Estimating Geo-Mechanical Strength of Reservoir Rocks from Well Logs for Safety Limits in Sand-Free Production

Dorcas S. Eyinla[1*] Michael A. Oladunjoye[2]
1.Department of Earth Sciences, Adekunle Ajasin University, Akungba Akoko, Ondo State, Nigeria
2.Department of Geology, University of Ibadan, Ibadan, Oyo State, Nigeria
* E-mail of the corresponding author: eyinladorcas@yahoo.com

Abstract
Hydrocarbon exploration and exploitation does not only require the knowledge of hydrocarbon in-place, however, mechanical competency of the reservoir rock must also be known. The direct method of inferring S- and P-wave velocities from seismic data usually has limitation of poor resolution because of the uncertainty in seismic inversion which may also affect other derivatives.
An analytical method is presented with the possibility of predicting shear wave velocity from wireline log data where S-wave sonic logs do not exist. By estimating S-wave velocity, formation geo-mechanical properties can be calculated using P-wave sonic and density logs with appropriate equations. Elastic constants such as Poisson Ratio, Young's, Shear and Bulk moduli which are the parameters for characterizing rock mechanical properties were estimated and used to predict the mechanical competency of the formation for hydrocarbon exploration. Well planning demands knowledge of these geo-mechanical properties which can be used to estimate the pressures required to initiate a fracture into a formation for the safety of the personnel and equipment, in particular minimizing the associated risks.
In this paper, firstly we investigate the possibility to predict the shear velocity from well logs, and then Elastic moduli calculated from the log data can therefore be used effectively in predicting safety limits in sand-free production from friable sandstones.
The results of this study shows that the combined modulus of strength (K) and the shear modulus (S) to compressibility (c) ratio (S/c) for the formation are relatively low. The average value of K is lower than the threshold value indicating the minimum value at which fluids may be produced safely at any rate but falls within the range which generally represents a condition in which the problem of sand production should not arise below a certain optimum flow rate. Average value of S/c ratio is lower but tends towards acceptable range.
Keywords: Mechanical competency, predicting shear wave velocity, elastic constants, well planning, combined modulus of strength, shear modulus to compressibility ratio.

1. Introduction

It has been discovered that in many developed oil fields, only compressional wave velocity may be available through old sonic logs or seismic velocity check shots. For practical purpose such as in amplitude variation with offset (AVO) analysis, seismic modeling, and engineering applications, shear wave velocities and moduli are needed. In these applications, it is important to extract, either empirically or theoretically, the needed shear wave velocities or moduli from available compressional velocities or moduli (Wang, 2000).

P-wave velocity (Vp) and S-wave velocity (Vs) show a linear correlation in water saturated sandstones as Vs = 0.79Vp – 0.79 (Han, 2004). Castagna (1985) proposed a method for shear velocity estimation in shaly sandstones from porosity ($\emptyset$) and clay content (V_{cl}) as $V_s = 4.89 - 7.07\emptyset - 2.04V_{cl}$. Also well log studies (Pickett, 1963; Nations, 1974; Kithas, 1976; Miller and Stewart, 1990) indicate a correlation between *Vp/Vs* values and lithology. Beyond lithology identification, elastic behavior of the material can be known. As a matter of fact, production of sand along with oil and gas is a formidable problem in many younger, unconsolidated rocks. The object of estimating formation strength on the basis of elastic constants is to determine whether the formation is strong enough to produce at high flow rates without sand. If the formation cannot sustain high flow rates without sand, it is beneficial to determine the optimum production rate which can be sustained without producing sand. There is considerable evidence that a good correlation exists between the intrinsic strength of the rock and its elastic constants. The *sonic* or *acoustic* log measures the travel time of an elastic wave through the formation. This information can also be used to derive the velocity of elastic waves through the formation.

2. Basic Theory

The velocity of the compressional wave depends upon the elastic properties of the rock (matrix plus fluid), so the measured slowness varies depending upon the composition and microstructure of the matrix, the type and distribution of the pore fluid and the porosity of the rock. The velocity of a P-wave in a material is directly proportional to the strength of the material and inversely proportional to the density of the material. Hence, the slowness of a P-wave in a material is inversely proportional to the strength of the material and directly

proportional to the density of the material.
Elastic properties of rocks are affected by some geological factors which include: Depth of burial, Lithology, Anisotropy and Diastrophism. The specific transit times are influenced by these geological factors as well as porosity. Texture and geological history determine the elastic properties more than the mineral composition. Crystalline rocks generally exhibit larger values of elastic moduli than fragmental rocks (Dresser Atlas).
Hooke's law describing the behavior of elastic materials states that within elastic limits, the resulting strain is proportional to the applied stress. Stress is the external force applied per unit area, while strain is the fractional-distortion which results because of the acting force. Three types of deformation can result, depending upon the mode of acting force. The modulus of elasticity is the ratio of stress to strain.
The elastic moduli are:

Young's Modulus, $Y = \frac{F/A}{dl/l}$ (1)

Bulk Modulus, B*:* this is the extent to which a material can withstand isotropic squeezing.

$B = \frac{F/A}{dv/v}$ (2)

Shear Modulus, S**:** this is the extent to which a material can withstand shearing.

$S = \frac{F/A}{\tan s}$ (3)

Where F/A is the force per unit area, dl/l, dv/v, and tan s are the fractional strains of length, volume, and shape, respectively.
Another important elastic constant, called Poisson's Ratio, is defined as the ratio of strain in a perpendicular direction to the strain in the direction of extensional force, such as:

Poisson's Ratio, $P = \frac{dx/x}{dy/y}$ (4)

Where x and y are the original dimensions, and dx and dy are the changes in x and y directions respectively, as the deforming stress acts in y direction.
Distances between adjacent molecules increase in order from solids to liquids to gases. Because of this, solids have little compressibility as compared to liquids and gases. In fact, the bulk modulus is the reciprocal of compressibility and is therefore sometimes referred to as the coefficient of incompressibility (Dresser Atlas).

2.1 Methodology
In terms of well logging parameters and in practical units, the relationship between Sonic wave Velocities and Elastic constants are established. The four elastic constants are expressed as:

$$\mathbf{Y} = \frac{\rho_b}{\Delta ts^2}\left(\frac{3\Delta ts^2 - 4\Delta tc^2}{\Delta ts^2 \times \Delta tc^2}\right) \times 1.34 \times 10^{10} \text{ psi} \quad (5)$$

$$\mathbf{B} = \rho_b \left(\frac{3\Delta ts^2 - 4\Delta tc^2}{3\Delta ts^2 \times \Delta tc^2}\right) \times 1.34 \times 10^{10} \text{ psi} \quad (6)$$

$$\mathbf{S} = \frac{\rho_b}{\Delta ts^2} \times 1.34 \times 10^{10} \text{ psi} \quad (7)$$

$$\mathbf{P} = 0.5\left(\frac{\Delta ts^2 - 2\Delta tc^2}{\Delta ts^2 - \Delta tc^2}\right) \times 1.34 \times 10^{10} \text{ psi} \quad (8)$$

The conversion factor included in the above equation accounts for the units of specific acoustic transit time being measured in µsec/ft instead of sec/cm, and elastic moduli being expressed in Ib/in.^{2} instead of dynes/cm^2.

Hence, $\frac{10^{12} \times 30.48 \times 14.22}{1000 \times 980} = 1.34 \times 10^{10}$

The shear modulus is the most important elastic parameter in comparing the strength of the different formations. A combined modulus of strength has been defined as:

$$\mathbf{K} = \mathbf{B} + \frac{4}{3}S \quad (9)$$

From equation 8 and 9, this can be expressed in well logging terms as:

$$K = \rho_b \left(\frac{3\Delta ts^2 - 4\Delta tc^2}{3\Delta ts^2 \times \Delta tc^2}\right) \times 1.34 \times 10^{10} \text{ psi} + \frac{4}{3}\left[\frac{\rho_b}{\Delta ts^2} \times 1.34 \times 10^{10} \text{ psi}\right]$$

$$K = \frac{\rho_b}{\Delta tc^2} \times 1.34 \times 10^{10} \text{ psi} \qquad (10)$$

This combined modulus compares favorably with known conditions of formation strength. Corrections to the log data for hydrocarbon effects are required before calculating the combined modulus values.

2.2 Pseudo Factor

The fraction of the total porosity occupied by disseminated shale is known as the q factor. The factor q is indicative of the producibility of reservoir rock (Dresser Atlas). Irrespective of the type of shale distribution, it is possible to derive a pseudo value based upon Densilog and Acoustilog porosity estimate (Dresser Atlas).

$$q = \frac{\emptyset_{AC} - \emptyset_D}{\emptyset_{AC}} \qquad (11)$$

Where q is the pseudo factor,

$\emptyset_{AC}$ and $\emptyset_D$ are the porosities estimated from Acoutilog and Densilog respectively.

3. Shear wave Velocity Prediction Method

There are several empirical equations (for example, Han et al., (1986) and Castagna et al., (1993)) to predict V_s from other logs. Most formations give transit times between $40\mu sec/ft$. and $140\mu sec/ft$., so these values are usually used as the scale. The reciprocal of velocity is the specific acoustic time, which is recorded on the Acoustilog in $\mu sec/ft$. The conversion equation between velocity and slowness is given as:

$$\Delta t = \frac{10^6}{V} \qquad (12)$$

(Δt is in microseconds per foot, and the velocity, V is in feet per second).

The knowledge of ρ_b and Δt_c is insufficient for computation of relative formation strength from all the parameters listed above. Measurement of Δt_s (which is the specific acoustic time for shear wave) is also very required. Since the log or the information about this is not available for the well, an alternative approach is to determine it indirectly by a correlation of another elastic constant, the Poisson's Ratio (P), to the pseudo q factor obtained from the Acoustilog and Densilog. All the other elastic constants can be computed after this.

Poisson's Ratio has been related to the shaliness index in the following manner (Dresser Atlas, 1982):

$$P = 0.125q + 0.27 \qquad (13)$$

After the Poisson Ratio has been computed using equation (13) above, equation (8) is used to estimate Δt_s, indirectly, and equation (5) through (7) is employed to compute the remaining elastic constants with the knowledge of Δt_s.

4. Results and Discussion

For this work, RT, Sonic log (DT), Deep resistivity log, Bulk Density (RHOB), Gamma Ray (GR) and Deep Laterolog (LLD) from Niger Delta Basin were made available (figure 1) from which the petrophysical parameters (Table 1) and Geo-mechanical Properties (Table 2) were derived.

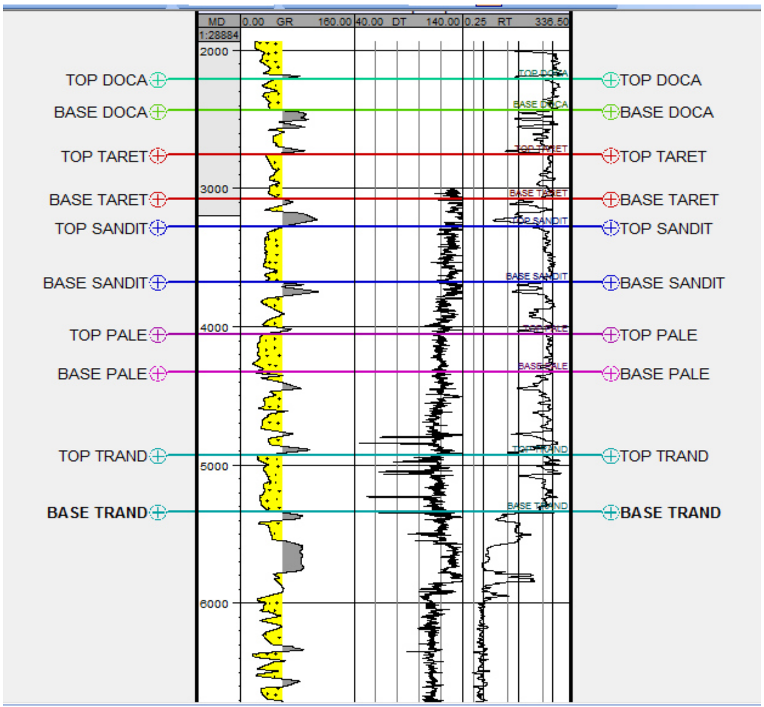

Figure 1: Well section for study

Table 1: Estimated Petrophysical Parameters

Sand unit	Top (m)	Bottom (m)	Thickness (m)	ρ_b (g/cm^3)	Δt_c (µsec/ft)	Δt_s (µsec/ft)	V_P (ft/sec)	V_S (ft/sec)	v_{sh} %	(φ_D) %	(φ_{AC}) %	q
DOCA	2207	2438	231	2.10	131	241	7630	4150	7.62	34	41	0.17
TARET	2755	3072	317	2.13	130	239	7690	4180	10.44	32	39	0.18
SANDIT	3277	3674	397	2.12	127	234	7870	4270	6.38	33	40	0.18
PALE	4088	4330	242	2.10	119	215	8400	4650	3.45	35	37	0.05
TRAND	4927	5336	409	2.15	113	201	8850	4980	3.67	32	33	0.03

Table 2: Estimated Geo-mechanical Parameters

Sand unit	Top (m)	Bottom (m)	Thickness (m)	P	S (psi) $\times 10^5$	K (psi) $\times 10^6$	B (psi) $\times 10^6$	Y (psi) $\times 10^6$	S/c (psi^2) $\times 10^{12}$
DOCA	2207	2438	231	0.29	4.84	1.64	1.00	1.25	0.51
TARET	2755	3072	317	0.29	5.0	1.69	1.02	1.29	0.53
SANDIT	3277	3674	397	0.29	5.19	1.76	1.07	1.34	0.57
PALE	4088	4330	242	0.28	6.09	1.99	1.18	1.56	0.80
TRAND	4927	5336	409	0.27	7.13	2.26	1.34	1.31	0.95

The result of this study shows that the velocity of Shear wave and that of Compressional wave give a linear relationship (figure 2).Also, the combined modulus of strength (K) and the shear modulus (S) to compressibility

(c) ratio (S/c) for the formation are relatively low. The average value of K derived lies between **1.64 $\times 10^6$psi and 2.26 $\times 10^6$psi** which is lower than the threshold value **of 3$\times 10^6$psi** indicating the minimum value at which fluids may be produced safely at any rate. Since experience in hydrocarbon-bearing tertiary sediments indicates that a value between $\mathbf{1.5 \times 10^6}$ ***psi*** and $\mathbf{3 \times 10^6}$ ***psi*** generally represents a condition in which the problem of sand production should not arise below a certain optimum flow rate, the value of K for this study lies within this range.
Sand control is necessary when S/c ratio is equal to or less than $\mathbf{0.7 \times 10^{12}}$ **psi^2.** The value of S/c ratio for the well ranges between $\mathbf{0.51 \times 10^{12}}$**psi^2** and $\mathbf{0.95 \times 10^{12}}$**psi^2** which indicates that the values tend towards the acceptable range down depth (figure 3).

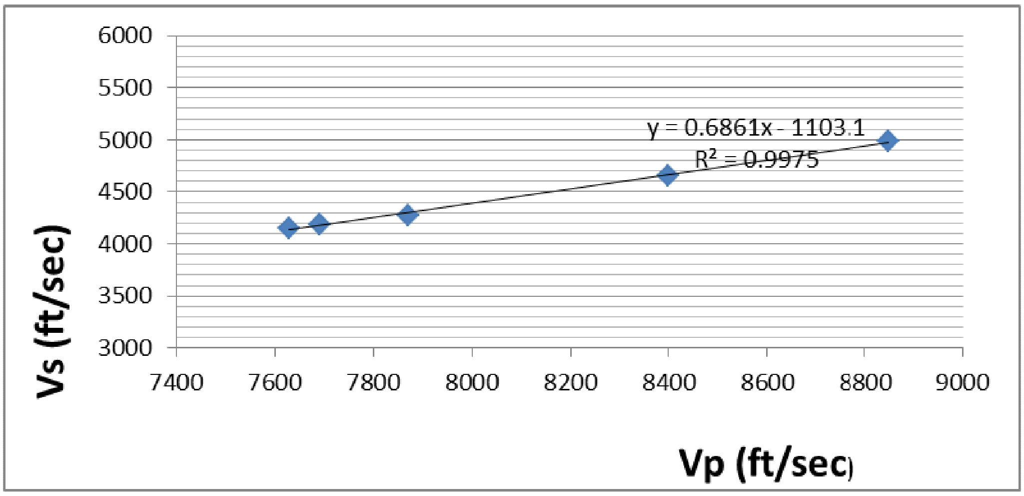

Figure 2: Linear relationship between Velocities of Shear wave and Compressional wave.

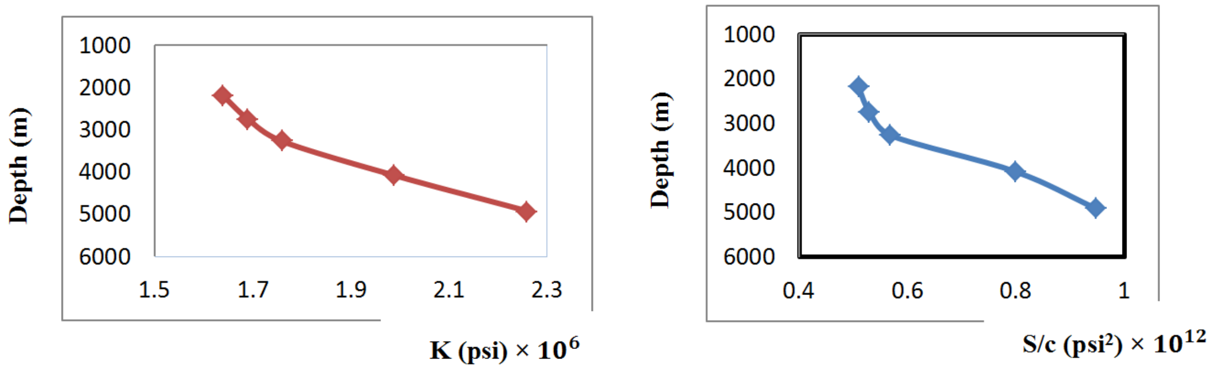

Figure 3: (a) Plot of K increasing with depth (b) Plot of S/c increasing with depth

5. Conclusion

This petrophysical study has investigated the use of well log data of acoustic properties for prediction of shear wave velocities. It has been clearly demonstrated that the S-wave velocity can be estimated from P-wave velocity, porosity and shale contents if the dipole sonic log is not available, and also, other rock mechanical properties can be estimated from the knowledge of all these. The combined modulus of strength as well as the Shear modulus to compressibility ratio can be predicted from the knowledge of these elastic moduli which in turn can help in the study of competency of the formation in terms of sand production. This information can reduce the risk involve in hydrocarbon exploration for the safety of the personnel and equipment, in particular minimizing the associated risk to the environment at large.

6. Acknowledgment

I would like to thank Exxon Mobil for the provision of valuable data and workstation for the execution of this research.

References

Castagna , J. P., etc., Relationships between compressional-wave and shear-wave velocity in clastic silicate rocks, Geophysics, Vol. 50, No.4 (April 1985), P.571-581.

Castagna, J. P., and Batzle, M. L., and Kan, T. K., 1993, Rock physics - The link between rock properties and AVO response, in Castagna, J. P., and Backus, M. M., Eds., Offset-dependent reflectivity-Theory and practice of AVO analysis: Soc. Expl. Geophys., 124-157.

Dresser Atlas (1982): Well logging and interpretation techniques. The course for home study,
Dresser Atlas Publication.

Han, D. and Nur, A., and Morgan, D., 1986, Effects of porosity and clay content on wave velocities in sandstones: Geophysics, 51, 2093-2107.

Han, De-Hua, Estimate Shear Velocity Based on Dry P-wave and Shear Modulus Relationship, SEG Int'l Exposition and 74th Annual Meeting, Dever Colorado, 10-15 Oct, 2004

Kithas, B.A., 1976, Lithology, gas detection, and rock properties from acoustic logging systems: Soc. Prof. Well Log Analysts 17th Annual Symp.

Krief, M., Garat ,J., Stellingwerf, J., and Venter, J.,1990, A petrophysical interpretation using the velocities of P and S waves (full waveform sonic): The Log Analyst, 31, 355-369.

Miller, S.L.M. and Stewart, R.R., 1990, Effects of lithology, porosity and shaliness on P- and S-wave velocities from sonic logs: Can. J. Expl. Geophys., 26, 94-103.

Nations, J.F., 1974, Lithology and porosity from acoustic shear and compressional wave transit time relationships: Soc. Prof. Well Log Analysts 15th Annual Symp.

Pickett, G.R., 1963, Acoustic character logs and their applications in formation evaluation: J. Petr.Tech., June, 659-667.

Wang, Z., 2000, The Gassmann equation revisited: Comparing laboratory data with Gassmann,s predictions: Geophysics, 56, 8- 23.