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Nigeria's Potential Growth and Output Gap: Application of Different Econometrics Filters

Baba N. Yaaba
Statistics Department, Central Bank of Nigeria, P.M.B.0187, Central Business District, Abuja, Nigeria.
Email: yaabakatcha@yahoo.com;bnyaaba@cbn.gov.ng

Abstract
The concept of potential output and the corresponding output gap had received considerable attention by both policy makers and academic researchers, particularly in the developed countries. This is a reflection of not only its theoretical significance, but also its policy relevance. Output gap is used to model price and wage inflation, in estimating fiscal balance and the impact of structural reforms on the economy, hence an important indicator of fiscal policy trust. Most importantly, to a central banker it is critical in modelling monetary policy decision making process, as it serves as an input into central banks economic projections which forms an integral part of monetary policy decision and the setting of monetary policy rates. This paper measures the potential output and the corresponding output gap for Nigeria using Hodrick-Prescott filter, Baxter-King filter and both fixed and full length Christiano-Fitzgerald filters. The methods yielded different results, but with strong similarities in their evolution over time. According to all the methods, on the average, the economy was over heated during the early part of the sample period (2004:Q1 to 2005:Q4) but operated below capacity between 2008:Q1 and 2009:Q4. Interestingly, a fairly strong and stable relationship exists between inflation and the estimated output gaps. With this noticeable connection, using output gap to compliment expert judgement, in monetary policy decision making, would conceptually be a good decision.
Keywords: Potential growth, output gap, econometric filtering, Nigeria

1. Introduction

The phenomenal attention given to the concept of potential output and the corresponding output gap by both policy makers and academic researchers particularly in the developed countries is a reflection of, not only its theoretical significance, but also, its policy relevance. Output gap is used in both price and wage inflation models, as well as monetary policy reaction functions. Besides, it is used in estimating fiscal balance and the impact of structural reforms on the economy, hence an important indicator of fiscal policy trust.

The most critical role of potential output and the corresponding output gap is in the policy decision making process of central banks, as it serves as an input into central banks economic projections which forms an integral part of monetary policy decision and the setting of not only monetary policy rates, but also cash reserves requirements.

Potential output is the rate of economic growth that is sustainable in the long run without triggering inflationary pressures. Output gap is the difference between the actual level of nation's output and the potential output. In other words, it is a deviation of the actual output from its equilibrium. A positive output gap is an indication of excess demand, which is positively related to inflation, while negative output gap is considered as spare capacity for the economy, hence puts a sustainable downward pressure on inflation. This relationship lures central banks to monitoring the variables in order to determine their magnitude, so as to understand the future evolution of prices and take pre-emptive policy measures.

However, despite the importance of these variables in the monetary policy decision making process, it has not been given adequate research attention in Nigeria.

This paper is, therefore, an attempt to measure the potential output and output gap for Nigeria using different filters in econometrics. To achieve this, the paper is organised into five sections. Following this introduction, section 2 briefly reviews relevant literature, as well as provides the rationale for focusing on the concept of potential growth. Section 3 discusses the methodology, highlighting the derivation of all the methodologies adopted in econometrics filtering. Section 4 presents the results while the last section concludes the paper.

2.0 Theoretical Background and Literature Review

2.1 Rationale for Potential Growth

According to Henriot (2008) two main reasons can be advanced in support of the concept of potential growth, namely: the usefulness of the concept to policy makers and public finance. In Nigeria for instance, the essence of monetary policy decisions are mostly price stability. To achieve this, central bank targets monetary growth in nominal terms. The target of nominal monetary growth is the product of controlling inflation vis-à-vis potential growth. The CBN does not directly control the GDP or price level. The growth of the money supply is an intermediate target, an objective that helps the CBN in achieving its ultimate policy objective of economic growth with stable prices.

Using the growth of money supply as an intermediate target lends credence to the relationship between changes in money and changes in income and prices. The yardstick for this assumption is the equation of exchange and quantity theory of money (QTM).

$$= \qquad (1)$$

Where M is the quantity of money, V is the velocity of money, P is the price level and Q is the quantity of output. If the money M increases and velocity V is constant, the nominal GDP (PQ) must increase, if the economy realises its potential growth (i.e. maximum level of Q), any increase in M causes an increase in P, but if the economy has a negative output gap (Q not up to maximum) increase in M may lead to a higher price level (P) as well as higher real GDP (Q). Therefore, CBN effort is geared towards setting money growth targets consistent with rising output and low inflation. The CBN ultimately strives to adjust M at a rate that supports the steady increase in Q with slow and steady increase in P. If CBN does not have the idea of the gap between Q and Q* (where Q* is the potential output), it might increase M beyond the rate that could support increase in Q that will be accompanied by rapid increase in P, hence inflationary pressure. A good example of this happened in Europe at the end of 1990s when they experienced robust economic growth. Analyst attributed the growth to strong potential of the economy, believing that information technology (IT) revolution in Europe has a lag compare to the US. Following this thinking, therefore, the European governments did not adopt any policy measure to control public expenditure. Unfortunately, the robust growth was the result of cyclical upswing not higher potential. Thus, in the subsequent years, gross domestic product (GDP) was negatively affected and government was forced to adopt restrictive fiscal policy measures to reduce fiscal imbalances after the internet crisis of 2000/2001. This is a clear indication of the importance of the assessment of potential growth for economic policy decisions.

2.2 Output Gap, Inflation and Monetary Policy

The link between the economy, monetary policy and information structure vis-à-vis output gap and inflation can be explored using the backward-looking sticky price model of Svensson (1997). This model is assumed to capture the actual behaviour of the economy, as well as reflects the views of most central banks about the transmission process of monetary policy.

The Economy

If output is given as y_t and inflation f_t. Let's assume, they are determined as:

$$y_t = z_t - \varphi r_t + g_t \qquad (2)$$

$$f_t = \lambda(y_t - z_t) + \mu_t \qquad (3)$$

Where z_t denotes potential output at period t, r_t is a real short-term interest rate, g_t is a demand shock and μ_t is a cost push shock. The intuition from the above equations is that the potential output z_t fundamentally determines actual output in the long run. Besides, demand shock and real interest rate also exact some influence on actual output.

If we assume that z_t exhibits a random walk due to the fact that demand and cost shocks are less persistent than changes in potential output, then an assumption of a stochastic process for the shocks can take the following form:

$$g_t = \mu g_{t-1} + \hat{g}_t \;\; 0 < \quad < 1; \; \hat{g}_t \sim N(0, \sigma^2{}_g) \qquad (4)$$

$$u_t = \rho \mu_{t-1} + \hat{u}_t \;\; 0 < \quad < 1; \; \hat{u}_t \sim N(0, \sigma^2{}_u) \qquad (5)$$

$$z_t = z_{t-1} + \hat{z}_t \qquad z_t \sim N(0, \sigma^2{}_z) \qquad (6)$$

The model depicts the characteristics of an optimally chosen monetary policy in macroeconomic terms in a situation of uncertainty with regards to sources of growth in the economy.

Monetary Policy

With the assumption that nominal interest rate is the sole instrument available to the monetary authority, coupled also with the assumption of temporary sticky prices, any desire for change in real rate by the monetary authority can be achieved by adjusting nominal rate. For ease of analysis, therefore, the policy maker is assigned the role of setting the real interest rate r_t. The interest rate is set at the beginning of time t prior to output and inflation y_t and π_t, respectively, to realise shocks at period t. To minimise the objective function, the following ensued:

$$L_t \equiv \frac{1}{2} E\left\{\sum_{j=0}^{\infty}\left[\alpha(X_{t+j})^2 + (f_{t+j})^2\right] \Big| \; J_{t-1}\right\} \quad \alpha > 0 \qquad (7)$$

Where $L_t \equiv y_t - z_t$ represent output gap and J_{t-1} is the information available at the beginning of time t at any chosen r_t. Following this, therefore, the first order condition for the discretionary monetary policy, (i.e. $\min_{rt}$, L_t) is:

$$X_{t|t-1} = -\frac{\lambda}{\alpha} f_{t|t-1} \qquad (8)$$

Where $X_{t|t-1}$ and $f_{t|t-1}$ are the expected value of the output gap and inflation, respectively. Note that J_{t+1}contains information including both actual output and inflation in period t-1.
With the above exploration in mind, the monetary authority arrives at the equilibrium outcome for the interest rate, inflation and output as follows:

$$r_t = \frac{1}{\varphi}\left[g_{t|t-1} + \frac{\lambda}{\alpha + \lambda^2}U_{t|t-1}\right] \quad (9)$$

$$y_t = z_t + \left(g_t - g_{t|t-1}\right) - \frac{\lambda}{\alpha + \lambda^2}U_{t|t-1} \quad (10)$$

$$f_t = \frac{\alpha}{\alpha + \lambda^2} + \lambda\left(g_t - g_{t|t-1}\right) + \frac{\lambda^2}{\alpha + \lambda^2}\left(U_t - U_{t|t-1}\right) \quad (11)$$

Optimal Monetary Policy Decision and the Information Pattern

From equation (9), it is obvious that the interest rate requires that the optimal real interest rate policy for period t+1 and r_{t+1}, entails forming some expectations about both the values of demand and cost-push shocks to be formed *apriori* by the policy makers (i.e. g_{t+1} and U_{t+1}). It is generally agreed that real information about these shocks are not readily available to the policy makers, but information about some noisy economic variables with which optimal forecast of at least the structure of the shocks, can be derived, are obtainable. It is only the precise stochastic source of fluctuations in output and inflation that the policy maker is ignorant of, but possesses the true structure of the economy: $\Omega \equiv \{\varphi, \lambda, \rho, \mu, \sigma^2{}_u, \sigma^2{}_g, \sigma^2{}_z\}$. At the beginning of period t+1 when r_{t+1} interest rate will be set, expectations about g_{t+1} and u_{t+1} (i.e. demand and cost-push shocks) are formed based on past information which include numbers on output and inflation up to the current period (t). Following Lippi (2003) the information obtainable at period t+1, can be summarised as follow:

$$J_t = \{\Omega, y_{t-i}, f_{t-i} | i = 0, 1, 2, \dots\} \quad (12)$$

If past output and inflation are equated to the two signals for the same period, $S_{1,t}$ and $S_{2,t}$, by re-arranging equations 10 and 11, we obtain:

$$S_{1,t} \equiv y_t + g_{t|t-1} + \frac{\lambda}{\alpha + \lambda^2}U_{t|t-1} = z_t + g_t \quad (13)$$

$$S_{2,t} \equiv f_t + \lambda g_{t|t-1} + \frac{\lambda^2}{\alpha + \lambda^2}U_{t|t-1} = \lambda g_t + u_t \quad (14)$$

Note that the variables left of the equations are observed independently, unlike those to the right. There is, however, no doubt that, $S_{1,t}$ and $S_{2,t}$, include noise on g_t and u_t, but it can undoubtedly be used to make inferences on g_{t+1} and u_{t+1} with the evidence that $g_{t+1|t} = \mu g_{t|t}$ and $u_{t+1|t} = \rho u_{t|t}$.

2.3. Empirical Literature

Since the seminal work of Burns and Mitchel (1946) many other studies have attempted to estimate business cycles using different concept and methodologies. The deviation of actual output from its long-run level provides an estimate about the cyclical position of the economy.
Various methodologies that have been adopted to estimate potential output and its corresponding output gap can best be categorised into two; namely: statistics approach and economic approach.
According to Somchai (2002), the economic approach relies more on economic theories when compared to the statistics approach. The most common method under economic approach is the production function (PF) approach. This approach has a closer link to economic theory. However, it also requires some assumptions of the functional forms of the production technology and return to scale, among others. The statistics approach on the other hand, uses a procedure of extracting trend and cyclical component of output, with the pure trend pattern taken for the potential output. Although, Hodrick-Prescott (1997) filter is the most outstanding method used in the estimation of potential output, under this approach, there are others, which include Band-pass filter, Wavelet-transform filter and Kalman filter, among others.
Somchai (2002) utilises HP filter to estimate potential output for Japan. He uses quarterly data from 1980:Q1 to 2002:Q3. In order to check for the consistency of the estimated output with the business cycle, he compares the result with Japan's coincident index and Tankan[1]. The final result shows that the estimated peak and troughs do not differ significantly from the coincident index and Tankan.
Leon (2007) applied HP filter for estimating potential output for South Africa. He adopted optimal filtering by Pedersen (1998, 2001 and 2002) to determine the optimal value of the smoothing constant as against 1600 smoothing constant suggested by Hodrick and Prescott (1997) for quarterly data. He argues that, the most appropriate censoring rule specifies business cycle frequencies as those which occur at frequencies of less than six years and that the optimal value of lambda is 352 for South Africa. He, therefore, concluded that optimal

[1] Tankan is an abbreviation for "Tanki Keizai Chousa" meaning "short term economic survey of enterprises.

filtering requires explicit thinking about the structure of the economy and the conceptualisation of the business cycle.

Takuji et al (2010) uses a combination of various method of estimating potential output including production function approach and some filtering approaches such as HP filter, Phillip curve approach and DSGE approach. He concludes that, the estimate of potential output can differ considerably depending on the method used. According to him, all the methods are subject to error and that the reliability of estimate is hampered whenever turbulences are rampant in the economy. All the approaches, however, gave the same indication – decline in the potential growth of Japan, but the magnitude varies.

Phurichai (2012) uses a relatively newer framework that combined a simple quantitative model with an investigative approach of growth for Cambodia. The result, according to him, is that Cambodia compares less favourably to other lower-income Asian countries in term of investment rate, which is constrained by the poor quality of its infrastructure.

Office of the Parliamentary Budget Officer (2010) estimated a potential GDP for Canadian economy by assessing trends in labour input and labour productivity. The result shows a downward trend in potential GDP for Canada over the reviewed period. The projected decline in potential growth was attributed to projected decline in the growth of trend labour input. The decline in labour input, according to Parliamentary Budget Officer (PBO) reflects slower growth of the working age population and a decline in the trend unemployment rate associated largely with the shifting age composition of the workforce.

Tino (2008) estimates potential output, the natural rate of unemployment and the core inflation rate for Euro Area using aggregated data. The empirical model includes Phillip curves linking inflation to unemployment. He used Okun's type relationship to link the output gap to cyclical unemployment. The model also accounts for new developments in unobserved components models. The results show that, there is a one-time large shift in the growth rate of potential output in 1974:Q1, indicating that the conventional approach of modelling potential output as either deterministic or a unit-root process with shocks occurring every period is inadequate to capture shifts.

Osman (2008) used different statistical methodologies to estimate potential output and output gaps of four East African countries of Kenya, Ethiopia, Tanzania and Uganda. The methodologies include linear method, HP filter, frequency domain filter and the unobserved component model. The results show similar assessments of the variables for all the countries. The corresponding output gaps are all in agreement with the historical boom-bust cycles of all the countries.

Moosa et al (2009) estimate potential GDP for the Romanian economy using a combination of production function approach and several statistical de-trending methods. The result indicates a continuous increase in the growth rate of the potential output until the third quarter of 2008, then a decline in 2008:Q4 and 2009:Q1. They identified technological progress as the main driving force of the potential growth in Romania.

Darvas and Vadas (2003) applied a univariate method to estimate and evaluate potential output for Hungary. They paid special attention to structural breaks. Considering the strengths and weaknesses of each of the methods adopted, they derived a single measure of potential output by weighing those methods that pass both the statistical and expertise criteria. The weights were derived on revisions of the output gap for all dates by recursively estimating the models.

Patricia et al (2009), applied linear trend model, HP filter and SVAR model on the Nigeria's GDP from 1980:Q1 to 2008:Q4. They showed that different techniques yield different results of potential output. They concluded that the result of SVAR model provides the most reliable predictor of inflation in Nigeria. They strongly suggested the use of professional judgement, as well as other intermediate indicators, to compliment output gap results in taking monetary policy decisions.

3. Model Specification and Estimation

It is a well-documented fact that potential output cannot directly be observed and therefore has to be estimated using information from other macroeconomic variables. This has led to the development of various techniques extensively used to estimate the variable. This, however, presents a considerable challenge for policy makers, since different measures of the unobservable potential output yields, in most cases, different results. If policy makers mistakenly adopt policies based on wrong estimates of the corresponding output gap, inadvertently they will induce instability in the economy.

Therefore, to ensure the robustness of the result, different econometrics filtering methodologies are employed in this study. The logic is that, since Nigeria is an emerging economy, one method may not be robust enough to capture the specifics of the economy. This will limit the error in result, ensuing from the weaknesses of the filtering methodologies.

Although, the software automatically generates both potential output and the corresponding output gap, in the alternative, however, with available information on potential output, the corresponding output gap can be calculated as follows:

$$\text{Output Gap} = \frac{\text{Actual Output} - \text{Potential Output}}{\text{Potential Output}} \text{ X } 100 \qquad (15)$$

The four econometrics filtering methodologies applied are: The Hodrick-Prescott filter (1997), band-pass filters - Baxter-King fixed length symmetric filter, Baxter and King (1999); and both fixed and full length Christiano-Fitzgerald filters, Christiano and Fitzgerald (2003). The derivations of the methods adopted are as follows:

The Hodrick-Prescott Filter

The filter minimizes the weighted sum of the square cycle, the square change in the growth rate of the potential growth. The HP filter extracts from y_t the trend component $\hat{y}$ and from u_t the cyclical component $\hat{u}$. The estimation of $\hat{y}$ is obtained through the minimization of the sum of squares of the transitory component subject to a penalty for the variation in the second difference in the growth component. That is $\hat{y}$ is the solution to the minimization problem:

$$\min_{\{\hat{y}_t{}^T\}_{t=1}} \sum_{t=1}^{T} \left[(y_t - \hat{y}_t)^2 + \lambda[(\hat{y}_{t+1} - \hat{y}_t) - (\hat{y}_t - \hat{y}_{t-1})]^2 \right] \qquad (16)$$

Where λ is a penalty parameter which is related to the smoothness of the estimated trend. Equation (16) can be re-formulated as follow:

$$\min_{\{\hat{y}_t{}^T\}_{t=1}} \sum_{t=1}^{T} \left[d_t{}^2 + \lambda(\Delta^2 \hat{y}_{t+1})^2 \right]$$

The minimization of equation (16) yields linear equations giving the series y_t as a function of its permanent component through a T X T matrix M:

$$y = M_{\hat{y}} \qquad (17)$$

Where y and $\hat{y}$ represent the series y_t and $\hat{y}$, respectively. The first order condition for the minimization of equation (5) gives:

$$d_1 = y_1 - \hat{y}_1 = \lambda(\hat{y}_1 - 2\hat{y}_2 + \hat{y}_3)$$
$$d_2 = y_2 - \hat{y}_2 = \lambda(-2\hat{y}_1 + 5\hat{y}_2 - 4\hat{y}_3 + \hat{y}_4)$$
$$d_1 = y_t - \hat{y}_t = \lambda(\hat{y}_{t-2} - 4\hat{y}_{t-1} + 6\hat{y}_t - 4\hat{y}_{t+1} + \hat{y}_{t+2}) \text{ for } t = 3, \dots, T-2$$
$$d_{T-1} = y_{T-1} - \hat{y}_{T-1} = \lambda(\hat{y}_{T-3} - 4\hat{y}_{T-2} + 5\hat{y}_{T-1} - 2\hat{y}_T)$$
$$d_T = y_T - \hat{y}_T = \lambda(\hat{y}_{T-2} - 2\hat{y}_{T-1} + \hat{y}_T)$$

The higher the value of λ, the smoother the estimates of the growth component $\hat{y}$, while the more volatile the value of the estimate of the transitory component d_t. If $\lambda \to \infty$, equation (16) is minimized if the estimated trend is a straight line (for which $\Delta^2\hat{y}_t$ are identically zero); however, if λ = 0 equation (16) is minimized if $\hat{y}_t = y_t$ for every t. Hodrick-Prescott filter (1980) suggests the following values for λ. (λ = 100, 1600 and 14400 for annual, quarterly and monthly data, respectively).

The Ideal Band-Pass Filter

Band-pass filters tries to eliminate both high frequency fluctuations arising mostly from the measurement errors and noise and low frequency fluctuations, which reflects the long-term growth component.

Drawing from the analysis of Hens and Kai (2011), consider a linear filter G(L) which is a linear transformation of a time series x_t with weights g_l at lag l.

$$G(L) = \sum_{l=a}^{b} g_l L^l, \qquad a \le 0 \le b, \qquad (18)$$

Where L is the lag operator $L^k = x_{t-k}$. To produce the filter series x_t, the filter is applied to y_t:

$$x_t = G(L)y_t = \sum_{l=a}^{b} g_l y_t \qquad (19)$$

The effect of the application of the filter is reflected in the frequency response function (FRF) of the filter. This is represented as:

$$G(e^{-i\omega}) = \sum_{l=a}^{b} g_l e^{i\omega l} \qquad (20)$$

The growth of the amplitudes of y_t is caused by the linear filter.

$$\text{Gain}(\omega) = \left|G\left(e^{-i\omega}\right)\right| \quad (21)$$

While the shift of its position with regards to

$$\text{Phase}(\omega) = \frac{\arg\left(G\left(e^{-i\omega}\right)\right)}{(2f)} \text{ at frequency } \omega \quad (22)$$

Equations (21) and (22) are respectively the gain shift and phase shift of the filter.
If $g_l = g - l$ for $l > 0$, implying that weights are symmetrical, the linear filter will not cause any phase shift.
However, since phase shift causing filters can lead to either wrong or spurious lead-lag relationships between/among variables, according to Hens and Kai (2011), it therefore, follows that the gain function of the ideal band pass filter is a perfect rectangular function, given as:

$$\text{Gain}(\omega) = \begin{cases} 1 & \text{for } \omega_1 \le \omega \le \omega_2 \\ 0 & \text{for } \omega < \omega_1 \text{ or } \omega > \omega_2 \end{cases} \quad (23)$$

Note that the phase shift function is a constant zero.
Now, to derive the weight of the ideal band pass filter, we have:

$$g_l = \begin{cases} \dfrac{\sin(\omega_2 l) - \sin(\omega_1 l)}{\pi l} & \text{for } l \neq 0 \\ \dfrac{\omega_2 - \omega_1}{f} & \text{for } l \neq 0 \end{cases} \quad (24)$$

However, according to Hens and Kai (2011), the ideal band pass filter is practically not feasible, because, of the infinite nature of the weights. In other words, to calculate such a filter, an infinite-order moving average is necessary which requires a data set of infinite length, which is practically not available. Therefore, some form of approximation is required, thus making the contributions of Baxter and King (1999) and Christiano and Fitzgerald (2003) highly relevant.

Baxter-King (BK) and Christiano-Fitzgerald (CF) Approximations

Following Hens and Kai (2011) the weight of the ideal filter (i.e. equation 24), in the Baxter-King approximation, are used up to certain lags, thereafter, the weights are truncated. A restriction is then added that the FRF at frequency $\omega = 0$, is exactly zero. In this case, the linear quadratic and stochastic trends up to order two are eradicated, making it more reasonable for economic analysis. The weight then becomes:

$$gl^* = \begin{cases} gl + \theta & \text{for } -a \le l \le a \\ 0 & \text{for } |l| > \end{cases}, \quad \theta = \frac{-\sum_{l=-q}^{a} g_l}{2a+1} \quad (25)$$

Where *gl* are the weights of the ideal filter. The beauty of the Baxter-King filter is that it causes no phase shift.
Christiano-Fitzgerald[1] approximation, on the other hand, uses alternative loss criterion, as well as, the assumptions on the underlying process of y_t they yielded, to adjust the weights to take account of the missing values. The extrapolation of the sample is done by using what is referred to as 'an assumed model' and the extrapolation overlaps the observed sample. Now, following Hens and Kai (2011), if we assumed a random walk for series y_t, the following simple adjustment is required:

$$\breve{g}_0 = \frac{g_0}{2} \quad (26)$$

$$\breve{g}_1 = \frac{-g_0}{2} \quad (27)$$

$$\breve{g}_l = \frac{-g_0}{2} - \sum_{k=1}^{l-1} g_l, l \ge 2 \quad (28)$$

Where g_l in equation (28) is the weight of the ideal filter as represented in equation (24). The $\breve{g}_l$ is the adjusted weights and are used on the end points y_1 and y_T. In between are the observations that are weighted by the unmodified weights g_l.

4. Empirical Result

The study used data on gross domestic product (GDP) from 2004:Q1 to 2011:Q4. The reason for the choice of the study period is that the quarterly GDP data dating back from 2004:Q1 are not real survey data. They are disaggregated annual data. From 2004:Q1 the quarterly GDP numbers are those obtained directly from the field, through the collaborative efforts of the Central Bank of Nigeria (CBN) and National Bureau of Statistics (NBS), hence will yield a relatively more accurate result than the disaggregated ones.
From figure 1, it is obvious that output gap for Nigeria, was quite hectic since 2004. For instance, during the first quarter of 2004, both Hodrick-Prescott and Christiano-Fitzgerad's full (CF-full) length symmetric filter estimates show that Nigeria economy operated significantly below its potential as the output gap estimates of the approaches were negative. This trend continues until the fourth quarter of the year. Output gap was positive for

[1] For detailed exploration on the derivation of BK and CF filter, see Hens and Kai (2011) and Christiano and Fitzgerald (2003)

all the methods in 2005:Q1and Q2, indicating that the economy out-performed its potential. Interestingly too, the estimates for both methods show that output gap recorded the lowest negative in 2004:Q2 and highest negative in 2004:Q1.

In the third and fourth quarters of 2005, the results from all the methods were negative (Table 2) with HP filter recording the highest negative gap of -4.1 per cent in 2005:Q3, when BK was -0.7 per cent, CF-fixed -3.9 per cent and CF-full -2.9 per cent. Similarly, during the third and fourth quarters of 2006, the actual output fell short of potential output as reported by all the methods. The gap, however, turned positive as from 2007:Q1 up to 2008:Q2, except for 2007:Q3 when it was negative to the tune of 0.9, 1.1, 4.5 and 1.8 per cent, for HP, BK, CF-fixed and CF-full, respectively.

Between 2008:Q3 and 2009:Q2, the economy performed below potential. During this period, the smallest negative gap recorded was -0.3 per cent in 2009:Q2 under the CF-full. However, BK, and CF-fixed show that the output gap was positive in 2009:Q1 and Q2. The remaining part of 2009, however, experienced economic overheating as the output gap turned positive.

Most importantly, there exists a consensus among the results emanated from all the techniques as to the under-utilisation of resources for production in 2010. Infact, BK turned out an output gap as low as -3.0 per cent in 2010:Q1 and CF-full recorded a negative output gap of -3.3 per cent in 2010:Q4.

Figures 1 to 5 depict the potential growth and the corresponding output gaps of each of the four methodologies adopted. In general, the methods (i.e. HP, CF-fixed, CF-full and BK) generate a similar path for output gap during the 2004:Q1 to 2011:Q4 period. Although, the amplitude varies, but the shape of the curves show basically the same cyclical behaviour.

Under both HP and CF-fixed approaches, the output gaps reached a positive peak of around 6.9 and 7.1 per cent, respectively, whereas within the same period BK and CF-full yielded a positive gap of 1.4 and 3.6 per cent, respectively. The maximum positive gap for BK and CF-full were 6.9 and 4.3 per cent in 2005:Q1 and 2006:Q1, respectively. The maximum negative output gaps recorded were -6.8, -6.1, -3.9 and -3.0 per cent for CF-fixed, HP, CF-full and BK in 2006:Q4, 2006:Q4, 2004:Q1 and 2010:Q1, respectively.

It can also be observed that while the potential output and the actual output for HP and CF-full closed-up in 2010:Q2 and 2008:Q1, output gap was just a little less than 1.0 per cent for the other methods throughout the sample period.

From Table 3, it is clear that CF-fixed filter yielded the most volatile estimate of the output gap throughout the period than the other methods. Similarly, it is the only method whose estimate differs visibly from the others.

The correlation across the four measures is represented in Table 4. CF-fixed tends to be more highly correlated with BK method, followed by the HP and CF-fixed; and HP and CF-full. The HP estimates is relatively less-correlated with BK. Captivatingly, the correlation among all the methods is positive.

Overall, following Table 5 the result suggest that, on the average, the economy was over heated during the early part of the sample period (i.e. 2004:Q1 to 2007:Q4) but operated below capacity between 2010:Q1 and 2011:Q4.

The strength of the concept of output gap lies heavily with its link to inflation[1]. Interestingly, Table 6 depicts the correlation between output gap and inflation in Nigeria. There appear to be fairly strong and stable relationships among inflation and the estimated output gaps, particularly HP and Christiano-Fitzgerald filters. The correlation between CF-fixed and inflation (IF) is over 92.0 per cent, while that of HP is about 88.0 per cent. CF-full is 87.0 per cent correlated with IF and BK recorded the lowest correlation of 72 per cent. With this noticeable connection, using an output gap, to compliment expert judgement, in monetary policy decision making would conceptually be a good decision.

5. Conclusion and Implication for Monetary Policy

This paper has presented estimates of the Nigeria's potential output and the corresponding output gap for the period 2004:Q1 to 2011:Q4 using different econometrics filtering methodologies. As expected, the methods yielded different results; they nevertheless show strong similarities in their evolution over time. In other words, a high degree of consistency evolved among all the methods (i.e. HP, CF-fixed, CF-full and BK). Estimates based on the CF-fixed filter proved to be more-volatile and less-similar to the others. All the methods, on the average, clearly confirmed that the economy was over heated between 2004:Q1 and 2007:Q4 but operated below capacity between 2010:Q1 and 2011:Q4. While HP estimate shows that the economy operates below capacity in the first quarter of 2011, BK and both CF-fixed and full indicates the opposite.

The major implications of the study on both monetary and fiscal policies are:

i. Monetary Policy

In countries where inflation targeting framework is operational, the output gap is one of the determinants of the degree of looseness or otherwise of the monetary policy that can enhance the achievement of the targeted

[1] The link between inflation and output gap is a potential issue the researcher wish to deeply explore in future research.

inflation at optimal growth. Although, the inflation targeting framework is not operational in Nigeria, but the twin objective of achieving price stability and promoting economic growth requires the knowledge of not only the growth rate but also the country's potential output and the corresponding gap. For instance, during the periods when the output gap estimates show that the economy was overheated, particularly in the last three quarters of 2011, the policy direction of the Bank in the first quarter of 2012, should be contractionary.

ii. Fiscal Policy

The positive output gap in the last three quarters of 2011 suggests that there is no room for expansionary fiscal policy. However, government, within this period, recorded a fiscal deficit of ₦267.1 billion and ₦133.9 billion in November and December 2011, respectively, making the fiscal deficit in the fourth quarter totalled ₦350.1 billion, about 3.5 per cent of the country's GDP for the quarter. This further inundated the economy with more liquidity. For Nigeria to achieve optimal growth with stable prices, there is need for government to consider the output potential of the economy in her fiscal policy stance.

In conclusion, therefore, there is the need for considering both potential output and the corresponding output gap in taking both monetary and fiscal policy decisions, since fiscal policy is likely to impact more on the economy than monetary policy which has limited scope.

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APPENDIX

Table 1: Signs of Output Gap Estimates

Qtr	HP	BK	CF-Fixed	CF-Full	Qtr	HP	BK	CF-Fixed	CF-Full
2004Q1	-			-	2008Q1	+	+	+	+
2004Q2	-			-	2008Q2	+	+	+	+
2004Q3	-			-	2008Q3	-	-	-	-
2004Q4	+				2008Q4	-	-	-	-
2005Q1	+	+	+	+	2009Q1	-	+	+	-
2005Q2	+	+	+	+	2009Q2	-	+	+	-
2005Q3	-	-	-	-	2009Q3	+	-	-	-
2005Q4	-	-	-	-	2009Q4	+	-	-	-
2006Q1	+	+	+	+	2010Q1	-	-	-	-
2006Q2	+	+	+	+	2010Q2	-	-	-	-
2006Q3	-	-	-	-	2010Q3	-	-	-	-
2006Q4	-	-	-	-	2010Q4	-	-	-	-
2007Q1	+	+	+	+	2011Q1	-			+
2007Q2	+	+	+	+	2011Q2	+			+
2007Q3	-	-	-	-	2011Q3	+			+
2007Q4	+	+	+	+	2011Q4	+			+

Source: *Authors calculation*

Table 2: Output gap as a percentage of Potential output

Qtr	HP	BK	CF-Fixed	CF-Full	Qtr	HP	BK	CF-Fixed	CF-Full
[redacted]						3.9	2.39	4.75	0.1
						0.9	1.85	3.08	-0.3
						-2.2	-1.64	-5.17	-0.3
						-2.5	-2.80	-0.63	-0.6
						-1.7	2.5	1.2	-2.0
						-1.1	1.9	6.2	-0.3
						1.2	-0.5	-0.4	-0.3
						0.6	-0.7	-0.9	-0.7
						-1.9	-3.0	-1.7	-2.9
						-1.0	-2.1	-2.4	-2.8
						-2.5	-1.3	-3.7	-3.7
						-1.3	-2.8	-2.9	-3.3
						-0.8			2.2
						0.1			1.8
						1.1			0.7
						4.3			0.5

Table 3: Summary Statistics of the Output Gap Estimates

	BK	CF_FIXED	CF_FULL	HP
Mean	0.00	0.20	-0.22	0.05
Median	-0.63	-0.51	-0.28	-0.85
Maximum	2.47	7.11	4.30	6.90
Minimum	-2.96	-6.75	-3.91	-6.10
Std. Dev.	2.01	4.35	2.47	3.16
Skewness	-0.06	0.07	0.24	0.24
Kurtosis	1.38	1.62	1.84	2.36
Jarque-Bera	2.64	1.91	2.09	0.86
Probability	0.27	0.38	0.35	0.65
Sum	0.06	4.72	-7.13	1.70
Sum Sq. Dev.	93.18	434.65	189.68	309.26
Observations	24	24	32	32

Source: *Authors calculation*
Note: *BK=Baxter-King, CF-Fixed =Fixed Length Christiano-Fitzgerald Filter, CF-Full=Full Length Christiano-Fitzgerald Filter and HP=Hodrick Prescott Filter*

Table 4 : Correlation among output gap estimates

	BK	CF_FIXED	CF_FULL	HP
BK	1.0000	0.8309	0.7188	0.7220
CF_FIXED		1.0000	0.7979	0.8494
CF_FULL			1.0000	0.8539
HP				1.0000

Source: *Authors' calculation*

Table 5: Average Growth Rate of Potential Output for Nigeria, 2004:Q1 to 2011:Q4

Period	HP	BK	CF-Fixed	CF-Full
2004Q1 - 2005Q4	0.44	0.14	0.68	-0.41
2006Q1 - 2007Q4	1.01	0.72	0.57	1.01
2008Q1 - 2009Q4	-0.10	0.36	1.02	-0.55
2010Q1 - 2011Q4	-0.26	-2.29	-2.67	-0.94

Source: *Authors calculation*

Table 6: Correlations among BK, CF-Fixed, CF-Full, HP Filters and Inflation for Nigeria, 2004:Q1 to 2011:Q4

	IF	BK	CF_FIXED	CF_FULL	HP
IF	1.000	-0.721	-0.925	-0.870	-0.886
BK		1.000	0.831	0.719	0.722
CF_FIXED			1.000	0.798	0.849
CF_FULL				1.000	0.854
HP					1.000

Source: *Authors' calculation*

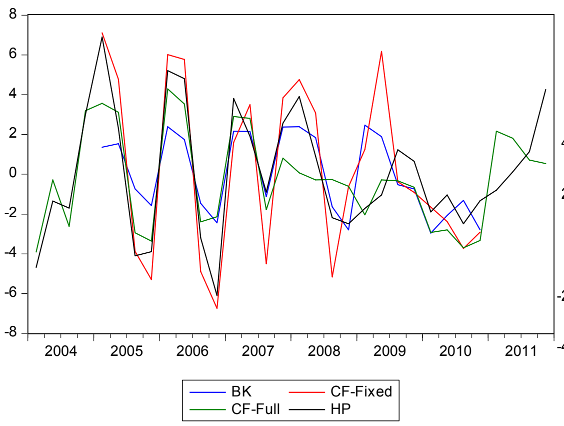

Figure 1: Combined Output Gap Estimates, 2004:Q1-2011:Q4

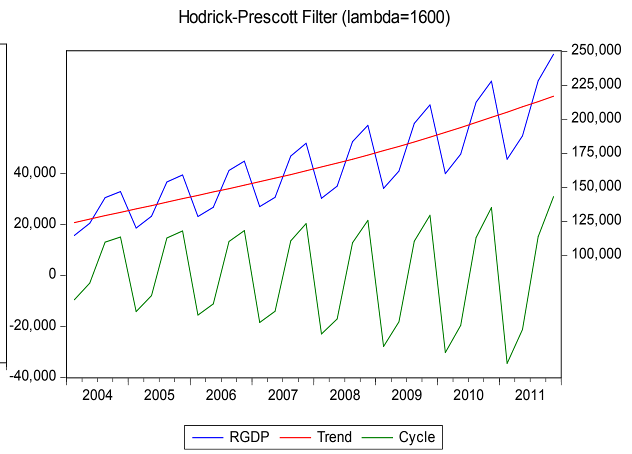

Figure 2: Real GDP, Potential Output and Output Gap (HP Filter)

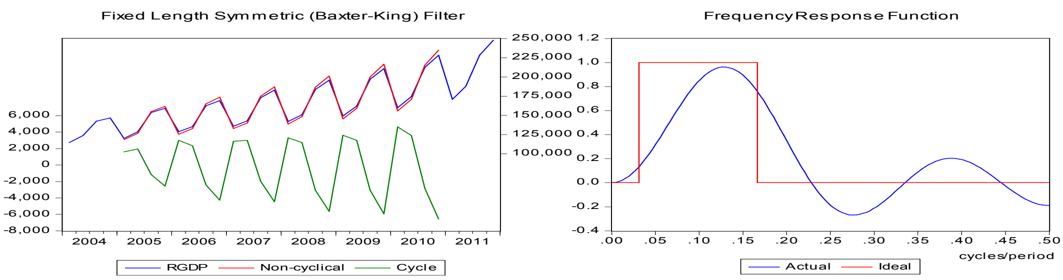

Figure 3: Real GDP, Potential Output and Output Gap (Fixed Length Symmetric Baxter-King Filter)

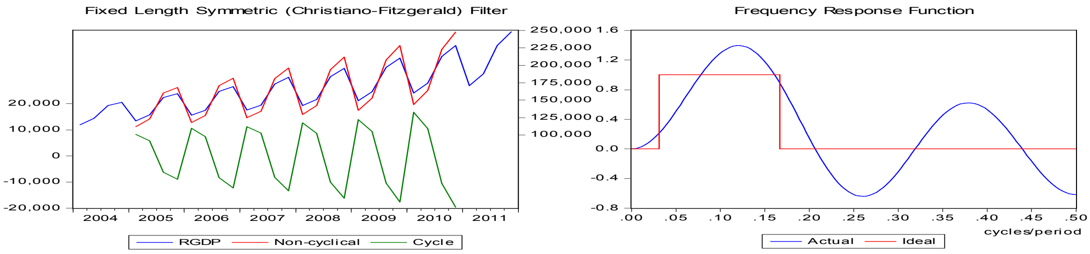

Figure 4: Real GDP, Potential Output and Output Gap (Fixed Length Symmetric CF Filter)

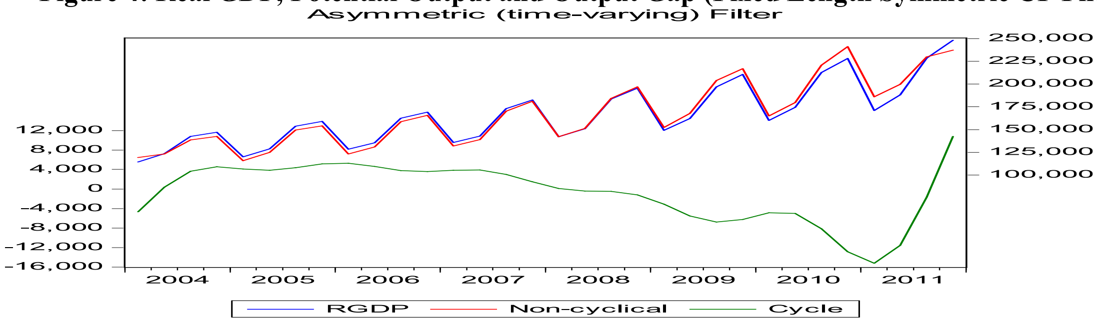

Figure 5: Real GDP, Potential Output and Output Gap (Full Length Symmetric CF Filter)