

Estimation of Garch Models for Nigerian Exchange Rates Under Non-Gaussian Innovations

Adedayo A. Adepoju [1] Olaoluwa S.Yaya[2] Oluwadare O. Ojo[3]
[123]Department of Statistics, University of Ibadan, Ibadan, Nigeria
Email: [1]pojuday@yahoo.com [2]o. s.olaoluwa@gmail.com [3]daruu208075@yahoo.com

Abstract

Financial series often displays evidence of leptokurticity and in that case, the empirical distribution often fails normality. GARCH models were initially based on normality assumption but estimated model based on this assumption cannot capture all the degree of leptokurticity in the return series. In this paper, we applied variants of GARCH models under non-normal innovations-t-distribution and Generalized Error Distribution (GED) on selected Nigeria exchange rates. The Berndt, Hall, Hall, Hausman (BHHH) numerical derivatives applied in the estimation of models converged faster and the time varied significantly across models. Asymmetric GARCH model with t-distribution (GARCH-t) was selected in most of the cases whereas for Nigeria-US Dollar exchange rate, GARCH-GED was specified. Both distributions showed evidence of leptokurticity in Naira exchange rate return series. The result is of practical importance to practitioners.

Key Words: GARCH, Exchange rate, Model specification, Non-Gaussian distribution.

1. INTRODUCTION

The recent economic crises in the world have awakened economist and financial econometricians towards monitoring the financial assets such as stocks and exchange rates, which are characterized by different forms of volatility. Researches have been concentrated been on the study and modelling of volatility.

Due to the fact that conditional distribution of the innovations of financial asset is normal, the unconditional distribution has fatter tails than the normal distribution, hence the usual time series models such as Vector Autoregressive (VAR) and Autoregressive Integrated Moving Average (ARIMA) that assume normality and homoscedasticity cannot be used to model volatility (Pinho and Santos, 2012). However, the magnitude of leptokurtosis introduced by the GARCH process does not always capture all the Leptokurtosis that is present in the high-frequency financial asset (Xekalaki and Degiannakis, 2010). Thus, there is a fair amount of evidence that the conditional distribution of ε_t is non-normal as well.

The problem of non-normality of innovations of these financial assets has been considered lately. Bollerslev (1987) proposed the standardized student t distribution; which is symmetric around zero. Nelson (1991) introduced Generalized Error Distribution (GED) which accounts for fat tail, which is a symmetric distribution.

Few articles on Nigerian naira exchange rates employed interpreting and estimating properties of the series in different dimension. These few ones include Olowe (2009), Shittu (2009), Awogbemi and Alagbe (2011) and Ezike and Amah (2011). The analysis of the Naira exchange rate returns indicate that the empirical distribution of returns in the foreign exchange rate market is non-normal and this have very thick tails (Olowe, 2009). Shittu (2009) applied the Intervention Analysis Approach (IAA) on the exchange rate and the diagnostic tests were satisfied at both points of the intervention. Awogbemi and Alagbe (2011) examined the volatility in the Naira-US dollar and Naira-UK pound exchange rates using GARCH model and obtained estimates of volatility persistence. Their results further showed evidence of asymmetries in the residual series, and this is an indication for asymmetric volatility models. Ezike and Amah (2011) checked for possible long run relationship between exchange rates, demand and supply of foreign exchange rate in the Dutch Auction Market (DAS) and obtained a significant relationship in the variables. All these authors have considered monthly data in their investigation, and based on the frequency of the data applied, characteristics of the series were not well captured. Secondly, they consider one or two Naira exchange rates out of many. Thirdly, though they have considered different analysis approach but their models did not assume different distributional forms different from the normal distribution. Fourthly, exchange rates are volatility series and are asymmetric at time, and daily data need to be applied to really examine these properties.

In this work, we consider modelling some Nigeria exchange rate returns series with Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models with normal and non-normal distribution innovations. The rest of the paper is structured as: Section 2 deals with the distributional assumptions of GARCH models and log likelihood functions; section 3 presents the data as well as the results of the model specification based on the model selection criteria and section 4 renders the conclusions remark.

2. GARCH MODELS AND DISTRIBUTIONAL ASSUMPTIONS

Following Bollerslev (1986), the Generalized Autoregressive Conditionally Heteroscedastic (GARCH) model of order (p, q) is given as:

$$\sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \qquad \varepsilon_t = \sigma_t z_t \tag{1}$$

where ε_t are the returns series of the financial asset; σ_t is the volatility at time t and z_t gives the assumed distribution. The parameters $w > 0$, $\alpha_i\left(i = 1, 2, ..., p\right)$, $\beta_i\left(i = 1, 2, ..., q\right)$ and for stationarity of the whole process, $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$.

The model in (1) is symmetric in the sense that the magnitude of the innovations (returns), ε_t is expected to predict the future volatility. The asymmetric specifications allow for the signs of the innovations (returns) to have impact on the volatility apart from the magnitude. The first asymmetric GARCH(p,q) model is Exponential GARCH (EGARCH)

$$\log \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k \left(\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right) \tag{2}$$

proposed in Nelson (1991). The GJR(p,q) model

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \left[\gamma_i d\left(\varepsilon_{t-i} < 0\right) \varepsilon_{t-i}^2\right] + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{3}$$

was proposed in Glosten *et* al.(1993). The APARCH(p,q) model of Ding et al.(1993) is given as,

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left(\left| \varepsilon_{t-i} \right| - \gamma_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^\delta \tag{4}$$

where, in the three models, w, $\alpha_i \left(i = 1, 2, ..., p\right)$, $\beta_i \left(i = 1, 2, ..., q\right)$ are the parameters. The γ_i are the asymmetric parameter and $\delta > 0$ in APARCH model is the Box and Cox (1964) power transformation.

The GARCH (p, q) model in (1-4) are specified with normal innovations z_t distributed as standardized normal,

$$z_t \sim N(0,1) \tag{5}$$

and this suggests approaching the estimation of GARCH (p,q) process via maximum likelihood estimation but in most cases, the distribution of the residuals (innovations) presents fatter tail than the normal distribution,

$$f\left(z_t\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} z_t^2 \right) \tag{6}$$

Thus there is a fair amount of evidence that the conditional distribution of ε_t is a non-normal as well.

Bollerslev (1987) circumvented the problem of non-normality of innovations of GARCH (p, q) model by proposing the standardized student t-distribution with $v > 2$ degree of freedom as,

$$f\left(z_t, v\right) = \frac{\Gamma\left(\left(v+1\right)/2\right)}{\Gamma\left(v/2\right)\sqrt{\pi\left(v-2\right)}} \left(1 + \frac{z_t^2}{v-2}\right)^{-(v+1)/2} \tag{7}$$

where $\Gamma(.)$ is the gamma function. This distribution is as well symmetric about zero and with $v > 2$. At $v > 4$, its kurtosis becomes 3(v-2)(v-4) which is larger than 3, the corresponding value for the normal distribution. As $v \rightarrow \infty$, the distribution converges to the standard normal distribution in (3).

Nelson (1991) proposed another competing distribution with standardized student t-distribution. This is the Generalized Error Distribution (GED) with distribution function,

$$f\left(z_t, v\right) = v \exp \frac{\left(-0.5 \left| z_t / \lambda \right|^v\right)}{\lambda . 2^{\left(1+v^{-1}\right)} \Gamma\left(v^{-1}\right)} \tag{8}$$

with $v > 0$, where v is the tail fatness parameter and $\lambda \cong \sqrt{2^{-2/v} \Gamma\left(v^{-1}\right) / \Gamma\left(3v^{-1}\right)}$.

At v =2, z_t becomes standardized normal and so the distribution reduces to normal distribution in (6). At v < 2, the GED distribution of z_t has thicker tails than the normal distribution; it has double exponential or Laplace distribution at v = 1, while at v > 2, the distribution has thinner tails than the normal distribution. As $v \to \infty$, the distribution becomes Uniform on the interval (- $\sqrt{3}$, $\sqrt{3}$).

Note that in the EGARCH(p,q) model, the component $E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| = \sqrt{\frac{2}{\pi}}$ under normally distributed innovations. For the Student t distribution and Generalized Error Distribution (GED), $E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| = \Gamma\left(\frac{v+1}{2}\right)^2 \frac{\sqrt{v-2}}{\sqrt{\pi}(v-1)\Gamma(v/2)}$ and $E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| = \lambda 2^{v-1}\frac{\Gamma(2v^{-1})}{\Gamma(v^{-1})}$ respectively.

2.1 KURTOSIS OF GARCH MODELS

For a GARCH(p,q) model, $\varepsilon_t = \sigma_t z_t$, where $E(z_t) = 0$, $Var(z_t) = 1$ and $E(z_t^4) = k_z + 3$ where k_z is the excess kurtosis of the innovation z_t. Also, $E(\varepsilon_t) = 0$,

$Var(\varepsilon_t) = E(\sigma^2) = w/(1-\alpha_1-\beta_1)$ in a GARCH(1,1) model,

$$\sigma_t^2 = w + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2, \quad \varepsilon_t = \sigma_t z_t. \tag{9}$$

then follows to write $E(\varepsilon_t^4) = E(\sigma_t^4)E(z_t^4)$. Under the assumption of independency, then, $E(\varepsilon_t^4) = (k_z+3)\sigma_t^4$. Squaring both sides of GARCH(1,1) model in (9) above leads to

$$\sigma_t^4 = w^2 + \alpha_1^2\varepsilon_{t-1}^4 + \beta_1^2\sigma_{t-1}^4 + 2w\alpha_1\varepsilon_{-1}^2 + 2w\beta_1\sigma_{t-1}^2 + 2\alpha_1\beta_1\sigma_{t-1}^2\varepsilon_{t-1}^2 \tag{10}$$

Taking the expectation of the resulting expression and using the assumptions stated above,

$$E(\sigma_t^4) = \frac{w^2(1+\alpha_1+\beta_1)}{(1-\alpha_1-\beta_1)\left[1-\alpha_1^2(k_z+2)-(\alpha_1+\beta_1)^2\right]} \tag{11}$$

The excess kurtosis of ε_t is then given as,

$$k_\varepsilon = \frac{E(\varepsilon_t^4)}{\left[E(\varepsilon_t^2)\right]^2} - 3$$

$$= \frac{(k_z+3)\left[1-(\alpha_1+\beta_1)^2\right]}{1-2\alpha_1^2-(\alpha_1+\beta_1)^2-k_z\alpha_1^2} - 3 \tag{12}$$

When z_t is normally distributed ($k_z = 0$),

$$k_\varepsilon = \frac{6\alpha_1^2}{1-2\alpha_1^2-(\alpha_1+\beta_1)^2} \tag{13}$$

When z_t is not normally distributed ($k_z \neq 0$),

$$k_\varepsilon = \frac{k_z - k_z(\alpha_1+\beta_1)^2 + 6\alpha_1^2 + 3k_z\alpha_1^2}{1-2\alpha_1^2-(\alpha_1+\beta_1)^2 - k_z\alpha_1^2} \tag{14}$$

In the two cases discussed, the coefficient α_1 is important in determining the tail behaviour of α_1, since in both, once $\alpha_1 = 0,\ k_\varepsilon = 0$. Hence, $k_\varepsilon = k_z$ for the non-normally distributed case and it implies the similarity of the tail behaviours of both ε_t and the standardized z_t. In the student *t*-distribution earlier discussed, $E\left(\varepsilon_t^4\right) = k_z + 3$ at degree of freedom $v > 4$ where k_z, the excess kurtosis is set at $k_z = 6/(v-4)$.

2.2 LOG-LIKELIHOOD OF THE DISTRIBUTIONS AND ESTIMATION APPROACH

The log-likelihood functions of the standardized distributions discussed above are presented here.

For normally distributed innovations, z_t the log-likelihood function is

$$l_t = \log\left(\frac{1}{\sqrt{2\pi}}\right) - \frac{\varepsilon_t^2}{2\sigma_t^2} - \frac{1}{2}\log\sigma_t^2, \tag{15}$$

where x_t denotes a $k \times 1$ vector of endogenous and exogenous explanatory variables in the information set I_{t-1} and N is the sample size of the time series. The full log-likelihood function is written as;

$$l_t = -\frac{1}{2}\left[N\log(2\pi) + \sum_{t=1}^{N}\frac{\varepsilon_t^2}{\sigma_t^2} + \sum_{t=1}^{N}\log\sigma_t^2\right] \tag{16}$$

where *N* is the sample size.`

In a similar way, the log likelihood function for the standardized t-distribution is

$$l_t = -\frac{1}{2}\left\{N\log\left(\frac{\pi(v-2)\Gamma(v/2)^2}{\Gamma((v+1)/2)^2}\right) + \sum_{t=1}^{N}\log\sigma_t^2 + (v+1)\sum_{t=1}^{N}\log\left[1+\frac{\varepsilon_t^2}{\sigma_t^2(v-1)}\right]\right\} \tag{17}$$

and that of GED is,

$$l_t = -\frac{1}{2}\left\{ N\log\left(\frac{\Gamma\left(v^{-1}\right)^3}{\Gamma\left(3v^{-1}\right)\left(v/2\right)^2} \right) + \sum_{t=1}^{N}\log\sigma_t^2 + \left(v+1\right)\sum_{t=1}^{N}\left(\frac{\Gamma\left(3v^{-1}\right)\varepsilon_t^2}{\sigma_t^2\Gamma\left(v^{-1}\right)} \right)^{v/2} \right\}$$

(18)

These likelihood functions gives in (7), (8) and (9) are then estimated using the numerical derivatives based on the fact that GARCH models lack closed form estimation. Berndt, Hall, Hall and Hausman (BHHH) algorithm of Berndt, et al (1974) is then used. This algorithm uses only first derivatives of the likelihood function and computes a set of parameter values as

$$\psi^{(i+1)} = \psi^{(i)} - \left(\sum_{t=1}^{N} \frac{\partial l_t^{(i)}}{\partial \psi} \cdot \frac{\partial l_t^{(i)}}{\partial \psi'} \right)^{-1} \frac{\partial l_N^{(i)}}{\partial \psi} \tag{19}$$

where l_t is the likelihood function. The initial parameter set is given as $\psi^{(0)}$ and the parameter set which maximize the likelihood function is denoted as $\psi^{(i+1)}$.

The estimation of GARCH (p, q) model with t-distribution and GED follow Quasi Maximum Likelihood Estimation (QMLE) since normality assumption is violated in both cases.

The best model is determined for the series by employing necessary criterion. The commonly used criteria suggested in Harvey (1989) are the Akaike Information (AIC) and Schwarz Bayesian Information Criterion (SBIC) proposed by Akaike 91974) and Schwarz (1978). The AIC and BIC are defined by:

$$AIC = -2l_t\left(\Theta\right) + 2k \tag{20}$$

and

$$SBIC = -2l_t\left(\Theta\right) + 2k\ln\left(N\right) \tag{21}$$

where l_t is any of the likelihood function defined above. The Θ is the parameter set in the AR-GARCH model, k is the number of parameters to be jointly estimated and N is the size of the time series.

3. DATA PRESENTATION, RESULTS AND DISCUSSION

The data considered in this work are the trading days Nigeria exchange rate with Euro, British pound, Japanese Yen and US dollars. These data span between 10/12/2001 to 14/12/2011. The data have been sourced from Central Bank of Nigeria website (www.cenbank.org).

The empirical analysis of these data is given in Yaya, Adepoju and Adeniyi (2012). Based on the results, the Naira-US Dollar exchange rate was the least volatile while Naira- British pound was the most volatile rate.

In the log return series, autocorrelation is only significant at first lag, therefore autoregressive model of order one [AR (1)] is the first estimated as the mean equation.

The best GARCH model is determined for each of the exchange rate returns based on the minimum AIC and SBIC, maximum log-likelihood estimates and normality of the GARCH residuals.

Table 1: Nigeria Naira-Euro Exchange Rate

Model	AIC	SBIC	Log-lik.	Skewness	Ex. Kurtosis	JB	Comp. Time(secs)
AR(1)-GARCH(1,1)-Normal	-8.404	-8.392	10249.7	-3.436	80.779	6.68E5	1.373
AR(1)-GJR(1,1)-Normal	-8.388	-8.374	10231.3	-3.182	70.647	5.11E5	2.278
AR(1)-EGARCH(1,1)-Normal	-8.358	-8.342	10196.1	-0.064	9.4796	9130.2	20.374
AR(1)-APARCH(1,1)-Normal	-8.426	-8.410	10278.8	-2.562	58.094	3.46E5	8.673
AR(1)-GARCH(1,1)-t	-8.722	-8.707	10637.8	-3.271	75.319	5.81E5	3.51
AR(1)-GJR(1,1)-t	-8.724	-8.707	10641.7	-3.280	77.109	6.08E5	3.9
AR(1)-EGARCH(1,1)-t	-8.687	-8.668	10597.1	-3.538	84.356	7.28E5	63.585
AR(1)-APARCH(1,1)-t	**-8.740**	**-8.721**	**10662.6**	**-0.866**	**126.87**	**1.64E6**	**9.22**
AR(1)-GARCH(1,1)-GED	-8.701	-8.687	10612.6	-3.373	79.567	6.48E5	2.995
AR(1)-GJR(1,1)-GED	-8.676	-8.660	10583.6	-2.089	47.190	2.28E5	4.134
AR(1)-EGARCH(1,1)-GED	-8.583	-8.564	10470.8	-4.107	97.602	9.75E5	21.559
AR(1)-APARCH(1,1)-GED	-8.675	-8.656	10583.0	NAN	NAN	NAN	4.587

In Table 1, APARCH(1,1) model with t-distribution was specified as the best model for Naira-Euro exchange rate based on the minimum AIC and SBIC values. This GARCH residuals of this model also presents longest tail (kurtosis = 126.87) among the other models.

Table 2: Nigeria Naira-British Pound Exchange Rate

Model	AIC	SBIC	Log-lik.	Skewness	Ex. Kurtosis	JB	Comp. Time(secs)
AR(1)-GARCH(1,1)-Normal	-8.701	-8.689	10611.8	-0.3595	7.379	5583.8	2.465
AR(1)-GJR(1,1)-Normal	-8.695	-8.680	10604.6	-0.3991	7.052	5116.6	2.839
AR(1)-EGARCH(1,1)-Normal	-8.590	-8.574	10478.6	-0.0621	10.805	11860	25.178
AR(1)-APARCH(1,1)-Normal	-8.695	-8.678	10606.8	NAN	NAN	NAN	3.494
AR(1)-GARCH(1,1)-t	-8.892	-8.878	10845.8	-0.3762	7.7926	6226.2	2.325
AR(1)-GJR(1,1)-t	-8.892	-8.875	10845.9	-0.3781	7.8562	6.327.8	2.683
AR(1)-EGARCH(1,1)-t	-8.842	-8.828	10786.1	0.2486	15.200	23496.0	22.839
AR(1)-APARCH(1,1)-t	**-8.898**	**-8.882**	**10854.9**	**-0.2562**	**8.7319**	**77772.0**	**9.001**
AR(1)-GARCH(1,1)-GED	-8.857	-8.843	10802.6	-0.2584	7.9064	6377.2	2.434
AR(1)-GJR(1,1)-GED	-8.856	-8.840	10802.7	-0.2618	7.9360	6425.6	3.323
AR(1)-EGARCH(1,1)-GED	-8.769	-8.750	10697.8	0.20702	14.353	20944.0	16.068
AR(1)-APARCH(1,1)-GED	-8.837	-8.818	10780.6	NAN	NAN	NAN	5.226

In Table 2, the best model for Naira-British Pound exchange rate is APARCH(1,1) with t-distribution of residuals.

Table 3: Nigeria Naira-Japanese Yen Exchange Rate

Model	AIC	SBIC	Log-lik.	Skewness	Ex. Kurtosis	JB	Comp. Time(secs)
AR(1)-GARCH(1,1)-Normal	-8.478	-8.46	10339.4	-0.737	17.580	31614	1.342
AR(1)-GJR(1,1)-Normal	-8.495	-8.481	10361.8	-0.520	13.084	17501	1.748
AR(1)-EGARCH(1,1)-Normal	-8.364	-8.347	10202.4	-0.413	17.640	31640	24.445
AR(1)-APARCH(1,1)-Normal	-8.484	-8.467	10348.7	NAN	NAN	NAN	4.165
AR(1)-GARCH(1,1)-t	-8.738	-8.724	10657.9	-4.077	102.48	1.07E6	2.309
AR(1)-GJR(1,1)-t	-8.740	-8.724	10661.5	-5.460	147.62	2.23E6	5.116
AR(1)-EGARCH(1,1)-t	-8.703	-8.684	10617.4	-4.626	119.58	1.46E6	33.009
AR(1)-APARCH(1,1)-t	**-8.750**	**-8.731**	**10674.7**	**-3.726**	**162.38**	**2.68E6**	**8.986**
AR(1)-GARCH(1,1)-GED	-8.716	-8.701	10630.5	-3.327	80.387	6.60E5	3.479
AR(1)-GJR(1,1)-GED	-8.715	-8.798	10630.6	-3.499	85.281	7.41E5	4.227
AR(1)-EGARCH(1,1)-GED	-8.622	-8.603	10518.1	-1.988	50.089	2.64E5	31.403
AR(1)-APARCH(1,1)-GED	-8.714	-8.695	10630.4	-4.042	101.64	1.11E5	7.144

In Table 3, the best model for Naira-Japanese Yen exchange rate is also APARCH (1,1) model with t-distribution in the residuals. This model presents kurtosis estimate of 162.38, which is the highest among the models estimated.

Table 4: Nigeria Naira-US Dollar Exchange Rate

Model	AIC	SBIC	Log-lik.	Skewness	Ex. Kurtosis	JB	Comp. Time(secs)
AR(1)-GARCH(1,1)-Normal	-9.507	-9.495	11593.8	5.462	209.34	4.46E6	1.326
AR(1)-GJR(1,1)-Normal	-9.525	-/9.510	11616.5	5.305	213.86	4.66E6	2.855
AR(1)-EGARCH(1,1)-Normal	-9.430	-9.413	11502.3	-3.462	119.84	1.46E6	60.106
AR(1)-APARCH(1,1)-Normal	-9.507	-9.491	11596.5	NAN	NAN	NAN	2.839
AR(1)-GARCH(1,1)-t	-15.837	-15.822	19311.0	NAN	NAN	NAN	19.921
AR(1)-GJR(1,1)-t	-16.131	-16.115	19671.1	NAN	NAN	NAN	23.540
AR(1)-EGARCH(1,1)-t	-15.343	-15.323	18710.7	NAN	NAN	NAN	75.722
AR(1)-APARCH(1,1)-t	-9.773	-9.753	11920.7	NAN	NAN	NAN	22.932
AR(1)-GARCH(1,1)-GED	**-27.421**	**-27.406**	**33431.6**	**-6.265**	**248.13**	**6.27E6**	**4.025**
AR(1)-GJR(1,1)-GED	-23.323	-23.307	-28438.4	7.005	128.87	1.71E6	3.276
AR(1)-EGARCH(1,1)-GED	-14.951	-14.932	18232.9	2.788	152.00	2.38E6	18.096
AR(1)-APARCH(1,1)-GED	-24.811	-24.792	30252.7	0.931	89.15	8.08E5	7.207

In Table 4, estimation of models for Naira-US dollar exchange rate posed more serious convergence problem due to more zeros in the return series as a result of series stability (less volatility). As it is observed in the results that normality tests were not computed for models with the t-distributions. Based on the Information Criteria, the best model here is GARCH(1,1) with GED. The model also records highest tail measure.

4. CONCLUSION

In this paper, variants of GARCH models for both symmetric and asymmetric types were considered in modelling daily Nigeria naira exchange rate returns series under non-normal GARCH distributions. Four common naira exchange rates selected were Naira-Euro, Naira-British Pound, Naira-Japanese Yen And Naira US Dollars exchange rates. GARCH models were estimated under both normality and non-normality assumptions of GARCH models. The t-distribution and Generalized Error Distribution (GED) were considered in the non-normally distributed case.

The complex log-likelihood from the Quasi Maximum Likelihood Estimation (QMLE) was simplified using the Berndt, Hall, Hall, Hausman (BHHH) numerical derivative to optimize the estimates of the parameters of the models. Computational time varied from model to model, and divergence was hardly experienced except in the case of Naira-US dollars exchange rate series in which return series gave more zeros as a result of series stability for some time periods.

Asymmetric GARCH models with t-distribution were specified for the series, except for Naira-US Dollars exchange rates, where GARCH model with GED was specified as the optimal model. More zeros in the return series of Naira-US dollars exchange rates affected the tail measure the series. Estimates of kurtosis for GARCH residuals also showed evidence for specifying GARCH variants with t-distribution.

This work can be generalized by considering all the available Nigeria naira exchange rates to confirm if the return series will always show longer tail in most cases.

REFERENCES

Akaike, H. (1974). “A new look at the statistical model identification”. IEEE Transactions on Automatic Control, 19 (6), 716-723.

Awogbemi C. and Alagbe, S. (2011). “Empirical Modeling of Nigerian Exchange Rate Volatility”. Mathematical Theory and Modeling, 1(3): 8-14.

Berndt, E., Hall, B., Hall, R. and Hausman, J. (1974). “Estimation and inference in nonlinear structural models”. Annals of Economic and Social Measurement 3: 653-665.

Bollerslev, T. (1986). “Generalized autoregressive conditional heteroscedasticity”. Journal of Econometrics 31: 307–27.

Bollerslev, T. (1987). “A conditional heteroscedastic time series model for speculative prices and rates of return”. Review of Economics and Statistics 69:542-547.

Ding, Z., Granger, C. W. J. and Engle, R. F. (1993). “A long memory property of stock market returns and a new model”. Journal of Empirical Finance, 1: 83-106.

Ezike, J. E. and Amah, P. N. (2011). “Time series variation of foreign exchange rate in the Nigerian Wholesale Dutch Auction Market”. Journal of Economics and International Finance, 3(13): 685-696.

Engle R. (1982). "Autoregressive conditional heteroscedasticity, with estimates of the variance of United Kingdom inflation". Econometrica 50: 987-1007.

Glosten, L. W., Jaganathan, R. and Runkle, D. E. (1993). "On the relation between the expected value and the volatility of the nominal excess return on stocks". Journal of Finance 48: 1779–801.

Harvey, A.C. (1989). Forecasting, Structural Time Series Models and the Kalman Filter. Cambridge University Press, Cambridge.

Nelson, D.B. (1991). "Conditional heteroscedasticity in asset returns: A new approach". Econometrica 59: 347-370.

Olowe, R. A. (2009). "Modelling Naira/Dollar Exchange Rate Volatility: Application of GARCH and asymmetric Models". International Review of Business Research Papers 5: 377- 398.

Pinho, F.M. and Santos, T. R. (2012). "The non-Gaussian State Space Volatility and APARCH models-A comparison study for important Financial indexes". A paper presented at 58th Annual Conference of African Econometric Society Conference in Kampala, Uganda.

Schwarz, G.E., 1978. Estimating the dimension of a model. Annals of Statistics, 6 (2): 461-464.

Shittu, O. I. (2009). "Modelling Exchange Rate in Nigeria in the Presence of Financial and Political Instability: An Intervention Analysis Approach". Middle Eastern Finance and Economics, Issue 5

Xekalaki, E. and Degiannakis, S. (2010). ARCH models for financial applications. *John* Wiley & Sons Ltd.

Yaya, O. S., Adepoju, A. A. and Adenuga, A. O. (2012): "Asymmetric GARCH model specifications for Nigerian exchange rates". To appear in CBN Economic and Financial Review.