

Optimum Crop Production and Income in Brong Ahafo Region

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Abstract
This paper seeks to present a proposed linear programming model to determine the best hectare allocation for optimum crop production to ensure food security and reduce poverty among farmers in the region. Five selected crops in fifteen sampled communities in the Brong Ahafo Region were used for this study. The net income per hectare for each crop was used in formulating the objective function and data on available arable land, mean annual rainfall and the area cultivated constitutes the constraints. The revised simplex scheme was employed to determine optimal basic variables.
Keywords: Linear Programming Model, Optimum Crop Production, Revised Simplex Scheme.

1. Introduction

The region is endowed with a vast tract of arable land, forestry, inland fisheries and clay deposits spanning over 23,734 km^2 (60% of land area) of arable land with about 9,746 km^2 under rain fed agriculture.

The study area consists of 15 municipalities/districts and five selected crops namely: maize, cassava, yam, cocoyam and plantain making a total of 7,397 hectares. The objective of this proposed model is to determine the minimum hector allocation for optimum crop production and the net income generated from the cultivation of these selected crops in the Brong Ahafo Region of Ghana. This paper presents the mathematical formulation of the problem and the solution using the QM software.

Singh et al. (2001) studied the optimal cropping pattern in the command area of Shahi distributaries in Uttar Pradesh. A linear programming model was formulated giving maximum net returns at different water availability level. The objective function of the model was subjected to the following constraints; total available water and land during different seasons, the minimum area under wheat and rice cultivation for local food requirement, farmers' socio-economic conditions and preference to grow a particular crop in a specific area.

Desai (1962) used linear programming technique to explore the possibilities of increasing farm production and income in the regions of Ahamadnagar and Nasik districts of Maharashtra state. It was realized that with the existing resources and technology, farm income and production could be increased substantially.

Chambers and Chames (1961), as well as Cohen and Hammer (1967; 1972), developed a series of sophisticated linear programming models for managing the balance sheet of larger banks, while Waterman and Gee (1981) and Fortson and Dince (1977) proposed less elegant formulations which were better suited for the small to medium-sized bank.

Dantzig et al., (1954) applied the simplex method to an instance with 49 cities by solving the TSP with linear programming. One of the earliest exact algorithms is due to Dantzig et al (1954), in which linear programming (LP) relaxation is used to solve the integer formulation by suitably chosen linear inequality to the list of constraints continuously. However, Miller et al. (1960) extended the idea by applying integer programming formulation of the TSP and its computational results of solving several small problems using Gomory's cutting-plane algorithm was reported. Lambert (1960) solved a 5-city example of the TSP using Gomory cutting planes. Dacey, (1960) reported a heuristic, whose solutions to the TSP were on average 4.8 percent longer than the optimal solutions.

Kanniappan and Ramachandran (1998) optimized for maximum plant residue production as a feedstock for electricity generation. They indicated that in their base year, three tons of surplus residues per hectare were available for electricity generation, whereas the optimal residue generation was four tons per hectare. Their model suggests that the optimal cropping pattern within the district should consist of rice, jowar, groundnut, sugarcane and vegetables cultivated under irrigation, with other crops such as gram and cotton cultivated under rain-fed conditions which will contribute to the larger biomass generation potential.

Ishtiaq et al. (2004) applied a linear programming model to calculate the crop acreage, production and income of the Faisalabad division. The study was conducted on 2702 thousand acres of the irrigated areas from the three districts. Crop included in the model were wheat, Basmati rice, IRRI rice, cotton, sugar cane, maize and potato. The results showed that cotton, maize and wheat gained acreage by about 5-10%, while main losers were Basmati rice, IRRI rice, sugarcane and potato. Overall optimal crop acreage increased by 1.88% while, optimal income was increased by around 2% as compared to the existing solutions.

He used one year data for his model and suggested that the model could be used in a number of situations and could be improved if at least five year average figures have been used in the model. He also added that if more recent cost estimates were used, the model would have been more realistic. On this basis we therefore used four year average values for both the objective and constraint functions in the model to make it more realistic.

In this paper we present a proposed linear programming model for the best hectare allocation which will give optimum crop production and net income in the region. For the robustness of the Model, the coefficients for both the objective function and constraints were average values estimated using a four year period data on the five selected crops from 15 communities giving rise to73 parameters.

2. Mathematical Formulation

The revised simplex method is a scheme for ordering the computations required of the simplex method so that unnecessary which is more efficient for execution on a computer to save computational effort.

The general linear programming model for the revised simplex method which uses matrix manipulations is given as:

Maximize: $z = \mathbf{cx}$

subject to: $\mathbf{Ax} \le \mathbf{b}$

and $x \ge 0$

where,

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

To obtain the augmented form of the problem, introduce the column vector of slack variables

$$\mathbf{x_s} = \begin{bmatrix} x_{n+1} \\ x_{n+2} \\ \vdots \\ x_{n+m} \end{bmatrix}$$

so that the constraints become

$$[\mathrm{A},\ \mathrm{I}] \begin{bmatrix} x \\ \vdots \\ x_s \end{bmatrix} = \mathbf{b} \text{ and } \begin{bmatrix} x \\ \vdots \\ x_s \end{bmatrix} \ge \mathbf{0}$$

Where **I** is the $m \times m$ identity matrix, and the null vector **0** now has $n + m$ elements. Given these variables to solve for the basic feasible solution, the resulting basic solution is the solution of the m equations

$$[\mathrm{A},\ \mathrm{I}] \begin{bmatrix} x \\ \vdots \\ x_s \end{bmatrix} = \mathbf{b}$$

in which the n non-basic variables from the $n + m$ elements of

$$\begin{bmatrix} x \\ \vdots \\ x_s \end{bmatrix}$$

are set equal to zero. Eliminating these n variables by equating them to zero leaves a set of m equations in m

unknowns (the basic variables). This set of equations can be denoted by $\mathbf{Bx_B = b}$ where the vector of basic variables

$$\mathbf{x_B} = \begin{bmatrix} x_{B1} \\ x_{\mathbf{B}2} \\ \vdots \\ x_{Bm} \end{bmatrix}$$

are obtained by eliminating the non-basic variables from

$$\begin{bmatrix} x \\ \vdots \\ x_s \end{bmatrix}$$

and the basis matrix

$$\mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1m} \\ B_{21} & B_{22} & \cdots & B_{2m} \\ \vdots & \vdots & & \vdots \\ B_{m1} & B_{m2} & \cdots & B_{mm} \end{bmatrix}$$

is obtained by eliminating the columns corresponding to coefficients of non-basic variables from [**A**, **I**]. (In addition, the elements of $\mathbf{x_B}$ and, therefore, the columns of **B** may be placed in a different order when the method is executed). The revised simplex method introduces only basic variables such that **B** is nonsingular, so that $\mathbf{B}^{-1}$ always will exist. Therefore, to solve

$$\mathbf{Bx_B = b} \text{ - - - - - - - - - - - - - - - (1)}$$

we multiplied by both sides $\mathbf{B}^{-1}$ to get

$$\mathbf{B^{-1}Bx_B = B^{-1}\, b} \text{ - - - - - - - - - - - (2)}$$

But

$$\mathbf{B^{-1}B = I} \text{ - - - - - - - - - - - - - -(3)}$$

Hence the desired solution for the basic variables is

$$\mathbf{x_B = B^{-1}b} \text{ - - - - - - - - - - - - - (4)}$$

Let $\mathbf{c_B}$ be the vector whose elements are the objective function coefficients (including zeros for slack variables) for the corresponding elements of $\mathbf{x_B}$.

The value of the objective function for this basic solution is then given by

$$Z = \mathbf{c_B B^{-1} b} \text{ - - - - - - - - - - - -(5)}$$

Applying equation to equation (5) yields

$$Z = \mathbf{c_B x_B} \text{ - - - - - - - - - - - - - - (6)}$$

The condition for optimality is given by:

$z_j - c_j \geq 0$ for $j = 1, 2, \ldots, n$

$x_i \geq 0$ for $i = 1, 2, \ldots, m$

3. Data Collection and Analysis

The data on arable land, land allocated for the various cropping activities, annual yield of crops and annual rainfall figures for the fifteen Districts/Municipalities for the four years (2006 - 2010) under consideration were collected from the regional MOFA office in Sunyani.

The decision variables for the selected crops (maize, cassava, yam, cocoyam, plantain) were indexed for the various districts as $x_{i,j}(for\ j = 1, 2, ..., 73 \text{ and } i = 1, 2, ..., 15)$.

The assumptions made during the formulation are:

- The contribution of each activity to the value of the objective function Z is proportional to the level of the activity x_j, as represented by the $c_j x_j$ term in the objective function.

- The contribution of each activity to the left-hand side of each functional constraint is proportional to the level of the activity x_j, as represented by the $a_{ij} x_j$ term in the constraint.
- Rainfall pattern and other weather conditions will be constant.

The areas planted to the selected crops in the various districts/municipalities are figures which have been reported by the Extension officers.

The average figures for the selected crops (2006-2010) were found and summarized below.

District/ Municipality	**Average Allocation per year(ha)**					**TOTAL**
	Maize	**Cassava**	**Yam**	**Cocoyam**	**Plantain**	
Sunyani	7,410	2,450	380	727	1,355	**12,322**
Asutifi	1,793	2,999	40	3,275	4,333	**12,440**
Wenchi	3,737	2,578	2,309	496	290	**9,410**
Dormaa	8,766	3,123	219	1,055	900	**14,063**
Berekum	2,207	2,389	541	1,245	914	**7,296**
Tano North	1,620	1,102	100	496	887	**4,205**
Tano South	1,834	1,937	178	572	926	**5,447**
Nkoranza	8,438	2,594	2,881	126	98	**14,137**
Techiman	3,673	4,187	3,040	625	1,685	**13,210**
Asunafo N.	1,220	1,299	17	927	1,884	**5,347**
Asunafo S.	1,091	2,556	24	1,863	2,121	**7,655**
Jaman S.	1,520	995	2,355	639	257	**5,766**
Kintampo N.	6,187	1,132	2,062	51	7	**9,439**
Kintampo S.	2,841	874	1,702	83	11	**5,511**
Pru	675	3,647	3,075	-	-	**7,397**
TOTAL	**53,012**	**33,862**	**18,923**	**12,180**	**15,668**	**133,645**

Source: Ministry of food and Agriculture, Sunyani –B/A

The reported yield for the various crops in the fifteen communities for the four years was collected and their averages were found, on crop basis. Under the current situation the region produces 4,847,031 metric tons of food. The breakdown is in the table below.

District/ Municipality	**Average yield per year(metric tons)**					**TOTAL**
	Maize	**Cassava**	**Yam**	**Cocoyam**	**Plantain**	
Sunyani	50,079	139,282	8,498	23,858	37,954	259,671
Asutifi	12,156	213,532	1,891	95,514	215,330	538,423
Wenchi	35,183	122,806	145,976	9,247	9,846	323,058
Dormaa	73,605	182,483	11,778	22,902	31,292	322,060
Berekum	19,801	158,686	14,410	37,293	31,066	261,256
Tano North	14,794	53,859	3,221	15,083	34,036	120,993
Tano South	17,152	152,005	7,583	14,930	34,707	226,377
Nkoranza	68,790	135,196	179,307	2,508	2,581	388,382
Techiman	38,503	425,186	230,882	16,777	49,086	760,434
Asunafo N.	8,884	60,711	908	29,296	95,560	195,359
Asunafo S.	9,030	195,572	1,245	56,193	125,977	388,017
Jaman S.	6,939	24,203	114,062	6,376	4,499	156,079
Kintampo N.	55,043	77,014	154,844	1,274	465	288,640
Kintampo S.	23,015	60,469	135,890	2,070	720	222,164
Pru	5,792	176,349	213,977	-	-	396,118
TOTAL	**438,766**	**2,177,353**	**1,224,472**	**333,321**	**673,119**	**4,847,031**

Source: Ministry of food and Agriculture, Sunyani –B/A

The net revenue per hectare for each crop was estimated by dividing the net income generated from each activity by the area allocated annually as shown below.

District/ Municipality	**Net income per hectare of crop (Gh¢/ha)**					
	Maize	**Cassava**	**Yam**	**Cocoyam**	**Plantain**	**TOTAL**
Sunyani	9896	10,261	13,297	12,628	13,148	**59,230**
Asutifi	14307	12,852	20,919	11,223	23,327	**82,628**
Wenchi	4,593	8,598	27,975	7,174	15,937	**64,277**
Dormaa	423,797	10,547	4,096	8,353	16,320	**463,113**
Berekum	11,786	11,989	4376	11,526	15,955	**55,632**
Tano North	4,455	8,822	14,254	11,701	18,012	**57,244**
Tano South	9,562	8,850	14,165	10,044	17,594	**60,215**
Nkoranza	3,977	9,407	27,540	7,660	12,361	**60,945**
Techiman	5,113	18,330	33,607	10,329	13,674	**81,053**
Asunafo N.	3,552	8,436	23,622	12,161	23,809	**71,580**
Asunafo S.	4,037	13,811	22,959	11,607	27,880	**80,294**
Jaman S.	2,227	4,391	21,432	3,839	8,217	**40,106**
Kintampo N.	4,340	31,148	33,229	9,612	12,280	**90,609**
Kintampo S.	3,952	30,703	35,330	9,598	12,488	**92,071**
Pru	4,186	8,728	30,792	-	-	**43,706**
TOTAL	**509,780**	**196,873**	**327,593**	**137,455**	**231,002**	**1,402,703**

Estimated based on FAO quoted food prices for years under review.

This data is used to formulate objective function. The number of hectares to be allocated for the j^{th} activity in the i^{th} Districts/Municipalities for optimum production and income. Since Z is the total net return the resulting linear programming model for this problem is:

$$\text{Maximize } Z = \sum_{i=1}^{15} c_{i,j} x_{i,j} \quad for\ j = 1,2,\ldots,73$$

Subject to the following constraints

a) Arable land:

$$\sum_{i=1}^{15} a_{i,j} x_{i,j} \le L_i \qquad for\ all\ \ j$$

b) Mean rainfall:

$$\sum_{i=1}^{15} w_{ij} x_{ij} \le W_i \qquad for\ all\ \ j$$

c) Hectare allocation :

$$\sum_{i=1}^{15} a_{ij} x_{ij} \le H_i \qquad for\ \ all\ \ j$$

$$x_{ij} \ge 0, \qquad for\ all\ \ i\ \ =\ 1,\ 2,\ \ldots,15$$

and all $j = 1, 2, \ldots, 73$. Where

c_{ij} = Net income (Gh¢/hectare) on the j^{th} activity in the i^{th} district/municipality.

x_{ij} = optimum hectares for j^{th} activity in the i^{th} district/municipality.

a_{ij} = the arable land allocated for j^{th} activity in the i^{th} town.

w_{ij} = the amount of water required for the j^{th} activity in the i^{th} town.

W_i = the total amount of water available in the i^{th} district/municipality.

L_i = the arable land available in the i^{th} district/municipality.

H_i = the total hectares allocated for all activities in the i^{th} district/municipality.

Thus we

$$
\begin{aligned}
\text{Maximize}Z = {} & (9896x_{1,1} + 10261x_{1,2} + 13297x_{1,3} + 12628x_{1,4} + 13148x_{1,5}) \\
& + (14307x_{2,1} + 12852x_{2,2} + 20919x_{2,3} + 11223x_{2,4} + 23327x_{2,5}) \\
& + (4593x_{3,1} + 8598x_{3,2} + 27975x_{3,3} + 7174x_{3,4} + 15937x_{3,5}) \\
& + (423797x_{4,1} + 10547x_{4,2} + 4096x_{4,3} + 8353x_{4,4} + 16320x_{4,5}) \\
& + (11786x_{5,1} + 11989x_{5,2} + 4376x_{5,3} + 11526x_{5,4} + 15955x_{5,5}) \\
& + (4455x_{6,1} + 8822x_{6,2} + 14254x_{6,3} + 11701x_{6,4} + 18012x_{6,5}) \\
& + (9562x_{7,1} + 8850x_{7,2} + 14165x_{7,3} + 10044x_{7,4} + 17594x_{7,5}) \\
& + (3977x_{8,1} + 9407x_{8,2} + 27540x_{8,3} + 7660x_{8,4} + 12361x_{8,5}) \\
& + (5113x_{9,1} + 18330x_{9,2} + 33607x_{9,3} + 10329x_{9,4} + 13674x_{9,5}) \\
& + (3552x_{10,1} + 8436x_{10,2} + 23622x_{10,3} + 12161x_{10,4} + 23809x_{10,5}) \\
& + (4037x_{11,1} + 13811x_{11,2} + 22959x_{11,3} + 11607x_{11,4} + 27880x_{11,5}) \\
& + (2227x_{12,1} + 4391x_{12,2} + 21432x_{12,3} + 3839x_{12,4} + 8217x_{12,5}) \\
& + (4340x_{13,1} + 31148x_{13,2} + 33229x_{13,3} + 9612x_{13,4} + 12280x_{13,5}) \\
& + (3952x_{14,1} + 30703x_{14,2} + 35330x_{14,3} + 9598x_{14,4} + 12488x_{14,5}) \\
& + (4186x_{15,1} + 8728x_{15,2} + 30792x_{15,3}).
\end{aligned}
$$

Subject to the following constraints

1. Arable land available in each district/municipality.

$$
\begin{aligned}
7410x_{1,1} + 2450x_{1,2} + 380x_{1,3} + 727x_{1,4} + 1355x_{1,5} &\le 7350000 \\
1793x_{2,1} + 2999x_{2,2} + 40x_{2,3} + 3275x_{2,4} + 4333x_{2,5} &\le 9843700 \\
3737x_{3,1} + 2578x_{3,2} + 2309x_{3,3} + 496x_{3,4} + 290x_{3,5} &\le 31232500 \\
8766x_{4,1} + 3123x_{4,2} + 219x_{4,3} + 1055x_{4,4} + 900x_{4,5} &\le 8674000 \\
2207x_{5,1} + 2389x_{5,2} + 541x_{5,3} + 1245x_{5,4} + 914x_{5,5} &\le 15430000 \\
1620x_{6,1} + 1102x_{6,2} + 100x_{6,3} + 496x_{6,4} + 887x_{6,5} &\le 7221500 \\
1834x_{7,1} + 1937x_{7,2} + 178x_{7,3} + 572x_{7,4} + 926x_{7,5} &\le 10623100 \\
8438x_{8,1} + 2594x_{8,2} + 2881x_{8,3} + 126x_{8,4} + 98x_{8,5} &\le 31140200 \\
3673x_{9,1} + 4187x_{9,2} + 3040x_{9,3} + 625x_{9,4} + 1685x_{9,5} &\le 6674300 \\
1220x_{10,1} + 1299x_{10,2} + 17x_{10,3} + 927x_{10,4} + 1884x_{10,5} &\le 10551500 \\
1091x_{11,1} + 2556x_{11,2} + 24x_{11,3} + 1863x_{11,4} + 2121x_{11,5} &\le 30237000 \\
1520x_{12,1} + 995x_{12,2} + 2355x_{12,3} + 639x_{12,4} + 257x_{12,5} &\le 6437500 \\
6187x_{13,1} + 1132x_{13,2} + 2062x_{13,3} + 51x_{13,4} + 7x_{13,5} &\le 51025400 \\
2841x_{14,1} + 874x_{14,2} + 1702x_{14,3} + 83x_{14,4} + 11x_{14,5} &\le 16530400 \\
675x_{15,1} + 3647x_{15,2} + 3075x_{15,3} &\le 19344200
\end{aligned}
$$

2. Water availability in each district/municipality

$$
\begin{aligned}
860x_{1,1} + 1291x_{1,2} + 1291x_{1,3} + 1291x_{1,4} + 2581x_{1,5} &\le 18173475 \\
948x_{2,1} + 1422x_{2,2} + 1422x_{2,3} + 1422x_{2,4} + 2844x_{2,5} &\le 20214188 \\
703x_{3,1} + 1054x_{3,2} + 1054x_{3,3} + 1054x_{3,4} + 2109x_{3,5} &\le 11339351 \\
875x_{4,1} + 1313x_{4,2} + 1313x_{4,3} + 1313x_{4,4} + 2625x_{4,5} &\le 21094125 \\
823x_{5,1} + 1234x_{5,2} + 1234x_{5,3} + 1234x_{5,4} + 2468x_{5,5} &\le 10286303 \\
890x_{6,1} + 1334x_{6,2} + 1334x_{6,3} + 1334x_{6,4} + 2669x_{6,5} &\le 6410719 \\
890x_{7,1} + 1334x_{7,2} + 1334x_{7,3} + 1334x_{7,4} + 2669x_{7,5} &\le 8307438 \\
583x_{8,1} + 875x_{8,2} + 875x_{8,3} + 875x_{8,4} + 1750x_{8,5} &\le 14135750 \\
846x_{9,1} + 1269x_{9,2} + 1269x_{9,3} + 1269x_{9,4} + 2538x_{9,5} &\le 19154863 \\
875x_{10,1} + 1313x_{10,2} + 1313x_{10,3} + 1313x_{10,4} + 2625x_{10,5} &\le 8020500 \\
875x_{11,1} + 1313x_{11,2} + 1313x_{11,3} + 1313x_{11,4} + 2625x_{11,5} &\le 11481000 \\
423x_{12,1} + 634x_{12,2} + 634x_{12,3} + 634x_{12,4} + 1269x_{12,5} &\le 4179988 \\
933x_{13,1} + 1400x_{13,2} + 1400x_{13,3} + 1400x_{13,4} + 2800x_{13,5} &\le 15101600 \\
642x_{14,1} + 963x_{14,2} + 963x_{14,3} + 963x_{14,4} + 1925x_{14,5} &\le 6061550 \\
933x_{15,1} + 1400x_{15,2} + 1400x_{15,3} &\le 11835600
\end{aligned}
$$

3. Maximum hectares allocated in each district/municipality.

$$7410x_{1,1} + 2450x_{1,2} + 380x_{1,3} + 727x_{1,4} + 1355x_{1,5} \le 12321$$
$$1793x_{2,1} + 2999x_{2,2} + 40x_{2,3} + 3275x_{2,4} + 4333x_{2,5} \le 12440$$
$$3737x_{3,1} + 2578x_{3,2} + 2309x_{3,3} + 496x_{3,4} + 290x_{3,5} \le 9410$$
$$8766x_{4,1} + 3123x_{4,2} + 219x_{4,3} + 1055x_{4,4} + 900x_{4,5} \le 14063$$
$$2207x_{5,1} + 2389x_{5,2} + 541x_{5,3} + 1245x_{5,4} + 914x_{5,5} \le 7295$$
$$1620x_{6,1} + 1102x_{6,2} + 100x_{6,3} + 496x_{6,4} + 887x_{6,5} \le 4204$$
$$1834x_{7,1} + 1937x_{7,2} + 178x_{7,3} + 572x_{7,4} + 926x_{7,5} \le 5448$$
$$8438x_{8,1} + 2594x_{8,2} + 2881x_{8,3} + 126x_{8,4} + 98x_{8,5} \le 14136$$
$$3673x_{9,1} + 4187x_{9,2} + 3040x_{9,3} + 625x_{9,4} + 1685x_{9,5} \le 13210$$
$$1220x_{10,1} + 1299x_{10,2} + 17x_{10,3} + 927x_{10,4} + 1884x_{10,5} \le 5347$$
$$1091x_{11,1} + 2556x_{11,2} + 24x_{11,3} + 1863x_{11,4} + 2121x_{11,5} \le 7654$$
$$1520x_{12,1} + 995x_{12,2} + 2355x_{12,3} + 639x_{12,4} + 257x_{12,5} \le 5766$$
$$6187x_{13,1} + 1132x_{13,2} + 2062x_{13,3} + 51x_{13,4} + 7x_{13,5} \le 9439$$
$$2841x_{14,1} + 874x_{14,2} + 1702x_{14,3} + 83x_{14,4} + 11x_{14,5} \le 5511$$
$$675x_{15,1} + 3647x_{15,2} + 3075x_{15,3} \le 7397$$

4. Results and Discussion

The QM software was used to generate the optimal solution from which the net income is calculated. The best crop allocation in hectare for the region is presented in the table below.

Table 4.8: Best crop allocation

District/ Municipality	Index No.	Crop	Optimal crop allocation ($x_{i,j}$) (ha)	Net income per hectare ($c_{i,j}$) (Gh¢/ha)	Expected Net income per year (Gh¢)	Expected crop yield per year (tons)
Sunyani	1	yam($x_{1,3}$)	32.42	13,297.00	275,505.16	431,088.74
Asutifi	2	yam($x_{2,3}$)	311.00	20,919.00	588,101.00	6,505,809.00
Wenchi	3	plantain($x_{3,5}$)	32.45	15,937.00	319,502.70	517,155.65
Dormaa	4	maize($x_{4,1}$)	1.60	423,797.00	117,768.00	678,075.20
Berekum	5	plantain($x_{5,5}$)	7.98	15,955.00	247,906.68	127,320.90
Tano North	6	yam($x_{6,3}$)	42.04	14,254.00	135,410.84	599,238.16
Tano South	7	yam($x_{7,3}$)	30.61	14,165.00	232,115.63	433,590.65
Nkoranza	8	plantain($x_{8,5}$)	144.24	12,361.00	372,283.44	1,782,950.64
Techiman	9	cocoyam($x_{9,4}$)	21.14	10,329.00	354,665.78	218,355.06
Asunafo N.	10	yam($x_{10,3}$)	314.53	23,622.00	285,593.24	7,429,827.66
Asunafo S.	11	yam($x_{11,3}$)	318.92	22,959.00	397,055.40	7,322,084.28
Jaman S.	12	plantain($x_{12,5}$)	22.44	8,217.00	100,957.56	184,389.48
Kintampo N.	13	plantain($x_{13,5}$)	1,348.43	12,280.00	627,019.95	16,558,720.40
Kintampo S.	14	plantain($x_{14,5}$)	501.00	12,488.00	360,720.00	6,256,488.00
Pru	15	yam($x_{15,3}$)	2.41	30,792.00	515,684.57	74,208.72
Optimal Value (Z)			**3,131.21**	**651,372.00**	**49,120,850**	**4,930,289.95**

In the current situation the region observes 4,847,023 tons of yield and Gh¢ 1,402,701 per year. Given that the rainfall is constant and the required hectares as prescribed above are allocated and managed properly, the region would observe a yield of 4,930,290 and Gh¢ 49,120,850 per year. This would improve the food security and poverty situations in the region. In other to obtain optimum production and income for the region, some variables [yam($x_{1,3}$), plantain($x_{3,5}$), maize($x_{4,1}$), plantain($x_{5,5}$), yam($x_{6,3}$), yam($x_{7,3}$), cocoyam($x_{9,4}$), plantain($x_{12,5}$), plantain($x_{13,5}$)] lost some hectares whiles other variables [yam($x_{2,3}$), yam($x_{10,3}$), yam($x_{11,3}$), yam($x_{15,3}$), plantain($x_{8,5}$), plantain($x_{14,5}$)] gained additional hectares.

Meaning that if we want to achieve optimum production and income, we need to increase the number of hectors allotted to the production of these crops which gained additional hectors.

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