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The Dynamics of Energy Attenuation inHIV/AIDS Patients

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Abstract
In this work, we used the constituted carrier wave (CCW) which we developed in a previous study to determine the dynamics of energy attenuation in HIV/AIDS patients. The CCW is the superposition of two incoherent waves, the 'parasitic wave' and the 'host wave' and it describes the coexistence of the parasite HIV and the human system which of course is now the host. We also used Fourier transform technique to determine quantitatively the spectrum of energy attenuationconveyed by the CCW. It is established in this study, that the reduced velocity of the CCW causes a delay in the energy transfer mechanism which eventually leads to starvation and weakening of the initial strength of the active cells that make up the human biological system. It is shown that in the absence of specific treatment, the HIV infection degenerates to acquire immunodeficiency syndrome (AIDS) after about92 months (8 years) and this is indicated by the reduced frequency of the energy spectrum. The energy of the CCW which now describes the new biological system of Man after infection with HIVfinally goes to zero - a phenomenon called death, when the multiplier approaches the critical value of 13070 and the time it takes to attain this value is about 126 months (10 years).
Keywords: Energy attenuation, 'host wave', 'parasitic wave', CCW, HIV/AIDS, 'third world approximation'.

1. Introduction.

After several years of intense experimental and theoretical studies of HIV/AIDS, there is still no adequate understanding of the formation of the HIV and possible cure to the epidemic disease. The human immunodeficiency virus (HIV) is among the most pressing health problem in the world today. Since its discovery, AIDS has caused nearly 30 million deaths as of 2009. It has been estimated that as of 2010 approximately 34 million people have contracted HIV globally and greater proportion of the population coming from Africa and Asian countries (Morgan et al., 2002 ; UNIADS, 2011).

The HIV fatal effect stems from the attack on a person's CD_4 cell counts. This result to the progressive depletion of the CD_4 cell counts which play a pivotal regulatory role in the immune response to infections and tumours. Infection by the human immunodeficiency virus (HIV) gradually evolves to acquire immunodeficiency syndrome (AIDS) and which finally leads to death (Anderson et el., 1986). In the absence of specific treatment, around half of the people infected with HIV develop AIDS within 10 years and the average survival time after infection with HIV is estimated to be 9 to 11 years depending on the subtype. After the diagnosis of AIDS, if treatment is not available, survival ranges between 6 and 19 months (UNIADS and WHO, 2007).

According to the literature of clinical diseases, the HIV feeds on and in the process kills the activecells that make up the human immune system. This is a very correct statement but not a unique understanding. There is also a cause (vibration) that gives the HIV its ownintrinsic characteristics, activity and existence.It is not the Human system that gives the HIV its life and existence, since the HIV itself is a living organism and with its own peculiar characteristics even before it enteredthe system of Man(Enaibe and Osafile, 2013).

In addition to the knowledge of the medical experts about HIV/AIDS, is the understanding that Man and the HIV are both active matter, as a result, they must have independent peculiar vibrations in order to exist. It is the vibration of the HIV that interferes with the vibration of Man (host) in the human blood circulating system after infection. The resultant interference of the two vibrations is parasitically destructive and it slows down or makes the biological system of Man to malfunction since the basic intrinsic parameters of the 'host wave' would have been altered and destroyed (Enaibe and Idiodi, 2013).

It is the vibration of the unknown force that causes life and existence. Therefore, for any active matter to exist it must possess vibration. The human heart stands as a transducer of this vibration while the blood stands as a means of conveying this vibration to all units of the human system. The cyclic heart contraction generates pulsatile blood flow and latent vibration. The latent vibration is sinusoidal and central in character, that is, it flows along the middle of the vascular blood vessels. It thus orients theactive particles of the blood and sets them into oscillating motion with a unified frequency as it passes.

Some waves in nature behave parasitically when they interfere with another one. Such waves as the nameimplies have the ability of transforming the initial characteristics and behaviour of the 'host wave' to its ownform and quality after a period of time. Under this circumstance, all the active constituents of the 'host wave' would have been completely eroded and the resulting wave which is now parasiticallymonochromatic, will eventually

attenuate to zero, since the 'parasitic wave' does not have its own physical parameters for sustaining a continuous independent existence(Enaibe et al., 2013).
Human blood is a liquid tissue composed of roughly 55% fluid plasma and 45% cells. The three main types of cells in blood are red blood cells, white blood cells and platelets. 92% of blood plasma is composed of water and the other 8% is composed of proteins, metabolites and ions (Cutnel and Johnson, 1998). The density of blood plasma is approximately 1025 kg/m^3 and the density of blood cells circulating in the blood is approximately 1125 kg/m^3. The Blood plasma and its contents are known as whole blood and the average density of whole blood for a human is about 1050 kg/m^3(Alexandra et al., 1998).
Blood viscosity is a measure of the resistance ofblood to flow, which is being deformed by either shear or extensional strain. The dynamic viscosity of the human blood at 37^{0}C is usually between 0.003kgm^{-1}s^{-1}and0.004 kgm^{-1}s^{-1}(Glenn, 2008). The viscosity of blood thus depends on the viscosity of the plasma, in combination with the particles. However, plasma can be considered as a Newtonian fluid, but blood cannot due to the particles which add non-idealities to the fluid.
When looking at viscoelastic behaviour of blood in general, it is necessary to also consider the effects of arteries, capillaries and veins. The viscosity of blood has a primary influence on flow in the larger arteries, while the elasticity of blood, which resides in the elastic deformability of red blood cells, has primary influence in the arterioles and the capillaries. The red blood cells occupy about half of the volume of blood and possess elastic properties. This elastic property is the largest contributing factor to the viscoelastic behaviour of blood (Canic et al. 2006 ; Burton, 1965). Due to the limited space between red blood cells, it is obvious that in order for blood to flow, significant cell to cell interaction will play a key role. This interaction and tendency for cells to aggregate is a major contributor to the viscoelastic behaviour of blood. The elasticity of red blood cells has a value that ranges between 0.108 – 2.146 dyn/cm x 10^{-3}with a median value of 0.692 x 10^{-3}dyn/cm or 6.92 x 10^{-7}N/m (Brandao et al., 2003).
Fourier series has long provided one of the principal methods of analysis for mathematical physics, engineering, and signal processing. It has spurred generalizations and applications that continue to develop right up to the present. While the original theory of Fourier series applies to periodic functions occurring in wave motion, such as with light and sound, its generalizations often relate to wider settings, such as the time-frequency analysis underlying the recent theories of wavelet analysis and local trigonometric analysis. Periodic functions arise in the study of wave motion, when a basic waveform repeats itself periodically (Walker, 1981).
This paper is outlined as follows. Section 1, illustrates the basic concept of the work under study. The mathematical theory is presented in section2. The results obtained are shown in section 3. While in section 4, we present the analytical discussion of the results obtained. The conclusion of this work is shown in section 5.This is immediately followed by appendix of some useful identities and a list of references.

2. Mathematical Theory

2.1 Dynamical theory of superposition of two incoherent waves.

That the HIV kills slowly with time shows that the wave functions of the HIV and that of the host were initially incoherent. As a result, the amplitude, angular frequency, wave number and the phase angle of the host wave which are the basic characteristics of waves and vibration were initially greater than those of the HIV. The interference of one wave y_2 say 'parasitic wave'on another one y_1 say 'host wave', could cause the 'hostwave' to decay to zero if they are out of phase. The decay process of y_1 can be gradual, over-damped orcritically damped depending on the rate in which the amplitude is brought to zero. However,the general understanding is that the combination of y_1 and y_2 would first yield a third stage called the resultantwave y, before the process of decay sets in. Now let us consider two incoherent waves defined by the below non-stationary displacement vectors

$$y_1 = a\beta \cos\left(\vec{k}\beta . \vec{r} - n\beta t - \varepsilon \beta\right) \qquad (2.1)$$

$$y_2 = b\lambda \cos\left(\vec{k}'\lambda . \vec{r} - n'\lambda t - \varepsilon' \lambda\right) \qquad (2.2)$$

$$y = y_1 + y_2 = a\beta \cos\left(\vec{k}\beta . \vec{r} - n\beta t - \varepsilon \beta\right) + b\lambda \cos\left(\vec{k}'\lambda . \vec{r} - n'\lambda t - \varepsilon' \lambda\right) \qquad (2.3)$$

where all the symbols have their usual wave related meaning. In this study, (2.1) is regarded as the 'host wave' whose propagation depends on the inbuilt multiplier β (= 0, 1, 2, . . . $\beta_{\max}$). While (2.2) represents a 'parasitic wave' with an inbuilt multiplier λ (= 0, 1, 2, . . . $\lambda_{\max}$). The inbuilt multipliers are both dimensionless and they have the ability of gradually raising the basic characteristics of both waves respectively with time. We have established in a previous paper [5] that when (2.2) is superposed on (2.1) we getafter some algebra that

$$y = \left\{(a^2 - b^2\lambda^2) - 2(a - b\lambda)^2 \cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right\}^{\frac{1}{2}} \cos\left(\vec{k} . \vec{r} - (n - n'\lambda)t - E(t)\right) \qquad (2.4)$$

In this study, equation (2.4) is regarded as the constituted carrier wave (CCW) and it is the equation that governs the dynamical behaviour of the coexistence of the HIV 'parasitic wave' in the human blood circulating system. We however set the multiplier β =1 to make it invariant with respect to time. From the equation once the multiplier λ raises the dynamical constituents of the HIV 'parasitic wave' to become equal to those of Man the 'host wave', then the CCW goes to zero and the host biological system ceases to exist.

The total phase angle of the CCW is represented by $E(t)$, $\vec{k}.\vec{r}$ is the coordinate position vector. However, in this work we made the displacement vector represented by the CCW independent of space by simply using $\vec{k}.\hat{r}$ instead of $\vec{k}.\vec{r}$ where $\hat{r} = \vec{r}/r$ is a unit vector.

$$E(t) = \tan^{-1}\left(\frac{a\sin\varepsilon + b\lambda\sin\left(\varepsilon'\lambda - (n-n'\lambda)t\right)}{a\cos\varepsilon + b\lambda\cos\left(\varepsilon'\lambda - (n-n'\lambda)t\right)}\right) \quad (2.5)$$

The variation of the total phase angle $E(t)$ with time gives the characteristic angular velocity $Z(t)$ of the CCW.

$$\frac{dE(t)}{dt} = -Z(t) = -(n-n'\lambda)\left(\frac{b^2\lambda^2 + ab\lambda\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)}\right) \quad (2.6)$$

We should understandthat $\vec{k}.\vec{r} = (k-k'\lambda)r\times$ $(\cos\varphi + \sin\varphi)$ or $\vec{k}.\hat{r} = (k-k'\lambda)$ $(\cos\varphi + \sin\varphi)$ is a two dimensional (2D) space vector and $\varphi = \pi - (\varepsilon - \varepsilon'\lambda)$ from the geometry of the two interfering waves. By definition: $(n-n'\lambda)$ the modulation angular frequency, the modulation propagation constant $(k-k'\lambda)$, the phase difference δ between the two interfering waves is $(\varepsilon-\varepsilon'\lambda)$, the interference term is $2(a-b\lambda)^2\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)$, while waves out of phase interfere destructively according to $(a-b\lambda)^2$ that is, if y comes out smaller than the larger of a and b ;and waves in-phase interfere constructively according to $(a+b\lambda)^2$, that is, y comes out larger than both.

In the regions where the amplitude of the wave is greater than either of the amplitude of the individual wave, we have constructive interference that means the path difference is $(\varepsilon+\varepsilon'\lambda)$, otherwise, it is destructive in which case the path difference is $(\varepsilon-\varepsilon'\lambda)$ .The CCW can be decomposed into two functions; function of the oscillating amplitude $f(A)$ and the function of the spatial oscillating phase $f(\theta)$. That is

$$f(A) = \left\{(a^2 - b^2\lambda^2) - 2(a-b\lambda)^2\cos\left((n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\right)\right\}^{\frac{1}{2}} \quad (2.7)$$

$$f(\theta) = \cos\left(\vec{k}.\hat{r} - (n-n'\lambda)t - E(t)\right) \quad (2.8)$$

As it is, (2.7) represents the maximum oscillating amplitude of the CCW and hence the displacement vector y is also a maximum. In this work, we are going to use two approaches as an approximation to (2.7). One approach would be to derive the velocity of the oscillating amplitude directly from it. The second approach is to determine the stationary behaviour of theoscillating amplitude with respect to the phase angle δ .

2.2 Application of the "third world approximation" on the oscillating amplitude $f(A)$ of the CCW.

Equation (2.7) is comprehensively valid in the macroscopic scale. However, if we implement the "third world approximation", then the function can be made valid for both macroscopic and microscopic scale. The "third world approximation" states that

$$\left(1+\xi f(\phi)\right)^{\pm n} = \frac{d}{d\phi}\left(1 + n\,\xi f(\phi) + \frac{n(n-1)}{2!}\left(\xi f(\phi)\right)^2 + \frac{n(n-1)(n-2)}{3!}\left(\xi f(\phi)\right)^3 + \dots\right) - n\frac{d}{d\phi}\left(\xi f(\phi)\right) \quad (2.9)$$

We should emphasize here that ϕ is a function of any variable which depends upon the dimension of the physical parameter we are investigating. However, in this study ϕ is taken as the time. In this approximation, the first term in the series or 'first world' is usually a constant while the rest of the series is based on the choice of the parameter under evaluation. For instance, the dimension of (2.7) is meters and if we apply (2.9) on it, then the first two terms, otherwise, the 'first world' and the 'second world' terms are both switched off leaving the third term or the 'third world' in m/s which is the dimension of velocity. Now let us rearrange (2.7) for the utilization of (2.9).

$$f(A) = \left(a^2 - b^2\lambda^2\right)^{\frac{1}{2}} \left\{ 1 - \frac{2\left(a - b\lambda\right)^2}{(a^2 - b^2\lambda^2)} \cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \right\}^{\frac{1}{2}} \quad (2.10)$$

It can be shown that after a careful implementation of (2.9) in (2.10) we obtain

$$\left\{ 1 - \frac{2\left(a - b\lambda\right)^2}{(a^2 - b^2\lambda^2)} \cos\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \right\}^{\frac{1}{2}} = \frac{\left(a - b\lambda\right)^4 (n - n'\lambda)}{2(a^2 - b^2\lambda^2)^{3/2}} \sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \quad (2.11)$$

Hence

$$v_m = f(A) = \frac{(a - b\lambda)^4 (n - n'\lambda)}{2(a^2 - b^2\lambda^2)^{3/2}} \sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) = Q(n - n'\lambda)\sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right) \quad (2.12)$$

For the purpose of linearity we have introduced a new parameter as $Q = (a - b\lambda)^4 / 2(a^2 - b^2\lambda^2)^{3/2}$.
Thus we have established that (2.12) represents the equation of the maximum velocity of the oscillating amplitude of the CCW and the unit is m/s or rad./s depending on the system under investigation.

2.3 Fourier series expansion of the maximum velocity of the oscillating amplitude of the CCW.

The cornerstone of Fourier theory is a theorem which states that almost any periodic function can be analyzed into a series of harmonic functions with periods τ, $\tau/2$, $\tau/3$, ... , where τ is the period of the function under analysis (Lain, 1995). Expansion of an oscillating function by Fourier series gives all modes of oscillation (fundamental and all overtones) which is extremely useful in physics.In particular, astronomical phenomena are usually periodic, as are animal heartbeats, tides and vibrating strings, so it makes sense to express them in terms of periodic functions.Now, by expanding the oscillating term of (2.12) in terms of Fourier series we get

$$F\left[v_m\right] = C_0 + C_1\left(\sin 2\left(1(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right) + C_2\left(\sin 2\left(2(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right) +$$
$$C_3\left(\sin 2\left(3(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right) + \ldots + C_\alpha\left(\sin 2\left(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right) (2.13)$$

$$F\left[v_m\right] = C_0 + \sum_{\alpha=1}^{\infty} C_\alpha\left(\sin 2\left(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right) (2.14)$$

The constant term C_0 may be thought of as a harmonic with zero frequency or the fundamental. Each term in the series has amplitude and a phase constant; by adjusting these we can expand the various harmonics vertically, or shift them horizontally, to make the superposition fit the function $F\left[v_m\right]$. Harmonic analysis consists essentially of finding C_α and $(\varepsilon - \varepsilon'\lambda)_\alpha$ for each value of α. From (2.14) it is however not always convenient to specify amplitude and phase (Lipson et al., 1996), we can decompose the last term as

$$C_\alpha\left(\sin 2\left(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)\right) = A_\alpha \cos 2\alpha(n - n'\lambda)t + B_\alpha \sin 2\alpha(n - n'\lambda)t \quad (2.15)$$

We can specify the modulation amplitudes A_α and
B_α as components of the variable phase angle as

$$\left.\begin{aligned} A_\alpha &= C_\alpha \cos 2(\varepsilon - \varepsilon'\lambda) \\ B_\alpha &= -C_\alpha \sin 2(\varepsilon - \varepsilon'\lambda) \end{aligned}\right\} \Rightarrow C_\alpha = \sqrt{A_\alpha^2 + B_\alpha^2} \quad (2.16)$$

The negative sign indicates complex conjugate of the real part and the inclusions will make the dynamic components of the phase angle real.
Thus (2.16) represents the amplitude of the nth harmonic. Where α is the Fourier index.From (2.15), if $\alpha = 0$

$$C_0 = -\frac{1}{\sin 2(\varepsilon - \varepsilon'\lambda)} A_0 \because (\sin(-x) = -\sin x) \quad (2.17)$$

2.4 Determination of the Fourier coefficients of the maximum velocityof the CCW.

The Fourier components C_α in (2.15) which is specified in (2.17) and (2.18) are given by the
Euler formulas

$$A_0 = \frac{1}{\tau}\int_0^\tau v_m \, dt = \frac{1}{\tau}\int_0^\tau Q(n - n'\lambda)\sin 2\left((n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)\right)dt \quad (2.18)$$

$$A_{\alpha} = \frac{1}{\tau}\int_0^{\tau} v_m \cos 2\big(\alpha(n-n'\lambda)t\big)dt = \frac{1}{\tau}\int_0^{\tau} Q(n-n'\lambda)\sin 2\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\cos 2\big(\alpha(n-n'\lambda)t\big)dt \qquad (2.19)$$

$$B_{\alpha} = \frac{1}{\tau}\int_0^{\tau} v_m \sin 2\big(\alpha(n-n'\lambda)t\big)dt = \frac{1}{\tau}\int_0^{\tau} Q(n-n'\lambda)\sin 2\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\sin 2\big(\alpha(n-n'\lambda)t\big)dt \qquad (2.20)$$

where τ is the period of the function under analysis, here it is the period of the latent vibration of the CCW generated by the beating of
the human heart. In this work, we define $\tau(n-n'\lambda) = 2\pi$. Let us now evaluate (2.18) for A_0. Direct integration and rearrangement gives

$$A_0 = \frac{Q}{2\tau}\left\{\cos 2(\varepsilon-\varepsilon'\lambda) - \cos 2\big((n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\big)\right\} \qquad (2.21)$$

$$A_0 = \frac{(a-b\lambda)^4(n-n'\lambda)}{8\pi(a^2-b^2\lambda^2)^{3/2}}\left\{\cos 2(\varepsilon-\varepsilon'\lambda) - \cos 2\big((2\pi-(\varepsilon-\varepsilon'\lambda)\big)\right\} \qquad (2.22)$$

The dimension of A_0 in m/s . Hence, when we substitute (2.22) into (2.17), we get

$$C_0 = -\frac{(a-b\lambda)^4(n-n'\lambda)}{8\pi(a^2-b^2\lambda^2)^{3/2}\sin 2(\varepsilon-\varepsilon'\lambda)}\left\{\cos 2(\varepsilon-\varepsilon'\lambda) - \cos 2\big((2\pi-(\varepsilon-\varepsilon'\lambda)\big)\right\} (2.23)$$

By invoking the rule of compound angles in trigonometry, see appendix, we can further simplify (2.23) to yield

$$C_0 = \left(\frac{(a-b\lambda)^4(n-n'\lambda)}{4\pi(a^2-b^2\lambda^2)^{3/2}\sin 2(\varepsilon-\varepsilon'\lambda)}\right)\left\{\sin 2(\pi)\sin 2\big((\varepsilon-\varepsilon'\lambda)-\pi\big)\right\} \qquad (2.24)$$

Hence, C_0 has the dimension of velocity and this represents the fundamental velocity of the CCW.
Now, let us use trigonometric identityto further reduce equation (2.19), so that

$$A_{\alpha} = \frac{Q(n-n'\lambda)}{2\tau}\int_0^{\tau}\left\{\sin 2\big((1+\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big) + \sin 2\big((1-\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\right\}dt \ (2.25)$$

$$A_{\alpha} = \frac{Q(n-n'\lambda)}{2\tau}\left\{-\left(\frac{1}{2(1+\alpha)(n-n'\lambda)}\Big(\cos 2\big((1+\alpha)(n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\big) - \cos 2(-(\varepsilon-\varepsilon'\lambda))\Big)\right)\right\} -$$
$$\frac{Q(n-n'\lambda)}{2\tau}\left\{-\left(\frac{1}{2(1-\alpha)(n-n'\lambda)}\Big(\cos 2\big((1-\alpha)(n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\big) - \cos 2(-(\varepsilon-\varepsilon'\lambda))\Big)\right)\right\} (2.26)$$

The second term on the right side of (2.26) is ignored since if $\alpha = 1$ according to the summation rule the expression in the parenthesis will otherwise be infinite and will not be a useful result in this work. Then

$$A_{\alpha} = \frac{(a-b\lambda)^4(n-n'\lambda)}{16\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)}\left\{\cos 2(\varepsilon-\varepsilon'\lambda) - \cos 2\big((1+\alpha)2\pi-(\varepsilon-\varepsilon'\lambda)\big)\right\} \ (2.27)$$

$$A_{\alpha} = -\frac{(a-b\lambda)^4(n-n'\lambda)}{8\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)}\left\{\sin 2\big((1+\alpha)\pi\big)\sin 2\big((\varepsilon-\varepsilon'\lambda)-(1+\alpha)\pi\big)\right\} \quad (2.28)$$

where $\alpha = 1, 2, 3, \ldots, \infty$, and therefore leaving the dimension of A_{α} in m/s .Finally by following the same step and procedure that led to (2.28) we can solve for B_{α} as follows.

$$B_{\alpha} = \frac{Q(n-n'\lambda)}{2\tau}\int_0^{\tau}\left\{\cos 2\big((1-\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big) - \cos 2\big((1+\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\right\}dt \ (2.29)$$

$$B_\alpha = \frac{Q(n-n'\lambda)}{2\tau}\left\{\frac{1}{2(1-\alpha)(n-n'\lambda)}\left(\sin 2\left((1-\alpha)(n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\right)-\sin 2(-(\varepsilon-\varepsilon'\lambda))\right\}-\right.$$

$$\frac{Q(n-n'\lambda)}{2\tau}\left\{\frac{1}{2(1+\alpha)(n-n'\lambda)}\left(\sin 2\left((1+\alpha)(n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\right)-\sin 2(-(\varepsilon-\varepsilon'\lambda))\right)\right\} \quad (2.30)$$

$$B_\alpha = -\frac{(a-b\lambda)^4(n-n'\lambda)}{16\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)}\left\{\sin 2(\varepsilon-\varepsilon'\lambda)+\sin 2\left((1+\alpha)2\pi-(\varepsilon-\varepsilon'\lambda)\right)\right\} \quad (2.31)$$

$$B_\alpha = -\frac{(a-b\lambda)^4(n-n'\lambda)}{8\pi(a^2-b^2\lambda^2)^{3/2}(1+\alpha)}\left\{\sin 2\left((1+\alpha)\pi\right)\cos 2\left((\varepsilon-\varepsilon'\lambda)-(1+\alpha)\pi\right)\right\} \quad (2.32)$$

The dimension of B_α is the same as m/s .Upon adding the squares of (2.28) and (2.32) the Fourier coefficients C_α in (2.16)becomes

$$C_\alpha = \left(\frac{(a-b\lambda)^4(n-n'\lambda)}{8\pi\sqrt{(a^2-b^2\lambda^2)^3}\,(1+\alpha)}\right)\sin 2\left((1+\alpha)\ \pi\right) \quad (2.33)$$

Then finally, we can now substitute (2.24) and (2.33) into (2.14) so that the Fourier analysis of the oscillatingamplitude of the CCW becomes

$$F[v_m] = \left(\frac{(a-b\lambda)^4(n-n'\lambda)\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{4\pi\sqrt{(a^2-b^2\lambda^2)^3}\,\sin 2(\varepsilon-\varepsilon'\lambda)}\right)+\frac{(a-b\lambda)^4(n-n'\lambda)}{8\pi\sqrt{(a^2-b^2\lambda^2)^3}}\sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\alpha)}\right)\times$$
$$\left\{\sin 2\left((1+\alpha)\ \pi\right)\sin 2\left(\alpha(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\} \quad (2.34)$$

Equation (2.34) represents the Fourier transform of the maximum velocity v_m of the CCW and the unit is m/s or rad. /s. The relevant unit to apply is self consistent. We should understand that since (2.34) represents the Fourier transform of the oscillating amplitude which is also the velocity of the CCW, and then it is also the maximum velocity v_m with which the CCW is being propagated. Hence

$$v_m = F[v_m] = \left(\frac{(a-b\lambda)^4(n-n'\lambda)\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{4\pi\sqrt{(a^2-b^2\lambda^2)^3}\,\sin 2(\varepsilon-\varepsilon'\lambda)}\right)+\frac{(a-b\lambda)^4(n-n'\lambda)}{8\pi\sqrt{(a^2-b^2\lambda^2)^3}}\sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\alpha)}\right)\times$$
$$\left\{\sin 2\left((1+\alpha)\ \pi\right)\sin 2\left(\alpha(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\} \quad (2.35)$$

However, in this work we are going to use the unit of is m/s and not rad. /s.

$$v_m^2 = \left(\frac{(a-b\lambda)^4(n-n'\lambda)\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{4\pi\sqrt{(a^2-b^2\lambda^2)^3}\,\sin 2(\varepsilon-\varepsilon'\lambda)}\right)^2+$$
$$\left(\frac{(a-b\lambda)^8(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{16\pi^2(a^2-b^2\lambda^2)^3\sin 2(\varepsilon-\varepsilon'\lambda)}\right)\times\sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\alpha)}\right)\left\{\sin 2\left((1+\alpha)\ \pi\right)\sin 2\left(\alpha(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}$$
$$+\frac{(a-b\lambda)^8(n-n'\lambda)^2}{64\pi^2(a^2-b^2\lambda^2)^3}\sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\alpha)}\right)^2\times$$
$$\left\{\sin^2\left(2(1+\alpha)\ \pi\right)\sin^2\left(2\alpha(n-n'\lambda)t-2(\varepsilon-\varepsilon'\lambda)\right)\right\} \quad (2.36)$$

The unit of the square of the maximum velocity v_m^2 of the CCW is therefore m^2/s^2.

2.5 Fourier series expansion of the spatial oscillating phase $f(\theta)$ of the CCW.

$$F[f(\theta)] = C_0 + C_1\cos\left(\vec{k}.\hat{r}-\left((n-n'\lambda)t+E(t)\right)\right)+C_2\cos\left(\vec{k}.\hat{r}-2\left((n-n'\lambda)t+E(t)\right)\right)+$$
$$C_3\cos\left(\vec{k}.\hat{r}-3\left((n-n'\lambda)t+E(t)\right)\right)+\ldots+C_\beta\cos\left(\vec{k}.\hat{r}-\beta\left((n-n'\lambda)t+E(t)\right)\right) \quad (2.37)$$

$$F[f(\theta)] = C_0 + \sum_{\beta=1}^{\infty} C_\beta \cos\left(\vec{k}.\hat{r} - \beta((n-n'\lambda)t + E(t))\right) \quad (2.38)$$

However, there is need to separate the function in the summation sign into two components.

$$C_\beta \cos\left(\vec{k}.\hat{r} - \beta((n-n'\lambda)t + E(t))\right) = A_\beta \cos\beta\left((n-n'\lambda)t + E(t)\right) + B_\beta \sin\beta\left((n-n'\lambda)t + E(t)\right) \quad (2.39)$$

With the assumption that

$$\left.\begin{aligned} A_\beta &= C_\beta \cos(\vec{k}..\hat{r}) \\ B_\beta &= -C_\beta \sin(\vec{k}..\hat{r}) \end{aligned}\right\} \Rightarrow C_\beta = \sqrt{A_\beta^2 + B_\beta^2} \qquad (2.40)$$

When we make the assumption that $\beta = 0$, then (2.38) approximates to

$$C_0 = \frac{1}{\cos(\vec{k}.\hat{r})} A_0 \qquad (2.41)$$

where A_0 , A_β and B_β are the Fourier coefficients of the series expansion of the CCW to be determined.

2.6 Determination of the Fourier coefficients of the spatial oscillating phase $f(\theta)$ of the CCW.

The Fourier coefficients of $F[f(\theta)]$ in (2.38) are given by the Euler formulas

$$A_0 = \frac{1}{\tau}\int_0^\tau f(\theta)\,dt = \frac{1}{\tau}\int_0^\tau \cos\left(\vec{k}.\hat{r} - (n-n'\lambda)t - E(t)\right)dt \quad (2.42)$$

$$A_\beta = \frac{1}{\tau}\int_0^\tau f(\theta)\cos\beta((n-n'\lambda)t + E(t))\,dt = \frac{1}{\tau}\int_0^\tau \cos\left(\vec{k}.\hat{r} - (n-n'\lambda)t - E(t)\right)\cos\beta\left((n-n'\lambda)t + E(t)\right)dt$$

(2.43)

$$B_\beta = \frac{1}{\tau}\int_0^\tau f(\theta)\sin\beta\left((n-n'\lambda)t + E(t)\right)dt = \frac{1}{\tau}\int_0^\tau \cos\left(\vec{k}.\hat{r} - (n-n'\lambda)t - E(t)\right)\sin\beta\left((n-n'\lambda)t + E(t)\right)dt$$

(2.44)

To integrate (2.42) we should know that the total phase angle E is also a function of time t . Thus by substitution method we simply write

$$u = \vec{k}.\hat{r} - (n-n'\lambda)t - E(t) \Rightarrow \frac{du}{dt} = -(n-n'\lambda) + Z(t) \Rightarrow dt = -\left(\frac{1}{(n-n'\lambda) - Z(t)}\right)du \quad (2.45)$$

$$A_0 = -\frac{1}{\tau}\left\{\frac{\sin\left(\vec{k}.\hat{r} - (n-n'\lambda)\tau - E(\tau)\right)}{((n-n'\lambda) - Z(\tau))} - \frac{\sin\left(\vec{k}.\hat{r} - E(0)\right)}{((n-n'\lambda) - Z(0))}\right\} (2.46)$$

$$A_0 = \frac{(n-n'\lambda)}{2\pi}\left\{\frac{\sin\left(\vec{k}.\hat{r} - E(0)\right)}{((n-n'\lambda) - Z(0))} - \frac{\sin\left(\vec{k}.\hat{r} - 2\pi - E(\tau)\right)}{((n-n'\lambda) - Z(\tau))}\right\} \quad (2.47)$$

$$C_0 = \frac{(n-n'\lambda)}{2\pi\cos(\vec{k}.\hat{r})}\left\{\frac{\sin\left(\vec{k}.\hat{r} - E(0)\right)}{((n-n'\lambda) - Z(0))} - \frac{\sin\left(\vec{k}.\hat{r} - 2\pi - E(\tau)\right)}{((n-n'\lambda) - Z(\tau))}\right\} \quad (2.48)$$

Also upon using trigonometric relations we can further reduce (2.43) as

$$A_\beta = \frac{1}{2\tau}\left\{\int_0^\tau \cos\left(\vec{k}.\hat{r} - (1-\beta)(n-n'\lambda)t - (1-\beta)E(t)\right)dt + \int_0^\tau \cos\left(\vec{k}.\hat{r} - (1+\beta)(n-n'\lambda)t - (1+\beta)E(t)\right)dt\right\}$$

(2.49)

$$A_\beta = \frac{1}{2\tau}\left\{-\left(\frac{\sin\left(\vec{k}.\hat{r} - (1-\beta)((n-n'\lambda)\tau + E(\tau))\right)}{(1-\beta)((n-n'\lambda) - Z(\tau))}\right) + \left(\frac{\sin\left(\vec{k}.\hat{r} - (1-\beta)E(0)\right)}{(1-\beta)((n-n'\lambda) - Z(0))}\right)\right\} +$$

$$\frac{1}{2\tau}\left\{-\left(\frac{\sin\left(\vec{k}.\hat{r} - (1+\beta)((n-n'\lambda)\tau + E(\tau))\right)}{(1+\beta)((n-n'\lambda) - Z(\tau))}\right) + \left(\frac{\sin\left(\vec{k}.\hat{r} - (1+\beta)E(0)\right)}{(1+\beta)((n-n'\lambda) - Z(0))}\right)\right\} \quad (2.50)$$

The first term on the right side of (2.50) is ignored since it becomes infinite if $\beta = 1$. As a result,

$$A_\beta = \frac{(n-n'\lambda)}{4\pi}\left\{\left(\frac{\sin\left(\vec{k}.\vec{r}-(1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda)-Z(0)\right)}\right)-\left(\frac{\sin\left(\vec{k}.\vec{r}-(1+\beta)\left(2\pi+E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda)-Z(\tau)\right)}\right)\right\} \quad (2.51)$$

Finally, when we follow the same arithmetic method that led to (2.51) we can solve for B_β in (2.44).

$$B_\beta = \frac{1}{2\tau}\left\{\int_0^\tau \sin\left(\vec{k}.\vec{r}-(1-\beta)\left((n-n'\lambda)t-E(t)\right)-(1-\beta)E(t)\right)-\right.$$

$$\left.\int_0^\tau \sin\left(\vec{k}.\vec{r}-(1+\beta)(n-n'\lambda)t-(1+\beta)E(t)\right)\right\}dt \quad (2.52)$$

$$B_\beta = \frac{(n-n'\lambda)}{4\pi}\left\{\left(\frac{\cos\left(\vec{k}.\vec{r}-(1+\beta)\left(2\pi+E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda)-Z(\tau)\right)}\right)-\left(\frac{\cos\left(\vec{k}.\vec{r}-(1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda)-Z(0)\right)}\right)\right\} \quad (2.53)$$

Eventually upon adding the squares of (2.51) and (2.53) according to (2.40) we obtain

$$C_\beta^2 = \left\{\frac{(n-n'\lambda)}{4\pi}\left(\frac{\sin\left(\vec{k}.\vec{r}-(1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda)-Z(0)\right)}\right)-\left(\frac{\sin\left(\vec{k}.\vec{r}-(1+\beta)\left(2\pi+E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda)-Z(\tau)\right)}\right)\right\}^2+$$

$$\left\{\frac{(n-n'\lambda)}{4\pi}\left(\frac{\cos\left(\vec{k}.\vec{r}-(1+\beta)\left(2\pi+E(\tau)\right)\right)}{(1+\beta)\left((n-n'\lambda)-Z(\tau)\right)}\right)-\left(\frac{\cos\left(\vec{k}.\vec{r}-(1+\beta)E(0)\right)}{(1+\beta)\left((n-n'\lambda)-Z(0)\right)}\right)\right\}^2 \quad (2.54)$$

$$C_\beta = \left(\frac{(n-n'\lambda)}{4\pi(1+\beta)}\right)\left\{\frac{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}{\left((n-n'\lambda)-Z(0)\right)^2\left((n-n'\lambda)-Z(\tau)\right)^2}-\frac{2\cos\left((1+\beta)(2\pi+E(\tau)-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)\left((n-n'\lambda)-Z(\tau)\right)}\right\}^{\frac{1}{2}} \quad (2.55)$$

Consequently, when we substitute the values of C_0 and C_β into (2.38) and after some simplifications we get

$$F\left[f(\theta)\right] = \frac{(n-n'\lambda)}{2\pi\cos\left(\vec{k}.\vec{r}\right)}\left\{\frac{\sin\left(\vec{k}.\vec{r}-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)}-\frac{\sin\left(\vec{k}.\vec{r}-2\pi-E(\tau)\right)}{\left((n-n'\lambda)-Z(\tau)\right)}\right\}+$$

$$\left(\frac{(n-n'\lambda)}{4\pi}\right)\sum_{\beta=1}^{\infty}\left(\frac{1}{1+\beta}\right)\left\{\frac{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}{\left((n-n'\lambda)-Z(0)\right)^2\left((n-n'\lambda)-Z(\tau)\right)^2}-\right.$$

$$\left.\frac{2\cos\left((1+\beta)\,2\pi+E(\tau)-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)\left((n-n'\lambda)-Z(\tau)\right)}\right\}^{\frac{1}{2}}\cos\left(\vec{k}.\vec{r}-\left(\beta(n-n')t+E(t)\right)\right) \quad (2.56)$$

Thus the Fourier transform of the spatial oscillating phase of the CCW is given by (2.56) and it has no unit.

2.7 Convolution theory of the Fourier transform of the oscillating amplitude and the spatial oscillatingphase of the CCW.

Now that we have separately determined the Fourier transform of the oscillating amplitude $F\left[v_m\right]$ and the spatial oscillating phase $F\left[f(\theta)\right]$ respectively the necessary requirement now is to convolute them in order to obtain a concise velocity equation of the CCW. Convolution here means multiplying (2.35) by (2.56) term by term. Let us represent the result of the convolution of these functions by H.

$$velocity\ v = H\left\{F\left[v_m\right];F\left[f(\theta)\right]\right\}=F\left[v_m\right]\otimes F\left[f(\theta)\right] \quad (2.57)$$

$$v = H\left\{F\left[v_m\right];F\left[f(\theta)\right]\right\}= \left(\frac{(a-b\lambda)^4(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{8\pi^2(a^2-b^2\lambda^2)^{3/2}\sin 2(\varepsilon-\varepsilon'\lambda)\cos\left(\vec{k}.\vec{r}\right)}\right)\times$$

$$\left\{\frac{\sin\left(\vec{k}.\vec{r}-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)}-\frac{\sin\left(\vec{k}.\vec{r}-2\pi-E(\tau)\right)}{\left((n-n'\lambda)-Z(\tau)\right)}\right\}+\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{16\pi^2(a^2-b^2\lambda^2)^{3/2}\sin 2(\varepsilon-\varepsilon'\lambda)}\right)\times$$

$$\sum_{\beta=1}^{\infty}\left(\frac{1}{1+\beta}\right)\left\{\frac{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}{\left((n-n'\lambda)-Z(0)\right)^2\left((n-n'\lambda)-Z(\tau)\right)^2}-\frac{2\cos\left((1+\beta)2\pi+E(\tau)-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)\left((n-n'\lambda)-Z(\tau)\right)}\right\}^{\frac{1}{2}}\times$$

$$\cos\left(\vec{k}.\vec{r}-\beta\left((n-n'\lambda)t+E(t)\right)\right)+\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{16\pi^2(a^2-b^2\lambda^2)^{3/2}\cos\left(\vec{k}.\vec{r}\right)}\right)\times$$

$$\left\{\frac{\sin\left(\vec{k}.\vec{r}-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)}-\frac{\sin\left(\vec{k}.\vec{r}-2\pi-E(\tau)\right)}{\left((n-n'\lambda)-Z(\tau)\right)}\right\}\times\sum_{\beta=1}^{\infty}\left(\frac{1}{(1+\beta)}\right)\left\{\sin 2\left((1+\beta)\ \pi\right)\sin 2\left(\beta(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}+$$

$$\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2}{32\pi^2(a^2-b^2\lambda^2)^{3/2}}\right)\sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\beta)}\right)^2\left\{\sin 2\left((1+\beta)\ \pi\right)\sin 2\left(\beta(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}\times$$

$$\left\{\frac{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}{\left((n-n'\lambda)-Z(0)\right)^2\left((n-n'\lambda)-Z(\tau)\right)^2}-\frac{2\cos\left((1+\beta)2\pi+E(\tau)-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)\left((n-n'\lambda)-Z(\tau)\right)}\right\}^{\frac{1}{2}}\times$$

$$\cos\left(\vec{k}.\vec{r}-\beta\left((n-n')t+E(t)\right)\right)\qquad(2.58)$$

In most cases, equation (2.58) may yield imaginary values and not absolute values due to the expression in the square root sign. In order to avoid such unnecessary complicationsthere is need for us to use Binomial expansion to find an approximation to the expression in the square root and once this is done we get

$$v=H\left\{F\left[v_m\right];F\left[f(\theta)\right]\right\}=\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{8\pi^2(a^2-b^2\lambda^2)^{3/2}\sin 2(\varepsilon-\varepsilon'\lambda)\cos\left(\vec{k}.\vec{r}\right)}\right)\times$$

$$\left\{\frac{\sin\left(\vec{k}.\vec{r}-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)}-\frac{\sin\left(\vec{k}.\vec{r}-2\pi-E(\tau)\right)}{\left((n-n'\lambda)-Z(\tau)\right)}\right\}+\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{16\pi^2(a^2-b^2\lambda^2)^{3/2}\sin 2(\varepsilon-\varepsilon'\lambda)}\right)\times$$

$$\sum_{\beta=1}^{\infty}\left(\frac{1}{1+\beta}\right)\left(\frac{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}{\left((n-n'\lambda)-Z(0)\right)^2\left((n-n'\lambda)-Z(\tau)\right)^2}\right)^{\frac{1}{2}}\times$$

$$\left\{1-\frac{\left((n-n'\lambda)-Z(0)\right)\left((n-n'\lambda)-Z(\tau)\right)\cos\left((1+\beta)\,2\pi+E(\tau)-E(0)\right)}{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}\right\}\times$$

$$\cos\left(\vec{k}.\vec{r}-\beta\left((n-n'\lambda)t+E(t)\right)\right)+\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2\sin 2(\pi)\sin 2\left((\varepsilon-\varepsilon'\lambda)-\pi\right)}{16\pi^2(a^2-b^2\lambda^2)^{3/2}\cos\left(\vec{k}.\vec{r}\right)}\right)\times$$

$$\left\{\frac{\sin\left(\vec{k}.\vec{r}-E(0)\right)}{\left((n-n'\lambda)-Z(0)\right)}-\frac{\sin\left(\vec{k}.\vec{r}-2\pi-E(\tau)\right)}{\left((n-n'\lambda)-Z(\tau)\right)}\right\}\times\sum_{\beta=1}^{\infty}\left(\frac{1}{(1+\beta)}\right)\left\{\sin 2\left((1+\beta)\ \pi\right)\times\right.$$

$$\left.\sin 2\left(\beta(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}+$$

$$\left(\frac{(a-b\lambda)^4(n-n'\lambda)^2}{32\pi^2(a^2-b^2\lambda^2)^{3/2}}\right)\sum_{\alpha=1}^{\infty}\left(\frac{1}{(1+\beta)}\right)^2\left(\frac{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}{\left((n-n'\lambda)-Z(0)\right)^2\left((n-n'\lambda)-Z(\tau)\right)^2}\right)^{\frac{1}{2}}\times$$

$$\left\{1-\frac{\left((n-n'\lambda)-Z(0)\right)\left((n-n'\lambda)-Z(\tau)\right)\cos\left((1+\beta)\,2\pi+E(\tau)-E(0)\right)}{\left((n-n'\lambda)-Z(\tau)\right)^2+\left((n-n'\lambda)-Z(0)\right)^2}\right\}\times$$

$$\cos\left(\vec{k}\,.\,\hat{r}-\beta\left((n-n'\lambda)t+E(t)\right)\right)\times\left\{\sin 2\left((1+\beta)\;\pi\right)\sin 2\left(\beta(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}\text{(2.59)}$$

Thus (2.59) is the Fourier transform of the actual velocity of the CCW, since it is the convolution of the oscillating amplitude and the spatial oscillating phase. In this work we only considered situation where the constraints are of equal weight, that is $\alpha=\beta$. Otherwise, if we apply the double summation rule as it stands, that means, we shall first allow α take the value of one and let β run from one to infinity, again we allow α take the value of two and let β run from one to infinity and the process is repeated. However, since both constraints are of the same source function we can equate them so as to save us computation time and unnecessary difficult task.

$$E(\tau)=\tan^{-1}\left(\frac{a\sin\varepsilon+b\lambda\sin(\varepsilon'\lambda-2\pi)}{a\cos\varepsilon+b\lambda\cos(\varepsilon'\lambda-2\pi)}\right)\;;\quad E(0)=\tan^{-1}\left(\frac{a\sin\varepsilon+b\lambda\sin(\varepsilon'\lambda)}{a\cos\varepsilon+b\lambda\cos(\varepsilon'\lambda)}\right)\text{(2.60)}$$

$$Z(t)=(n-n'\lambda)\left(\frac{b^2\lambda^2+ab\lambda\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)}{a^2+b^2\lambda^2+2ab\lambda\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)}\right)\text{(2.61)}$$

$$Z(0)=(n-n'\lambda)\left(\frac{b^2\lambda^2+ab\lambda\cos(\varepsilon-\varepsilon'\lambda)}{a^2+b^2\lambda^2+2ab\lambda\cos(\varepsilon-\varepsilon'\lambda)}\right);Z(\tau)=(n-n'\lambda)\left(\frac{b^2\lambda^2+ab\lambda\cos\left((\varepsilon-\varepsilon'\lambda)+2\pi\right)}{a^2+b^2\lambda^2+2ab\lambda\cos\left((\varepsilon-\varepsilon'\lambda)+2\pi\right)}\right)\text{(2.62)}$$

(2.63)

However, in the absence of HIV/AIDS in which case $\lambda=0$ we get

$$E(\tau)=E(0)=\varepsilon\;;\;Z(t)=Z(\tau)=Z(0)=0\;;\;k\,.\,\hat{r}=k\;(\cos\phi+\sin\phi)\,;\;\phi=\pi-\varepsilon\,. \qquad (2.64)$$

Hence, ordinarily (2.59) is the equation of the Fourier transform of the actual velocity of the CCW. This velocity can no longer be a maximum since it is convoluted with the spatial oscillating phase.

2.8 Fourier series expansion of the maximum oscillating amplitude $f(A)$ of the CCW in terms of maximum displacement y_m

We know that if the spatial oscillating phase of the CCW is equal to one, that is $f(\theta)=1$, then the oscillating amplitude $f(A)$ of the CCW becomes a maximum. Hence we can write for the maximum displacement vector

$$y_m=f(A)=\left\{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}^{\frac{1}{2}}\quad(2.65)$$

Now let us expand the maximum oscillating amplitude which is now the maximum value of the displacement vector of the CCW in terms of Fourier series. The process is similar to those of (2.13) – (2.36).

$$y_m=F\left[f(A)\right]=C_0+C_1\left(\cos\left(1(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)+C_2\left(\cos\left(2(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)+$$
$$C_3\left(\cos\left(3(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)+\ldots+C_\alpha\left(\cos\left(\alpha(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)\quad(2.66)$$

$$y_m=F\left[f(A)\right]=C_0+\sum_{\alpha=1}^{\infty}C_\alpha\left(\cos\left(\alpha(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)\qquad(2.67)$$

Accordingly, we can further decompose(2.67) as follows.

$$C_\alpha\left(\cos\left(\alpha(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)=A_\alpha\cos\alpha(n-n'\lambda)t+B_\alpha\sin\alpha(n-n'\lambda)t\qquad(2.68)$$

We can specify the modulation amplitudes A_α and B_α as components of the variable phase angle as

$$\left.\begin{aligned}A_\alpha&=C_\alpha\cos(\varepsilon-\varepsilon'\lambda)\\B_\alpha&=-C_\alpha\sin(\varepsilon-\varepsilon'\lambda)\end{aligned}\right\}\Rightarrow C_\alpha=\sqrt{A_\alpha^2+B_\alpha^2}\qquad(2.69)$$

Thus (2.69) represents the amplitude of the nth harmonic and α is the Fourier index. From (2.68), if $\alpha=0$

$$C_0=\frac{1}{\cos(\varepsilon-\varepsilon'\lambda)}A_0\because\;(\cos(-x)=\cos x\,)\qquad(2.70)$$

We can now conveniently replace (2.68) and (2.69) into (2.67) and get as follows.

$$y_m=F\left[f(A)\right]=\frac{1}{\cos(\varepsilon-\varepsilon'\lambda)}A_0+\sum_{\alpha=1}^{\infty}C_\alpha\left(A_\alpha\cos\left(\alpha(n-n'\lambda)t\right)+B_\alpha\sin\left(\alpha(n-n'\lambda)t\right)\right)\qquad(2.71)$$

Thus generally we can use either of (2.67) or (2.71) to discuss the maximum displacement vector in relation to energy attenuation process in HIV/AIDS patients. In case (2.71) were to be used, then we should understand that it comprises of three parts; the fundamental, even (symmetric) and the odd (antisymmetric) functions respectively. Although, we can converge both the even and the odd functions which appear in the summation sign. However, for clarity of purpose and for a more direct computation we shall restrict our work to (2.67).

2.9 Determination of the Fourier coefficients of the maximum oscillating amplitude of the CCW.

The Fourier components C_α in (2.67) which is specified in (2.69) and (2.70) are given by the Euler formulas

$$A_0 = \frac{1}{\tau}\int_0^\tau f(A)\,dt \quad (2.72)$$

$$A_\alpha = \frac{1}{\tau}\int_0^\tau f(A)\cos\big(\alpha(n-n'\lambda)t\big)dt \quad (2.73)$$

$$B_\alpha = \frac{1}{\tau}\int_0^\tau f(A)\sin\big(\alpha(n-n'\lambda)t\big)dt \quad (2.74)$$

Now let us substitute (2.65) into (2.72) and solve the resulting equation for A_0

$$A_0 = \frac{1}{\tau}\int_0^\tau \left\{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\right\}^{\frac{1}{2}}dt \quad (2.75)$$

$$A_0 = \left\{\frac{\left[(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\right]^{\frac{3}{2}}}{6\pi(a-b\lambda)^2\sin\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)}\right\}_0^\tau \quad (2.76)$$

$$A_0 = \left\{\frac{\left[(a^2-b^2\lambda^2)-2(a-b\lambda)^2\big(\cos\big(2\pi-(\varepsilon-\varepsilon'\lambda)\big)-\cos(\varepsilon-\varepsilon'\lambda)\big)\right]^{\frac{3}{2}}}{6\pi(a-b\lambda)^2\left[\sin\big(2\pi-(\varepsilon-\varepsilon'\lambda)\big)+\sin\big((\varepsilon-\varepsilon'\lambda)\big)\right]}\right\} \quad (2.77)$$

$$A_0 = \left\{\frac{\left[(a^2-b^2\lambda^2)+4(a-b\lambda)^2\big(\sin(\pi)\sin(\pi-(\varepsilon-\varepsilon'\lambda))\big)\right]^{\frac{3}{2}}}{12\pi(a-b\lambda)^2\sin(\pi)\cos(\pi-(\varepsilon-\varepsilon'\lambda))}\right\} \quad (2.78)$$

$$C_0 = \left\{\frac{\left[(a^2-b^2\lambda^2)+4(a-b\lambda)^2\big(\sin(\pi)\sin(\pi-(\varepsilon-\varepsilon'\lambda))\big)\right]^{\frac{3}{2}}}{12\pi(a-b\lambda)^2\sin(\pi)\cos(\pi-(\varepsilon-\varepsilon'\lambda))\cos(\varepsilon-\varepsilon'\lambda)}\right\} \quad (2.79)$$

Also when we substitute (2.65) into (2.73) we get

$$A_\alpha = \frac{1}{\tau}\int_0^\tau \left\{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\right\}^{\frac{1}{2}}\cos\big(\alpha(n-n'\lambda)t\big)dt \quad (2.80)$$

For similarity and easy computation there is the need for us to find an approximation to the term in the square root parenthesis of (2.80). The approximation should be the stationary state of the maximum oscillating amplitude so that the initial unit of the CCW is not altered in the course of the approximation. This approximation would simply mean to differentiate the resulting binomial expansion with respect to the phase angle δ. Thus

$$\left\{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\right\}^{\frac{1}{2}} =$$
$$\left(a^2-b^2\lambda^2\right)^{\frac{1}{2}}\frac{d}{d(\varepsilon-\varepsilon'\lambda)}\left\{1-2\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\right\}^{\frac{1}{2}} \quad (2.81)$$

$$\left\{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)\right\}^{\frac{1}{2}} =$$
$$\left(a^2-b^2\lambda^2\right)^{\frac{1}{2}}\frac{d}{d(\varepsilon-\varepsilon'\lambda)}\left\{1-\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\cos\big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)+\ldots\right\} \quad (2.82)$$

$$\left\{(a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}^{\frac{1}{2}} = -\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)^{1/2}}\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\text{(2.83)}$$

$$f(A) = -\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)^{1/2}}\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\text{(2.84)}$$

Thus the oscillating amplitude still retain the initial unit of meters (m) which is length.
Now upon replacing (2.84) in (2.80) we realize

$$A_\alpha = -\frac{(a-b\lambda)^2}{\tau(a^2-b^2\lambda^2)^{1/2}}\int_0^\tau \sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\cos\left(\alpha(n-n'\lambda)t\right)dt \quad (2.85)$$

$$A_\alpha = -\frac{(a-b\lambda)^2}{\tau(a^2-b^2\lambda^2)^{1/2}}\left\{\frac{1}{2}\int_0^\tau \sin\left((1+\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)+\frac{1}{2}\int_0^\tau \sin\left((1-\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}dt$$

(2.86)

$$A_\alpha = \frac{(a-b\lambda)^2}{2\tau(a^2-b^2\lambda^2)^{1/2}}\left\{\left(\frac{1}{1+\alpha}\right)\left(\frac{1}{(n-n'\lambda)}\right)\cos\left((1+\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}_0^\tau +$$

$$\frac{(a-b\lambda)^2}{2\tau(a^2-b^2\lambda^2)^{1/2}}\left\{\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{(n-n'\lambda)}\right)\cos\left((1+\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}_0^\tau \quad (2.87)$$

The last term of (2.87) is ignored since the equation becomes infinite if $\alpha = 1$ and further negative values may not also be useful. Thus by substituting the boundary values and using $\tau(n-n'\lambda) = 2\pi$ we get

$$A_\alpha = \frac{(a-b\lambda)^2}{2\tau(a^2-b^2\lambda^2)^{1/2}}\left(\frac{1}{1+\alpha}\right)\left(\frac{1}{(n-n'\lambda)}\right)\left\{\cos\left((1+\alpha)2\pi-(\varepsilon-\varepsilon'\lambda)\right)-\cos\left((\varepsilon-\varepsilon'\lambda)\right)\right\} \quad (2.88)$$

$$A_\alpha = -\frac{(a-b\lambda)^2}{2\pi(a^2-b^2\lambda^2)^{1/2}}\left(\frac{1}{1+\alpha}\right)\left\{\sin\left((1+\alpha)\pi-(\varepsilon-\varepsilon'\lambda)\right)\sin(1+\alpha)\pi\right\} \quad (2.89)$$

Finally, when we substitute (2.84) into (2.74) we get

$$B_\alpha = -\frac{(a-b\lambda)^2}{\tau(a^2-b^2\lambda^2)^{1/2}}\int_0^\tau \sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\sin\left(\alpha(n-n'\lambda)t\right)dt \quad (2.90)$$

$$B_\alpha = -\frac{(a-b\lambda)^2}{\tau(a^2-b^2\lambda^2)^{1/2}}\left\{\frac{1}{2}\int_0^\tau \cos\left((1-\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)-\frac{1}{2}\int_0^\tau \cos\left((1+\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}dt \quad (2.91)$$

$$B_\alpha = -\frac{(a-b\lambda)^2}{2\tau(a^2-b^2\lambda^2)^{1/2}}\left\{\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{(n-n'\lambda)}\right)\sin\left((1+\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}_0^\tau +$$

$$\frac{(a-b\lambda)^2}{2\tau(a^2-b^2\lambda^2)^{1/2}}\left\{\left(\frac{1}{1+\alpha}\right)\left(\frac{1}{(n-n'\lambda)}\right)\sin\left((1+\alpha)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}_0^\tau \quad (2.92)$$

The first term of (2.92) is ignored since the equation becomes infinite if $\alpha = 1$ which is not a useful result. Thus

$$B_\alpha = \frac{(a-b\lambda)^2}{4\pi(a^2-b^2\lambda^2)^{1/2}}\left\{\left(\frac{1}{1+\alpha}\right)\sin\left((1+\alpha)2\pi-(\varepsilon-\varepsilon'\lambda)\right)+\sin\left((\varepsilon-\varepsilon'\lambda)\right)\right\} \quad (2.93)$$

$$B_\alpha = \frac{(a-b\lambda)^2}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\left\{\left(\frac{1}{1+\alpha}\right)\sin\left((1+\alpha)\pi\right)\cos\left((1+\alpha)\pi-(\varepsilon-\varepsilon'\lambda)\right)\right\} \quad (2.94)$$

$$C_\alpha^2 = A_\alpha^2 + B_\alpha^2 \quad (2.95)$$

$$C_\alpha = \frac{(a-b\lambda)^2}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\left\{\left(\frac{1}{1+\alpha}\right)\sin\left((1+\alpha)\pi\right)\right\} \quad (2.96)$$

Then finally upon the substitution of (2.79) and (2.96) into (2.67) we get after some simplification that

$$y_m = F[f(A)] = \left\{ \frac{[(a^2 - b^2\lambda^2) + 4(a - b\lambda)^2(\sin(\pi)\sin(\pi - (\varepsilon - \varepsilon'\lambda)))]^{\frac{3}{2}}}{12\pi(a - b\lambda)^2 \sin(\pi)\cos(\pi - (\varepsilon - \varepsilon'\lambda))\cos(\varepsilon - \varepsilon'\lambda)} \right\} +$$

$$\frac{(a - b\lambda)^2}{2\pi\sqrt{(a^2 - b^2\lambda^2)}} \sum_{\alpha=1}^{\infty} \left(\frac{1}{1+\alpha}\right) \sin((1+\alpha)\pi) \cos(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \quad (2.97)$$

Hence the dimension of the Fourier transform of the maximum oscillating amplitude which is the same as the maximum displacement vector y_m is meters (m). Please see appendix for the relevant identities used to obtain these results. The square of the maximum displacement vector is

$$y_m^2 = \left\{ \frac{[(a^2 - b^2\lambda^2) + 4(a - b\lambda)^2(\sin(\pi)\sin(\pi - (\varepsilon - \varepsilon'\lambda)))]^{\frac{3}{2}}}{12\pi(a - b\lambda)^2 \sin(\pi)\cos(\pi - (\varepsilon - \varepsilon'\lambda))\cos(\varepsilon - \varepsilon'\lambda)} \right\}^2 +$$

$$\left\{ \frac{[(a^2 - b^2\lambda^2) + 4(a - b\lambda)^2(\sin(\pi)\sin(\pi - (\varepsilon - \varepsilon'\lambda)))]^{\frac{3}{2}}}{12\pi^2\sqrt{(a^2 - b^2\lambda^2)} \sin(\pi)\cos(\pi - (\varepsilon - \varepsilon'\lambda))\cos(\varepsilon - \varepsilon'\lambda)} \right\} \times$$

$$\sum_{\alpha=1}^{\infty} \left(\frac{1}{1+\alpha}\right) \sin((1+\alpha)\pi) \times \cos(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) +$$

$$\frac{(a - b\lambda)^4}{4\pi^2(a^2 - b^2\lambda^2)} \sum_{\alpha=1}^{\infty} \left(\frac{1}{1+\alpha}\right)^2 \sin^2((1+\alpha)\pi) \cos^2(\alpha(n - n'\lambda)t - (\varepsilon - \varepsilon'\lambda)) \quad (2.98)$$

Thus the unit of the square of the maximum displacement y_m^2 of the CCW is therefore m^2. Here $\alpha = 1, 2, 3, \ldots,$ 13070. However, in this work, we shall only consider the condition in which α = 13070 which also corresponds to the maximum value of the multiplier λ.

2.10 Evaluation of the Energy attenuation equation of the CCW

In natural systems, we can rarely find pure wave which propagates free from energy-loss mechanisms. But if these losses are not too serious we can describe the total propagation in time by a given force law $f(t)$. The propagating CCW in the micro-vascular blood circulating system is affected by three major factors: (i) the damping effect of the mass m of the surrounding blood (ii) the damping effect of the dynamic viscosity η of blood and (iii) the damping effect of the elastic μ property of blood which rest in the red blood cells. Then the force law equation governing the propagation of the CCW in the human system is given by

$$f(t) = m\frac{\partial^2 y}{\partial t^2} + \eta\frac{\partial y^2}{dt} + \mu y \qquad (2.99)$$

$$f(t) = \rho V \frac{\partial^2 y}{\partial t^2} + 2\eta y\frac{\partial y}{dt} + \mu y \quad (2.100)$$

That is, $V = n\pi r^2 l$ is the volume of blood in the capillary which is considered to be cylindrical. The capillary is the region of space available for the exchange of energy within the human system. The notation r, n and l are the radius, number and the length of the capillary respectively. However, the influence of gravity on the flow process of the CCW in the human blood system is neglected. We should also know that we are going to neglect the variables of the capillary as it may cause indeterminate results since we cannot have exact values of the number and length of the capillary. This we can achieve when we apply only the maximum values of the velocity and the displacement vector. That means, we are making the force and energy equations of the CCW independent of space.

$$f(t) = \rho V \frac{\partial}{\partial t}\left(\frac{\partial y}{\partial t}\right) + 2\eta y\frac{\partial y}{dt} + \mu y \quad (2.101)$$

$$\int f(t)dt = \rho V \int \frac{\partial}{\partial t}\left(\frac{\partial y}{\partial t}\right)dt + 2\eta y\int\left(\frac{\partial y}{\partial t}\right)dt + \mu\int ydt \quad (2.102)$$

$$\text{Impulse} = \rho V \left(\frac{\partial y}{\partial t}\right) + 2\eta y^2 + \mu\int ydt \quad (2.103)$$

$$\text{Im}pulse = \rho V \; v + 2\eta y^2 + \mu \int y dt \quad (2.104)$$

We can now multiply through (2.104) by the velocity v in order to convert the impulse to energy. Hence

$$\text{Im}pulse \times velocity \; v = \rho V \; v^2 + 2\eta v y^2 + \mu \int v \, y \, dt \quad (2.105)$$

$$Energy\,(E) = \rho V \; v^2 + 2\eta v y^2 + \mu \int \frac{\partial y}{\partial t} y \, dt \quad (2.106)$$

$$Energy\,(E) = \rho V \; v^2 + 2\eta v y^2 + \mu y^2 \quad (2.107)$$

For the energy to be a maximum then the spatial oscillating part of the CCW must be equal to one and the CCW would only have the oscillating amplitude which will now be maximum also. The velocity of the CCW is also a maximum for maximum oscillating amplitude. Hence

$$Energy\,(E) = \rho V \; v_m^2 + 2\eta v_m y_m^2 + \mu y_m^2 \quad (2.108)$$

Hence (2.108) is the energy equation which governs the propagation of the CCW in the human blood circulating system. The first two terms in the equation represent the kinetic energy term while the last term represent the potential energy term. The addition of these two terms gives the total energy of the propagating CCW. The unit of the energy is Joules or kgm^2/s^2 or Nm. We should understand here that, v_m, v_m^2 and y_m^2 comprises of two parts, the fundamental and the nth harmonic components of the Fourier transform given by (2.35), (2.36) and (2.98) respectively. Thus we are also expected to have the fundamental and the nth harmonic energies. We also assume in this work that the volume of the blood vessel, $V = 1m^3$. The unit volume we used in our calculation makes the equation invariant with respect to the volume of the blood vessel. The graphs of the energy emanating from (2.108) are presented in section 3.

2.11 Calculated values of the dynamical characteristics of the latent vibration of Man represented by the 'host wave' and those of the HIV represented by the 'parasitic wave'.

We have in a previous study (Enaibe and Osafile, 2013)presented a model for determining the dynamical characteristics of HIV/AIDS in the human blood circulating system. Our work assumes that the physical dynamic components of the HIV responsible for their destructive tendency are $b\lambda$, $n'\lambda$, $\varepsilon'\lambda$ and $k'\lambda$ been influenced by the multiplicative factor λ whose physical range of interest is $0 \le \lambda \le 13070$. In this study, we calculated values of the amplitude $b = 1.60 \times 10^{-10} m$, angular frequency $n' = 1.91 \times 10^{-11} rad./s$, phase angle $\varepsilon' = 0.0000466 rad$, the wave number or the spatial frequency $k' = 0.0127 rad./m$ of the HIV parameters and with a slow varying interval of the multiplier $\lambda = 0, 1, 2, 3, \ldots, 13070$. While the dynamical characteristics of the latent vibration of the human blood circulating system caused by the beating of the human heart also from the calculation are; amplitude $a = 2.1 \times 10^{-6} m$, angular frequency $n = 2.51 \times 10^{-7} rad./s$, phase angle $\varepsilon = 0.6109 rad$, wave number $k = 166 rad./m$. We also established in the study that the average survival time for HIV/AIDS patient is about 10 years (126 months or about 328479340 seconds) counting from the moment the HIV is contacted. However we classified the time interval in seconds as $0 \le t \le 328479340 s$, with a slow varying time interval $t = 0, 1, 2, 3, \ldots, 328479340 s$.

We used table scientific calculator and Microsoft excel to compute our results. Also the GNUPLOT 3.7 version was used to plot the corresponding graphs.

3. Results.

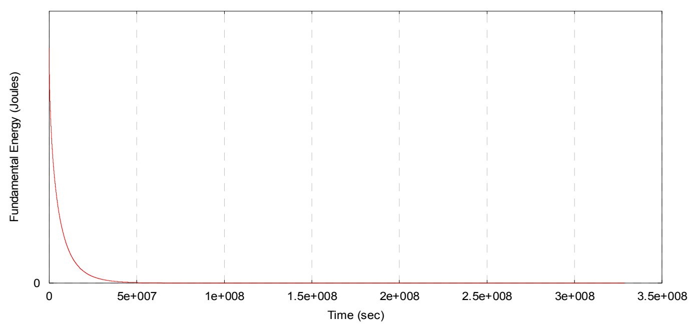

Fig 3.1: Represents the multiplier [0 – 13070] and time [0 – 126 months], β =0

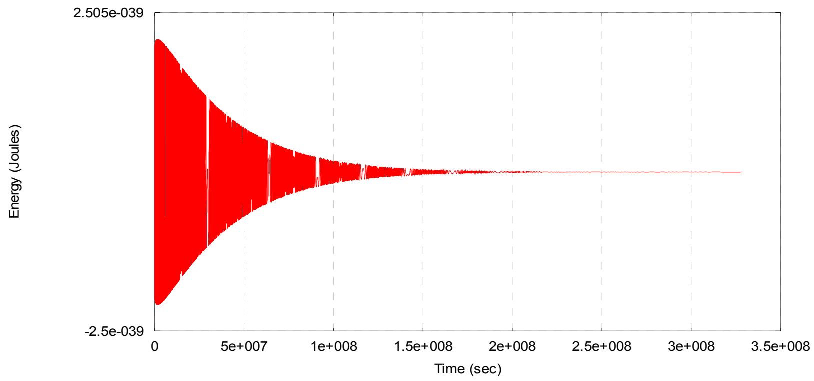

Fig 3.2: Represents the multiplier λ [0 – 13070] and time [0 – 126 months], β = 13070.

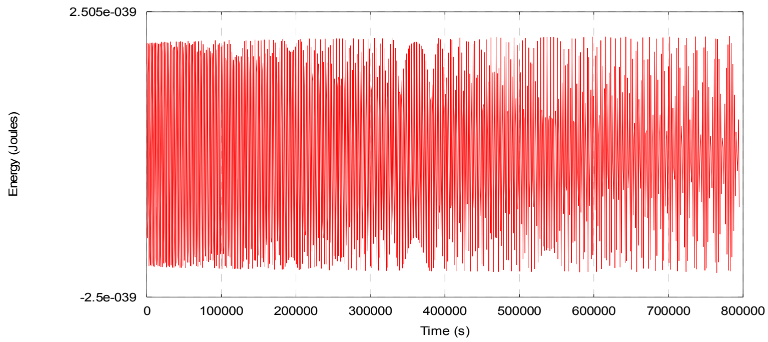

Fig 3.3: Represents the multiplier λ [0 – 1500] and time [0 – 9.2 days], β = 13070.

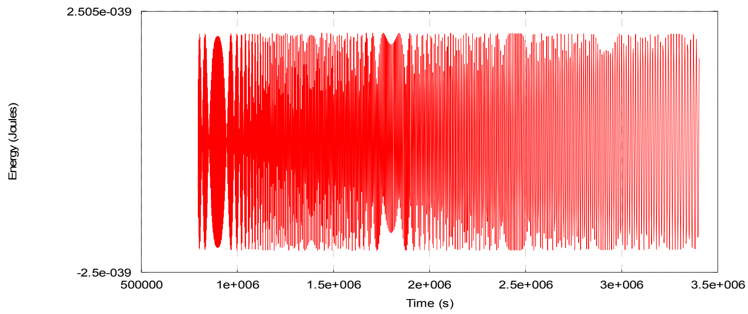

Fig 3.4: Represents the multiplier λ [1500 – 3000] and time [9.3 – 1.3 months], β = 13070.

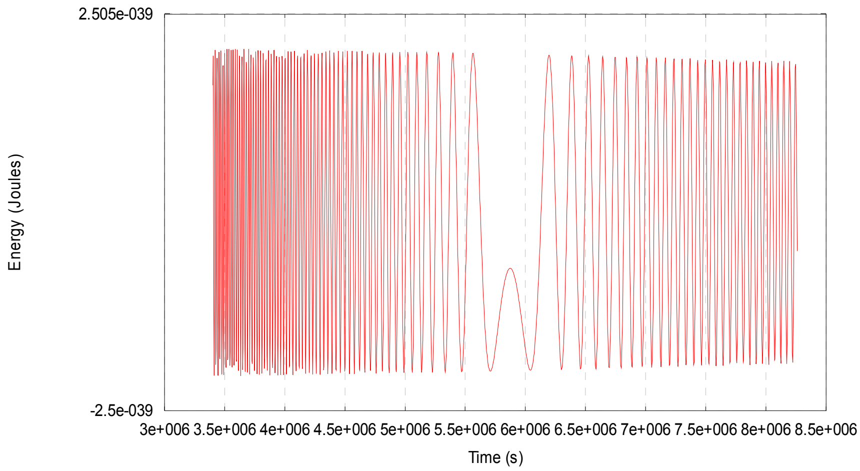

Fig 3.5: Represents the multiplier λ [3000 – 4500] and time [1.3 – 3 months], β = 13070.

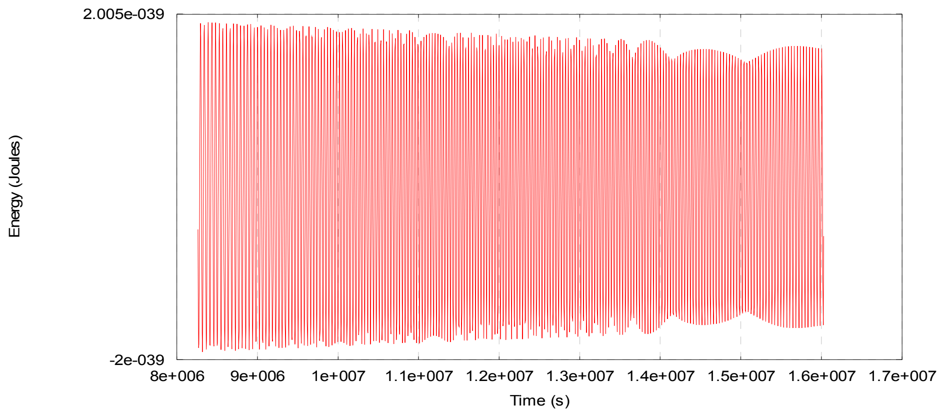

Fig 3.6: Represents the multiplier λ [4500 – 6000] and time [3 – 6 months], β = 13070

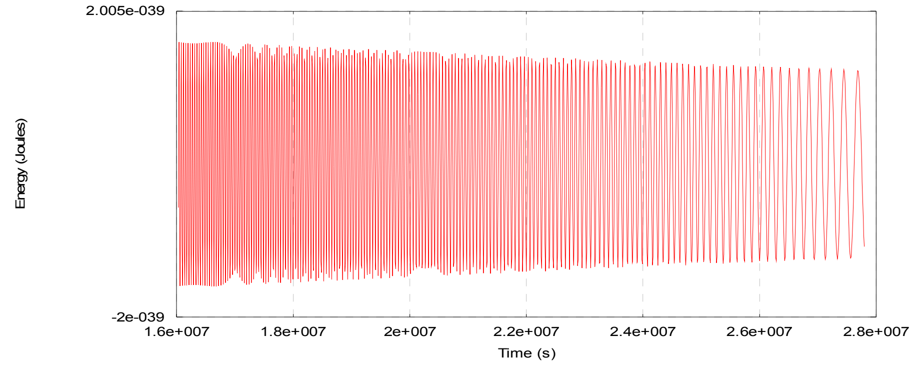

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Fig 3.7: Represents the multiplier [6000 – 7500] and time [6 – 10.7 months], β = 13070.

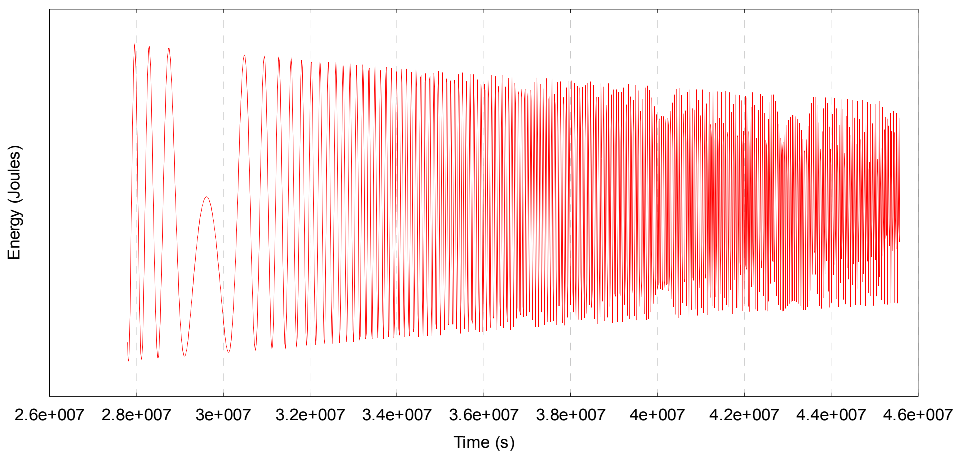

Fig 3.8: Represents the multiplier λ [7500 – 9000] and time [10.7 – 17.5 months], β = 13070.

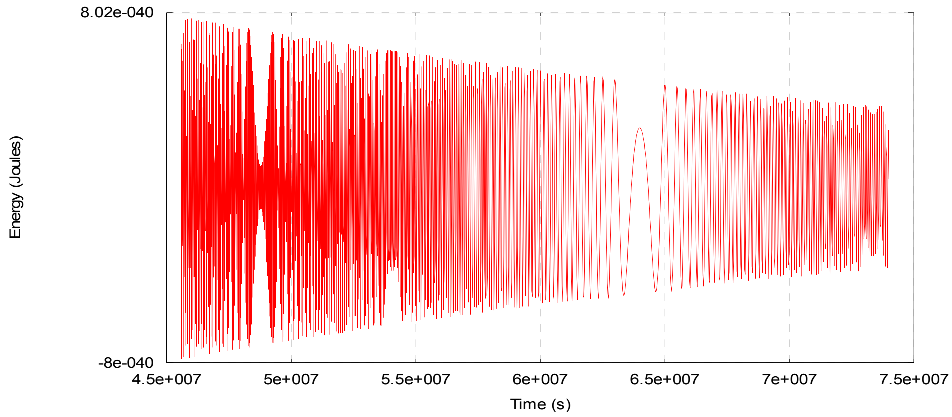

Fig 3.9: Represents the multiplier λ [9000 – 10500] and time [17.5 – 28.5 months], β = 13070.

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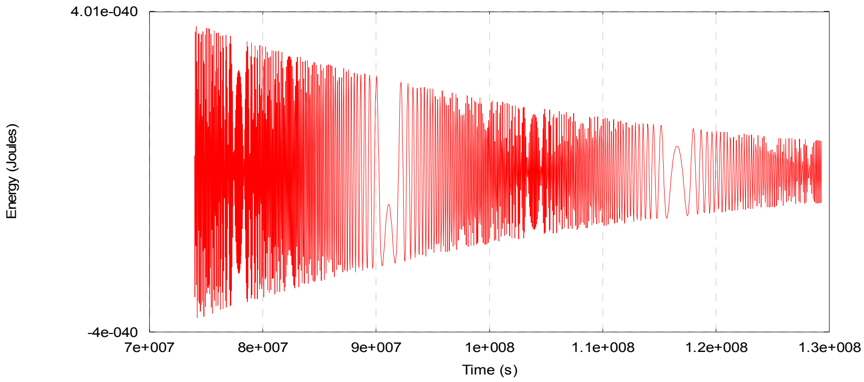

Fig 3.10: Represents the multiplier λ [10500 – 12000] and time [28.5 – 49.5 months], β = 13070.

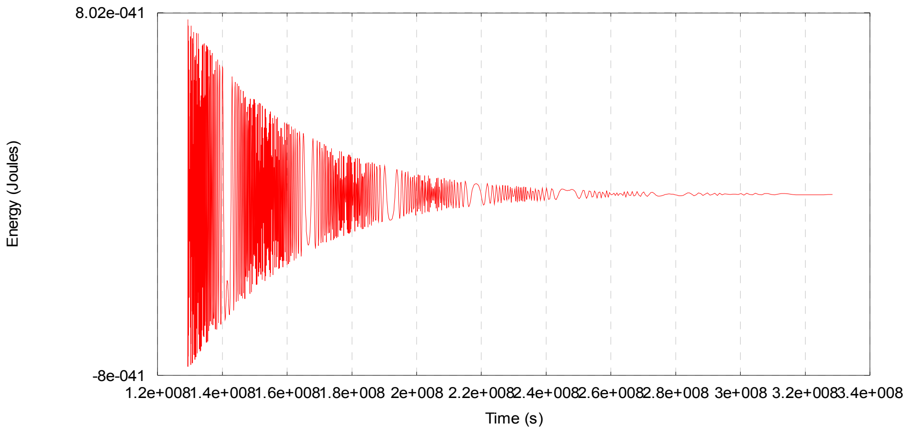

Fig 3.11: Represents the multiplier λ [12000 – 13070] and time [49.8 – 126 months], β = 13070.

4. Discussion.

The graph of the total energy spectrum of the constituted carrier wave given by equation (2.108) is represented by figs 3.1 – 3.11. Obviously, fig. 3.1 represents the fundamental total energy or the first harmonic analysis of the attenuation process of the total energy initially possessed by the CCW. In this case, the Fourier index β =0 and it is clear from the figure that the decay process follows a steady and exponential form. Generally, the figures comprises of energy spectrums of the 'host wave' which is blurred in nature and the 'parasitic wave' which is made up of unblurred sharp lines. The figures show subsequent depletion in the blurred nature of the 'host wave' been constantly eroded by the interfering HIV 'parasitic wave'.

Also, fig. 3.2 represents the attenuation process of the total energy of the CCW in which case the Fourier index β =13070. Note that while the multiplier is raised by an interval of 1500, we also focused only on when the Fourier index is a maximum because this is the region of most relevant interest to our work. Due to the numerous waveforms involved when β =13070, for every value of the multiplier, fig. 3.2 could not really reflect all the possible energy waveforms during the period of time (0 – 126 months) that the CCW lasted. The figure displayed a straight line beyond a certain value of the time. This of course is the reason for classifying the multiplier so that all the possible energy waveforms available to the CCW can beunveiled.

It is shown in fig. 3.2 that the energy spectrum of the CCW fluctuates between the interval of positive and negative energies value of $\pm$ 2.505 x 10^{-39} Joules. At time t = 0 λ=0, that is, just before the HIV is contacted the 'host wave' has initial maximum positive total energy of about 1.59074 x 10^{-39} Joules. Thereafter, the energy fluctuates within the given interval and finally the total energy reduces to a negative value of -8.76961 x 10^{-44} Joules at time t = 126 months. Positive total energy means attraction and hence constructive interference between the 'host wave' and the 'parasitic wave', while negative total energy means repulsion and hence destructive interference between them. It is the reduction in the total energy of the CCW that causes a delay or a slow down process in the energy transfer mechanism which eventually leads to constant loss of energy in HIV/AIDS patients.

It is obvious from fig. 3.3, that between 0 – 9.2 days or around 350000 s (4 days)after infection with HIV, the energy spectrum of the CCW show something different from usual which indicates the presence of strange manifestations of a velocity-like body. However, this situation is renormalized to a continuous group velocity with high component frequencies. This is synonymous with the fact that the process of constant energy degeneracy in the host system after the HIV infection is not immediate, and that the host system would by itself tends to annul the destructive effect of the interfering parasite HIV.

It is observed that there are certain regions of remarkable separations or anomalies in the energy spectrum of the CCW as shown in figs. 3.5, 3.8, 3.9, 3.10 and 3.11. These regions are remarkably characterised by a reduced frequencies of the components of the CCW. The time for these remarkable separations in the energy spectrumis2.3, 11.5, 18.9, 24.6, 34.7, 45, 54, 63.6, 72 and 84.8 months respectively. To be consistent with theliterature of clinical diseases the first separation time in the energy spectrum which is 2.3 months,is regarded as the window period. The window period signifies the time when the human biological system is now reacting fully to the presence of the HIV parasite due to the noticeabledamage it would havedone to the energy transfer mechanism of the human system. While the other separation time also indicates the advanced stage of the effect of the interfering HIV parasite in the energy profiles of the CCW.

The spectrum of the total energy becomes parasitically monochromatic beyond 2.4 x 10^8 s or about 8 years as shown in fig. 3.11. This however, indicates the predominance of the HIV active components in the CCW. Thus in this region all the active components of the 'host wave' contained in the CCW would have been completely eroded by the interfering HIV 'parasitic wave' thereby rendering the immune system of the host ineffective and therefore non restorable. This situation depicts the possible period of time when the HIV infection degenerates to AIDS. Finally, the energy of the CCW is brought to rest after 126 months (10 years) as shown in fig. 3.11 and once this stage is reached the phenomenon called death of the host occurs.

5. Conclusion.

The energy attenuation of the CCW which describes the coexistence of the HIV/AIDS and the human system is not instantaneous but gradual. Initially, the biological system of Man tends to annul the destructive influence of the HIV starting from the moment an individual contacted it. In the absence of specific treatment, the HIV infection degenerates to AIDS after about92 months (8 years). This period involves a steady decay process in the energy spectrum of the CCW and this result to a rapid weakening of the initial strength of the active constituents of the host biological system. The energy of the CCW that describes the biological system of Man finally goes to zero - a phenomenon called death, when the multiplier approaches the critical value of 13070 and the time it takes to attain this value is about 126 months (10 years).

Appendix

The following is the list of some useful identities which we implemented in the study.

(1) $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$; (2) $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

(3) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$; (4) $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

(5) $2\sin x\cos y = \sin(x+y) + \sin(x-y)$; (6) $2\cos x\sin y = \sin(x+y) - \sin(x-y)$

(7) $2\cos x\cos y = \cos(x+y) + \cos(x-y)$; (8) $2\sin x\sin y = \cos(x-y) - \cos(x+y)$

(9) $\sin(x \pm y) = \sin x\ \cos y \pm \cos x\ \sin y$; (10) $\cos(x \pm y) = \cos x\ \cos y \mp \sin x\ \sin y$

(11) $\sin 2x = 2\sin x\cos x$; (12) $\sin(-x) = -\sin x$ (odd and antisymmetric function)

(13) $\cos(-x) = \cos x$ (even and symmetric function)

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