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“Planted-shared" Contract Farming, Optimal Production Sharing Rules and Sustainable Development

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Abstract
This article studies the optimality properties of "planted-shared" contract farming compatible with a sustainable development issue. Using a dynamic principal-agent model with bilateral engagement and taking into account the land fertility, we show that in the absence of an agricultural credit market, the optimal remuneration of the agent depends both on his productive performance and a land fertility index. In addition, the optimal long-term contract highlights an intertemporal smoothing of the production sharing index and the land fertility index. This reflects not only the incentive of the agent to redistribute effort and compensation throughout the duration of the contract but also its intertemporal incentive to maintain the land, which is a guarantee of sustainability of agriculture. Public authorities could promote such contracts not only to meet the strong demand for securing "planted-shared" contracts, but also to promote the efficient management of natural resources.
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1. Introduction

The various international meetings (Johannesburg, Copenhagen, Rio, Paris, Morocco) put issues related to the environment and sustainable development at the heart of any process of growth and development. Sustainable development involves reconciling economic logic and environmental concerns, with a view to intra and intergenerational social equity. It aims to change unsustainable patterns of consumption and production, the eradication of poverty, and the protection and management of natural resources for economic and social development. In sub-Saharan African (SSA) countries, where agriculture is still the basis of the economy, agricultural growth has severely affected countries' natural capital, including forests, agricultural land and the micro-climate. This makes the development of sustainable agriculture urgent. From this point of view, the management of rural lands serving mainly agricultural activities becomes a major issue of sustainable development. With land monetarization, rural lands are becoming an indispensable resource in the fight against poverty in rural areas; this is all the more because abundant land with no immediate utility is easily sold in return for significant income (Paulme, 1962; Lena, 1981; Lesourd, 1982). These monetarized land transfers thus make it possible to valorize the land availabilities whose exploitation is constrained by the lack of family labor power (Chauveau and Richard, 1983).

The need for land control and the arrival of migrants lead to a change in tenure systems characterized by the shift from sale to lease of land (Chauveau and Richard, 1983). Landowners tend more and more to delegate land use rights to individuals through different contractual forms. Among these, tenancy contracts (renting land for a set price) and sharecropping (renting land with production sharing agreement and possibly a fixed sum to pay) remain the best known. In addition to these two forms of agricultural contract, other contractual mechanisms coexist side by side: the short-term land-for-work exchange (land rent is paid in the form of labor), the loan without any specific conditions (duration of a culture cycle or of indefinite duration), the loan with explicit compensation (transitional device between free loan and rental). All of these direct tenure practices concern more non-perennial crops such as food crops, pineapple, cotton, etc. (Colin, 2008). The perpetuation of the use of land by long cycle crops, 20 to 40 years (cocoa, coffee, palm, cashew, rubber) has allowed the development of another form of contract in African forest area: the so-called contracts "Planted-shared". These can be defined, in generic terms, as an institutional arrangement by which an operator gains access to a long-term right of use, or even a right of land ownership, by developing land through realization of a permanent plantation and by surrendering to the landowner part of the plantation created.

With the long-term horizon of "plant-shared" contracts, two types of concerns emerge. The first is relational and concerns the tensions and conflicts that may arise between the transferor and the operator due to clauses of ambiguous duration or terms of sharing of production. In general, there are several ways of sharing production: half the rate of sharing, whether it is planting or sharing the crop; third-party sharing, one-third for the owner and two-thirds for the transferor; two-thirds sharing for the farmer for a few years followed by a split to half beyond. But the questioning of the rights of one or the other on this point explains the strong demand for securing these contracts through formalization and public validation. The second concern is about sustainable development and concerns the issue of land maintenance. It is therefore necessary to encourage the emergence of a rural land market that is more equitable, better organized and more compatible with sustainable agriculture through the provision of appropriate legal and contractual tools for "planted-shared" contract farming. This will help to improve not only the relationship between owners of land rights and non-holders of these rights, but also to improve the management of natural resources.

Our objective in this paper is to determine the characteristics of a "planted-shared" contract farming that satisfies the contracting parties by resolving potential conflicts while promoting land maintenance for sustainable agriculture. Theoretically, this amounts to characterizing an optimal long-term contract with commitment, compatible with a concern for the maintenance of the land. This problem has been little studied in the literature on contract farming. Because most agrarian contracts studied in the literature are spot contracts, the long-term commitment is not possible because it is not credible (Dubois, 1999). We develop a dynamic contract model with bilateral engagement via the principal-agent paradigm which is inspired by both a model of spot contract farming of Dubois (1999), integrating the dimension of the land fertility and a dynamic model of Laffont and Martimort (2002) dealing with non-agricultural contracts. We manage to show that taking into account the endogenous evolution of land fertility in the principal-agent model, the optimal "planted-shared" contract, a dynamic contract with bilateral commitment, arbitrates between sharing risk, incentives to produce and land maintenance. The remainder of the article is organized as follows: after having reviewed some elements of the economic literature on contract farming (section 2), we present, under the principal-agent paradigm, the characteristics of an optimal long-term bilateral commitment contract that provides the right incentives, equitably shares the risks and ensures land maintenance for more sustainable agriculture (section 3). Finally we discuss the results, make recommendations and conclude the article (section 4).

2. Literature Review on Contract Farming

Early economic work on contract farming does not include land maintenance concerns. The issues of sustainable agriculture are therefore not taken into account. This is to explain the choice of contractual form by invoking reasons related to risk sharing, transaction costs or various types of constraints (financial constraints, labor constraints, etc.). The contracts farming that were the first to be studied are the agreements for the delegation of land use between a landowner and a farmer, especially share-cropping and tenant farming. The theoretical study of these contracts is based on the transaction cost theory (Williamson, 1971) and the risk and information theory (Laffont and Tirole, 1986; Allen and Lueck, 1995). The various possible sources of transaction costs (costs of observation and measurement of inputs and outputs, cost of effort) theoretically and empirically explain the choice of contracts. The risk and information theory studies contracts between a farmer and firms using the principal-agent paradigm. These contracts are seen as a way for the firm to delegate tasks to the farmer while managing the risk sharing arrangements and providing the right incentives. Cheung (1969) explains the choice of the contract by an arbitrage between risk sharing and minimization of transaction costs. In doing so, he is the first to explicitly use the concept of transaction cost in the analysis of agrarian contracts. In line with this approach, models consider sharecropping as a contract that reduces, under certain circumstances, transaction costs compared to tenancy and direct tenure with hired labor (Allen and Lueck, 1995, 1999). Datta et al (1986) explain the choice of the agricultural contract after an evaluation of the costs borne by the landowner because of the risks of opportunistic behavior on the part of the tenant: risk of overexploitation of the land resource (case of tenant farming), risk of less effort at work of the maneuver (case of the direct tenure with salaried employment), opportunistic behavior of the tenant relative to his investment at work (risk lower than in the case of the wage earner), to the mining of the soil, to fraud during production sharing (case of sharecropping).

Other models, using the principal-agent approach, attribute not only transaction costs but risk to the explanation of contractual choices. The pioneer in this field is Stiglitz (1974). Here, because of the problem of moral hazard generated by the non-observability of the agent's actions, the optimal contract of sharecropping results from an arbitrage between incentives and risk sharing between a risk-neutral landowner and a risk averse farmer. In addition to transaction costs and risk sharing, research work identifies other reasons for the choice of contract farming. Eswaran and Kotwal (1985) explain the choice of sharecropping by its ability to coordinate off-market the relative benefits of cost-sharing and complementarity of resources contributed by each actor. In their model, landowners have better technical and economic management capacity while tenants have a better ability to supervise family work and reduce opportunistic behaviors when pooling the workforce. The interest of these two actors for the production strongly reduces the risk of moral hazard, which is not limited any more to the investment in the work of the tenant but integrates other risks: degradation of the land, fraud during the production sharing. According to Laffont and Matoussi (1995), the choice of the type of contract (lease, share at half, third-party sharing or quarter share) is determined by a growing level of financial constraints on the tenant, along with a growing disincentive, attenuated by a logic of repeated games. Shetty (1988) shows that the choice of contract depends on the assets held by the keeper. In this case, the landowner will not delegate the use of land to a tenant with a low level of capital accumulation, because no guarantee can be made in case of default of payment of the annuity. These producers are therefore forced to engage in sharecropping contracts, while the better-off tenants enter into tenancy farming contracts and generate higher profits.

Agrarian contracts are also a substitute for the credit market. This is the case of interlinked sharecropping/credit contract where the landowner grants credit to the tenant, which is in fact guaranteed by the production of the parcel of land under sharecropping (Hayami and Otsuka, 1993; Jaynes, 1982). This type of contract induces behaviors that cause the landowner to encourage the tenant to incur debt when the latter, by modifying the terms of the loan agreement, may encourage the owner to work more (Braverman and Stiglitz, 1982). In addition, in the context of a non-monetarized economy, the interlinked contract can reduce the costs of enforcement (Bardhan, 1991). As can be seen, the forms of agricultural contracts emerge as an optimal response to the presence of risk, transaction costs and imperfections in the capital market (financial constraints) and the rural labor market (lack of skilled labor). The imperfections of the insurance market can also explain the interest of sharecropping for both the tenant and the owner. Sharecropping also works as a substitute for the insurance market. Households that access sharecropping contracts to share production risks are better insured than others (Dubois, 2000). Thus, these contracts allow households to better insure against risks. In fact, they make it possible to supplement the markets by providing risk averse households with contingent assets that no combination of the other accessible goods makes it possible to obtain.

In recent years, we have seen the emergence of new types of contract farming: production and/or marketing agreements between an agribusiness firm and a farmer. In this type of agreement, a farmer agrees to deliver certain quantities of agricultural products of the required quality standards and at a fixed time, to a firm that undertakes to buy them at a pre-determined price while providing inputs and technical support for production. The theoretical study of these contracts is based, as before, on the transaction cost approach and the risk and information approach. In the transaction cost approach, agreements between farmers and agribusiness firms, both risk-neutral, are similar to vertical coordination in the agricultural supply chain and can be analyzed in detail using the tools of vertical integration (Hennessy, 1996; Leathers, 1999). This research work analyzes the reasons justifying the adoption of such an industrial organization and the benefits it brings to the parties involved in the contract. The risk and information economy approach uses the tools of contract theory to determine optimal contracts when informational rents play an important role. Various types of contracts are studied, ranging from the tomato sector (Alexander et al, 2000; Hueth and Ligon, 2002) to the broiler industry (Knoeber and Thuman, 1995; Goodhue, 2000). Glover et al (1994) show that the predetermination of prices in contracts between farmers and a firm confronted with the volatility of spot market prices, induces an optimal sharing of risks, and improves productive efficiency. Other studies analyze contracts in the agricultural supply chain in the event that one party does not have perfect information on one or more characteristics of the other (Knoeber and Thuman 1995; Goodhue 2000; Hueth and Ligon, 2002). In addition to the optimal allocation of risks, the improvement of productive efficiency and the reduction of information rents, agricultural contracts have other virtues: the stabilization of the supply chain of perishable products when the number of buyers and potential sellers is weak (MacDonald and Korb, 2011), the introduction of new agricultural products and new technologies (Boehlje et al, 1998), etc.

All of these studies mentioned above have three essential characteristics: firstly, contracts farming are essentially linear contracts; secondly, these contracts farming do not include sustainable agriculture issues; and thirdly, these contracts are analyzed as spot contracts, that is to say, static contracts. Concerning the linearity of contracts farming, the studies explain this by the need to be as close as possible to the real world (Stiglitz 1974; Eswaran and Kotwal 1985; Laffont and Matoussi 1995; Dubois 1999). The linear sharing rule between the landowner and the landlord is for the owner to offer the agent a fixed share of the production. However, in general, linear contracts are not considered optimal. The linear sharing rule is less powerful than the second-best optimal contract. For example, Laffont and Matoussi (1995) show that the linear contract, even if it provides sufficient incentives for effort, is an inefficient way of extracting rent from agents. While in the second-best optimal contract, the yield on agent output is zero in the bad state of nature, it is positive with the linear sharing rule. Thus, with a linear sharing rule, it is difficult to punish agents for poor performance. Agents always have a strictly positive informational rent. But, under certain assumptions about contractual capacities and preferences, it is possible to bring out, at an optimum, the linearity of the contract farming (Holmstrom and Milgrom, 1987).

Regarding the issues of sustainable agriculture, we note that the concerns of land maintenance are absent from land leases (tenant farming, sharecropping) or production and marketing contracts between an agribusiness firm and farmers. The choice of the optimal contractual form is based on reasons related to risk sharing, transaction costs or market imperfections. However, by showing that taking into account the maintenance of the land may be sufficient to explain the choice of contract without the need to invoke reasons related to risk sharing, transaction costs or market imperfections, Dubois (1999) introduces issues of sustainable agriculture into the analysis of contracts farming. More specifically, he shows that taking into account the evolution of soil fertility endogenously in a principal-agent model, the optimal contract arbitrates between risk sharing, incentives to produce and maintenance of the land. The model of Dubois (1999) focuses on agricultural spot contracts and shows that these static contracts are characterized by a memory effect linked to the evolution of the fertility of the land. The terms of payment therefore take into account the trade-off between the incentive to produce and the incentive to invest in the fertility of the land, to maintain the land. The contingent payment that the agent receives each period depends on the evolution of the fertility of the land. If effort and fertility are complementary in the production function, making more effort to produce more can increase the fertility of the land. But when effort and fertility are substitutes in the production function, there is a tradeoff between the incentive to produce and the incentive to maintain the land. Making more effort to produce more reduces the fertility of the land. In this case where more effort lowers fertility, the optimal spot contract must be less incentive to effort.

As regards the nature of contracts farming as spot contracts, it must be emphasized that even when the relationship between the principal and the agent is considered over a long period of time, it is only a question of repetition of spot contracts. Such a repetition of the static optimal contract is never optimal (Rogerson, 1985). But the importance of these static contractual forms is due to the fact that the long-term commitment is not possible because it is not credible (Dubois, 1999). However, with the "planted-shared" contracts practiced mainly in forest areas in Africa, there is an example of an agricultural contract that is not a static spot contract. These "planted-shared" contracts are real dynamic contracts farming that can be analyzed within the framework of the theory of long-term contracts via the principal-agent paradigm with bilateral engagement of the actors. Long-term contracts have been studied in contexts other than agriculture. In these studies, the optimal long-term contract is usually a contract with memory that smooths the agent's income by transferring a portion of the payment to the second period, which imposes a commitment from the principal (Lambert, 1983; 1985a, Chiappori et al, 1994). It also happens that the optimal long-term contract is a contract without memory where the remuneration of the agent in the second period depends only on the second period. Formally, if τ^{ij} is the second period remuneration for a second period performance j and a first period performance i, we have $\tau^{ij} = \tau^{kj}$ for all $i, k = 1, \dots, n$.

The "planted-shared" contracts specific to perennial crops are here analyzed as long-term linear contracts and integrating concerns for the maintenance of the land. For these crops, in general, effort and fertility are substitutes in the production function. As a result, there is a trade-off between the incentive to produce and the incentive to maintain the land. The more the effort reduces fertility, the less the optimal long-term contract must be incentive to effort. Starting from these hypotheses, we will try in the following to apprehend the characteristics of such linear long-term contracts. The challenge is, as has been pointed out, to prevent and resolve rural land conflicts and improve the management of natural resources.

3. The dynamic principal-agent model

3.1. Assumptions of the model

We consider a principal-agent relationship that takes place over two periods. During each period the agent (the operator) chooses a level of effort, then the nature determines the resulting performance: the level of production. Production is observable and verifiable. It depends on the fertility of the land, the effort of the agent and the climatic hazard according to a following production function drawn from Otsuka, Chuma and Hayami (1992) and Dubois (1999):

$$y_t = v_t f(x_{t-1}, e_t) \quad (1)$$

The function $f(.)$ is increasing concave in its arguments. v_t is a positive random variable reflecting the climatic hazard. x_{t-1} is the land fertility index at the end of the period $t-1$. The effort e_t is unobservable by the principal (the transferor of the earth), the monitoring costs being assumed to be too high. The land fertility index x_t is observable and verifiable, so contracts are contingent on the future fertility of the land. Suppose the effort e can only take two values in $\{0,1\}$. The costs of the effort are noted $C(1) = C$ and $C(0) = 0$. At each period, the effort of the agent makes it possible to reach a stochastic performance $\tilde{y}_t = \overline{y}$ (resp. $\underline{y}$) with probability $P^i(e_t)$ (resp. $(1 - P^i(e_t))$. We denote $P_1^i = P^i(1)$ and $P_0^i = P^i(0)$ and $\Delta P^i = P_1^i - P_0^i$. So $P^i(e)$ is the probability to obtain the result i when the level of effort is e.These probabilities are assumed to be identical for both periods.The performances are supposed to be independent from one period to the next. The payment of each period is contingent on the present and past performances. In the first period, the agent receives τ^i if the observed result is i; likewise, in the second period, he receives a remuneration τ^{ij} if he has obtained the result i in the first period then the result j in second period. We assume that there is no agricultural credit market. While noting a the sharing rate between the principal and the agent, let us consider linear contracts of the form:

$$\tau^i = a^i y_1 + b \quad \text{where } y_1 \in \left\{\underline{y}_1, \overline{y}_1\right\} \text{ and } \tau^i \in \left\{\underline{\tau}^i, \overline{\tau}^i\right\} \quad (2)$$

$$\tau^{ij} = a^{ij} y_2 + b \quad \text{where } y_2 \in \left\{\underline{y}_2, \overline{y}_2\right\} \text{ and } \tau^{ij} \in \left\{\underline{\tau}^{ij}, \overline{\tau}^{ij}\right\} \quad (3)$$

The contract between the principal and the agent relates to the payment of remuneration at the end of each period. The remuneration is contingent on the information available, namely the performance of the first period for the first remuneration and the two performances for the second. The two co-contractors can engage in a long-term contract with bilateral commitment. It is assumed that there is no actualization, which simplifies the calculations without changing the conclusions. The principal is supposed to be risk-neutral, and its objective-function is written:

$$V = P_1^i\left(\overline{y}_1 - \overline{\tau}^i + P_1^i\left(\overline{y}_2 - \overline{\tau}^{ij}(\overline{y}_2)\right) + \left(1 - P_1^i\right)\left(\underline{y}_2 - \underline{\tau}^{ij}(\overline{y}_2)\right)\right)$$
$$+ \left(1 - P_1^i\right)\left(\underline{y}_1 - \underline{\tau}^i + P_1^i\left(\overline{y}_2 - \overline{\tau}^{ij}(\underline{y}_2)\right) + \left(1 - P_1^i\right)\left(\underline{y}_2 - \underline{\tau}^{ij}\left(\underline{y}_2\right)\right)\right) \quad (4)$$

It is assumed that the agent is risk averse, without which the problem of moral hazard is trivial. Its Von Neumann-Morgenstern utility function $u\,(.)$ is therefore strictly increasing and concave. Moreover, it is assumed that the set of possible values of payment and utility is unbounded. The utility of the agent is: $U(\tau_1(y_1), \tau_2(y_2), e_1, e_2) = u(\tau_1(y_1)) - C(e_1) + u(\tau_2(y_2)) - C(e_2)$. In the stationary framework, the separability conditions on the function of utility and independence of the probability distributions are sufficient to avoid introducing an adverse selection dimension and to maintain a pure moral hazard model. The principal decides to concretize the effort e_1 in the first period and the effort e_2^i in the second period if the result i has been observed in the past. He will seek to minimize the costs necessary to induce the agent to follow such behavior. The incentive constraint of the agent in the second period is, for any result i obtained before:

$$P_1^i u_2\left(\overline{\tau}^{ij}(y_2)\right) + \left(1 - P_1^i\right)u_2\left(\underline{\tau}^{ij}(y_2)\right) - C \geq P_0^i u_2\left(\overline{\tau}^{ij}(y_2)\right) + \left(1 - P_0^i\right)u_2\left(\underline{\tau}^{ij}(y_2)\right) \quad (5)$$

By adopting the following notations: $u(\tau^i(y_1)) = u_1$; $u(\tau^i(\overline{y}_1)) = \overline{u}_1$ and $u\left(\tau^i(\underline{y}_1)\right) = \underline{u}_1$; and $u(\tau^{ij}(y_2)) = u_2(y_1)$; $u_2\left(\overline{\tau}^{ij}(y_2)\right) = \overline{u}_2(y_1)$ and $u_2\left(\underline{\tau}^{ij}(y_2)\right) = \underline{u}_2(y_1)$, the above constraint is written:

$$P_1^i\overline{u}_2(y_1) + \left(1 - P_1^i\right)\underline{u}_2(y_1) - C \geq P_0^i\overline{u}_2(y_1) + \left(1 - P_0^i\right)\underline{u}_2(y_1) \;\Rightarrow\; \overline{u}_2(y_1) - \underline{u}_2(y_1) \geq \frac{C}{\Delta P^i} \quad (6)$$

Similarly, in the first period, assuming the satisfied second-period constraint, the incentive constraint of the agent is written:

$$\overline{u}_1 + P_1^i\overline{u}_2(\overline{y}) + \left(1 - P_1^i\right)\underline{u}_2(\overline{y}) - \left(\underline{u}_1 + \left(P_1^i\overline{u}_2\left(\underline{y}\right) + \left(1 - P_1^i\right)\underline{u}_2\left(\underline{y}\right)\right)\right) \geq \frac{C}{\Delta P^i} \quad (7)$$

In the class of contract verifying these inequalities, the agent's second-period participation constraint is written, for any result i obtained in the past:

$$P_1^i\overline{u}_2(y_1) + \left(1 - P_1^i\right)\underline{u}_2(y_1) - C \geq u_2(y_1) \quad (8)$$

This constraint reflects the lack of commitment of the agent. Imposing that it is verified implies that the agent has not engaged in a long-term relationship. Therefore, the bilateral commitment assumption requires removing this constraint.

The intertemporal participation constraint is:

$$P_1^i\left(\overline{u}_1 + \left(P_1^i\overline{u}_2(\overline{y}) + \left(1 - P_1^i\right)\underline{u}_2(\overline{y})\right)\right) + \left(1 - P_1^i\right)\left(\underline{u}_1 + \left(P_1^i\overline{u}_2\left(\underline{y}\right) + \left(1 - P_1^i\right)\underline{u}_2\left(\underline{y}\right)\right)\right) - 2C \geq 0 \quad (9)$$

The main objective is to determine the long-term contract (τ^i, τ^{ij}) which maximizes its expected utility, guarantees the agent's good incentives for effort, and ensures the proper sharing of risks and the best maintenance of the land. Noting h the inverse function of the utility function of the agent, we put: $\tau^i(\overline{y}_1) = \overline{\tau}^i = u^{-1}(\overline{u}_1) = h(\overline{u}_1)$; $\underline{\tau}^i = h(\underline{u}_1)$; $\overline{\tau}^{ij}(y_2) = h(\overline{u}_2(y_1))$; $\underline{\tau}^{ij}(y_2) = h(\underline{u}_2(y_1))$. The principal program consists in maximizing the objective-function V of the principal under the constraints of participation and incentive. This program which we will note P is written as follows:

$$Max_{\{(\overline{u}_1,\underline{u}_1)(u_2(\overline{y}),u_2(\underline{y}))\}}V = P_1^i\left(\overline{y}_1 - h(\overline{u}_1) + P_1^i\left(\overline{y}_1 - h(\overline{u}_2(\overline{y}))\right) + \left(1 - P_1^i\right)\left(\underline{y}_1 - h(\underline{u}_2(\overline{y}))\right)\right) +$$
$$\left(1 - P_1^i\right)\left(\underline{y}_1 - h(\underline{u}_1) + P_1^i\left(\overline{y}_1 - h\left(\overline{u}_2(\underline{y})\right)\right) + \left(1 - P_1^i\right)\left(\underline{y}_1 - h\left(\underline{u}_2(\underline{y})\right)\right)\right)$$

Subject to constraints (6), (7) and (9)

3.2. Resolution of the program of the principal

To solve the optimization program P, there are two steps. The first step is to optimize the principal pay-off in the second period to determine its contract continuation value $V_2(u_2(y_1))$. This value is in fact the value of the program $P_2(y_1)$ which optimizes the objective function:

$$Max_{\{\overline{u}_2(y_1),\underline{u}_2(y_1)\}}P_1^i\left(\overline{y}_2 - h(\overline{u}_2(y_1))\right) + \left(1 - P_1^i\right)\left(\underline{y}_2 - h(\underline{u}_2(y_1))\right) \quad (10)$$

under the constraints (6), (7) and (9). The constraints (6) and (9) are saturated to the optimum and the following optimal solutions are obtained: $\overline{u}_2^D(y_1) = C + u_2(y_1) + \left(1 - P_1^i\right)\frac{C}{\Delta P^i} \Rightarrow \underline{u}_2^D(y_1) = C + u_2(y_1) - P_1^i\frac{C}{\Delta P^i}$. The second best cost to implement the high effort in period 2 following the promise of second-period utility $u_2(y_1)$

is : $C_2^{SB}(u_2(y_1)) = P_1^i h\left(C + u_2(y_1) + (1 - P_1^i)\frac{C}{\Delta P^i}\right) + (1 - P_1^i) h\left(C + u_2(y_1) - P_1^i \frac{C}{\Delta P^i}\right) \Rightarrow C_2^{SB}(u_2(y_1)) = P_1^i h(\bar{u}_2^D(y_1)) + (1 - P_1^i) h\left(\underline{u}_2^D(y_1)\right)$. And so, we have as contract continuation for the principal: $V_2(u_2(y_1)) = P_1^i \bar{y}_2 + (1 - P_1^i)\underline{y}_2 - C_2^{SB}(u_2(y_1))$.

The second step is to return to the resolution of the initial program P by rewriting it in the form of the following program P':

$$\begin{cases} Max_{\{(\bar{u}_1, \underline{u}_1)(u_2(\bar{y}), u_2(\underline{y}))\}} P_1^i \left(\bar{y}_1 - h(\bar{u}_1)\right) + (1 - P_1^i)\left(\underline{y}_1 - h(\underline{u}_1)\right) + \left(P_1^i V_2(u_2(\bar{y})) + (1 - P_1^i) V_2\left(u_2\left(\underline{y}\right)\right)\right) \\ s.t \quad \bar{u}_1 + u_2(\bar{y}) + \left(\underline{u}_1 + u_2\left(\underline{y}\right)\right) \geq \dfrac{C}{\Delta P^i} \\ \quad P_1^i(\bar{u}_1 + u_2(\bar{y})) + (1 - P_1^i)\left(\underline{u}_1 + u_2\left(\underline{y}\right)\right) \geq C \end{cases}$$

These two constraints are nothing more than rewrites of the constraints (6) and (7). Let the multipliers λ and μ of these constraints. Since the problem P' is a concave problem with linear constraints, the first-order conditions of Kuhn and Tucker are necessary and sufficient to characterize the optimality:

$$\begin{cases} P_1^i h'(\bar{u}_1^D) = \lambda + \mu P_1^i & (11) \\ (1 - P_1^i) h'(\underline{u}_1^D) = -\lambda + \mu(1 - P_1^i) & (12) \\ P_1^i C_2^{SB'}(u_2^D(\bar{y})) = \lambda + \mu P_1^i & (13) \\ (1 - P_1^i) C_2^{SB'}\left(u_2^D\left(\underline{y}\right)\right) = -\lambda + \mu(1 - P_1^i) & (14) \end{cases}$$

The resolution of this system of equations makes it possible to obtain as results the following pairs $(\bar{u}_1^D, \underline{u}_1^D)$et $\left(u_2^D(\bar{y}), u_2^D(\underline{y})\right)$ (proof in *appendix 1*) :

$$h'(\bar{u}_1^D) = P_1^i h'(\bar{u}_2^D(\bar{y})) + (1 - P_1^i) h'\left(\underline{u}_2^D(\bar{y})\right) \quad (15)$$

$$h'(\underline{u}_1^D) = P_1^i h'\left(\bar{u}_2^D\left(\underline{y}\right)\right) + (1 - P_1^i) h'\left(\underline{u}_2^D\left(\underline{y}\right)\right) \quad (16)$$

Noting $E_{\tilde{y}_2}(.)$, the expectation operator with respect to the second-period production distribution induced by a high effort on that date, and $\tilde{u}_2^D$ the random variable of the second-period utilities, the two equations above satisfy ***the martingale property*** and therefore simplify to:

$$h'(u_1^D(y_1)) = E_{\tilde{y}_2}(h'(\tilde{u}_2^D(y_1)) \text{ for all } y_1 \in \left\{\underline{y}, \bar{y}\right\} \quad (17)$$

In the mathematical theory of measurement, the martingale property reflects the tendency of a random variable to remain centered around its previous value as in a balanced game. Recalling that $h(u_t^D(y_t))$ is the agent's pay in t, it follows that $h'(u_t^D(y_t))$ is his marginal remuneration per additional unit of production, the martingale property given by equation (17) is interpreted as follows: the agent's optimal incentive contract smoothes his income in an intertemporal way. Thus, a great performance in the first period is not entirely paid in the first period; it is spread until the second period. Such a smoothing of earnings is consistent with the assumption of no agricultural credit market. By further refining equation (17), it is possible to highlight a smoothing effect of a production sharing index and a land fertility index.

Smoothing effect of the production sharing index: To see it, let's first remember that $h(u_1^D(y_1)) = \tau^i(y_1) = a^i y_1 + b$ and $h(\tilde{u}_2^D(y_1)) = \tilde{\tau}^{ij}(y_2) = \tilde{a}^{ij} y_2 + b$ (linear contract hypothesis). In addition, the following notations are considered: $A^i = \frac{1}{a^i}$; $A^{ij} = \frac{1}{\tilde{a}^{ij}}$. If a^i and $\tilde{a}^{ij}$ are sharing rates, then A^i and $A^{ij}$are the inverse of the share rates that will be called share indices. We then show that we can write a martingale solution on the production sharing indices in the following way (proof in *appendix 2*):

$$A^i = M \times E_{\tilde{y}_2}(A^{ij}) \quad \text{where } M \equiv E_{\tilde{y}_2}\left[\frac{h'_y(\tilde{u}_2^D(y_1))}{h'_y\,(u_1^D(y_1))}\right] \quad (18)$$

With $M \gtreqless 1$, it come that:

$$A^i \gtreqless E_{\tilde{y}_2}(A^{ij}) \quad (19)$$

If $M = 1$, we have $A^i = E_{\tilde{y}_2}(A^{ij})$: the optimal sharing index is a martingale.

If $M > 1$, we have $A^i > E_{\tilde{y}_2}(A^{ij})$: the optimal sharing index is a supermartingale.

If $M < 1$, we have $A^i < E_{\tilde{y}_2}(A^{ij})$: the optimal sharing index is a submartingale.

Smoothing effect of the land fertility index: let us remember first that $h(u_1^D(y_1)) = \tau^i(y_1) = a^i y_1 + b$ and $h(\tilde{u}_2^D(y_1)) = \tilde{\tau}^{ij}(y_2) = \tilde{a}^{ij} y_2 + b$ (linear contract hypothesis), and that $y_1 = v_1 f(x_0, e_1)$ and $y_2 = v_2 f(x_1, e_2)$. Consider here a Cobb-Douglas form: $f(x_0, e_1) = {x_0}^{\alpha} {e_1}^{1-\alpha}$. In addition, the following notations are considered:

$X_0 = \frac{1}{{x_0}^{\alpha-1}}$; $X_1 = \frac{1}{{x_1}^{\alpha-1}}$. If x_0 and x_1 are land fertility indices, then X_0 and X_1 are also land fertility indices. We then show that we can write a martingale solution on the fertility indices of the earth in the following way (proof in *appendix 3*):

$$X_0 = N \times E_{\tilde{y}_2}(X_1) \quad \text{où } N \equiv E_{\tilde{y}_2}\left[\frac{v_1 {e_1}^{1-\alpha}}{v_2 {e_2}^{1-\alpha}} \frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{h'_x\left(u_1^D(y_1)\right)}\right] \tag{20}$$

With $N \gtrless 1$, it come that:

$$X_0 \gtrless E_{\tilde{y}_2}(X_1) \tag{21}$$

If $N = 1$, we have $X_0 = E_{\tilde{y}_2}(X_1)$: the optimal land fertility index is a martingale.
If $N > 1$, we have $X_0 > E_{\tilde{y}_2}(X_1)$: the optimal land fertility index is a supermartingale.
If $N < 1$, we have $X_0 < E_{\tilde{y}_2}(X_1)$: the optimal land fertility index is a submartingale.

3.3. Results interpretation

The relation (18) shows that the optimal sharing index at the end of each culture cycle is a martingale; it depends on the past history of relationships. This property reflects the intertemporal smoothing effect of the agent's income, in the absence of a credit market. The optimal "planted-shared" contract, a long-term contract, highlights a memory effect linked to the consideration of present and past performances in payment terms. In a nutshell, the optimal long-term contract is a memory contract. The results are summarized in the following proposition:

Proposition 1: Under the hypothesis of the absence of an agricultural credit market, the optimal "planted-shared" contract is a long-term memory contract. More precisely, the optimal production sharing index is a martingale: $A^i = E_{\tilde{y}_2}(A^{ij})$, which reflects an intertemporal smoothing effect of the agent's income.

Intuitively, we can explain this result as follows: a good performance in the first period translates, given effort, favorable random circumstances. The corresponding gain supplement, necessary to ensure a correct incentive, is transient. He has no reason to reproduce in the future. The agent will therefore seek to distribute this gain over time, which, in this context, is only possible by a memory contract. In other words, if the agent performs well in the first period, corresponding to a high payment, then the optimal long-term contract smooths his income by transferring part of the payment to the second period; this imposes a commitment of the principal. This result obtained in the context of a long-term contract farming is consistent with that obtained for other types of contract (Lambert, 1983; Rogerson, 1985a; Chiappori et al, 1994).

To see it formally, we start from equation (18): $A^i = E_{\tilde{y}_2}(A^{ij}) E_{\tilde{y}_2}\left[\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\left(u_1^D(y_1)\right)}\right]$. Because of the smoothing of the remuneration, we have $\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\left(u_1^D(y_1)\right)} \approx 1 \Rightarrow A^i \approx E_{\tilde{y}_2}(A^{ij})$. From there, it comes that the agent smooths the sharing index at the optimum, that is to say that this index is a martingale ($A^i \approx E_{\tilde{y}_2}(A^{ij})$). Let's see what happens when the agent does not smooth the production sharing index. This is to analyze the implications of the fact that this index is not a martingale. Is one always at the optimum in this case? Let us first state that the sharing index is a supermartingale (case where $A^i > E_{\tilde{y}_2}(A^{ij})$). Because of the smoothing of the remuneration, we have $\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\left(u_1^D(y_1)\right)} > 1$. The agent is better paid in the second period than in the first period if he makes an extra effort in each period. The long-term contract is not incentive to effort on an intertemporal basis. When the sharing index is a submartingale (case where $A^i < E_{\tilde{y}_2}(A^{ij})$), we have $\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\left(u_1^D(y_1)\right)} < 1$: The agent is better paid in the first period than in the second period if he makes an extra effort in each period. Again, the long-term contract is not incentive to effort on an intertemporal basis. We form the following proposition:

Proposition 2: The martingale property of the production sharing index is a necessary not only necessary but also sufficient condition of the optimality of the "planted-shared" long-term contract farming. In other words, the necessary and sufficient condition of the optimality of this contract is the intertemporal smoothing of the sharing index.

The relation (21) shows, on the one hand, that the optimal land fertility index at the end of each culture cycle is a martingale (case where $X_0 = E_{\tilde{y}_2}(X_1)$), and on the other hand, that the remuneration of the agent depends on both the production and the fertility of the land. Thus, not only the optimal contract "planted-shared", a long-term contract, highlights a memory effect related to the taking into account of the present and past land fertility in the terms of payment, but also it incites the agent to smooth, in an intertemporal perspective, the fertility of the land. Such an intertemporal smoothing of the land fertility, a guarantee of sustainable agriculture, results from the trade-off between the incentive to produce and the incentive to maintain the land. Indeed, the agent would like to increase his effort to obtain more production and therefore more compensation, but at the same time, increasing the effort reduces the fertility of the land, which tends to reduce the remuneration. Thus, the more the effort reduces fertility, the less the optimal long-term contract must be incentive to the effort. As the agent wants to smooth his pay, he smooths his effort, which induces a smoothing of the fertility of the land. To see it formally, we start from equation (20): $X_0 = E_{\tilde{y}_2}(X_1)\, E_{\tilde{y}_2}\left[\frac{v_1 {e_1}^{1-\alpha}}{v_2 {e_2}^{1-\alpha}} \frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{h'_x\left(u_1^D(y_1)\right)}\right]$. Because of the smoothing of the remuneration, we have $\frac{h'_x\left(u_1^D(y_1)\right)}{h'_x\left(\tilde{u}_2^D(y_1)\right)} \approx 1 \Rightarrow X_0 \approx E_{\tilde{y}_2}(X_1)\, E_{\tilde{y}_2}\left[\frac{v_1 {e_1}^{1-\alpha}}{v_2 {e_2}^{1-\alpha}}\right]$. From there, it comes that the agent smooths the optimal index of fertility to the optimum, that is to say that this index is a martingale ($X_0 \approx E_{\tilde{y}_2}(X_1)$) if it smooths its effort, namely $\frac{{e_1}^{1-\alpha}}{{e_2}^{1-\alpha}} \approx 1$. It is assumed here implicitly that the climatic hazard remains constant over time ($v_1 = v_2$). The results are summarized in the following proposition:

Proposition 3: Under the hypothesis of the absence of an agricultural credit market, the optimal "planted-shared" contract is a long-term memory contract that takes into account the present and past fertility of the land in terms of payment. Specifically, the optimal land fertility index is a martingale, which reflects an intertemporal smoothing effect of fertility and thus the maintenance of the earth.

This result can be explained intuitively, as follows: a good performance in the first period reflects consistent efforts and therefore a lower fertility of the land for the second period. The good remuneration received by the agent in the first period because of his effort gives him a correct incentive. But the agent perceives this additional gain as transitory, having no reason to reproduce in the future. Indeed, the agent knows that in the second period, a high effort will not necessarily produce a good performance due to a lower fertility of the earth. The agent will therefore seek to distribute this gain over time, by accepting a memory contract. He therefore has an interest in smoothing his effort intertemporally. The optimal long-term contract thus tends, via the intertemporal smoothing of the effort, to transfer a part of the fertility of the earth towards the second period. Such an optimal contract, through its incentive to maintain the land, is compatible with sustainable agriculture. Let's see what happens when the agent does not smooth the land fertility index. This is to analyze the implications of the fact that this index is not a martingale. Is one always at the optimum in this case? Let us first assume that the fertility index of the earth is an supermartingale (case where $X_0 > E_{\tilde{y}_2}(X_1)$). Because of the smoothing of the remuneration, we will have $\frac{{e_1}^{1-\alpha}}{{e_2}^{1-\alpha}} > 1$. The agent makes a lot of effort in the first period and little effort in the second half. The long-term contract is not incentive to effort on an intertemporal basis.

When the land fertility index is a submartingale (case where $X_0 < E_{\tilde{y}_2}(X_1)$), we have $\frac{{e_1}^{1-\alpha}}{{e_2}^{1-\alpha}} < 1$: the agent makes little effort in the first period and a lot of effort in the second half. Again, the long-term contract is not incentive to effort on an intertemporal basis. We form the following proposition:

Proposition 4: The martingale property of the land fertility is a condition not only necessary but also sufficient of the optimality of the "plantation-shared" long-term agricultural contract. In other words, the necessary and sufficient condition of the optimality of this contract is the intertemporal smoothing of the fertility of the land, that is to say its compatibility with sustainable agriculture.

4. Concluding remarks

In this article, we analyzed the optimality properties of "planted-shared" contracts farming that are compatible with a sustainable development issue. This question is all the more important since these contracts, dealing with long-cycle crops, must take into account the willingness of the sellers to preserve the fertility of their land throughout the duration of the contract. Using a dynamic contract model with bilateral engagement via the principal-agent paradigm, we have shown that in the absence of an agricultural credit market, the optimal long-term contract is a memory contract, essentially responding to a need of intertemporal smoothing of both the production sharing index and the land fertility index. Due to the intertemporal smoothing effect of the production sharing index, sharing rules such as "half-share rate over the entire contract period", "third-party sharing rate over the entire period of the "contracts" are those that are close to optimal "planted-shared" contracts contract in the long term. The smoothing effect of the land fertility index reflects the idea of a compatibility of the long-term optimal contract farming with sustainable agriculture.

Faced with the strong demand for securing "planted-shared" contracts, public authorities could respond by legal formalization and public validation by promoting such optimal contracts. These can improve not only the relationship between owners of land rights and non-holders of these rights, but also promote the efficient management of natural resources; this for a rural land market that is better organized and more compatible with sustainable agriculture. The model we have developed can be enriched by raising the hypothesis that effort and fertility are substitutes in the production function. Such an assumption implicitly implies that the agent does not use fertilizers. In this case, the maintenance of the land consists in efficiently dosing its use. But one can very well imagine a case where the agent uses fertilizers or certain specific cultural practices to maintain the land. In this case, it can both increase its production effort and the fertility of the land. Effort and fertility are no longer substitutes but rather complements in the production function. It would be interesting to determine the characteristics of the optimal long-term contract in this eventuality. We leave this question for future research.

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Appendix 1:
We consider the system of equations below characterizing the optimality of the program (P')

$$\begin{cases} P_1^i h'(\bar{u}_1^D) = \lambda + \mu P_1^i & (a) \\ (1-P_1^i)h'(\underline{u}_1^D) = -\lambda + \mu(1-P_1^i) & (b) \\ P_1^i C_2^{SB'}(u_2^D(\bar{y})) = \lambda + \mu P_1^i & (c) \\ (1-P_1^i)C_2^{SB'}\left(u_2^D\left(\underline{y}\right)\right) = -\lambda + \mu(1-P_1^i) & (d) \end{cases}$$

Where λ and μ are the multipliers of respectively constraint (6) and (7). The sum of (a) and (b) gives $\mu = P_1^i h'(\bar{u}_1^D) + (1-P_1^i)h'(\underline{u}_1^D) > 0 \Rightarrow$ the intertemporal participation constraint is necessarily saturated. It comes: $\lambda = P_1^i(1-P_1^i)(h'(\bar{u}_1^D) - h'(\underline{u}_1^D))$. Form (c) and (d), we have: $\lambda = P_1^i(1-P_1^i)\left(C_2^{SB'}(u_2^D(\bar{y})) - C_2^{SB'}\left(u_2^D\left(\underline{y}\right)\right)\right)$. We obtain:

$$h'(\bar{u}_1^D) - h'(\underline{u}_1^D) = C_2^{SB'}(u_2^D(\bar{y})) - C_2^{SB'}\left(u_2^D\left(\underline{y}\right)\right) \qquad (e)$$

By identifying (a) and (c), it comes: $h'(\bar{u}_1^D) = C_2^{SB'}(u_2^D(\bar{y})) \qquad (f)$

By identifying (b) and (d), it comes: $h'(\underline{u}_1^D) = C_2^{SB\prime}\left(u_2^D\left(\underline{y}\right)\right)$ (g)

$C_2^{SB}\left(u_2(\overline{y})\right) = P_1^i h\left(\bar{u}_2^D(\overline{y})\right) + (1-P_1^i) h\left(\underline{u}_2^D(\overline{y})\right) \Rightarrow C_2^{SB\prime}\left(u_2(\overline{y})\right) = P_1^i h'\left(\bar{u}_2^D(\overline{y})\right) + (1-P_1^i) h'\left(\underline{u}_2^D(\overline{y})\right)$ (h)

and $C_2^{SB\prime}\left(u_2\left(\underline{y}\right)\right) = P_1^i h'\left(\bar{u}_2^D\left(\underline{y}\right)\right) + (1-P_1^i) h'\left(\underline{u}_2^D\left(\underline{y}\right)\right)$ (i)

by inserting (h) into (f) and (i) into (g), we obtain respectively:

$$h'(\bar{u}_1^D) = P_1^i h'^{\left(\bar{u}_2^D(\overline{y})\right)} + (1-P_1^i) h'^{\left(\underline{u}_2^D(\overline{y})\right)};\quad h'(\underline{u}_1^D) = P_1^i h'\left(\bar{u}_2^D\left(\underline{y}\right)\right) + (1-P_1^i) h'\left(\underline{u}_2^D\left(\underline{y}\right)\right)$$

These two equations satisfy the martingale property; they simplify to:

$$h'(u_1^D(y_1)) = E_{\tilde{y}_2}(h'(\tilde{u}_2^D(y_1)) \text{ for all } y_1 \in \left\{\underline{y}, \overline{y}\right\} \quad \text{QED}$$

Appendix 2: proof of the martingale property about the production sharing index

Remembering that $h(u_1^D(y_1)) = \tau^i(y_1) = a^i y_1 + b$ and $h\left(\tilde{u}_2^D(y_1)\right) = \tilde{\tau}^{ij}(y_2) = \tilde{a}^{ij} y_2 + b$ (linear contract hypothesis); one can form the martingale solution about production sharing. Indeed:
$h'_y(u_t^D(v_t f(x_{t-1}, e_t))) = h'_u(u_t^D(v_t f(x_{t-1}, e_t))) \times u'_y(y_t) \Rightarrow h'_y = h'_u \times u'_\tau \times \tau'_y$ as $\tau'_y = a$
$\Rightarrow h'_u = {h'_y}/{a.u'_y}$. we have: $u\left(\tau_1(y_1)\right) = u(a^i y_1 + b\ )$ and so:

$$h'_u(u_1^D(y_1)) = \frac{h'_y\ (u_1^D(y_1))}{a^i.u'_y};\quad h'_u\left(\tilde{u}_2^D(y_1)\right) = \frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{\tilde{a}^{ij}.u'_y}$$

The martingale solution is so written:

$$\frac{h'_y\ (u_1^D(y_1))}{a^i.u'_y} = E_{\tilde{y}_2}\left(\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{\tilde{a}^{ij}.u'_y}\right) \Rightarrow \frac{1}{a^i} = E_{\tilde{y}_2}\left(\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\ (u_1^D(y_1))}\frac{1}{\tilde{a}^{ij}}\right)$$

We can write, postulating the independence of the two terms under the expectation operator:

$$E_{\tilde{y}_2}\left(\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\ (u_1^D(y_1))}\frac{1}{\tilde{a}^{ij}}\right) = E_{\tilde{y}_2}\left(\frac{1}{\tilde{a}^{ij}}\right) \times E_{\tilde{y}_2}\left[\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\ (u_1^D(y_1))}\right]$$

Ultimately, the martingale solution is written :

$$\frac{1}{a^i} = E_{\tilde{y}_2}\left(\frac{1}{\tilde{a}^{ij}}\right) \times E_{\tilde{y}_2}\left[\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\ (u_1^D(y_1))}\right]$$

We consider the following notations: $A^i = \frac{1}{a^i}$; $A^{ij} = \frac{1}{\tilde{a}^{ij}}$; $M \equiv E_{\tilde{y}_2}\left[\frac{h'_y\left(\tilde{u}_2^D(y_1)\right)}{h'_y\ (u_1^D(y_1))}\right]$. If a^i and $\tilde{a}^{ij}$ are production sharing rates, then A^i and A^{ij} are the inverses of theses sharing rates which we call production sharing indices. With $M \gtrless 1$, it comes: $A^i \gtrless E_{\tilde{y}_2}(A^{ij})$.

If $M = 1$, we have $A^i = E_{\tilde{y}_2}(A^{ij})$: the optimal sharing index is a martingale.
If $M > 1$, we have $A^i > E_{\tilde{y}_2}(A^{ij})$: the optimal sharing index is a supermartingale.
If $M < 1$, we have $A^i < E_{\tilde{y}_2}(A^{ij})$: the optimal sharing index is a submartingale.

Appendix 3: Proof of the martingale property about the land fertility index

Remembering that $h(u_1^D(y_1)) = \tau^i(y_1) = a^i y_1 + b$ and $h\left(\tilde{u}_2^D(y_1)\right) = \tilde{\tau}^{ij}(y_2) = \tilde{a}^{ij} y_2 + b$ (linear contract hypothesis), and that $y_1 = v_1 f(x_0, e_1)$ and $y_2 = v_2 f(x_1, e_2)$, let's consider a Cobb-Douglas specification: $f(x_0, e_1) = {x_0}^{\alpha} {e_1}^{1-\alpha}$; one can form the martingale solution about land fertility. Indeed: $h(u_1^D(y_1)) = a^i v_1 f(x_0, e_1) + b$; $h\left(\tilde{u}_2^D(y_1)\right) = \tilde{a}^{ij} v_2 f(x_1, e_2) + b$; $h(u_t^D(y_t)) = a^i v_t f(x_{t-1}, e_t) + b$. We obtain:

$$h'_x(u_t^D(v_t f(x_{t-1}, e_t))) = h'_u(u_t^D(v_t f(x_{t-1}, e_t))) \times u'_f(v_t f(x_{t-1}, e_t)) \times f'_x(x_{t-1}, e_t)$$

$h'_x = h'_u \times u'_f \times f'_x$ with $u'_f = v_t$; from where $h'_x = v_t.h'_u.f'_x \Rightarrow h'_u = {h'_x}/{v_t f'_x}$. We have : $v_t f'_x = v_t \alpha {x_0}^{\alpha-1} {e_1}^{1-\alpha}$, and so:

$$h'_u(u_1^D(y_1)) = \frac{h'_x\ (u_1^D(y_1))}{v_1 \alpha {x_0}^{\alpha-1} {e_1}^{1-\alpha}};\quad h'_u\left(\tilde{u}_2^D(y_1)\right) = \frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{v_2 \alpha {x_1}^{\alpha-1} {e_2}^{1-\alpha}}$$

The martingale solution is so written:

$$\frac{h'_x\ (u_1^D(y_1))}{v_1 \alpha {x_0}^{\alpha-1} {e_1}^{1-\alpha}} = E_{\tilde{y}_2}\left(\frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{v_2 \alpha {x_1}^{\alpha-1} {e_2}^{1-\alpha}}\right) \Rightarrow \frac{1}{{x_0}^{\alpha-1}} = E_{\tilde{y}_2}\left(\frac{v_1 {e_1}^{1-\alpha}}{v_2 {e_2}^{1-\alpha}}\frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{h'_x\ (u_1^D(y_1))}\frac{1}{{x_1}^{\alpha-1}}\right)$$

Knowing that fertility and effort are two independent real random variables, it comes that:

$$E_{\tilde{y}_2}\left[\frac{v_1 {e_1}^{1-\alpha}}{v_2 {e_2}^{1-\alpha}} \frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{h'_x\ (u_1^D(y_1))} \frac{1}{{x_1}^{\alpha-1}}\right] = E_{\tilde{y}_2}\left(\frac{1}{{x_1}^{\alpha-1}}\right) \times E_{\tilde{y}_2}\left[\frac{v_1 {e_1}^{1-\alpha}}{v_2 {e_2}^{1-\alpha}} \frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{h'_x\ (u_1^D(y_1))}\right]$$

Ultimately, the martingale solution is written :

$$\frac{1}{{x_0}^{\alpha-1}} = E_{\tilde{y}_2}\left(\frac{1}{{x_1}^{\alpha-1}}\right) \times E_{\tilde{y}_2}\left[\frac{v_1 {e_1}^{1-\alpha}}{v_2 {e_2}^{1-\alpha}} \frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{h'_x\ (u_1^D(y_1))}\right]$$

We consider the following notations: $X_0 = \frac{1}{{x_0}^{\alpha-1}}$; $X_1 = \frac{1}{{x_1}^{\alpha-1}}$; $N \equiv E_{\tilde{y}_2}\left[\frac{v_1 {e_1}^{1-\alpha}}{v_2 {e_2}^{1-\alpha}} \frac{h'_x\left(\tilde{u}_2^D(y_1)\right)}{h'_x\ (u_1^D(y_1))}\right]$. If x_0 and x_1 are land fertility indices, then X_0 and X_1 are also land fertility indices. With $N \gtreqless 1$, it comes that: $X_0 \gtreqless E_{\tilde{y}_2}(X_1)$.

If $N = 1$, we have $X_0 = E_{\tilde{y}_2}(X_1)$: the optimal land fertility index is a martingale.

If $N > 1$, we have $X_0 > E_{\tilde{y}_2}(X_1)$: the optimal land fertility index is a supermartingale.

If $N < 1$, we have $X_0 < E_{\tilde{y}_2}(X_1)$: the optimal land fertility index is a submartingale