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Performance Evaluation of Automated Teller Machine (ATM) in Nigerian Banking Institution: A Case Study of First Bank of Nigeria Plc. Ibadan

Adeniran, Adefemi T.
Department of Mathematical Sciences, Augustine University Ilara-Epe
P.M.B. 1010 Ilara-Epe 106101, Lagos State, Nigeria

Ojo, J. F.*
Department of Statistics, University of Ibadan, Ibadan, Nigeria

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Abstract
This paper evaluates the performance of ATM machine in Nigeria banking sector using queue theory model. A few simple queues-models were analysed in terms of steady-state derivation. Theoretical formulations and results (with real-life dataset) were established for queue models with Poisson arrivals and exponential service durations. Derivation and calculation of some performance measure including the average queue length, average waiting time in the queue and in the system, and the probability of encountering the system in certain states such as empty, full having an available server or having to wait a certain time to be served were explored under single and multi-server. FIFO (first in, first out) queue discipline was adopted.

Keywords: Steady state, waiting-time, queue-discipline, multi-severs, FIFO (first in, first out).

1. Introduction

The need for dynamism in the business environment and growing competition among business firms have forced companies to constantly evolve new ideas in order to succeed and to ensure sustainable relevance (Mian *et al.*, 2012), (Sowunmi *et al.*, 2015). The realization of this fact has made some organizations in Nigeria, especially those interested in attaining and maintaining excellent performance, to pay close attention to training and development, inventions and innovations and so on. In addition, Sowunmi *et al.* (2015), Haghighi and Mishev (2016) contends that the competition in Nigerian banking sector is getting more intense, partly due to regulatory imperatives of universal banking and also due to customers' awareness of their rights. Bank customers have become increasingly demanding, as they require high quality, low priced, immediate service delivery and additional improvement of value from their chosen banks (Olaniyi, 2004). Service delivery in banks is personal, customers are either served immediately or join a queue (waiting line) if the system is busy (Odirichukwu *et al.*, 2014). A queue occurs where facilities are limited and cannot satisfy demand made against them at a particular period. However, most customers are not comfortable with waiting or queuing, the danger of keeping them in a queue is that their waiting time may amount to or could become a cost to them (Olaniyi, 2004). According to Elaglam (1978), Yussuff *et al.* (2016) while it may or may not be actually calculable, there is at least an economic lost attribute to each person while remaining in the queue. A queue for the purpose of this study is the aggregation of customer awaiting a service function; it is an everyday occurrence and results when the number of calling units exceeds the number of available service centre (Olaniyi, 2004). This has become an integral of any service, which refers to the whole time from arrival of inputs to their departure.

Nowadays, long queuing that customers usually experienced in branches of many banks in Nigeria is not just a disappointment to Nigerians, but also decelerate the growth and development rate of the country economy. Because of long queue in bank, customers jockey, jostle, balk, disgruntle, renege, fail to meet financial need of their clients, banks overcrowded due to small service capacity, which leads to inadequate ventilation, and consequently facilitate easy spread of communicable diseases. In facts, if long queue is known for a particular bank, the bank in question might lose good number of his customers for the fact that customers would prefer a place where they do not queue (wait) at all or where they wait for lesser period. The common experience in Nigeria is that most banks do not have the facilities and capacities to service the number of customers without much delay (Yusuff, *et al.*, 2015). To rectify these and other associated problems that occur because of long queue in Nigeria banking sector, Automated Teller Machine (ATM) were incorporated to reduce the customers' queue-length, reduce bankers' workload and bring economic-balance between the waiting time and service cost.

Automated Teller Machine (ATM) is a computerized electronic machine banking outlet, which allows customers to perform several banking operations or transactions such as withdrawing cash, making deposits, paying bills, obtaining bank statements, effect cash transfers and the likes without the aid of a teller or branch representative. John Shepherd-Barron and his team at De La Rue Instruments Ltd. invented this device (ATM). The Automated Teller Machine (ATM) was introduced into Nigeria market in 1989, first installed by National

Cash Registers (NCR) for the defunct Society General Bank in 1987, First Bank Plc in 1991 (Adeoti, 2013). Today, there are over three million ATMs worldwide as well as more than 200 different kinds of transaction possible on these highly interconnected terminals (ATM Industry Association (ATMIA) 50th Anniversary Factsheet, 2015). At the time of its invention, the ATM was an unproven device which people didn't even know they needed, far less would come to rely on (Wurim, 2013), (Ivo and Resing, 2015). It has subsequently revolutionized society and helped bring about the 24/7 self-service culture we know today with its convenient access to financial services beyond banking hours, such as cash withdrawals, balance enquiries and a growing range of value added services (Moutinho and Smith, (2000)). The ATM has come to facilitate banking services by reducing the number of customer awaiting the teller service and put customer in control of their cash for the first time (Adeoti, 2011). Funds transfer can be done through ATM from one account to another at the push of a button, essential information relating to a transaction could be made available thousands of miles away within minutes (Moutinho, 2000), (Odunukwe, 2013), (Odirichukwu *et al*., 2014).

In spite of this great contributions of ATM machine to financial banking sector, dispense error is very small compare to long queue problem at ATM service-point that customers are experiencing in every branches of many banks in Nigeria. Now, it is very crucial to ask the question "does ATM actually solve the problem or serve the purpose of its establishment?" This research is therefore, directed towards examining critically, the extent of waiting and service cost in Nigeria ATM service system. This we do by exploring customers' arrival rate and services rates of the machine, determine the average waiting time spent on ATM queue and average total time spent in the system. We draw a relationship between increasing the number of channels (ATM machines) or changing the operational system and the effect it has on the average waiting time of the customers and service cost. A major aspect of mathematical theory of probability that deals with this phenomenon of waiting is called queuing theory (Janos Sztrik, 2010). Queuing theory is concerned with the design and planning of service facilities to meet a randomly fluctuating demand for service in order to minimize congestion and maintain economic balance between service cost and waiting cost (Taha (2007), Haastrup (2008)). Therefore, this study is reviewing the applicability of queuing theory in evaluating the performance of Automated Teller Machine (ATM) in Nigerian banking sector.

2. Methodology

The data used for this research was collected at First Bank Plc., University of Ibadan Branch, Ibadan, Nigeria. The services of bank considered were restricted to only customer activities with ATM. The following notations were used: n = number of customers in a system both waiting and in service, λ= average number of customer arriving per unit of time, μ = average number of customers being served per unit of time, c = number of parallel servers, ρ = traffic intensity and P_n or $P_n(t)$ = probability that there are n = customers in the system at any time t, both waiting and in service. The usual assumptions that the arrivals are independent of each other; services are also independent; and the mean arrival and service rates do not change over time were preserved. Since arrivals and services occur in accordance with a Poisson process, the time intervals between arrivals/services (i.e. the waiting time between successive arrival/service) follow exponential distribution and as such the mean inter-arrival time is represented as $\frac{1}{\lambda}$ and similarly the time intervals between services is $\frac{1}{\mu}$ (Taha (2007), Kothari, (2008), Haastrup, (2008), Haghighi, (2016)).

2.1 Model I-(M/M/1): (∞/FIFO) Queuing System

This study considers one of the simplest queue model ($M|M|1$): ($\infty|FIFO$). In this model, we have Poisson arrival, Poisson service, exponential inter-arrival/service time, single channel, infinite system capacity and first in first out queue discipline. Let $P_n(t)$ be the probability that there are n customers in the system at time t, then the difference equation for $P_n(t)$ is

$$P_{n(t+\Delta t)} - P_n(t) = -(\lambda+\mu)\Delta t P_n(t) + \mu\Delta P_{n+1}(t) + \lambda\Delta t P_{n-1}(t) + 0(\Delta t) \quad (1)$$

when $n = 0$

$$P_{0(t+\Delta t)} - P_0(t) = -\lambda\Delta t P_0(t) + \mu\Delta P_1(t) + 0(\Delta t) \quad (2)$$

then

$$\lim_{\Delta t\to 0}\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = -(\lambda+\mu)P_0(t) + \mu P_{(n+1)}(t) + \lambda P_{n-1}(t) + 0(\Delta t);$$

and

$$\lim_{\Delta t\to 0}\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = -\lambda P_0(t) + \mu P_1(t) + 0(\Delta t)$$

so that we have

$$\frac{d}{dt}P_n(t) = -(\lambda+\mu)P_n(t) + \mu P_{n+1}(t) + \lambda P_{n-1}(t);\ n \geq 1 \quad (3)$$

and

$$\frac{d}{dt}P_0(t) = -\lambda P_n(t) + \mu P_1(t) \tag{4}$$

Equations (3) and (4) are known as difference equations in n and t. The steady state solution for P_n in the system at an arbitrary point of time is obtained by taking the limit of (3) $as\ t \to \infty$. If the steady-state exists ($\lambda < \mu$, as $t \to \infty$), then

$$P_n(t) \to P_n\ and\ \frac{d}{dt}P_n(t) \to 0\ as\ t \to \infty$$

$If\ \lambda = \mu$, there exists no queue. If $\frac{\lambda}{\mu} > 1$, we have an explosive state. Using the steady state condition, that is $\frac{d}{dt} = 0$, equations (3) and (4) becomes

$$0 = -(\lambda + \mu)P_n + \mu P_{n+1} + \lambda P_{n-1}; \qquad \geq 1\ and \tag{5}$$

$$0 = -\lambda P_0 + \mu P_1 \tag{6}$$

From equation (6), $P_1 = \frac{\lambda}{\mu}P_0$. Also from equation (5)

$$P_{n+1} = \frac{\lambda + \mu}{\mu}P_n - \frac{\lambda}{\mu}P_{n-1}, \qquad n \geq 1 \tag{7}$$

Using iterative procedure on equation (7), we obtain the following results

$$P_2 = \frac{\lambda + \mu}{\mu}P_1 - \frac{\lambda}{\mu}P_0 = \left(\frac{\lambda}{\mu}\right)^2 P_0$$

$$P_3 = \frac{\lambda + \mu}{\mu}P_2 - \frac{\lambda}{\mu}P_1 = \left(\frac{\lambda}{\mu}\right)^3 P_0$$

In general,

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \tag{8}$$

To give more support to equation (8), by principle of mathematical induction, equation (7) can be re-written as

$$P_{n+1} = \frac{\lambda + \mu}{\mu}\left(\frac{\lambda}{\mu}\right)^n P_o - \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu}\right)^{n-1} P_0$$

$$P_{n+1} = \frac{\lambda + \mu}{\mu}\left(\frac{\lambda}{\mu}\right)^n P_o - \left(\frac{\lambda}{\mu}\right)^n P_0$$

Which gives

$$P_{n+1} = \left(\frac{\lambda}{\mu}\right)^{n+1} P_0 \tag{9}$$

Therefore, this result holds for all values of $n \in Z^+$.

Using the boundary condition $\sum_{n=0}^{\infty} P_n = 1$, then equation (11) yields

$$1 = \sum_{n=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^n P_0$$

$$1 = P_0\left[\left(\frac{\lambda}{\mu}\right)^0 + \left(\frac{\lambda}{\mu}\right)^1 + \left(\frac{\lambda}{\mu}\right)^2 + \cdots\right]$$

$$1 = P_0\left[\frac{1}{1-\frac{\lambda}{\mu}}\right], \text{ sum of geometric series where } \frac{\lambda}{\mu} = \rho < 1$$

$$1 = P_0\left[\frac{1}{1-\rho}\right]$$

This implies that $P_0 = 1 - \rho$, resulting in the steady state

$$P_n = \rho^n(1-\rho); \rho < 1\ and\ n \geq 0 \tag{10}$$

Equation (10) is the probability distribution of queue length.

2.1.1 Characteristics of Model I

(i) Probability of queue size greater or equal to n

$$\mathrm{p}(\geq \mathrm{n}) = \sum_{\mathrm{k=n}}^{\infty}\mathrm{P_k} = \sum_{\mathrm{k=n}}^{\infty}(1-\rho)\rho^{\mathrm{k}}$$

$$= (1-\rho)\rho^{\mathrm{n}}\sum_{\mathrm{k=n}}^{\infty}\rho^{\mathrm{k-n}}$$

$$= (1-\rho)\rho^{\mathrm{n}}\sum_{\mathrm{k-=n}}^{\infty}\rho^{\mathrm{k-n}}$$

$$= (1-\rho)\rho^n[\rho^0 + \rho^1 + \rho^3 + \cdots] \text{ using } S_\infty = \frac{a}{1-r}$$

$$p(\geq n) = (1-\rho)\rho^n\left[\frac{1}{1-\rho}\right] = \rho^n$$

(ii) Average number of customers in the system $[E(n)]$

$$E(n) = \sum_{n=0}^{\infty} nP_n = \sum_{n=0}^{\infty} n(1-\rho)\rho^n$$

$$= \rho(1-\rho)\sum_{n=0}^{\infty} n\rho^{n-1}$$

$$= \rho(1-\rho)\frac{d}{d\rho}\sum_{n=0}^{\infty}\rho^n$$

$$= \rho(1-\rho)\frac{d}{d\rho}\left[\frac{1}{1-\rho}\right]$$

$$E(n) = \rho(1-\rho)\left[\frac{1}{(1-\rho)^2}\right]$$

Therefore, the average queue length or average number of customers in the system is

$$E(n) = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \tag{11}$$

(iii) Average queue length$E(m)$, where $m = n-1$ (that is, number of customers in queue minus customers in service).

$$E(m) = \sum_{m=0}^{\infty} mP_n$$

$$= \sum_{n-1=0}^{\infty}(n-1)P_n$$

$$= \sum_{n-1}^{\infty} nP_n - \sum_{n-1}^{\infty} P_n$$

$$= \sum_{n-0}^{\infty} nP_n - \left[\sum_{n=0}^{\infty} P_n - P_0\right]$$

$$E(m) = \frac{\rho}{1-\rho} - [1-(1-\rho)]$$

$$E(m) = \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)} \tag{12}$$

(iv) Average length of non-empty queue $E(m|m>0)$

$$E(m/m>0) = \frac{E(m)}{P(m>0)}$$

$$P(m>0) = P(n-1>0) = P(n>1)$$

$$P(m>0) = P(n>1) = \sum_{n=0}^{\infty} P_n - P_0 - P_1$$

Recall that $P_n = (1-\rho)\rho^n$

$$P(m>0) = 1-(1-\rho)-(1-\rho)\rho = \rho^2 = \left(\frac{\lambda}{\mu}\right)^2$$

Therefore,

$$E(m/m>0) = \frac{\lambda^2}{\mu(\mu-\lambda)} \div \left(\frac{\lambda}{\mu}\right)^2$$

$$E(m/m>0) = \frac{\lambda^2}{\mu(\mu-\lambda)} \times \frac{\mu^2}{\lambda^2}$$

$$E(m/m>0) = \frac{\mu}{\mu-\lambda} \tag{13}$$

(v) The Fluctuation (Variance) of queue length is given by $Var(n) = E(n^2) - [E(n)]^2$.
By algebraic transformations, $n^2 = n(n-1) + n \Rightarrow E(n^2) = E[n(n-1)] + E(n)$ Therefore,

$$E[n(n-1)] = \sum_{n=0}^{\infty} n(n-1)P_n$$
$$= \sum_{n=0}^{\infty} n(n-1)(1-\rho)\rho^n$$
$$= (1-\rho\hat{}2 \sum_{n=2}^{\infty} n(n-1)p^{n-2}$$
$$E[n(n-1)] = (1-\rho)\rho^2 \frac{d^2}{d\rho^2}\left(\frac{p^2}{1-\rho}\right) = \frac{2\rho^2}{(1-\rho)^2}$$

Therefore,

$$Var(n) = E(n^2) - [E(n)]^2$$
$$= E[n(n-1)] + E(n) - [E(n)]^2$$
$$Var(n) = \frac{2\rho^2}{(1-\rho)^2} + \frac{\rho}{1-\rho} - \left[\frac{\rho}{1-\rho}\right]^2$$
$$Var(n) = \frac{\lambda\mu}{(\mu-\lambda)^2} \tag{14}$$

(vi) Waiting Time Distribution for Model I: Waiting time is mostly a continuous random variable and there is a non-zero probability of delay being zero. Denote time spent in queue by W. Let $\psi_w(t)$ be the cumulative probability distribution so that from a complex randomness of the Poisson, we have

$$\psi_w(0) = P(w=0) = P_0 = 1-\rho = P(No\ customer\ on\ the\ system\ upon\ arrival)$$

$\psi_w(0) = P(w = 0) = P0 = 1 - \rho = P$ (No customer on the system upon arrival)

To find $\psi_w(t)$ for $t > 0$, we suppose there will be n customers in the system upon arrival. For a customer to go into service at time between 0 and$t$, it means all the customers must have been served at time t. Therefore,

$\psi\_n\ (t) = P[(n - 1)$ Customers are served at time t].P [one customer being served in time dt]

$$\psi_{n(t)} = \frac{(\mu t)^{n-1}e^{-\mu t}}{(n-1)!}\mu dt$$

The waiting time is therefore

$$\psi_w(t) = P[w \le t] = \sum_{n-1}^{\infty} P_n \int_0^t \psi_n(t) + \psi_w(0)$$
$$= \sum_{n=1}^{\infty}(1-\rho)\rho^n \int_0^t \frac{(\mu t)^{n-1}e^{-\mu t}}{(n-1)!}\ \mu dt + (1-\rho)$$
$$= (1-\rho)\rho \int_0^t \mu\, e^{-\mu t} \sum_{n-1}^{\infty} \frac{(\mu t\rho)^{n-1}}{(n-1)!}\, dt + (1-\rho)$$
$$= \mu\rho(1-\rho) \int_0^t e^{-\mu(1-\rho)t} dt + (1-\rho)$$
$$= \mu\rho(1-\rho)\left[\frac{e^{-\mu(1-\rho)t}}{-\mu(1-\rho)}\right]_0^t + (1-\rho)$$
$$\psi_w(t) = 1 - \rho^{e^{-\mu(1-\rho)t}}, \qquad t > 0$$

Hence, the distribution of waiting time in queue is

$$\psi_w(t) = \begin{cases} 1-\rho; & t = 0 \\ 1-\rho e^{-\mu(1-\rho)t}; & t > 0 \end{cases} \tag{15}$$

(vii) Average waiting time of a customer (in the queue)

$$E(w) = \int_0^{\infty} t \frac{d}{dt}\ \psi_w(t)dt$$
$$= \int_0^{\infty} t\frac{d}{dt}\left[1-\rho e^{-\mu(1-\rho)t}\right]dt$$
$$= \rho\mu(1-\rho)\int_0^{\infty} t^{2-1}\cdot e^{-\mu(1-\rho)t}dt$$
$$= \rho\mu(1-\rho)\frac{\Gamma(2)}{[\mu(1-\rho)]^2}$$

$$E(w) = \frac{\rho}{\mu(1-\rho)} = \frac{\lambda}{\mu(\mu-\lambda)}$$

(viii) Average waiting time of an arrival that has to wait: Recall that $P(w > 0) = 1 - P(w = 0) = \frac{\lambda}{\mu}$

$$E(w|w) > 0 = \frac{E(w)}{P(w > 0)}$$

$$E(w|w > 0) = \frac{\frac{\lambda}{\mu(\mu-\lambda)}}{\left(\frac{\lambda}{\mu}\right)} = \frac{1}{\mu-\lambda}$$

(ix) For the busy period distribution, suppose v is the random variable denoting the total time that a customer had to spend in the system including service. This makes the cumulative density function to be

$$\psi(w|w > 0) = \frac{\psi'(w)}{P(w > 0)} \tag{16}$$

Where,
$\psi'(w) = \frac{d}{dt}\psi_w(t) = \frac{d}{dt}\left(1 - \rho e^{-\mu(1-\rho)t}\right) = \rho\mu(1-\rho)^{-\mu(1-\rho)t}$; and $P(w > 0) = \rho$
Therefore,

$$\psi(w|w > 0) = \frac{\rho\mu(1-\rho)^{-\mu(1-\rho)t}}{\rho}$$

$$\psi(w|w > 0) = \mu\left(1 - \frac{\lambda}{\mu}\right)^{-\mu\left(1-\frac{\lambda}{\mu}\right)t}$$

$$\psi(w|w > 0) = (\mu-\lambda)e^{-(\mu-\lambda)t}, \qquad t > 0 \tag{17}$$

(x) Average waiting time that a customer spends in the system including service

$$E(v) = \int_0^\infty t\psi(w|w > 0)dt$$

$$= \int_0^\infty t(\mu-\lambda)e^{-(\mu-\lambda)t}dt$$

$$E(v) = (\mu-\lambda)\int_0^\infty t\, e^{-(\mu-\lambda)t}dt$$

Let $x = (\mu-\lambda)t, \implies t = \frac{x}{\mu-\lambda}, \implies dt = \frac{dx}{\mu-\lambda}$. Therefore

$$E(v) = (\mu-\lambda)\int_0^\infty \frac{x}{\mu-\lambda}e^{-x}\frac{dx}{\mu-\lambda}$$

$$E(v) = \frac{1}{\mu-\lambda}\int_0^\infty xe^{-x}dx = \frac{1}{\mu-\lambda}\Gamma(2)$$

$$E(v) = \frac{1}{\mu-\lambda}$$

2.2 Generalization of Model I-(M/M/C): (∞/FIFO) Queuing System

This system deals with queue(s) served by parallel service channels (servers). Here each server has an independently and identically distributed exponential service-time distribution. This is the generalization of the Birth-Death process in the M/M/1 system. In this case, we consider service channels more than 1 (i.e. c). It is a birth-death process in that the arrival is Poisson and the service is exponential. Thus, the mean arrival rate is given by $\lambda_n = \lambda\ \forall n$. The service rate is $c\mu$. This is because there are c channels each dispensing at the rate of μ. If $n \le c$ (where customers are less than the number of servers, no queue), so only n of the c servers will be busy making the mean service rate to be $n\mu$. However, if $n > c$, then queue is formed. Therefore

$$\mu_n = \begin{cases} n\mu; & 1 \le n < c \\ c\mu; & n \ge c \end{cases} \tag{19}$$

Thus, the steady state solution for the probability of n customer at time t using values of λ_n and μ_n becomes

$$P_n = \begin{cases} \frac{\lambda^n P_0}{n\mu(n-1)\mu\cdots(1)\mu}; & 1 \le n < c \\ \frac{\lambda^n}{(c\mu)(c\mu)\cdots(c\mu)(c-1)\mu(c-2)\mu\cdots(1)\mu}; & n \ge c \end{cases}$$

$$P_n = \begin{cases} \dfrac{\lambda^n P_0 \mu^{-n}}{n!}; & 1 \le n < c \\ \dfrac{\lambda^n P_0}{c^{n-c}\mu^n}; & n \ge c \end{cases}$$

$$P_n = \begin{cases} \dfrac{\rho^n P_0}{n!}; & n < c \\ \dfrac{\rho^n P_0}{c^{n-c} c!}; & n \le c \end{cases}$$

Using the boundary condition $\sum_{n=0}^{\infty} P_n =$

$$1 = \sum_{n=0}^{c-1} P_n + \sum_{n=0}^{\infty} P_n$$

$$= \sum_{n=0}^{c-1} \frac{1}{n!}\rho^n + \quad \rho^c \sum_{n=c}^{\infty} \frac{1}{c^n -^c c!}\rho^n P_0$$

$$1 = \left[\sum_{n=0}^{c-1} \frac{1}{n!}\rho^n \; + \rho^c \sum_{n=c}^{\infty} \frac{1}{c!}\left(\frac{\rho}{\mu}\right)^{n-c}\right] P_0$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \frac{1}{1 - \frac{\lambda}{c\mu}}\right]^{-1}$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{c^n}{n!}\left(\frac{\lambda}{c\mu}\right)^n + \frac{c^c}{c!}\left(\frac{\lambda}{c\mu}\right)\frac{1}{1 - \frac{\lambda}{c\mu}}^{n}\right]^{-1} \tag{20}$$

or

$$P_0 = \left[\frac{(c\rho)^n}{n!} + \frac{(c\rho)^n}{c!\,(1-\rho)}\right]^{-1}; with \;\; \rho = \frac{\lambda}{c\mu} \tag{21}$$

Equation (21) is only valid if $\frac{\lambda}{c\mu} < 1$ what this means is that the mean arrival must be less than the mean maximum potential service rate of the system. P_0 becomes same as model I of the M/M/1 system if $c = 1$.

2.2.1 Characteristics of M/M/C: ∞/FIFO Queuing System

The most commonly used measures of performance in a queuing situation are

(i) The probability that an arrival has to wait $P(n \ge c)$

$$P(n \ge c) = \sum_{n=c}^{\infty} P_n = \sum_{n=c}^{\infty} \frac{1}{c!\,c^{n-c}}\left(\frac{\lambda}{\mu}\right) P_0$$

$$P(n \ge c) = \frac{\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!\,(c\mu - \lambda))} P_0$$

$$P(n \ge c) = \frac{c^c\left(\frac{\lambda}{c\mu}\right)^c}{c!\left(1 - \frac{\lambda}{c\mu}\right)} P_0 = \frac{(c\rho)^c}{c!\,(1-\rho)} P_0 \tag{22}$$

(ii) Probability that the arrival enters the service without having to wait $P(n < c)$

$$P(n < c) = 1 - P(n \ge c) = 1 - \frac{c^c\left(\frac{\lambda}{c\mu}\right)^c}{c!\left(1 - \frac{\lambda}{c\mu}\right)} P_0 = 1 - \frac{(c\rho)^c}{c!\,(1-\rho)} P_0 \tag{23}$$

(iii) Expected number of customers in queue (L_q) (average queue length)

$$L_q = \sum_{n=c}^{\infty} (n-c) P_n = \sum_{x=0}^{\infty} x P_{x+c} \;\; for \; x = n - c$$

$$L_q = \sum_{n=0}^{\infty} x \frac{1}{c!\,c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n P_0 = \sum_{n=0}^{\infty} x \frac{1}{c!\,c^x}\left(\frac{\lambda}{\mu}\right)^{c+x} P_0$$

$$L_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c P_0 \sum_{x=0}^{\infty} x\left(\frac{\lambda}{c\mu}\right)^x$$

$$L_q = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{\lambda}{c\mu}\right) P_0 \frac{\partial}{\partial\left(\frac{\lambda}{c\mu}\right)} \sum_{x=0}^{\infty} \left(\frac{\lambda}{c\mu}\right)^x$$

$$L_q = \frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!} P_0 \frac{\partial}{\partial\left(\frac{\lambda}{c\mu}\right)}\left[1 + \left(\frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^2 + \left(\frac{\lambda}{\mu}\right)^3 + \cdots\right]$$

Using sum to infinity of a geometric series gives

$$L_q = \frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!} P_0 \times \frac{\partial}{\partial\left(\frac{\lambda}{c\mu}\right)}\left[\frac{1}{\left(1-\frac{\lambda}{c\mu}\right)}\right]$$

$$L_q = \frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!} P_0 \times \frac{1}{\left(1-\frac{\lambda}{c\mu}\right)^2}$$

$$L_q = \frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2} P_0 = \frac{c^c \rho^{c+1}}{c!\,(1-\rho)^2} P_0 \tag{24}$$

(iv) Expected number of customers in the system (L_s) or $E(n)$ is

$$L_s = L_q + \frac{\lambda}{\mu} = \frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2} P_0 + \frac{\lambda}{\mu} = \frac{c^c \rho^{c+1}}{c!\,(1-\rho)^2} P_0 + \frac{\lambda}{\mu} \tag{25}$$

(v) Expected waiting time (an arrival spent) in the queue $E(W_q)$ is

$$E(W_q) = \frac{1}{\lambda} L_q = \frac{1}{\lambda}\left[\frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2} P_0\right] = \frac{1}{\lambda}\left[\frac{c^c \rho^{c+1}}{c!\,(1-\rho)^2} P_0\right] \tag{26}$$

(vi) Expected waiting time (an arrival spent) in the system $E(W_s)$ is given by $E(W_s) = E(W_q) + \frac{1}{\mu}$

$$E(W_s) = E(W_q) + \frac{1}{\mu} = \frac{1}{\lambda}\left[\frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2} P_0\right] + \frac{1}{\mu} = \frac{1}{\lambda}\left[\frac{c^c \rho^{c+1}}{c!\,(1-\rho)^2} P_0\right] + \frac{1}{\mu} \tag{27}$$

(vii) Expected number of busy servers ($\bar{c}$)

$$\bar{c} = \left[\frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2} P_0 + \frac{\lambda}{\mu}\right] - \frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2} P_0 = \frac{\lambda}{\mu} \tag{28}$$

(viii) Expected number of idle server ($\bar{c}'$) is number of server minus expected number of busy servers

$$\bar{c}' = c - \bar{c} \tag{29}$$

(ix) Efficiency of (M/M/C) model = $\frac{average\ number\ of\ customers\ served}{total\ number\ of\ customers}$ (30)

This multiple-server queue model is pertinent to determine how many servers are actually needed and at what wage/cost in order to maximize financial efficiency.

3. Results and Discussion

3.1. Arrivals Duration Analysis

We are interested in time intervals between customers' consecutive arrivals (inter-arrivals). We have distributed data over the inter-arrivals into 10 seconds amplitude classes, starting from 1 to 10 seconds, 11 to 20 seconds and more.

Table 1: Inter Arrival Times for Day 1

Inter Arrival Time(sec)	Number of customers(f_i)	Mid-Point (x_i)	*fixi*
1-10	45	5.5	247.5
11-20	30	15.5	465
21-30	20	25.5	510
31-40	10	35.5	710
41-50	12	45.5	546
51-60	9	55.5	499.5
61-70	3	65.5	196.5
71-80	2	75.5	151
81-90	1	85.5	85.5
Total	**132**		**3056**

$$\bar{x}_1 = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{3056}{132} = 23.1515$$

Table 2: Inter Arrival Times for Day 2

Inter Arrival Time(sec)	Number of customers (f_i)	Mid-Point (x_i)	$f_i x_i$
1-10	49	5.5	269.5
11-20	24	15.5	372
21-30	20	25.5	510
31-40	15	35.5	532.5
41-50	9	45.5	409.5
51-60	7	55.5	388.5
61-70	4	65.5	262
71-80	2	75.5	151
81-90	2	85.5	171
Total	**132**		**3066**

$$\bar{x}_2 = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{3066}{132} = 23.2272$$

Table 3: Inter Arrival Times for Day 3

Inter Arrival Time(sec)	Number of customers (f_i)	Mid-Point (x_i)	$f_i x_i$
1-10	50	5.5	275
11-20	29	15.5	449.5
21-30	18	25.5	459
31-40	13	35.5	461.5
41-50	9	45.5	409.5
51-60	7	55.5	388.5
61-70	4	65.5	262
71-80	2	75.5	151
81-90	0	85.5	0
Total	**132**		**2856**

$$\bar{x}_3 = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{3126}{132} = 21.6364$$

The empirical convoluted weighted average of arrival rate is

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

$$\bar{x} = \frac{(132 \times 23.1515) + (132 \times 23.2272) + (132 \times 21.6364)}{132 + 132 + 132}$$

$$\bar{x} = 22.6717$$

Fitting inter-arrival durations to an exponential law, the mean inter-arrival time (λ) is:

$$\lambda = \frac{1}{\bar{x}} = \frac{1}{22.6716} = 0.04410785 \; per\ seconds$$

$$\lambda = 0.04410785 \times 60 = 2.646471 \approx 2.6465\ per\ minutes$$

Service Duration Analysis

Table 4: Mean Service Durations for Day 1

Inter Service Time(sec)	Number of Customers served (f_i)	Mid-Point (y_i)	$f_i y_i$
31-60	7	45.5	318.5
61-90	12	75.5	906
91-120	11	105.5	1160.5
121-150	19	135.5	2574.5
151-180	5	165.5	827.5
181-210	6	195.5	1173
211-240	4	225.5	902
241-270	3	255.5	766.5
271-300	2	285.5	571
Total	**69**		**9199.5**

$$\bar{y}_1 = \frac{\sum_{i=1}^{n} f_i y_i}{\sum_{i=1}^{n} f_i} = \frac{9199.5}{69} = 133.3261$$

Table 5: Mean Service Durations for Day 2

Inter Service Time(sec)	Number of Customers served (f_i)	Mid-Point (y_i)	$f_i y_i$
31-60	8	45.5	364
61-90	13	75.5	981.5
91-120	8	105.5	844
121-150	11	135.5	1490.5
151-180	15	165.5	2482.5
181-210	7	195.5	1368.5
211-240	3	225.5	676.5
241-270	2	255.5	511
271-300	1	285.5	285.5
Total	**68**		**9004**

$$\bar{y}_2 = \frac{\sum_{i=1}^{n} f_i y_i}{\sum_{i=1}^{n} f_i} = \frac{900.4}{68} = 132.4118$$

Table 6: Mean Service Durations for Day 3

Inter Service Time(sec)	Number of Customers served (f_i)	Mid-Point (y_i)	$f_i y_i$
31-60	12	45.5	546
61-90	6	75.5	453
91-120	13	105.5	1371.5
121-150	3	135.5	406.5
151-180	8	165.5	1324
181-210	17	195.5	3323.5
211-240	2	225.5	451
241-270	7	255.5	1788.5
271-300	2	285.5	571
Total	**70**		**10235**

$$\bar{y}_3 = \frac{\sum_{i=1}^{n} f_i y_i}{\sum_{i=1}^{n} f_i} = \frac{10235}{70} = 146.2143$$

The empirical combined or overall weighted mean service rate is

$$\bar{y} = \frac{n_1\bar{y}_1 + n_2\bar{y}_2 + n_3\bar{y}_3}{n_1 + n_2 + n_3}$$

$$\bar{y} = \frac{(69 \times 133.3261) + (68 \times 132.4118) + (70 \times 146.2143)}{69 + 68 + 70}$$

$$\bar{y} = 137.3841$$

Fitting service durations to an exponential law, the mean service time (μ) is:

$$\mu = \frac{1}{\bar{y}} = \frac{1}{137.3841} = 0.007278863 \text{ served customers per seconds}$$

$$\mu = 0.007278863 \times 60 = 0.4367318 \approx 0.4367 \text{ served customers per minute}$$

With $\lambda = 2.6465, \mu = 0.4367$ and $c = 6$ (number of ATM machines), traffic intensity (ρ) is

$$\rho = \frac{\lambda}{c\mu} = \frac{2.6465}{6 \times 0.4367} = 1.010037 \approx 1.0100$$

Since $\rho > 1$this implies that, the mean arrival is greater than the mean maximum potential service rate of the system. That is, arrival come at a faster rate than the server can accommodate. The expected queue length increase without limit and a steady state does occur. For steady state condition $(\rho < 1)$ to prevail, we increase number of servers. At $c = 7$,

$$\rho = \frac{2.6465}{7 \times 0.4367} = 0.8657463 \approx 0.8657$$

which indicates the steady state condition.

Calculation of Measures of Effectiveness

At $c = 7$, we have

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{c^n}{c\mu}\left(\frac{\lambda}{c\mu}\right)^n + \frac{c^c}{c!}\left(\frac{\lambda}{c\mu}\right)^c \frac{1}{\left(1 - \frac{\lambda}{c\mu}\right)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{7-1} \frac{7^n}{n!}\left(\frac{2.6465}{7 \times 0.4367}\right)^n + \frac{7^7}{7!}\left(\frac{2.6465}{7 \times 0.4367}\right)^7 \times \frac{1}{\left(1 - \frac{2.6465}{7 \times 0.4367}\right)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{6} \frac{(6.060224)^n}{n!} + 443.674\right]^{-1}$$

$$P_0 = [255.6391 + 443.674]^{-1} = 0.001429975$$

(i) Probability that an arrival has to wait $P(n > c)$

$$P(n \geq c) = \frac{c^c \left(\frac{\lambda}{c\mu}\right)^c}{c!\left(1 - \frac{\lambda}{c\mu}\right)} P_0$$

$$= \frac{7^7 \left(\frac{2.6465}{7 \times 0.4367}\right)^7}{7!\left(1 - \frac{2.6465}{7 \times 0.4367}\right)} \times 0.001429975$$

$$P(n > c) = 0.6344427$$

(ii) Probability that the arrival enters the service without having to wait $P(n < c)$

$$P(n \geq c) = \frac{c^c \left(\frac{\lambda}{c\mu}\right)^c}{c!\left(1 - \frac{\lambda}{c\mu}\right)} P_0$$

$$= \frac{7^7 \left(\frac{2.6465}{7 \times 0.4367}\right)^7}{7!\left(1 - \frac{2.6465}{7 \times 0.4367}\right)} \times 0.001429975$$

$$P(n < c) = 0.3655573$$

(iii) Expected number of customers in queue (L_q) (average queue length)

$$L_q = \frac{c^c \left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1 - \frac{\lambda}{c\mu}\right)^2} P_0$$

$$= \frac{7^7 \left(\frac{2.6465}{7 \times 0.4367}\right)^{7+1}}{7!\left(1 - \frac{2.6465}{7 \times 0.4367}\right)^2} \times 0.001429975$$

$$L_q = 4.091258$$

(iv) Expected number of customers in the system (L_s) or $E(n)$

$$L_s = \frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2}P_0 + \frac{\lambda}{\mu}$$

$$= \frac{7^7\left(\frac{2.6465}{7\times 0.4367}\right)^7}{7!\left(1-\frac{2.6465}{7\times 0.4367}\right)} \times 0.001429975 \ + \frac{2.6465}{0.4367}$$

$$L_s = 4.091258 + \frac{2.6465}{0.4367} = 10.15148$$

(v) Expected waiting time (an arrival spent) in queue $E(W_q)$

$$E(W_q) = \frac{1}{\lambda}\left[\frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2}P_0\right]$$

$$E(W_q) = \frac{1}{2.6465}\left[\frac{7^7\left(\frac{2.6465}{7\times 0.4367}\right)^7}{7!\left(1-\frac{2.6465}{7\times 0.4367}\right)} \times 0.001429975\right]$$

$$E(W_q) = 1.545913$$

(vi) Expected waiting time (an arrival spent) in the system $E(W_s)$

$$E(W_s) = \frac{1}{\lambda}\left[\frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2}P_0\right] + \frac{1}{\mu}$$

$$E(W_q) = \frac{1}{2.6465}\left[\frac{7^7\left(\frac{2.6465}{7\times 0.4367}\right)^7}{7!\left(1-\frac{2.6465}{7\times 0.4367}\right)} \times 0.001429975\right] + \frac{1}{0.4367}$$

$$E(W_q) = 1.545913 \ + \frac{1}{0.4367} = 3.835815$$

(vii) Expected number of busy servers ($\bar{c}$)

$$\bar{c} = \left[\frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2}P_0\right] - \frac{c^c\left(\frac{\lambda}{c\mu}\right)^{c+1}}{c!\left(1-\frac{\lambda}{c\mu}\right)^2}P_0 = \frac{\lambda}{\mu}$$

$$\bar{c} = \frac{2.6465}{0.4367} = 6.060224$$

(viii) Expected number of idle server (c')

$$c' = c - \bar{c} = 7 - 6.060224 = 0.939776$$

The traffic intensity, average queue length, waiting time both queuing and in the system, expected number of busy server(s) for different number of servers were given in the table below.

Table 7: Average Queue Length and Waiting time in queue and in the system for different number of ATM Machines (servers)

c	P	$P(n \geq c)$	$P(n < c)$	L_q	L_s	W_q	W_s	Remark
1	6.060224	limitless	limitless	limitless	limitless	limitless	limitless	explosive
2	3.030112	limitless	limitless	limitless	limitless	limitless	limitless	explosive
3	2.020075	limitless	limitless	limitless	limitless	limitless	limitless	explosive
4	1.515056	limitless	limitless	limitless	limitless	Limitless	limitless	explosive
5	1.212045	limitless	limitless	limitless	limitless	limitless	limitless	explosive
6	1.010037	limitless	limitless	limitless	limitless	limitless	limitless	explosive
7	0.8657463	0.6344427	0.3655573	4.091259	10.15148	1.545913	3.835815	steady state
8	0.757528	0.3712217	0.6287783	1.159767	7.219991	0.4382266	2.728128	steady state
9	0.6733583	0.2051806	0.7948194	0.4229713	6.483196	0.1598229	2.449724	steady state
10	0.6060224	0.1068387	0.8931613	0.1643409	6.224565	0.06209746	2.351999	steady state
11	0.5509295	0.05232528	0.9476747	0.06419379	6.124418	0.02425611	2.314158	steady state
12	0.5050187	0.02409116	0.9759088	0.02457968	6.084804	0.00928762	2.299189	steady state
13	0.4661711	0.01043162	0.9895684	0.00910951	6.069334	0.0034421	2.293344	steady state
14	0.4328732	0.00425304	0.995747	0.003246242	6.063471	0.00122662	2.291128	steady state
15	0.404015	0.001635431	0.9983646	0.001108649	6.061333	0.00041891	2.29032	steady state

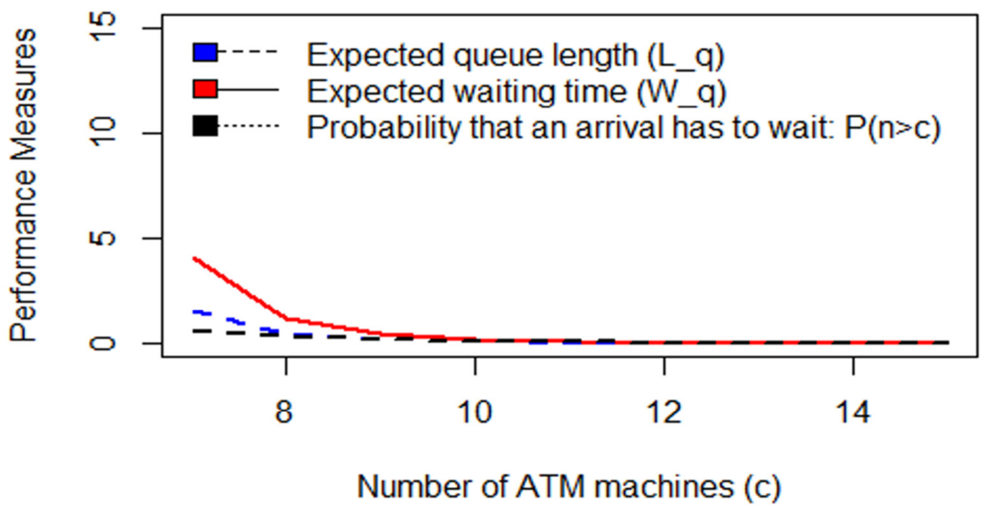

Figure 1: The graph of average queue length, average waiting time and probability of an arrival has to wait against number of servers (ATM machines)

Conclusion

The mean arrival rate of customer to the First Bank of Nigeria (UI Branch) and the mean service rate of the machines were found to be 2.6465 ≈ 3 per minute and 0.4367 per minute, respectively. Based on the effective six (6) ATM machines, the traffic intensity was calculated to be 1.010037, which implied that the mean arrival rate was greater than maximum potential service rate of the servers. From table (7), it was discovered that for number of servers (c) less than seven (7) the system was in explosive state, the queue length and the waiting time increased without limit. For number of servers greater than ≥ 7, average queue length and the average waiting time decreases as the number of ATM machines (servers) increases leading to a steady state of the system. Figure 1 also revealed that the expected queue length, expected waiting time, and the probability that an arrival has to wait decreases as the number of ATM machines increases. Since arrival rate could not be controlled, we recommend installation of more functional and effective ATM machines. In addition, more of on-line banking should be encouraged.

We also suggest that with the knowledge of probability theory, input and output models and Spectral theory for the differential equations of simple birth and death processes and more robust queuing model(s) can be explored. These will capture other unstable variables such as variation due to day, time of the day, bank location and the likes to reflect the real world situations.

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