Adaptive Extended Kalman Filter for Orbit Estimation of GEO Satellites

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Abstract

The aim of this paper is to develop Adaptive Extended Kalman Filter (AEKF) algorithm for the precise orbit estimation of GEO satellites (viz., GSAT-10 – Geostationary satellite and IRNSS-1A – Geosynchronous satellite) using two-way CDMA range measurements data from different ranging stations located in India. It brings forward the effectiveness of AEKF algorithm over Extended Kalman Filter (EKF) algorithm. EKF algorithm is adapted by updating process noise covariance (Q), measure of uncertainty in state dynamics during the time interval between measurement updates and measurement noise covariance (R), function of measurement update based on measurement residual. This paper addresses the modeling of all errors in measurement domain and the computation of measurement residual using observed and modeled measurement ranges for all stations. The filter incorporates non-linear model for measurement update, non-linear dynamic model for time update and estimation is carried out at every second. This paper also elaborates the development of indigenous full force propagation model considering all the perturbations during orbit prediction period for GEO Satellites. Adaptation of EKF algorithm in precise orbit estimation is done primarily for making the algorithm more robust by countering the uncertainties in process and measurement noises, resolving the problem of manual tuning of the filter and also by keeping the error covariance (P) consistent with real performance. Adaptation of Q is implemented based on the error in system states with respect to estimated states while Adaptation of R is implemented based on the error in observed measurements with respect to measurements obtained from estimated state vectors (aposteriori measurement expectation). Analysis of the estimated results using the above proposed method is carried out by comparison of Station-wise range residues for both the methods (AEKF and EKF). Consistency of obtained orbit for GEO Satellites are validated using overlapping technique for both AEKF and EKF methods, orbit estimated from these methods are also validated by comparing with batch least squares method and filter behavior is continuously monitored during data gaps by observing error covariance(P) for both the methods.

Keywords: Kalman Filtering, Process Noise Covariance, Measurement Noise Covariance, Orbit Estimation, CDMA

1. Introduction

1.1 IRNSS 1A

Indian Regional Navigational Satellite System (IRNSS) envisages establishment of Indian Regional Navigational Satellite System using a combination of GEO and GSO Spacecrafts [1]. The IRNSS System provides navigation solution all time, all weather, anywhere within India and a region extending about 1500 km around India, expected to provide position accuracy better than 20m. IRNSS-1A is placed in GSO orbit with longitude crossing of 55° with RAAN 139° and inclination of 27°. IRNSS system consists of Space segment, Ground segment and User segment [1] as shown in Figure. 1.

1.2 GSAT 10

GSAT-10 is an Indian communication satellite placed in geostationary orbit at 83.0° East, from where it will provide communication services in India. GSAT-10 carries 30 transponders (12 Ku-band, 12 C-band and six Extended C-Band), which will provide vital augmentation to INSAT/GSAT transponder capacity. Bend-pipe transponder is used for broadcasting signal corrections through messages for improving the accuracy of GPS based position estimation (of better than seven metres), using GAGAN payload. The SBAS/GAGAN messages will be used by Airports Authority of India for civil aviation requirements.

1.3 Orbit Estimation

The Orbit determination of an artificial satellite is a non-linear problem of estimation of parameters by processing a set of information which will completely specify the satellite trajectory in space. The receivers compute the distance on the basis of time taken for the signal to travel between satellite and receiver, hence the psuedorange computed by the receiver consists of various errors and biases [2]. Hence, the estimation of orbit from the psuedorange data is largely dependent on the precise knowledge of time apart from other errors and biases. The major steps involved in the orbit estimation process are:

1. All satellite and receiver related delays like Transponder delay, Antenna-calibration delay and Cable delay are calibrated and removed [3].
2. Generation of propagated ephemeris from initial guess state vectors using full force Orbit propagation model [4].
3. Inclusion of range-rate error corrections, light-time delay corrections and ionospheric delay corrections [5].
4. Removal of tropospheric delay using meteorological parameters viz. pressure, humidity and temperature [6].
5. Computation of Range residues and partial derivatives for design matrix [7].

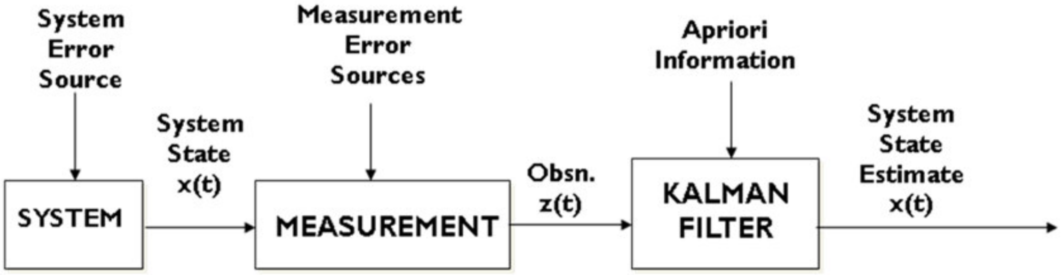
1.4 Prerequisites and Assumptions

- Precise Station Coordinates (< 5cm)
- Ground station reference receiver hardware delays are well calibrated & the uncertainties should be <1ns
- Satellite Antenna Phase centre offsets variations (<2mm)
- Initial guess state vectors from Batch estimation (<1km)
- Onboard hardware delay, uncertainties and variation (<1ns)
- Antenna to receiver cable delays are available to an accuracy of <1ns
- Noises are white and Gaussian in measurements
- S/C attitude profiles are known

2. Methodology

2.1 Kalman Filter Estimation (KF)

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of noisy measurements. The equations for KF fall into two groups: time update equations and measurement update equations [8]. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback—i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posterior estimate. The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. Indeed the final estimation algorithm resembles that of a predictor-corrector algorithm for solving numerical problem.



$$x_{j+1} = \Phi_j x_j + w_j \quad \text{Propagation state equation} \tag{1}$$

$$z_j = H_j x_j + v_j \quad \text{Measurement Model equation} \tag{2}$$

Where,

x State vector - Describes completely the dynamics of the system.
Φ State-transition matrix- to propagate states from one time to next
w Process noise with covariance Q - accounts for modeling errors also
z Measurement process
H Measurement model/sensor dynamics matrix
v Measurement noise with covariance R

The estimated parameters are

(x, y, z) ➢ Satellite Position
(v_x, v_y, v_z) ➢ Satellite Velocity

Denoted as: $X(t) = (x, y, z, V_x, V_y, V_z, C_0, C_1)$ (3)

The residue, $\Delta Z = \rho_T - \rho_C = H \Delta X + w$ (4)

ρ_C = Iono-free smooth range

ρ_T = Geometric range including all measurement errors

w = Measurements Noise

H = Partial derivative of measurement with respected to the parameters and

$$H = \begin{bmatrix} \frac{\partial \rho}{\partial X_S} & \frac{\partial \rho}{\partial Y_S} & \frac{\partial \rho}{\partial Z_S} & \frac{\partial \rho}{\partial V_X} & \frac{\partial \rho}{\partial V_Y} & \frac{\partial \rho}{\partial V_Z} & \frac{\partial \rho}{\partial C_0} & \frac{\partial \rho}{\partial C_1} \end{bmatrix} \tag{5}$$

Predicted estimates and covariances are found using time update equations and corrected estimates and covariances are found using measurement update equations.

Time Update:

$$\tilde{X}(t+h) = \Phi(t+h,t)\bar{X}(t) + v(t) \tag{6}$$

$$\tilde{P}(t+h) = \Phi(t+h,t)\bar{P}(t)\Phi^T(t+h,t) + Q \tag{7}$$

Kalman Gain:

$$K = \tilde{P}(t+h)H^T\left[H\tilde{P}(t+h)H^T + R\right]^{-1} \tag{8}$$

Measurement Update:

$$\bar{X}(t+h) = \tilde{X}(t+h) + K\Delta Z \tag{9}$$

$$\bar{P}(t+h) = \left[I - KH\right]\tilde{P}(t+h) \tag{10}$$

Where,

$\tilde{P}(t+h)$ = Predicted covariance Matrix at $t+h$

$\bar{P}(t)$ = Measurement updated Covariance Matrix t

Q = Process noise covariance

R = Measurement noise covariance

$$\Phi\left(t+h,t\right) = \frac{\left(X + \Delta X\right)(t+h) - X(t+h)}{\Delta X} \tag{11}$$

Where, $\Phi\left(t+h,t\right)$ is state transaction matrix from t to $t+h$

2.2 Adaptive Kalman Filter Estimation (AEKF)

Adaptive Kalman filtering [9] involves the adaptation of covariance matrices Q and R –

2.2.1 Process Noise Adaptation

Adaptation of Q is based on the error in system states with respect to estimated states given by,

$$Q^* = \Delta x_k \Delta x_k^T + P_k^- - P_k^+ - Q_k^- \tag{12}$$

$$\Delta x_k = x_k^+ - x_k^- \tag{13}$$

where,

x_k^+ = aposteriori state estimate

x_k^- = apriori state estimate

P_k^+ = aposteriori state covariance estimate

P_k^- = a priori state covariance estimate

Q_k^- = Current expected process noise covariance, and

$$Q_k^+ = Q_k^- + \frac{1}{L_Q}(Q^* - Q_k^-) \tag{14}$$

Where, L_Q is process noise window size.

2.2.2 Measurement Noise Adaptation

Adaptation of R is based on the error in measurements with respect to measurements obtained from estimated state vectors (a-posteriori measurement expectation).

$$R^* = \Delta y \Delta y^T - H_k P_k^+ H_k^T \tag{15}$$

$$\Delta y = y_k - y_k^+ \tag{16}$$

Where, H_k is the observation matrix at time k. H becomes Jacobian of nonlinear measurement function for non-linear systems. Δy is a measure of the covariance of the measurement residual. y_k is the actual set of measurements taken this time-step. y_k^+ is the a posteriori measurement expectation.

$$y_k^+ = h_k (x_k^+) \tag{17}$$

$$R_k^+ = R_k^- + \frac{1}{L_Q}(R^* - R_k^-) \tag{18}$$

Where, L_R is measurement noise window size.

2.3 Propagation Model

The equation of perturbation forces acting on the satellite is

$$a = \left(a_x, a_y, a_z\right) \tag{19}$$

$$a = \frac{\mu r}{r^3} + a_{asp} + a_{sun} + a_{moon} + a_{srp} + a_{tide} + a_{planet} + a_{rel} \tag{20}$$

Acceleration due to Earth's Oblateness, Sun, Moon's attraction, solar radiation pressure, solid and ocean tides, other planets attractions and Relativistic effect respectively. In the time update of the filter state variables from previous epoch to current epoch using Runge Kutta 4th order integration method,

$$X(t) = \left(x, y, z, v_x, v_y, v_z\right)^T \tag{21}$$

(22)

Where,

$$X(t+h) = X(t) + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \tag{23}$$

$$\dot{X}(t) = \left(v_x, v_y, v_z, a_x, a_y, a_z\right)^T \tag{24}$$

$$k_1 = \dot{X}(t)$$

$$k_2 = \dot{X}(t + \frac{h}{2}) + \frac{k_1}{2} \tag{25}$$

$$k_3 = \dot{X}(t + \frac{h}{2}) + \frac{k_2}{2} \tag{26}$$

$$k_4 = \dot{X}(t+h) + \frac{k_3}{2} \tag{27}$$

2.4 Range Residue Computation

The governing range equation for each line of sight measurement is given in

$$\rho_{isc} = \rho_g + (\delta t_r - \delta t_s)C + \Delta\rho_{tro} + \Delta\rho_{sag} + \Delta\rho_{rel} + \Delta\rho_{ants} + \Delta\rho_{antr} \tag{28}$$

ρ_{isc} = Iono free smooth range

ρ_g = Geometric distance between the satellite and corresponding station

$$\rho_g = \sqrt{\left(X_S - X_R\right)^2 + \left(Y_S - Y_R\right)^2 + \left(Z_S - Z_R\right)^2} \tag{29}$$

$\delta t_r, \delta t_s$ = Station and satellite clock offset from the GPS time respectively.

$\Delta\rho_{tro}$ = Tropospheric delay using Saastamoinen model

$\Delta\rho_{sag}$ = Sagnac effect due to the satellite position at the transmission time and

$$\Delta\rho_{sag} = \frac{(X_S - X_R) \times V_S}{C} \tag{30}$$

$\Delta\rho_{rel}$ = Relative effect due to the motion of the satellite and station and

$$\Delta\rho_{rel} = \frac{2 \times X_S \times V_S}{C} \tag{31}$$

$\Delta\rho_{ants}, \Delta\rho_{antr}$ = Antenna phase centre offset of the satellite and station respectively.

3. Inputs

3.1 GSAT10

Measurements from 4 Two-way ranging CDMA stations – Bhopal, Hassan, Jodhpur and Shillong (1sec interval) for 5 days from 05/07/2013 to 09/07/2013 has been used as input for orbit estimation. Data availability is continuous and data gaps are found at around 10 hrs and around 90 hrs as shown in Figure 2.

3.2 IRNSS 1A

Measurements from 4 Two-way ranging CDMA stations – Bhopal, Hassan, Jodhpur and Shillong (1sec interval) from 01/10/2013 to 24/11/2013 has been used as input for orbit estimation. Table 1 shows the percentage of valid data available for 2 months from 01/10/2013 to 24/11/2013.

4. Results and Analysis

Analysis of the estimated results is carried out by comparison of Station-wise range residues for both the methods (AEKF and EKF). Consistency of obtained orbit for GSAT-10 is validated by comparing the estimated orbit with orbit of the satellite broadcasted through SBAS message, whereas for IRNSS-1A Satellite is validated using overlapping technique for both AEKF and EKF methods, orbit estimated from these methods are also validated by comparing with batch least squares method and filter behavior is continuously monitored during data gaps by observing error covariance (P) for both the methods.

4.1Range Residue Comparison

Figure 3 and 4 shows the comparison of range residues between EKF and AEKF methods for GSAT -10 Satellite for 5 days from 05/07/2013 to 09/07/2013 for 3 CDMA Stations except Shillong station data, which is not used for estimation because of poor measurements quality. Figure 5 and 6 shows the comparison of range residues between EKF and AEKF methods for IRNSS-1A Satellite for 23 days from 01/11/2013 to 23/11/2013 using 4 CDMA Stations. It is evident from the above plots range residue level reduces when AEKF algorithm is used for estimation than EKF algorithm. Residue bounds for all stations are within ~2m in case of EKF and within ~1m in case of AEKF.

4.2 Process and Measurement Noise Adaptation

Figure 7 and 8 shows the process and measurement noise covariance using AEKF method for GSAT -10 Satellite for 5 days from 05/07/2013 to 09/07/2013. Adaptation of process and measurement noises can be observed from the above plots. Figure 9 and 10 shows the process and measurement noise covariance using AEKF method for IRNSS-1A Satellite for 25 days from 01/11/2013 to 25/11/2013. Adaptation of process and measurement noises can be observed from the below plots.

4.3 Orbit Validation

Figure 11 and 12 shows the difference in orbital estimates between EKF and GAGAN - Space Based Augmentation System (SBAS) Broadcast ephemeris and AEKF and SBAS Broadcast for GSAT -10 Satellite for 5 days from 05/07/2013 to 09/07/2013. It is evident from the above plots that the estimate differences are less in AEKF (RSS in position < 30 m) compared to EKF method (RSS in position< 60m). It is also evident from the above plots that estimated orbit is holding well even after data gaps in AEKF based estimation. Figure 13, 14 shows the difference in orbital estimates between EKF and Batch Least Squares and AEKF and Batch Least Squares for IRNSS-1A Satellite using one-way range measurements for 10 days from 02/11/2013 to 11/11/2013. It is evident from the above plots that the maximum RSS difference in position is found well within 30 m which evaluates the correctness of orbit estimated and also from the above plots it is found that consistency of orbit is better in AEKF method over EKF method. Figure 15 shows the difference in orbital estimates between AEKF method and Batch Least Squares method using LASER range measurements for 7 days from 03/07/2014 to 09/07/2014 in X, Y, Z directions and also RSS in position error. It is evident from the above plots that the AEKF estimated orbit using two-way range measurements is well aligned with more precise LASER range measurements based orbit within 10m.

5. Conclusion

This paper compares and analyses the orbital results of GSAT-10 and IRNSS-1A Satellites using both EKF and AEKF methods of estimation with two-way CDMA range measurements as input. It is evident from the results presented in this paper, that AEKF algorithm works better than EKF algorithm even in the presence of more data gaps and also when the measurements are of poor quality and noisy. The performance of the filter could have been better and the orbit estimation would have been more consistent, if the measurements gap reduces, if the measurements time stamping have been done more precise and also if full force propagation modeling is made more accurate with the computation of more precise accelerations. After the launch of more satellites into IRNSS Space Segment, range modeling can be further improved and more accurate results can be achieved.

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A S Ganeshan holds a Master's degree in Aerospace Engineering from IISC, Bangalore. He has been one of the pioneers in the satellite based navigation. Presently he is the Director of Space Navigation Group and Program Director, Satellite Navigation Program. He has several papers in national/international symposia and journals. He is a recipient of IETE award, ISRO award for performance and various other recognitions. Besides navigation, space debris model and research, re-entry dynamics and interplanetary missions are his other areas of interest.

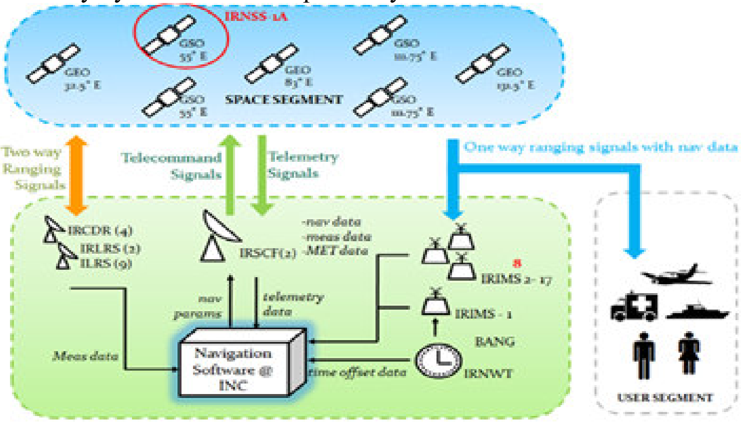


Figure 1. IRNSS Architecture – Space, Ground and User segments

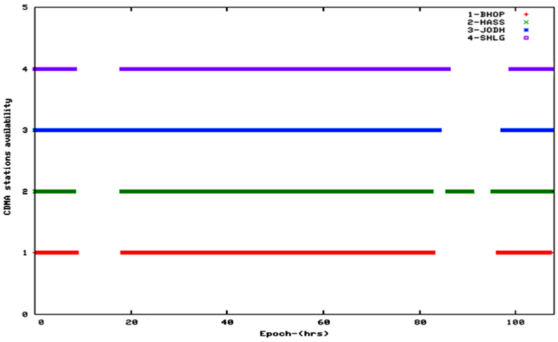


Figure 1. Data availability of CDMA Stations for GSAT-10 from 05/07/2013 to 09/07/2013

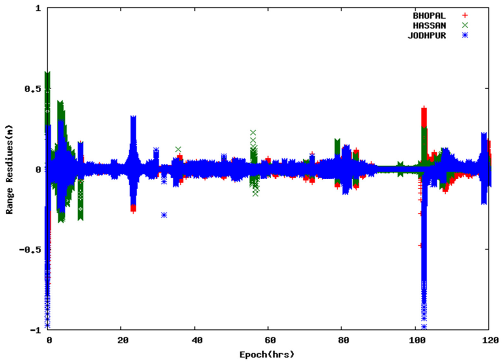


Figure 2. Range Residues for EKF - GSAT10

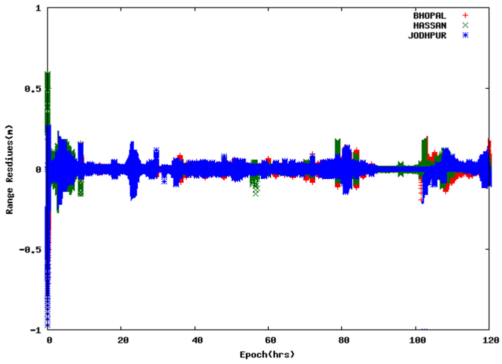
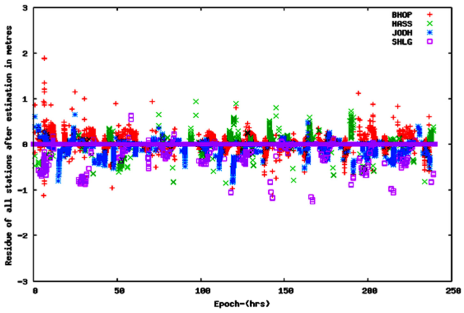


Figure 3. Range Residues for AEKF - GSAT10



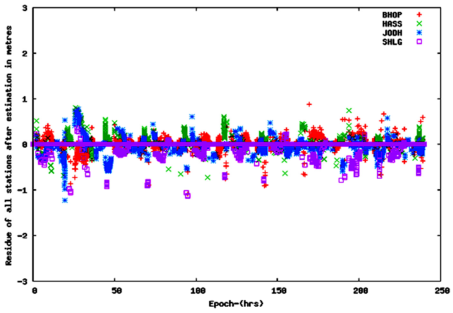


Figure 4. Range Residues for EKF-IRNSS 1A

Figure 5. Range Residues for AEKF – IRNSS1A

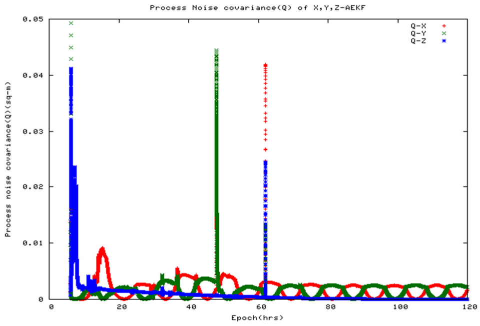


Figure 7. Process Noise Adaptation using AEKF for GSAT10

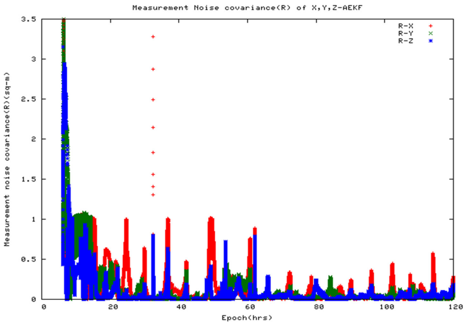
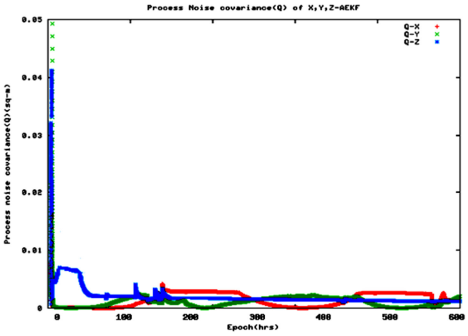


Figure 8. Measurement Noise Adaptation using AEKF for GSAT10



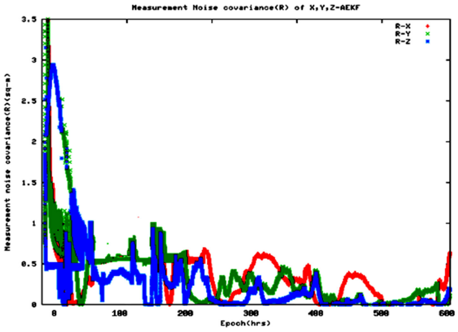


Figure 9. Process noise Adaptation using AEKF for IRNSS-1A

Figure10. Measurement noise Adaptation using AEKF for IRNSS-1A

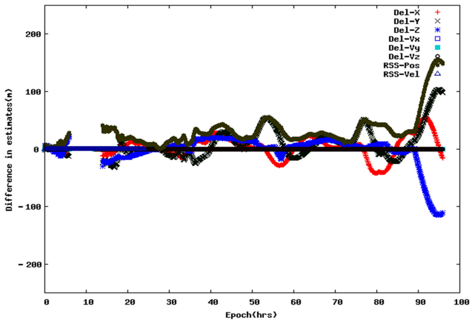


Figure 11. Difference in orbit estimates between EKF and SBAS Broadcast for GSAT-10

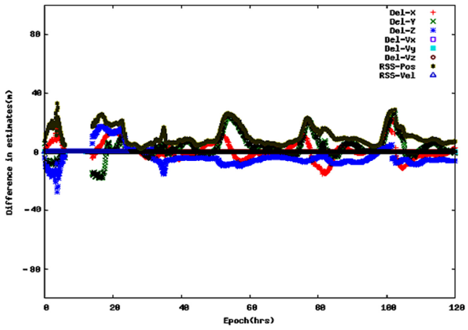


Figure 12. Difference in orbit estimates between AEKF and SBAS Broadcast for GSAT-10

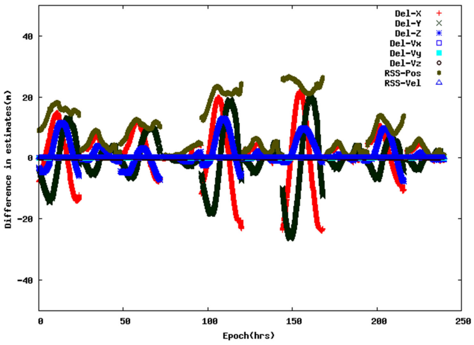


Figure 13. Difference in orbit estimates between EKF and Batch for IRNSS-1A

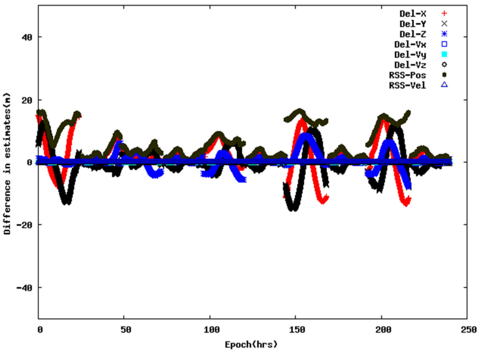


Figure 14. Difference in orbit estimates between AEKF and Batch for IRNSS-1A

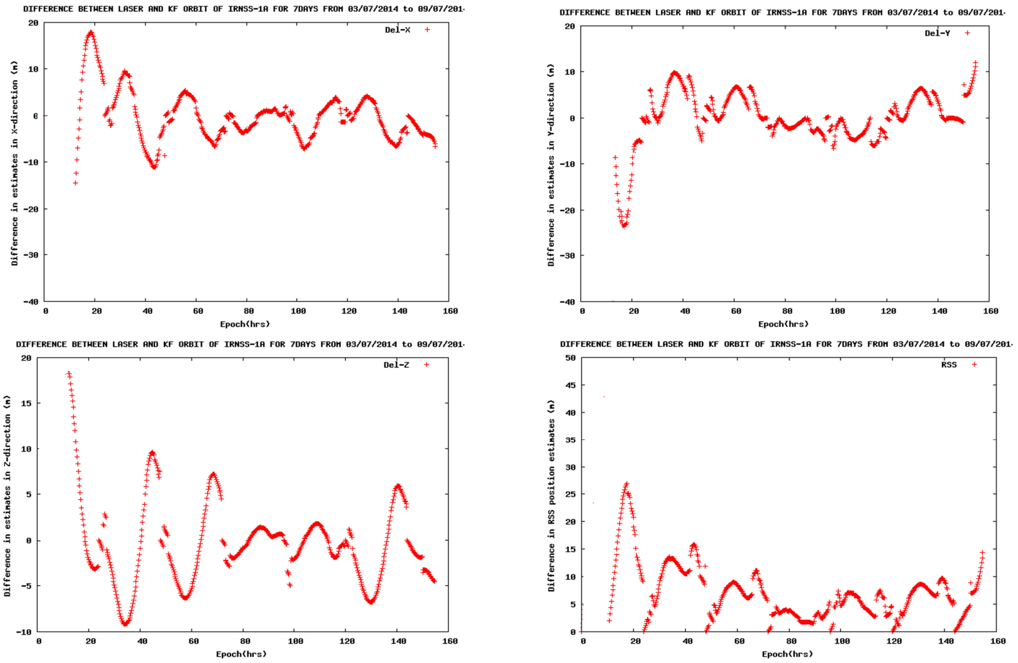


Figure 15. Difference in orbit estimates between AEKF and Batch using LASER Measurements for IRNSS-1A

Table 1. Summary of Station-wise range data statistics from 01/10/2013 to 24/11/2013

S.No	Station Name	Data Availability	Expected Data (20hrs)	Valid Data Available
1	**Bhopal**	**All days ~ (16-18hrs) (66-75 %)**	**83%**	**56-64%**
2	**Hassan**	**All days ~ (16-18hrs) (66-75 %)**	**83%**	**48-57%**
3	**Jodhpur**	**All days ~ (16-18hrs) (66-75 %)**	**83%**	**48-57%**
4	**Shillong**	**All days ~ (16-18hrs) (66-75 %)**	**83%**	**60-72%**