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Would Exchange Rate Converge in Nigeria?A Stochastic-Markov Transition Process Analysis

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Abstract
This paper examined if the Nigerian exchange rate would converge in the long run thereby looking at the exchange rate switches or transition from a particular state to another. This was done via the iterations of the Chapman-Kolmogorov equations of the Markov model, It was discovered that convergence occurred in the long run as shown by our markov model. It suggests that appreciation and depreciation of the naira via dollar rate would be stable as indicated by the probability values.
Keywords: Markov, Transition probabilities, Exchange rate, Chapman-Kolmogorov

1. Introduction

The introduction of structural adjustment programme (SAP) in 1986 was to promote growth and bring about positive economic indicators; but instead of achieving its aims and objectives, it led to Naira devaluation against Dollar and other currencies like Pound Sterling, Deutshe Mark, Swiss Franc and the Dutch Guilder in 1986.On 1st January 1999, Euro became the official exchange rate for Germany, France, and Netherlands.The Japanese YEN and the West Africa CFA Franc were less than one Naira until in the year 2002, when it increased to more than one Naira for the YEN but still less than a Naira for the West Africa CFA Franc till date (CBN, 2011).In trying to achieve both exchange rate stability and objectives of the exchange rate policies, a lot of modifications have been made, modifications such as transforming the Second Tier Foreign Exchange Market (SFEM) into the Foreign Exchange Market (FEM) and later to what is known as the Autonomous Foreign Exchange Market (AFEM) and to the Interbank Foreign Exchange Market (IFEM) and to the Dutch Auction System (DAS) and then to the Wholesale Dutch Auction System (WDAS).

Moreso, the depreciation of naira against dollar and some other curriencies like euro, yen etc didn't stop but it continued to depreciate and thus the value of naira is steadingly losing its value and power. In 1995, the Central Bank of Nigeria (CBN) intervened six times in the autonomous foreign exchange market (AFEM), meeting in full the US$1.748billiondemanded by the market. The inability of some end-users to effectively back their foreign exchange demand with naira deposit at the CBN, led to the allocation of the US$1.748billion., this action stabilized both the autonomous foreign exchange market and the parallel market rates; converging and stabilizing at US$1 to N82.3 and US$1 to N83.7 respectively.

The CBN attributed this to its"guided depreciation" policy adopted at the beginning of that year which allowed it to intervene periodically at the AFEM at market-determined rates (Nwidobie**, 2011**). A dual exchange rate system was maintained in 1996 with an official rate of N22 to a dollar and the AFEM rate averaging N82.5/US$1. The CBN intervention policy of 1995 was retained in 1996 to further stabilize the naira exchange. To enhance the naira rate stability, the CBN continued the suspension of the use of bills of collection and open accounts for import financing: the requirement that all imports into the country be accompanied by duly completed form M as well as import duty reports (IDRS) (Nwidobie**, 2011**). More so, in 1999, the foreign exchange management in Nigeria transited from the autonomous foreign exchange market to the interbank foreign exchange market (IFEM). During the year, the CBN intervened in the foreign exchange market 43 times against 51times in1998. IFEM rate in the year averaged N92.3/US$1; while the bureau-de-change rate (BDC) averaged N99.26/US$1, reducing the parallel market premium to 3.2%. The exchange rate of the naira depreciated in all segments of the foreign exchange market in 2000. At the IFEM, the naira depreciated on the average by 6.5% to N101.65/US$1. The rate was relatively stable during the first nine months of the year, but depreciated thereafter against the US$. A higher level of depreciation was experienced in the parallel market; falling by 10.7%. In 2001, the naira depreciated in both the IFEM and the BDC. At the IFEM, the naira exchanged at N111.96/US$1 (Nwidobie**,** 2011; Obadan, 2006).

Moreover, Mordi (2006) pointed out that in ensuring exchange rate stability, the DAS was re-introduced on July 22, 2002 so as to serve the triple purposes of reducing the parallel market premium, conserve the dwindling external reserves and achieve a realistic exchange rate for the naira. It is interesting to note that with all these exchange rate controls and various modifications, the major shortcomings in the short run and in the long run according to Sanni (2006) was the failure of the system to achieve internal and external balance in the short run and guarantee external equilibrium in the long run. Overvaluation of the currency will hinder the achievement of internal balance (Sanni, 2006). Despite the several modifications and exchange rate control system introduced and used over the years, exchange rate movements from 1986 through 2012 still exhibit an unhealthy development and volatile nature. This paper intends to look at exchange rate switches or transition from a particular state to another and also examine if the states arrived at would actually converge in the long run via the iterations of the Chapman-Kolmogorov equations, the speed at which it converges if convergence would actually occur, if convergence occurs that means there would be a single probability value for the switching states indicating that exchange rate behaviour would be in stability but if it diverges, it means there are more than one probability value.

1.1 Literature Review

The Markov Chain model, developed by the Russian mathematician Andrei, A. Markov in 1905, is a particular class of probabilistic model (also known as stochastic processes), in which it assumes that there is a one-stage dependence of events, with each event depending immediately on the preceding event, but not on the other prior events. A process of this type is said to be a Markov Chain process, Markov Chain or Markov process.(Sharma, 2009). The application of Markov chain type models to understanding regime switching behaviour of macroeconomic and financial time series can be traced to the pioneering work of Hamilton (1989) and Engel and Hamilton (1990).

Markov-switching models have been widely used to study economic problems in which there are occasional structural shifts in fundamentals. (Farmer, Waggoner and Zha, 2009).Bollen, Gray, and Whaley (2000) applied the regime switching techniques to understanding the different exchange rate policy regimes and in 2005, Bazdresch and Werner employed the regime switching techniques to examining appreciation and depreciation episodes of the nominal exchange rate of the Mexican Peso.Vigfusson, (1996) used the regime switching techniques to study existence of chartist and fundamentalist regimes and alsoNorden (1996) employed the regime switching techniques to understand the regime switching behaviour of exchange rate bubbles.

Masson, (2000) took another dimension from the usual trend of looking at the appreciation and depreciation in the exchange rate regime, he considered instead the three categories of exchange rate, that is, fix, intermediate, and float. He was of the opinion that for a given Markov chain, one has to calculate the long run distribution of regimes by repeatedly applying the transition matrix and also comparing the current distribution to the long run distribution. An interesting possibility according to him, for instance, would be that the invariant distribution implied much greater regime polarization than what prevails now (even if the hollowing out hypothesis is not strictly true). As is well known from the persistence in the use of reserve currencies, exchange rate regimes are slow to change, so that the effects of a new economic environment (involving for instance capital account liberalization) might take a long time to be visible in the number of countries in each regime category. Thus, a trend toward polarization might not yet be evident in the actual regime distribution though it would show up in the invariant distribution (and in the transition matrix). Parikakis, George and Syriopoulos, Theodore (2000) applied the Markov Switching Models in capturing volatility dynamics in exchange rates and to also evaluate their forecasting ability. They identified that increased volatilities in four Euro-based exchange rates are due to underlying structural changes and that currencies are closely related to each other, especially in high volatility periods.

Cheung and Erlandsson (2004) presented a systematic and extensive empirical study on the presence of Markov Switching dynamics in three Dollar-based exchange rates. A Monte Carlo approach was adopted to circumvent the statistical inference problem inherent so as to test the regime switching behaviour. They went further in considering two data frequencies, two sample periods, and various specifications and in conclusion, their test rejected neither the random walk nor Markov switching and they also proved that data frequency in addition to sample size is crucial for determining the number of regimes.

1.1.2 Methodology and Sources of Data

The monthly exchange rate data used would be the naira versus dollar exchange rate and it would run from 2002 to 2011. The source of the data would be the Central Bank of Nigeria statistical bulletin and the scilab version 5.3.2 would be used to run the analysis. Given that the observed monthly exchange rate data will follow different state spaces or regimes S_t. Consider that S_t is a random variable that can assume only integer values {0, 1, 2, …, N}. A two state Markov chain was assumed such that when $S_t=1$, the process was said to come from regime 1 (appreciation). When $S_t=2$, the process was said to come from regime 2 (depreciation). The probabilities of switching amongst the possible various regimes or states are captured and defined by the transition probabilities P_{ij}. The $P_{ij}$is the probability of moving from state i to state j or P_{ji},is the probability of moving from state j to state i, this implies that i is the ith row element and j is the jth column element of the stochastic matrix P.

Suppose that the probability that S_t equals some particular value j depends on the past only through its most recent value S_{t-1}. The transition probabilities (P_{ij}) becomes:

$$P\{s_t = j / s_{t-1} = i, s_{t-2} = k, \ldots\} = P\{ s_t = j / s_{t-1} = i \} = P_{ij} \qquad (1)$$

And the transition probability matrix becomes:

$$p = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \qquad (2)$$

1.1.3 The Chapman-Kolmogorov Equations

The n-step transition probability that a process currently in state i will be in state j after n additional transitions is:

$$P_{ij}^{(n)} = \Pr(X_n = j / X_0 = i) \qquad (3)$$

n, i, j≥ 0

note that $P_{ij}^{(i)} = P_{ij}$ and $P_{ij}^{(0)} = \begin{cases} 1 & if\ i=j \\ 0 & otherwise \end{cases}$ (4)

then the C-K equations:

$$P_{ij}^{(n+m)} = \sum_{k=0}^{\infty} P_{ik}^{n} P_{kj}^{m} \qquad (5)$$

If it will take the state i to go to j in n+m steps with an intermediate stop in state k after n steps; then sum over all possible k-values. Then the n-step transition matrix is:

$$P^{(n)} = \begin{bmatrix} P_{11}^{(n)} & P_{12}^{(n)} \\ P_{21}^{(n)} & P_{22}^{(n)} \end{bmatrix} \qquad (6)$$

The C-K equations imply that $P^{(n+m)}=P^{(n)}P^{(m)}$

1.1.4 . The Steady State or Long Run Behaviour For Markov Processes

Given a Markov chain with a regular transition matrix P , and πdenoting the limit of P^n as n approaches infinity, then;

$$P^n X \rightarrow \pi X = \rho \qquad (7)$$

where;

X is an initial steady state vector

and the system therefore approaches a fixed state vector ρcalled the **steady-state vector of the system.**

From C-K equations, it can be seen that $P^{n+1}=PP^n$, where m=1and n can take any other value, one (1) inclusive. Then both P^{n+1} and P^n approaches π, so that the C-K equations can transcend to

$$\pi = P\pi \qquad (8)$$

Note that any column of this matrix equation gives $P\rho = \rho$. Therefore, the steady-state vector of a regular Markov chain with transition matrix P is the unique probability vector ρ satisfying the equation below

$$P\rho = \rho \qquad (9)$$

Alternatively, equation (9) can be computed without using limit but by linear systems equation and having to recall the eigenvector and eigenvalue of a square matrix:

Let the square matrix A be the transition probability matrix (TPM), it can be inferred that the number λ is an eigenvalue of A if there exists a nonzero vector X satisfying the equation below:

$$AX = \lambda X \qquad (10)$$

In this case, X is an eigenvector of A corresponding to the eigenvalue λ.

Note that a steady-state vector of a regular Markov chain is an eigenvector for the transition matrix corresponding to the eigenvalue 1.

Recall also that the eigenvalues of the transition probability matrix A are the solutions to the equation :

$$\det(A - \lambda I) = 0 \qquad (11)$$

where I is the identity matrix of the same size as A. If λ is an eigenvalue of A, then an eigenvector corresponding to λ is a non-zero solution to the homogeneous system $(A - \lambda I)X=0$.

1.1.5. Analysis of Results

The different transition probability matrices would be seen in the table below

TABLE 1

Year	Switching regimes	Tpm	Year	Switching regimes	Tpm
2002	aa	0	2007	aa	0.7
2002	ad	1	2007	ad	0.3
2002	da	0.27	2007	da	1
2002	dd	0.73	2007	dd	0
2003	aa	0	2008	aa	0.7
2003	ad	1	2008	ad	0.3
2003	da	0.33	2008	da	0
2003	dd	0.67	2008	dd	1
2004	aa	0.86	2009	aa	0
2004	ad	0.14	2009	ad	1
2004	da	0.2	2009	da	0.3
2004	dd	0.8	2009	dd	0.7
2005	aa	0.44	2010	aa	0.33
2005	ad	0.56	2010	ad	0.67
2005	da	1	2010	da	0.5
2005	dd	0	2010	dd	0.5
2006	aa	0.62	2011	aa	0.33
2006	ad	0.38	2011	ad	0.67
2006	da	1	2011	da	0.22
2006	dd	0	2011	dd	0.78

It can be seen from the above transition probability table that there was a zero probability value for exchange rate that switched from an appreciation to appreciation state and the years affected are; 2002, 2003 and 2009. It simply implies that in those years (2002, 2003 and 2009) that naira never moved from an appreciation state to another appreciation state but rather instead of maintaining the state of appreciation, it would always depreciate against dollar. We can also seethat depreciation to depreciation state has a zero probability for 2005, 2006 and 2007, it also implies that instead of depreciating continuously, it would rather appreciate even if it were for some time. A zero probability value for depreciation to appreciation also occurred in 2008.

The pooled transition probability matrix ($TPM_{2002-2011}$) can be seen below

$$TPM_{2002-2011}=\begin{bmatrix}0.54 & 0.46\\0.47 & 0.53\end{bmatrix}$$

0.54 is the pooled probability transition value for appreciation to appreciation, 0.46 for appreciation to depreciation, 0.47 for depreciation to appreciation and 0.53 for depreciation to depreciation states. The Chapman-Kolmogorov equation was applied to the monthly exchange rate data which already has been transformed to the pooled transition probability matrix to check for convergence and the iterations can be seen in the table below;

TABLE 2

First iteration(n=1)	$\begin{bmatrix}0.54 & 0.46\\0.47 & 0.53\end{bmatrix}$
Second iteration (n=2)	$\begin{bmatrix}0.5078 & 0.4922\\0.5029 & 0.4971\end{bmatrix}$
Third iteration (n=3)	$\begin{bmatrix}0.505546 & 0.494454\\0.505203 & 0.494797\end{bmatrix}$
Fourth iteration (n=4)	$\begin{bmatrix}0.5053882 & 0.4946118\\0.5053642 & 0.4946358\end{bmatrix}$
Fifth iteration (n=5)	$\begin{bmatrix}0.5053772 & 0.4946228\\0.5053755 & 0.4946245\end{bmatrix}$
Sixth iteration (n=6)	$\begin{bmatrix}0.5053764 & 0.4946236\\0.5053763 & 0.4946237\end{bmatrix}$
Seventh iteration (n=7)	$\begin{bmatrix}0.5053763 & 0.4946237\\0.5053763 & 0.4946237\end{bmatrix}$

From the iterations above, it can be seen that convergence was attained at the seventh iteration which we can also call the seventh year after 2011 that is 2018. The convergence probability value of 0.5053763 is for both the appreciation to appreciation state and likewise the depreciation to appreciation state and the convergence probability value of 0.4946237 is for appreciation to depreciation state and depreciation to depreciation state. This implies that even if we take n to the 500^{th} iteration or 1000^{th} iteration, that the same valuesof 0.5053763 and 0.4946237 as that obtained in the seventh iteration would be achieved so that if exchange rate was in an appreciation state at a certain period t and it moves to another state of appreciation at another period t+1, it would still maintain a probability value of 0.5053763 even if the exchange rate were to move from a depreciation state at time t to another state of appreciation it would still maintain the probability value of 0.5053763 this is because it will switch to an appreciation state from any of the two possible state. The probability value of 0.4946237 simply implies that in whatever state it is coming from to the depreciation state it would maintain that single probability value. The case of the long term probability or steady state was also looked into and the following eigen vectors and eigen values ware obtained and a striking property of the markov chain is that the sum of the components must equal one, that is, the property of having an eigen value of one and getting the correspondent eigen vector. It is expected that if that property hold, that the probability values obtained from the analysis are the steady states probability.

Eigen vectors=R= $\begin{matrix} 0.7071068 & -0.6994630 \\ 0.7071068 & 0.7146688 \end{matrix}$

Eigen values=D= $\begin{matrix} 1 & 0 \\ 0 & 0.07 \end{matrix}$

R are eigenvectors corresponding to eigenvalues on the diagonal of D in position D(1,1). We divide the eigenvector R(1,:) by the sum of the components so that the result will have components summing to one. The resulting probability values below are called the steady state or stationary values of the exchange rate in the long term

Steady state probability =0.502711718 0.49728282

This implies that 50.3% of the time would be on the appreciation state of the naira via dollar exchange rate and 49.7% would be on the depreciation state of the naira via dollar exchange rate, any other probability value other these wouldn't give the stationary level or we can say cannot bring about the needed equilibrium in the long term.

1.16. Conclusion and Recommendations

In this study, we have explored some of the ways in which the theory of markov analysis can be applied as it concerns exchange rate behaviour but we actually have not exhausted the entire theory of markov analysis but we have been able to add to the existing literature.

This paper examined if the Nigerian exchange rate would converge in the long run considering the global economic meltdown. It was discovered that convergence would occur in the long run via the Chapman Kolmogorov equations and iterations. The convergence in the long run as shown by our markov model only suggests that appreciation and depreciation of the naira via dollar rate would be stable as indicated by the probability values.

Finally, further studies can be done on this area as it concerns investigating the mean time it takes a state to return to a state it just left and how long it stays on a state before leaving for another state.

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