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Prediction Using Estimators of Linear Regression Model with Autocorrelated Error Terms and Correlated Stochastic Uniform Regressors

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Abstract

Prediction remains one of the fundamental reasons for regression analysis. However, the Classical Linear Regression Model is formulated under some assumptions which are not always satisfied especially in business, economic and social sciences leading to the development of many estimators. This work, therefore, attempts to examine the performances of the Ordinary Least Square estimator (OLS), Cochrane-Orcutt estimator (COR), Maximum Likelihood estimator (ML) and the estimators based on Principal Component analysis (PC) in prediction of linear regression model under the violations of assumption of non – stochastic regressors, independent regressors and error terms. With stochastic uniform variables as regressors, Monte - Carlo experiments were conducted over the levels of autocorrelation (ρ), correlation between regressors (multicollinearity - λ) and sample sizes, and best estimators for prediction purposes are identified using the goodness of fit statistics of the estimators. Results show that the performances of COR and ML at each level of multicollinearity over the levels of autocorrelation have a convex – like pattern while that of OLS, PR1 and PR2 are concave – like. Also, as the level of multicollinearity increases the estimators especially the COR and ML estimators perform much better at all the levels of autocorrelation. Furthermore, results show that except when the sample size is small (n=10), the performances of the COR and ML estimators are generally best and almost the same, even though at low level of autocorrelation the PC estimator either performs better than or competes with the best estimator when $\lambda \leq -0.49$ and $\lambda \geq 0.6$. When the sample size is small (n =10), the COR estimator is best except when the autocorrelation level is low and $\lambda \leq -0.4$ or $\lambda \geq 0.2$. At these instances, the PR2 estimator is best. Moreover, at low level of autocorrelation in all the sample sizes, the OLS estimator competes with the best estimator in all the levels of multicollinearity.

.**Keywords:** Prediction, Estimators, Linear Regression Model, Autocorrelation, Multicollinearity

1.0 Introduction

Linear regression model is probably the most widely used statistical technique for solving functional relationship problems among variables. It helps to explain observations of a dependent variable,*y*, with observed values of one or more independent variables, X_1, X_2,...,X_p. In an attempt to explain the dependent variable, prediction of its values often becomes very essential and necessary. However, the linear regression model is formulated under some basic assumptions. Among these assumptions are regressors being assumed to be fixed (non-stochastic) and independent; and that the error terms are assumed to be independent. Consequently, various methods of estimation of the parameter model have been developed.

The assumption of non-stochastic regressors is not always satisfied especially in business, economic and social sciences because their regressors are often generated by stochastic process beyond their control. Many authors including Neter and Wasserman (1974), Fomby et al. (1984), Maddala (2002) have given situations and instances where these assumptions may be violated and also discussed their consequences on the Ordinary Least Square (OLS) estimator when used to estimate the model parameters. When regressors are stochastic and independent of the error terms; the OLS estimator is still unbiased and has minimum variance even though it is not Best Linear Unbiased Estimator (BLUE). Also, the traditional hypothesis testing remains valid if the error terms are further assumed to be normal. However, modification is required in the area of confidence interval and power of the test and the power of the test calculated for each sample.

The violation of the assumption of independent regressors leads to multcollinearity as found in business and economics data. With strongly interrelated regressors, interpretation given to the regression coefficients may no longer be valid because the assumption under which the regression model is built has been violated. Although the estimates of the regression coefficients provided by the OLS estimator is still unbiased as long as multicollinearity is not perfect, the regression coefficients have large sampling errors which affect both the inference and forecasting that is based on the model (Chartterjee et al., 2000). Various methods have been

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Vol.3, No.13, 2012

developed to estimate the model parameters when muitcollinearity is present in a data set. These estimators include Ridge Regression estimator developed by Hoerl (1962) and Hoerl and Kennard (1970), Estimator based on Principal Component Regression suggested by Massy (1965), Marquardt (1970) and Bock, Yancey and Judge (1973), Naes and Marten (1988), and method of Partial Least Squares developed by Hermon Wold in the 1960s (Helland, 1988, Helland, 1990, Phatak and Jony 1997).

When all the assumptions of the Classical Linear Regression Model hold except that the error terms are not homoscedastic (i.e. E $(U^1U) = \sigma^2 I_n$) but are heteroscedastic (i.e. $E(U^1U) = \sigma^2\Omega$), the resulting model is the Generalized Least Squares (GLS) Model. Aitken (1935) has shown that the GLS estimator β of β given as $\beta = (X^1\Omega^1X)^{-1} X^1\Omega^1Y$ is efficient among the class of linear unbiased estimators of β with variance – covariance matrix of β given as $\hat{V}(\beta) = \sigma^2(X^1\Omega^1X)^{-1}$ where Ω is assumed to be known. However, Ω is not always known, it is often estimated by Ω to have what is known as Feasible GLS estimator.

When the assumption of independence of error terms is violated as it is often found in time series data, the problem of autocorrelation arises. Several authors have worked on this violation especially in terms of the parameter estimation of the linear regression model when the error term follows autoregressive of orders one. The OLS estimator is inefficient even though unbiased. Its predicted values are also inefficient and the sampling variances of the autocorrelated error terms are known to be underestimated causing the t and the F tests to be invalid (Johnston, 1984; Fomby et al., 1984; Chartterjee, 2000; Maddala, 2002). To compensate for the lost of efficiency, several feasible GLS estimators have been developed. These include the estimator provided by Cochrane and Orcutt (1949), Paris and Winstern (1954), Hildreth and Lu (1960), Durbin (1960), Theil (1971), the Maximum Likelihood and the Maximum Likelihood Grid (Beach and Mackinnon, 1978), and Thornton (1982). Chipman (1979), Kramer (1980), Kleiber (2001), Iyaniwura and Nwabueze (2004), Nwabueze (2005a, b, c), Ayinde and Ipinyomi (2007) and many other authors have not only observed the asymptotic equivalence of these estimators but have also noted that that their performances and efficiency depend on the structure of the regressor used. Rao and Griliches (1969) did one of the earliest Monte-Carlo investigations on the small sample properties of several two-stage regression methods in the context of autocorrelation error. Other recent works done on these estimators and the violations of the assumptions of classical linear regression model include that of Ayinde and Oyejola (2007), Ayinde (2007a,b), Ayinde and Olaomi (2008), Ayinde (2008), and Ayinde and Iyaniwura (2008).

In spite of these several works on these estimators, none has actually been done on their prediction. Very fundamentally, prediction is one of the basic reasons for regression analysis. Therefore, this paper does not only examine the predictive potential of some of these estimators but also does it under some violations of assumption of regression model making the model much closer to reality.

2.0 Materials and Methods

Consider the linear regression model is of the form:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t \quad (1)$$

Where $u_t = \rho u_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$, t = 1, 2, 3,...n and $X_i \sim U(0,1)$ i= 1, 2, 3 are stochastic and correlated.

For Monte-Carlo simulation study, the parameters of equation (1) were specified and fixed as $\beta_0 = 4$, $\beta_1 = 2.5$, $\beta_2 = 1.8$ and $\beta_3 = 0.6$. The levels of multicollinearity among the independent variables were sixteen (16) and specified as: $\lambda(x_{12}) = \lambda(x_{13}) = \lambda(x_{23}) = -0.49, -0.4, -0.3, \ldots, 0.8, 0.9, 0.99.$ The levels of autocorrelation is twenty-one (21) and are specified as: $\rho = -0.99, -0.9, -0.8, \ldots, 0.8, 0.9, 0.99.$ Furthermore, the experiment was replicated in 1000 times (R =1000) under Six (6) levels of sample sizes (n =10, 15, 20, 30, 50, 100).

The correlated stochastic uniform regressors were generated by first using the equations provided by Ayinde (2007) and Ayinde and Adegboye (2010) to generate normally distributed random variables with specified intercorrelation. With P=3, the equations give:

$X_1 = \mu_1 + \sigma_1 Z_1$

$X_2 = \mu_2 + \rho_{12}\, \sigma_2 Z_1 +$ (2)

$X_3 = \mu_3 + \rho_{13}\, \sigma_3 Z_1 + \frac{m_{23}}{\sqrt{m_{22}}} Z_2 + \sqrt{n_{33}} Z_{3\,3}$

Where $m_{22} = \sigma_2^2(1-\rho_{12}^2)$, $m_{23} = \sigma_2\sigma_3(\rho_{23}-\rho_{12}\rho_{13})$ and $n_{33} = m_{33} - \frac{m_{23}^2}{m_{22}}$; and $Z_i \sim N(0, 1)$ i = 1,2,3.

(The inter-correlation matrix has to be positive definite and hence, the correlations among the independent variable were taken as prescribed earlier). In the study, we assumed $X_i \sim N(0, 1)$, i = 1, 2, 3. We further utilized the properties of random variables that cumulative distribution function of Normal distribution produces U (0, 1) without affecting the correlation among the variables (Schumann, 2009) to generate $X_i \sim U(0,1)$ i = 1, 2 and 3.

The error terms were generated using one of the distributional properties of the autocorrelated error terms ($u_t \sim N(0, \frac{\sigma_\varepsilon^2}{1-\rho^2})$) and the AR(1) equation as follows:

$$u_1 = \frac{\varepsilon_1}{\sqrt{1-\rho^2}} \tag{3}$$

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad t = 2,3,4,\ldots n \tag{4}$$

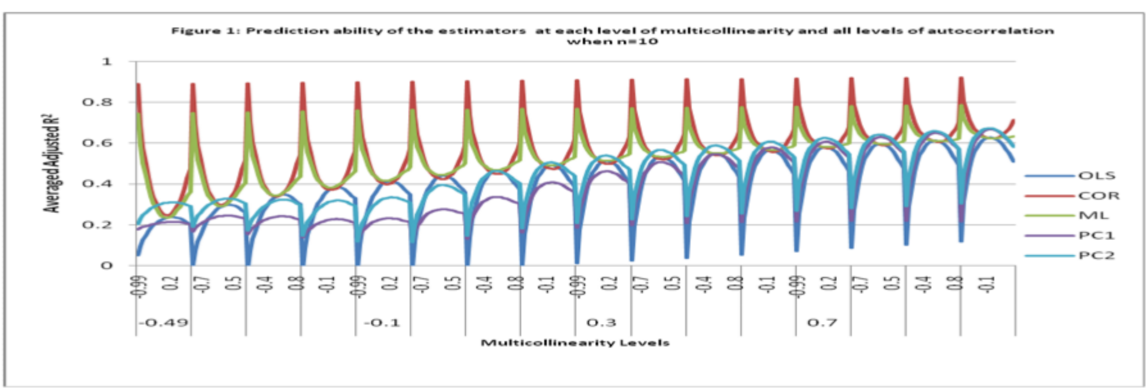
Since some of these estimators have now been incorporated into the Time Series Processor (TSP 5.0, 2005) software, a computer program was written using the software to estimate the Adjusted Coefficient of Determination of the model ($\bar{R}^2$) the Ordinary Least Square (OLS) estimator, Cochrane orcutt (COR) estimator, Maximum Likelihood estimator and the estimator based on Principal Component Analysis (PRN). The Adjusted Coefficient of Determination of the model was averaged over the numbers of replications. i.e.

$$\bar{\bar{R}} = \frac{1}{R}\sum_{i=1}^{R}\bar{R}_i^2 \tag{5}$$

The two possible PCs (PC1 and PC2) of the Principal Component Analysis were used. Each provides its separate Adjusted Coefficient of Determination. An estimator is best if its Adjusted Coefficient of Determination is closest to unity.

3.0 Results and Discussion

The full summary of the simulated results of each estimator at different level of sample size, muticollinearity, and autocorrelation is contained in the work of Olasemi (2011). The graphical representations of the results when n=10, 15, 20, 30, 50 and 100 are respectively presented in Figure 1, 2, 3, 4, 5 and 6.



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Vol.3, No.13, 2012

Figure 2: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=15

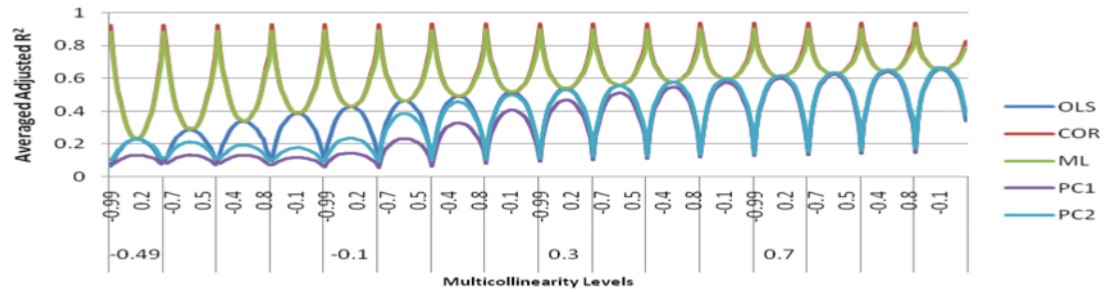


Figure 3: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=20

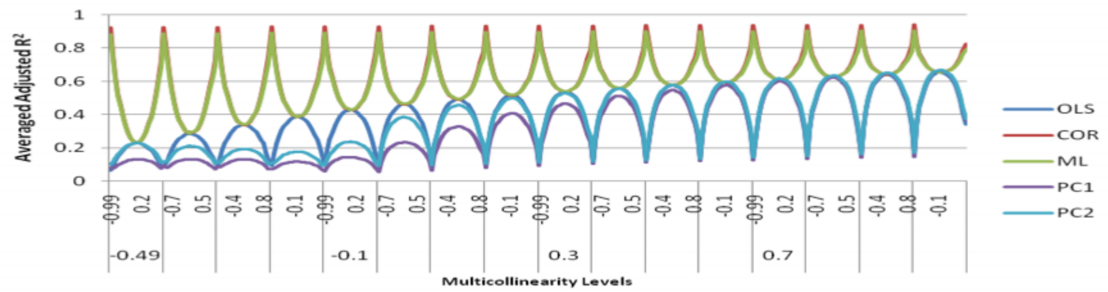


Figure 4: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=30

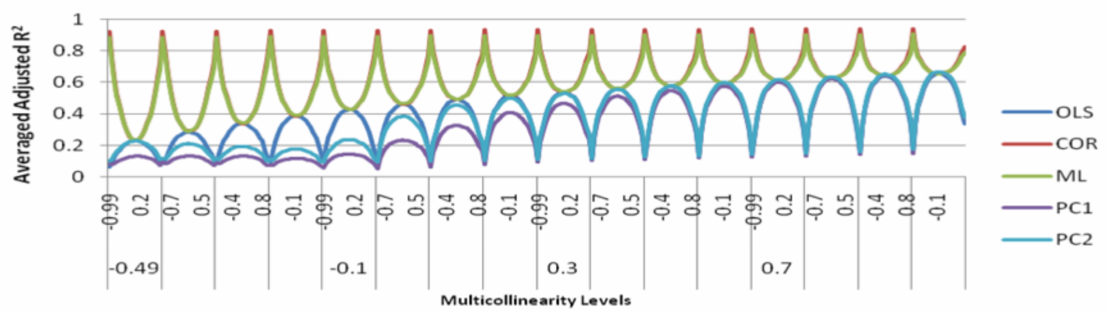
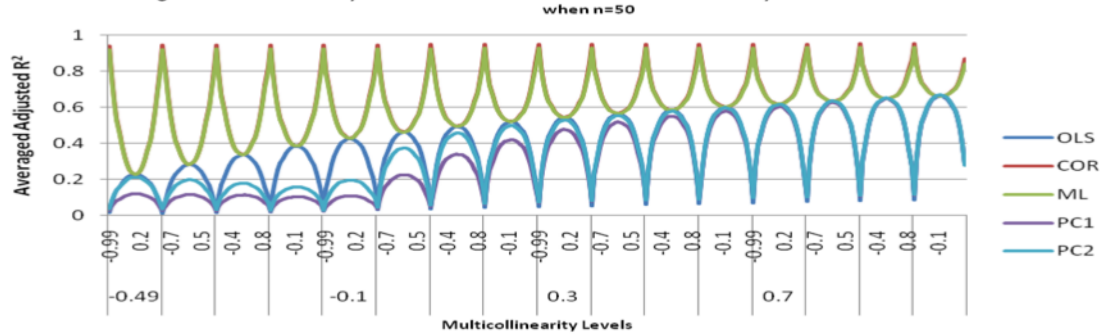


Figure 5: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=50



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Vol.3, No.13, 2012

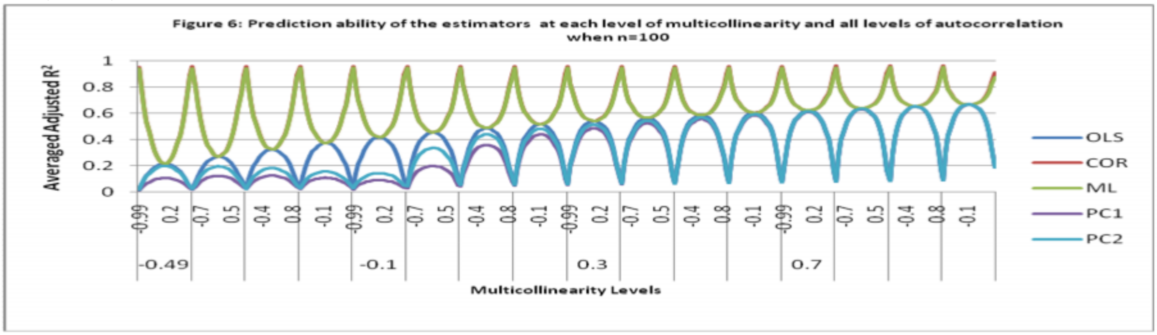

Figure 6: Prediction ability of the estimators at each level of multicollinearity and all levels of autocorrelation when n=100

From these figures, it can be generally observed that the performances of COR and ML at each level of multicollinearity over the levels of autocorrelation have a convex – like pattern while that of OLS, PR1 and PR2 are concave – like. Also, as the level of multicollinearity increases the estimators perform much better at all the levels of autocorrelation even though the values of their averaged adjusted coefficient of determination are not high at low and moderate levels of autocorrelation except when $\lambda \geq 0.6$. At these instances, COR and ML estimators are much better at all the levels of autocorrelation. Furthermore except when the sample size is small (n=10), the performances of the COR and ML estimators are generally best and almost the same, even though at low level of autocorrelation the PR2 estimator either performs better than or competes with the best estimator when $\lambda \leq -0.49$ and $\lambda \geq 0.6$. When the sample size is small (n =10), the COR estimator is best except when the autocorrelation level is low and $\lambda \leq -0.4$ or $\lambda \geq 0.2$. At these instances, the PR2 estimator is best. Moreover, at low level of autocorrelation in all the sample sizes, the OLS estimator competes with the best estimator in all the levels of multicollinearity.

Very specifically in term of identification of the best estimator, Table 1, 2, 3, 4, 5 and 6 respectively summarize the best estimator for prediction at all the levels of autocorrelation and multicollinearity when the sample size is10, 15, 20, 30, 50, 100.

Table 1: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=10.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	ML	ML	ML	ML	ML	COR	PC2	PC2	PC2	PC2	PC2	PC2
-0.2	PC2	COR	COR	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2
-0.1	PC2	PC2	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2
0	PC2	PC2	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2
0.1	PC2	PC2	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2
0.2	PC2	PC2	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2
0.3	PC2	PC2	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2
0.4	PC2	PC2	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2
0.5	PC2	COR	COR	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2	PC2	PC2	PC2	PC2
0.6	COR	COR	COR	COR	COR	ML	ML	COR	COR	COR	COR	PR2	PR2	PR2	PR2	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

From Table 1 when n = 10, COR estimator is best for prediction at all levels of multicollinearity especially when $\rho \leq -0.4$ and $\rho \geq 0.7$. When $-0.2 \leq \rho \leq 0.5$ and $\lambda \leq -0.4$ or $\lambda \geq 0.2$, the PR2 is best. The ML estimator is only best when $-0.2 \leq \rho \leq 0.5$ and $-0.3 \leq \lambda \leq 0.1$.

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Vol.3, No.13, 2012

Table 2: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=15.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2
0	PC2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2	PC2	PC2
0.1	PC2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2	PC2	PC2
0.2	PC2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2	PC2	PC2
0.3	PC2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2	PC2	PC2
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

From the Table 2, it can be seen that the COR estimator is generally best for prediction at all levels of multicollinearity and autocorrelation except $0 \le \rho \le 0.3$ when $\lambda \le -0.49$ or $\lambda \ge 0.6$ At these instances and occasionally when autocorrelation level is low and multicollinearity level is very high or tends to unity, the PR2 is best .

Table 3: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=20.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML
-0.1	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0	PC2	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0.1	PC2	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0.2	PC2	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0.3	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

When n = 20 as revealed in Table 3, the COR estimator is best except at $-0.2 \le \rho \le 0.3$. At these instances, the best estimator is PC2 when $\lambda \le -0.49$ or $\lambda \ge 0.7$; otherwise, the best estimator is often ML and sparsely COR even though the two of them compete very favorably well. (See figure 3).

Table 4: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=30.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	COR	COR
-0.2	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML
-0.1	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0.1	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2	PC2	PC2
0.2	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	PC2	PC2
0.3	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	ML	COR
0.4	COR	COR	COR	COR	COR	ML	ML	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

From Table 4, it can be observed that the best estimator for prediction when n = 30 is generally COR except when the $|\rho| \leq 0.3$ At these instances, the estimator based on using PC2 is best when $0.1 \leq \rho \leq 0.2$ and $\lambda \geq 0.7$; otherwise, the best estimator is often ML and sparsely COR even though the two of them still compete very favorably well. (See figure 4).

Table 5: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=50.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.1	ML	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
0	ML	ML	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2
0.1	ML	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2
0.2	ML	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

From Table 5 when n= 50, the best estimator for prediction is still COR except when $|\rho| \leq 0.1$ and $\lambda \geq 0.9$ and when $-0.1 \leq \rho \leq 0.2$ and $\lambda \leq -0.49$. At the former estimator based on using PC2 is best while ML is best at the latter.

Table 6: The Best Estimator for Prediction at different level of Multicollinearity and Autocorrelation when n=100.

ρ	λ															
	-0.49	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.99**
-0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.-6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
-0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2	PC2
0.1	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	PC2
0.2	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.3	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.4	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.5	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.6	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.7	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.8	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.9	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR
0.99	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR	COR

When the sample size is large (n =100), the best estimator is generally COR except when $0 \le \rho \le 0.1$ and $\lambda \ge 0.9$. At these instances, the estimator based on using PC2 is best.

4.0 Conclusions

The effect of two major problems, Multicollinearity and autocorrelation, on the predictive ability of the OLS, COR, ML and PC estimators of linear regression model has been jointly examined in this paper. Results reveal the pattern of performances of COR and ML at each level of multicollinearity over the levels of autocorrelation to be generally and evidently convex especially when $n \ge 30$ and $\lambda < 0$ while that of OLS and PC is generally concave. Moreover, the COR and ML estimators perform equivalently and better; and their performances become much better as multicollinearity increases. The COR estimator is generally the best estimator for prediction except at high level of multicollinearity and low levels of autocorrelation. At these instances, the PC estimator is either best or competes with the COR estimator. Moreover when the sample size is small (n=10) and multicollinearity level is not high, the OLS estimator is best at low level of autocorrelation whereas the ML is best at moderate levels of autocorrelation.

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