative effect of the jth predictor on the mean. Increasing x_i by one unit multiplies the mean by a factor $\exp(\beta_j)$.

3. Empirical Analysis

The analysis was carried out using the R-console version 2.15.3 statistical package. Time plot of quarterly assisted recorded childbirths can be seen in figure 1. Random fluctuations in the timeplot series seem to be roughly constant in size over time. The timeplot showed an upward trend in the second quarter (April to June) of 2007.

3.1 Determination of Smooth Parameters

Table 1 shows the parameter estimates for the models. The multiplicative model gave the estimated values of alpha, beta and gamma as 0.47, 0.00, and 0.6, respectively. The value of alpha (0.47) is relatively low, indicating that the estimate of the level at the current time point is based upon both recent observations and some observations in the more distant past. The value of beta is 0.00, indicating that the estimate of the slope β of the trend component is not updated over the time series, but set equal to its initial value. This makes good intuitive sense, as the level changes quite a bit over the time series, but the slope β of the trend component remains roughly the same. In contrast, the value of gamma (0.6) is high; indicating that the estimate of the seasonal component at the current time point is based upon more recent observations. The additive model's value of gamma (0.30) is low, indicating that the estimate of the seasonal component forecasts is based on both recent and less recent observations (although somewhat more weight is placed on recent observations). The alpha and beta were 0.45 and 0.00 respectively. From Table 2, the parameter in addictive model and multiplicative model for the first and third quarters for seasonal components of the forecasts was low as compared with the second and the fourth quarter.

3.2 Forecast Model Adequacy Selection

*Th*e two prediction models were compared using the **RMSE**, **RMSEP, MAPE and MASE.** The values for the multiplicative model were 188.080, 31.10, 0.7086 and 6.2951 for **RMSE,** RMSEP**, MASE and MAPE** respectively, and the additive model gave value of 201.825 49.59, 0.7106 and 6.3098 as its respective values for the estimators. The Ljung-Box test statistic was 20.46(p-value 0.6705) and 21.2 (p-value = 0.53) for the two models. For both models there was little evidence of non-zero autocorrelation in the in-sample forecast errors. Shapiro-Wilks normality test results for the multiplicative model gave a non significant p-value of 0.07358 suggesting that the additive model residuals are normally distributed but gave a significant p-value of 0.02668 for the additive model, suggesting that the model's residuals are not normally distributed. Hence multiplicative model predicted better than the additive model. Comparing the predicted quarterly childbirths of 2011 with the observed recovered deliveries, we observe that the predicted values are close to the observed values with these values falling inside the confidence interval. Hence, we can say that, multiplicative model is adequate to forecast quarterly assisted childbirths at the KATH. Table 3 below summarizes the forecasting results of the quarterly childbirths over the period 2011 to 2012 with 95% confidence interval.

3.4. Regression Model

With 132 degrees of freedom Poisson model had a deviance of 3691.6 while the negative binomial model had a deviance of 145.42 following the chi-square distribution with eleven (11) degrees of freedom. From Table 4 below it can be seen that the Negative binomial model had smaller value for AIC and BIC as compared with the Poisson model. The dispersion parameter for Negative Binomial model indicated neither over nor under dispersion since is not far from 1, while Poisson model was over dispersed. Hence the negative binomial model is considered adequate. The months of April (p-value = 0.009), May (p-value = **0.0004**), October (p-value = **0.0004**) and November (p-value = **0.012877**) recorded significant assisted childbirth during the period under study for the adequate model. The month of January was used as the base

4. Discussion and conclusion

The time plot shows a gradual upward trend in quarterly reordered childbirths in the hospital which started in the second quarter of 2007. During this period under study, the highest quarterly childbirths of 3792 was recorded in the second quarter of 2010 and the lowest quarterly childbirths of 2239 in the first quarter of 2003. This quarterly increase can be seen throughout the time-plot. The forecast for 2011 and 2012 exhibit a steady increasing trend in the number of quarterly reordered childbirths as showed by the multiplicative model time plot. The negative binomial regression model was also fitted to determine the months with significant assisted delivery. The negative binomial regression indicated April, May, October and November as the months that recorded high incidence of assisted childbirths. The month of May was found to be the most significant month during which the highest frequencies of assisted deliveries occur.

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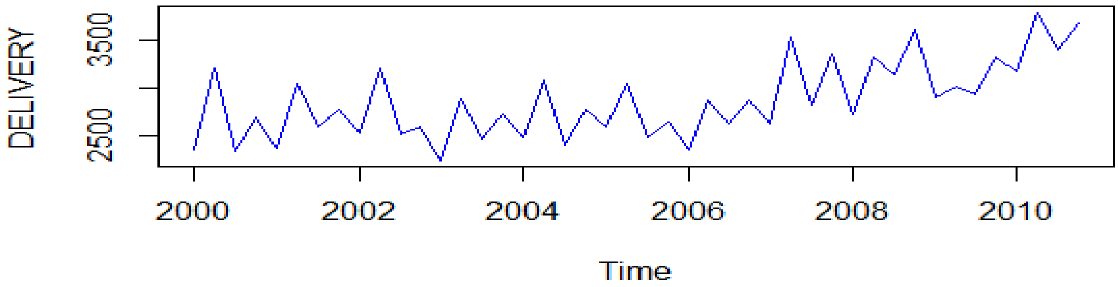

Figure 1. Time plot of quarterly assisted recorded childbirth.

Table 1. Smooth Parameters

	Alpha(α -level)	Beta(β -trend)	Gamma(γ -seasonal)
Additive model	0.45	0.00	0.30
Multiplicative model	0.47	0.00	0.60

Table.2. Model Coefficients

	QUT1	QUT2	QUT3	QUT4	Alpha	Trend
Addictive model	-238.991	316.709	-117.340	172.603	3494.431	11.650
Multiplicative model	0.917	1.090	0.964	1.0589	3489.326	11.650

Table 3. Quarterly Assisted childbirth Forecast

Period	2011 Q1	2011 Q2	2011 Q3	2011 Q4	2012 Q1	2012 Q2	2012 Q3
Forecast	3367.089	3834.440	3412.040	3713.635	3313.689	3881.040	3458.640
Lo 95%	2876.493	3406.280	2949.356	3218.829	2766.902	3306.817	2858.234
Hi 95	3657.686	4262.601	3874.725	4208.440	3860.477	4455.264	4059.047
Actual	3360	3385	3232	3388			

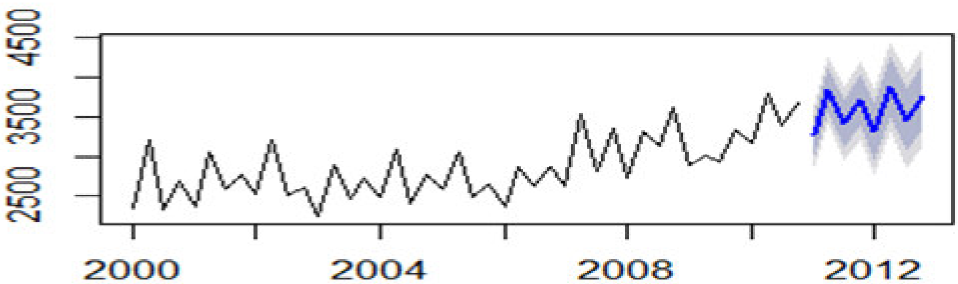

Figure 4. Forecast plot

Table 4. Penalty function statistics of candidate models.

Model	AIC	BIC	Dispersion Parameter
Negative binomial	1938.223	1899.616	1.1016
Poisson	4966.2	5001.837	27.96667

Table 5. Negative binomial regression model parameter estimates

Coefficients	**Estimate**	**Standard Error**	**z value**	**pr(>|z|)**
Intercept	6.77403	0.04936	137.240	2e-16
February	-0.08547	0.06986	-1.223	0.221187
March	0.09064	0.06974	1.300	0.193738
April	**0.18100**	**0.06969**	**2.597**	**0.009398**
May	**0.24325**	**0.06966**	**3.492**	**0.000479**
June	0.12064	0.06973	1.730	0.083605
July	0.07211	0.06976	1.034	0.301242
August	0.03353	0.06978	0.481	0.630840
September	0.04943	0.06977	0.709	0.478624
October	**0.15148**	**0.06971**	**2.173**	**0.029776**
November	**0.17334**	**0.06970**	**2.487**	**0.012877**
December	0.01794	0.06979	0.257	0.797175