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Simultaneous Optimization of Vertical Electrical Sounding and Magnetotelluric Data Using a Genetic Algorithm

Maxwell Adarkwa Quaquah* Elvis Boateng
Energy Systems Engineering Department, Koforidua Technical University, Koforidua, Ghana
* E-mail of the corresponding author: maxwell.quaquah@ktu.edu.gh

Abstract
There are many research studies that highlight the benefit of combining more than one geophysical method to delineate the subsurface. However, only a small number of studies discuss the use of genetic algorithm to simultaneously invert magnetotelluric (MT) and vertical electrical sounding (VES) data. The purpose of this research is to evaluate the efficacy of using the genetic algorithm technique to simultaneously optimize and invert MT and VES data. For this study, a GA inversion code was written in MATLAB, consisting of two parts: a forward program and an inverse program. The inverse program has an inherent forward model, which it uses to produce an apparent resistivity (from the corresponding input model parameter). The goal of GA inversion is to generate the best model parameter (that is, thickness and resistivity) whose apparent resistivity curve matches the field data's apparent resistivity curve or the synthetic data's apparent resistivity curve. The forward and GA inversion programs were tested on synthetic and real MT and VES datasets. A total of 34 MT and 15 VES soundings were acquired from different geothermal fields in Tuscany, Italy. The theoretical apparent resistivity values of the model parameter are extremely similar to the measured (experimental) apparent resistivity values, according to analysis of the inversion data. This is indicated by the low root mean square error. Results from the simultaneous inversion of the synthetic MT and VES models revealed negligible (less than 0.4 percent) errors in resistivity and thickness for each layer. In both cases, the error recorded by the application on field VES data and field MT data was less than 5 percent and less than 19 percent, respectively. This shows that the GA inverse technique produces accurate estimations of subsurface characteristics.
Keywords: Simultaneous Inversion, Genetic Algorithm, Vertical Electrical Sounding, Magnetotelluric, Optimization method
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1.0 Introduction

Electrical resistivity and electromagnetic geophysical prospecting methods are among the most commonly used geophysical methods in the exploration industry. They are used for detecting geothermal and hydrocarbon reservoirs. An example of the electrical resistivity method is vertical electrical sounding (VES), and magnetotellurics (MT) is an example of the electromagnetic geophysical method. These methods give data that is used to image the subsurface resistivity (Snieder & Trampert, 1999).

In the case of horizontal layering, VES can give information about the subsurface, such as its lithology, fluid content, and other geological properties. However, it is limited in its ability to determine lateral resistivity variations in shallow subsurface, at a more or less fixed depth of investigation. The equipotential method usually doesn't work because it's hard to figure out how to analyze the diagnostic features. This method involves mapping the equipotential lines on the surface of the earth when current is put into the ground through two-point electrodes.

This shortcoming can be addressed when a combination of MT and VES is employed. A clear delineation of subsurface anomalies is possible by determining both lateral and vertical features. It is often essential to combine two or more geophysical methods. This helps to appropriately delineate varying geological strata. When both resistivity and electromagnetic geophysical methods are used on the same field, it is easier to make accurate models of the subsurface. When only one geophysical method is used, it can be hard to tell electrical anomalies apart based on their properties. Thus, through a simultaneous inversion, ambiguities in the interpretation process are reduced, and a good interpretation of the subsurface can be made. Also, since MT and VES result in finding the same physical parameters of the subsurface (apparent resistivity and thickness) through different procedures of data processing, interpretations are faster and simpler. Based on these, a simultaneous inversion of MT and VES data is feasible.

Also, a simultaneous inversion of MT and resistivity is important because MT uses telluric currents (no need for current sources or long cables, greater depths of investigation, etc.), the magnetotelluric method of prospecting resolves the effects of individual layers better than traditional resistivity methods (like VES), and it's a great tool for looking into deeper depths while resistivity mostly deals with shallow depths in basins.

Shallow geothermal reservoirs have different properties compared to deep ones. Thus, for subsurface geothermal reservoirs, some physical properties of the earth are of immense essence to observe, like the effects of reservoir heterogeneities (fluid content, salinity, and lithology), temperature, and frequency effects on electrical conductivity. Shallow depth examination can be done with the resistivity geophysical method, while deep depth can be delineated with the magnetotelluric method.

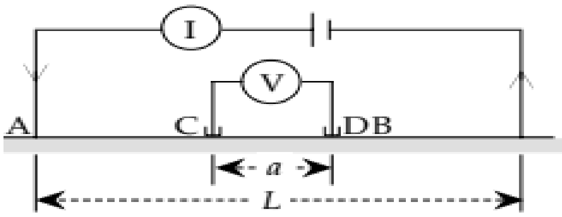
The genetic algorithm (GA) has proven to be a very powerful optimization approach in the field of geophysics as it produces better data-fitting models from real data. Hence, the GA method has found many applications in geophysical problems such as seismic ray tracing, earthquake location, non-linear data fitting, and seismic topography. These are well documented. Hamimu et al. (2015) used a genetic algorithm (GA) optimization technique encoded using a binary string of MATLAB source code for VES forward modeling and its inversion to find the best correlation between the synthetic data's experimental apparent resistivity and theoretical (predicted) apparent resistivity curves, recording very low errors. LI et al. (2008) applied the GA optimization technique to invert 1-D CSAMT data with minimum structural constraint by inverting both apparent resistivity and phase data. The GA inversion codes were tested with synthetic data and field data. A root-mean-square error (misfit) of 1% was recorded when the GA inversion was performed on the 1D CSAMT field data. This indicates that the GA inverted resistivity models fit the true models well. Pavlova and Reid (2010) applied genetic inversion of 3D seismic to generate porosity variations in a formation. The real data were from porosity logs, which were smoothed to match the frequency of seismic data. Minimal errors were recorded at the wells between the smoothed and inverted porosities by using genetic algorithm inversion.

1.1 Theory of VES

Theoretically, vertical electrical sounding (VES) determines electrical resistivity by direct current (DC). In the VES method, which is a surface geophysical technique, an electrical current is injected into the ground through two electrodes, and surface voltages are measured to determine the direction and quantity of current flow in the subsurface. The data is used to image the subsurface resistivity. To increase the depth of investigation, the spacing (distance) between the two electrodes is increased (Kearey et al. 2013).

VES data is commonly used in a variety of geophysical investigations, including groundwater exploration, mineral exploration, and environmental studies. In groundwater exploration, VES data can be used to identify the depth and thickness of aquifers and their hydraulic properties, which can help in the planning and management of groundwater resources. In mineral exploration, VES data can help to delineate the geological structures and ore deposits in the subsurface, which can guide the drilling and mining operations. In environmental studies, VES data can help assess the contamination and soil properties of a site.

VES data can be analyzed using various inversion techniques to derive the subsurface model from the resistivity measurements. These techniques involve solving the inverse problem by calculating the subsurface resistivity distribution from the measured resistivity data. The accuracy and resolution of the subsurface model depend on the quality of the data and the inversion technique used. Apparent resistivity, a weighted average of the earth materials' resistance to current flow, is calculated using actual current and voltage measurements. This apparent resistivity can be referred to as measured, real, field, observed, or experimental apparent resistivity. For this study, the field data for VES was acquired using Schlumberger configuration (Figure 1), and the apparent resistivity (ρ_a.) was computed using the following Equation 1 (Lowrie, 2007):



(1)

Where ρ_a represents the apparent resistivity, The voltage between the two electrodes of the circuit is denoted by V. The two electrodes inject a direct current, denoted by I, into the ground. The current electrodes are separated by a distance of L, while the potential electrodes are separated by a distance of a .

1.2 Theory of MT

Magnetotelluric (MT) data is a geophysical method used to study the electrical resistivity of the subsurface. It involves measuring the natural electromagnetic fields, which are generated by the Earth's ionosphere and magnetosphere in response to the changing magnetic field of the Earth. The resistivity of the subsurface materials can be calculated from the measured variations in the electromagnetic fields. MT data can tell us about the lithology, fluid content, and mineralization of the subsurface, among other things (Kearey et al., 2013).

MT data is commonly used in geophysical investigations, including mineral exploration, groundwater mapping, and crustal imaging. In mineral exploration, MT data can help identify potential mineral deposits by mapping the geological structures and identifying conductive zones. In groundwater mapping, MT data can help delineate the aquifers and estimate their hydraulic properties. In crustal imaging, MT data can help study the deep crustal structure and understand the tectonic processes.

MT data can be processed and analyzed using various inversion techniques to derive the subsurface model from the resistivity measurements. These techniques involve solving the inverse problem, by calculating the subsurface resistivity distribution from the measured electromagnetic data. The accuracy and resolution of the subsurface model depend on the quality of the data and the inversion technique used (Jones, 2012). The equation for calculating the apparent resistivity for MT data is shown below (Equation 2).

$$\rho = \frac{\mu_0 |E_y|^2}{\omega |B_x|^2} \tag{2}$$

Where : ρ = apparent resistivity, μ_o = permeability constant, ω = angular frequency
E_y = electric field along y – axis, B_x = magnetic field along x − axis.

1.3 Forward modelling

MT and VES geophysical methods measure apparent resistivity data on the field. These measured (field) data can be determined with the aid of a mathematical equation and an input model (such as resistivity and thickness for VES and MT). Hence, finding the apparent resistivity (output) data with the aid of model parameters describes a forward modeling process (Figure 2).

The forward modeling process for MT and VES methods will require model parameters of resistivity and thickness to generate apparent resistivity curves.

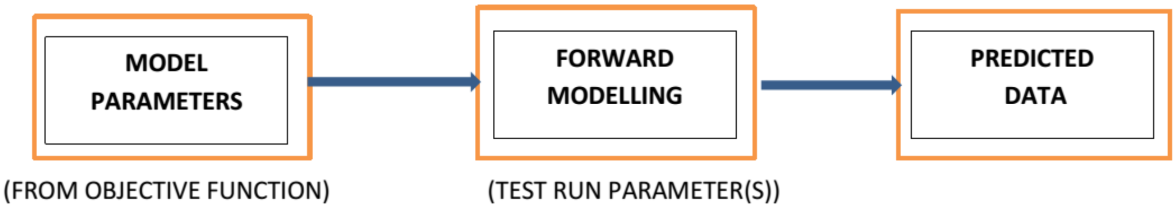


Figure 2 - Block diagram for forward modeling

So, forward modeling is a computational technique used in geophysics to simulate the response of subsurface geological structures to various physical measurements. It involves creating a mathematical model that represents the physical properties of the Earth's subsurface, such as its electrical conductivity, magnetic susceptibility, seismic velocity, or gravity distribution.

Forward modeling of VES (Vertical Electrical Sounding) data usually involves the following steps: first, defining the geometry and properties of the subsurface model. This involves creating a layered model that represents the different geological units or formations present in the subsurface. Each layer is assigned a resistivity value and layer thickness based on available geological information or assumptions. The second step is to use appropriate mathematical equations, such as the Wenner, Schlumberger, or Dipole-Dipole equations, to calculate the apparent resistivity at each electrode spacing for the given subsurface model. Noise or measurement errors are introduced into the calculated apparent resistivity values to simulate the realistic variability that can occur in field measurements. This helps make the forward modeling results more representative of actual survey conditions.

In this research, the inverse filter developed by Ghosh (Ghosh, 1971b) was implemented in the forward modeling software to quickly compute apparent resistivity curves. This method is able to compute apparent resistivity curves for known layer parameters such as thickness and resistivity. Ghosh's method is based on the application of a linear filter through a resistivity transform function, known as the kernel function (Koefoed, 1979). Using Ghosh's inverse method involves two major steps. The first step is to determine the resistivity transform function, T(m), for an N-layer subsurface system using Pekeris recurrence relations (Equation 3).

$$T_i = \frac{T_{i+1} + \rho_i \tanh(mh_i)}{1 + T_{i+1}(\tanh(mh_i)/\rho_i)} \tag{3}$$

Where T_i represents $T_{(m)}$ at the surface of the Earth, ρ_i represents the resistivity of the i^{th} layer, h_i represents the thickness of the i^{th} layer, and T_N represents ρ_N for the deepest layer.

The second stage involves the convolution of sampled resistivity transfer values with Ghosh's inverse filter coefficients, given by Equation 4.

$$\rho_{ak} = \sum_j a_j T_{k-j} \ ; \quad k = 0, 1, 2, 3, \dots . N \tag{4}$$

Where ρ_{ak} is the apparent resistivity transform at point k, k is the sample point where convolution is carried out, a_j is the filter coefficient. These equations are used to solve the forward modeling problem.

It's worth noting that the forward model is just a series of test runs to help generate the theoretical apparent resistivity curve from a known or assumed model parameter. This theoretical apparent resistivity curve would have been generated due to certain model parameters (assumed by the genetic algorithm) that are supplied to the forward model. The objective function statistically estimates the error between the measured (field) apparent resistivity and the theoretical apparent resistivity from the forward model.

1.4 Inversion

Apparent resistivity data are measured on the field through geophysical MT and VES geophysical methods. The measured apparent resistivity represents unknown earth models. The process of finding the earth model (thickness and resistivity) that describes the measured apparent resistivity data is termed "inversion".

All inversion models have a forward model inherent in them. Any assumed model (resistivity and thickness) can be used to generate an apparent resistivity curve by using the mathematical equation of forward models. This curve generated from the inversion process is compared to the apparent resistivity curve, which can be generated from the field data. A statistical function called the root-mean-square (rms) value helps to estimate the error between the measured apparent resistivity curve and that of the inversion's "apparent resistivity curve." If a low rms value is recorded, then the inversion's "apparent resistivity curve" can represent the measured field data. Thus, the model (resistivity and thickness) that generated the inversion's "apparent resistivity curve" is representative of the subsurface formation's layers.

The process of predicting models (resistivity and thickness) that can best describe the subsurface can be enhanced through optimization techniques such as genetic algorithms, simulated annealing, swarm optimization, neighborhood algorithms and particle swarm optimization. These are all stochastic data optimization techniques (Sen & Stoffa, 2013).

1.4.1 GA inversion technique

A genetic algorithm can be described as a computer program that mimics the process of evolution in nature. It applies the principle of natural selection of the best-fit individual from an initial population set to migrate to the next generation (of population) through genetic factors of crossover, mutation, inheritance, and selection (Figure 3).

1.4.1.1 Operators and Parameter Setting of Genetic Algorithm

GA operates through the reproduction of "genetic operators" (parameters); thus, it conducts a neighborhood search to obtain a local or global minimum. This step involves selecting models for mating based on their fitness (objective function). A rank selection scheme is used to assign a probability to each model, and a new population is randomly extracted with computed probabilities. The genetic section of the inversion algorithm is carried out by the genetic operators, namely selection, crossover, and mutation (Klinger et al., 2008). Hence, better models with a lower misfit when compared to the measured field data have a higher chance of being included in the next generation and contributing to the reproduction of new models (offspring). Thus, in each generation, the best models are identified, resulting in an optimized final model parameter.

1.4.1.2 Objective function

It is a mathematical description of a linear programming problem in which the objective is to maximize or minimize a particular numerical value. The objective function describes the problem's optimization goal and how much each parameter (variable) contributes to it (Imperial.ac.uk, Department of Computing, 1996).

The root-mean-square error (RMS) serves as the objective function to be reduced, and it is represented as:

$$\sigma_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - Z_i)^2} \tag{5}$$

Where; σ = Objective function, N = Total number of field data, X_i = Measured i^{th} real data

Z_i= The i^{th} predicted model

For the individual model parameters, we defined an objective function (σ), that was the average of root-means-squares of the differences between the real group (apparent resistivity [ohm-m] and phase [m]) and the calculated group apparent resistivity (apparent resistivity [ohm-m] and phase [rad]) .

The optimization takes place within a constrain referred to as the search limit (space), consisting of upper and lower limits of the model parameters. These are the maximum resistivity and thickness (ρ_{max}, h_{max}) and the minimum resistivity and thickness (ρ_{min} , h_{min}). These model parameters are vector values.

In this work, GA optimization was used to retrieve the true resistivity and thickness of each layer model.

These parameters were encoded as the gene type in the GA to be used for the genetic operation. For the synthetic earth model, a four-layer formation model was assumed, which consists of seven model parameters, namely, four (4) apparent resistivities and three (3) thicknesses. These were defined in the search area. An 8-bit binary string of 28 was used to encode each parameter in the search area; this allowed for the transfer of binary information between the boundary parameters and any parameter of interest in the search region. To facilitate the genetic procedures, a "chromosome" was created by concatenating the binary strings representing all of the parameters in a subsurface structural model. Each chromosome was used individually to get the best possible answer, which represents a model of resistivity and thickness (Stoffa and Sen, 1991).

1.4.1.3 Crossover

This entails swapping of the substrings of two individuals in order to produce a new offspring, thereby varying the programming of a chromosome from one generation to the next (MathsIsFun, 2016). This approach is likely to produce even better individuals by mixing good individuals, which are put into the next generation of the population (Imperial.ac.uk, Department of Computing, 1996). For our work we varied the cross-over fraction to obtain optimized model parameters.

1.4.1.4 Mutation

It provides GA-based inversion with a mechanism to avoid being trapped in local optima convergence, so the searching process goes to other areas of the search space. That's why GA has the distinct characteristic of always providing global (error minima) optimization for every problem. To achieve this objective, which is global convergence, mutation has to diversify the generation created by randomly changing the gene in each chromosome; hence, it adds new characteristics that diversify the chromosomes into the next generation, resulting in identifying the best gene, which at the last generation gives the least RMS recorded for all the population runs. In this work, a constant mutation factor of 10% was maintained.

1.4.1.5 Stopping condition

GA optimization is known to be an iterative process. The iteration can end whenever the repetition process reaches a predetermined number of generations or by timing the iterations so as not to exceed a specified time (seconds). Other conditions that could be employed to terminate the iteration are when all the models (individuals) within a population have the same root-mean-square (fitness) value or when the root-mean-square is below a specified value.

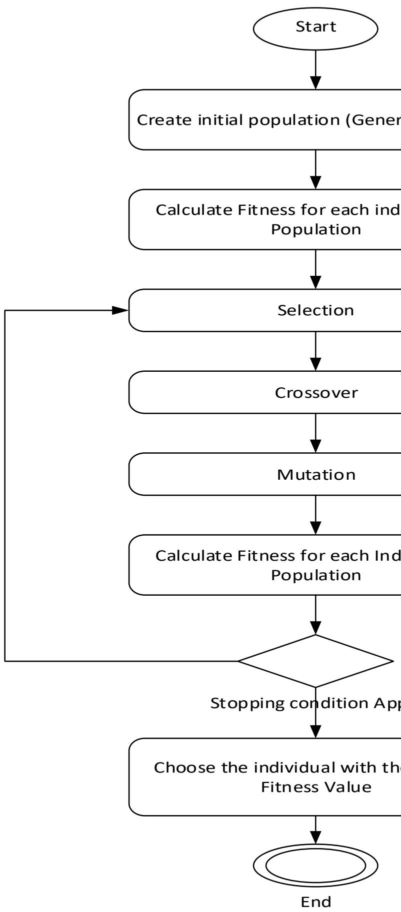


Figure 3: Flow diagram of Genetic Algorithm optimization [Source: Wirsansky, 2020]

2.0 Methodology

2.1 Simulation Work Flow

The simulation work for this paper consisted of four major stages: the main genetic algorithm codes (MAIN), the modeling of extracted synthetic and field data (MODEL), and the objective function, which is used to verify the validity of each predicted model's retrieved true resistivity and thickness (of each layer) in relation to the measured field data. The final stage is the depth plot, which provides a subsurface view of the optimized true resistivity and thickness of the formation (Figure 4).

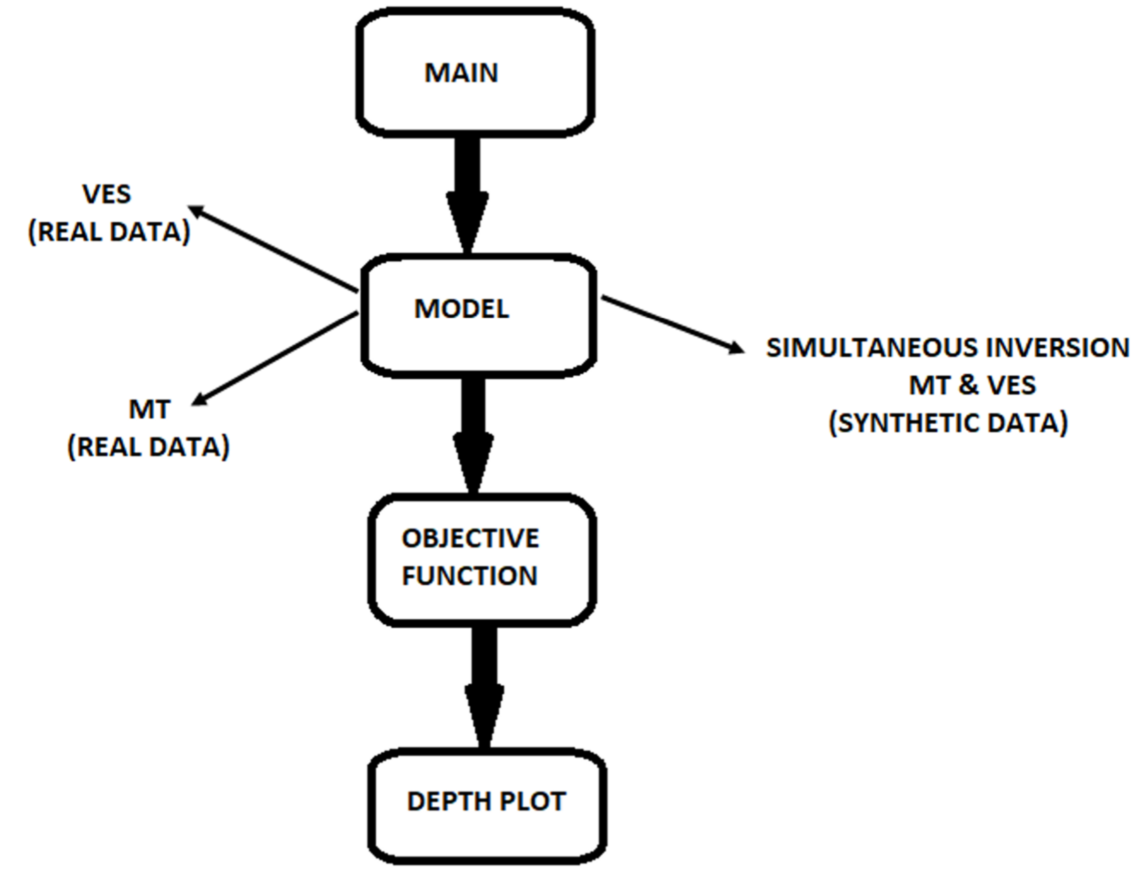


Figure 4. GA inversion simulation stages

2.2 Data acquisition and usage

Fifteen (15) VES data points were acquired through a single sounding in Italy. The VES soundings were acquired with a Schlumberger array. Acquisition of the VES soundings was carried out with a maximum AB/2 distance of 200 m electrode spacing. VES sounding plots are represented by the apparent resistivity (ohm-meter) and phase (degree).

Also, 34 MT sounding data were acquired from a geothermal field in Toscany, Italy. The electrical resistivity structure of the earth was imaged using electromagnetic (EM) fields produced by the planet's natural environment. At an MT location, the MT instrument captures the horizontal electric and magnetic field components as time series. This data was used to generate a period-dependent impedance tensor. Robust processing techniques were used to process the time series to obtain tensor impedances like impedance skewness and ellipticity (Egbert & Booker, 1986). This aids in knowing the noise effect on the real data and the dimensionality to apply in processing the MT data collected.

Magneto-telluric (MT) sounding plots are represented by means of two curves, namely those indicating apparent resistivity (ohm-meter) and phase (degree) as functions of period (time) or frequency. In the preparation of a depth model plot for the case of four formations (as done for the synthetic model in this work), seven (7) random parameters must be taken into account, including four resistivities and three thicknesses, each of which varies from zero to infinity.

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2.2.1 Data Used

Two main types of data were used in this work: synthetic data and real (field) data. The synthetic data described a four-layered (synthetic) earth model that was created with known thickness and corresponding resistivities (Table 3.1). It was used to test the effectiveness of the simultaneous MT and VES inversion models developed.

The real data consisted of MT and VES sounding data (taken from different fields in Tuscany, Italy). After the successful run of the simultaneous MT and VES inversion models on the synthetic data, the same simultaneous GA inversion codes were implemented on the field MT and VES data. The simultaneous GA inversion codes produced the MT and VES inversion models. The generated models described the subsurface layer properties by retrieving their thicknesses and corresponding resistivities. A depth plot was generated using the thicknesses and resistivities (Figure 5).

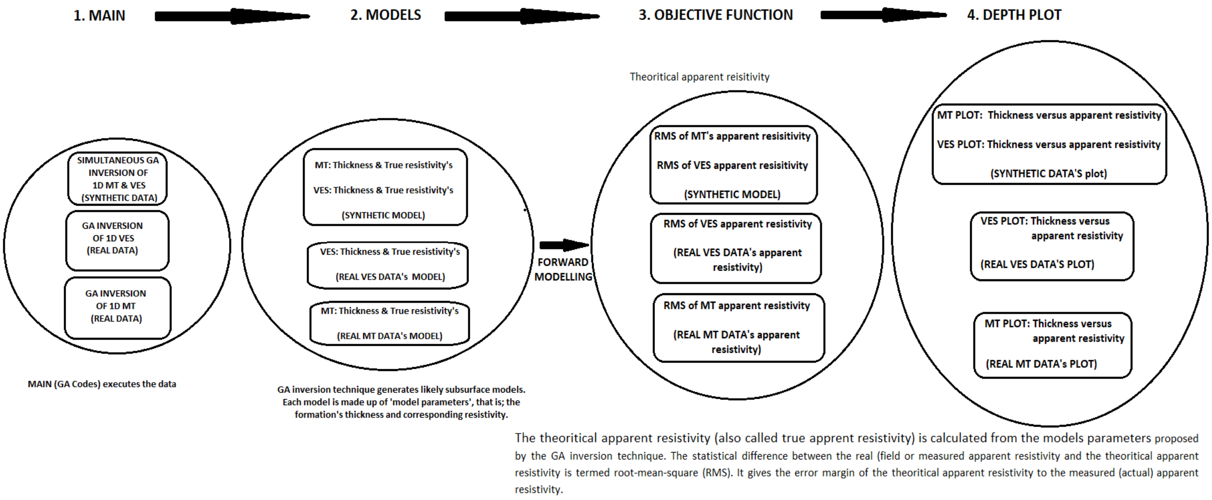


Figure 5: GA inversion process, data used and program output.

2.3 Description of the genetic algorithm used in the study

First, the model parameters to be determined, namely, the true resistivity and thickness of each layer of the formation, were encoded on strings of binary digits and stored in the computer's memory. The GA codes create a population made up of individuals (model parameters), which are stored in the computer's memory. The model parameters (likely solutions) evolve through the natural selection process, such as mutation, cross-over, and inheritance.

Each model (that is, true resistivity and thickness) is tested statistically and assigned a value to rate its closeness to measured data. This is achieved by using the objective function. This performs a statistical analysis by calculating the root-mean-square. Its value quantifies the errors in the estimated model from the field data (Snieder & Trampert, 1999; Forrest, S., 1993).

After rating all the models (individuals) within a population based on the objective function criteria, models with their theoretical apparent resistivity curves much similar to that of the measured or field data's apparent resistivity curves are kept and allowed into the next generation of models, made up of new populations (new models). 10% of the models in the new population were mutated (that is, flipping individual model parameters). Other model parameters were affected by crossovers (that is, recombination of model parameters to generate new offspring in the new population). These evolution factors optimize the final solution or model.

In this work, the GA inversion process terminates whenever the number of renewed generations reaches a predetermined number, which ranges from 50 to 200. Determining which parameters to vary and use in the GA, such as population size and crossover probability, can be challenging, since no specific criteria exist for such determinations. Thus, several trial runs were conducted. GA optimization parameters such as selection, mutation, cross-over, population, and generation size, which were input values, were diversified until the set of parameters that were appropriate with respect to convergence rate and misfit value were chosen. A satisfactory root-mean-square error was obtained for the simultaneous inversion of MT and VES synthetic models (Table 6) and the individual VES and MT data (Tables 7 and Table 8).

2.4 GA inversion Application on Synthetic Data

The developed simultaneous genetic algorithm inversion of MT and VES codes were tested on a four-layered synthetic earth model. The testing of the GA codes helps validate the GA inversion method before it is applied to a real data set. The MT synthetic data used for this earth model were the generated resistivity, thickness, frequency, and phase. For the VES synthetic data, the generated resistivity, thickness, and electrode spacing are the parameters used. The synthetic data were assumed to be measured in a horizontally layered space and were generated from the forward model.

2.5 GA Input Data for Synthetic VES & MT Data

A four-layered structural (earth) model was assumed to generate the synthetic data (Tables 5 and 6). The upper three strata are positioned above the basement, which has an infinite thickness and a resistivity of 20 ohms/m. In the simultaneous GA inversion, we searched for the best combination of MT and VES resistivity and thickness for each layer. The total number of unknown parameters were four resistivities and three thicknesses. The upper and lower limits were placed in the search spaces of the model parameters (Table 1 and Table 2).

We set constant upper and lower boundary conditions for the resistivity and thickness parameters, ensuring that the range is large enough to include all parameter values for the formation. As the GA inversion method is a stochastic procedure, it uses random numbers and identifies models near a global minimum solution. Several inversion runs were executed, where each inversion had a different population size, crossover fraction, and generation number. For each run, an optimal model was determined for the unknown parameters of resistivity and thickness (Table 5).

Table 1: Structural model of MT data used in the test and search space assuming a four-layer model

	Synthetic Model			Search Space	
Layer Number	Resistivity (ohm-m)	Thickness (m)	Frequency (Hz)	Resistivity (ohm-m)	Thickness (m)
1	50	100	1-10000	0.01-150	0.1 -200
2	10	100		0.01-100	0.1- 200
3	100	200		0.01-100	0.1-250
4	20	∞		0.01-100	

Table 2: Structural model of VES data used in the test and search space assuming a four-layer model.

	Synthetic Model			Search Space	
Layer Number	Resistivity (ohm-m)	Thickness (m)	Electrode Spacing (m)	Resistivity (ohm-m)	Thickness (m)
1	50	100	1-10000	0.01-150	0.1 -150
2	10	100		0.01-100	0.1- 150
3	100	200		0.01-100	0.1-250
4	20	∞		0.01-100	

2.6 GA Input Data for VES & MT real Data

Table 3 and Table 4 show the input parameters for the GA-based inversion that were applied for the 1D modeling of MT and VES real data, respectively. For this GA-based inversion, we searched for an optimal resistivity and thickness for each layer. The total number of unknown parameters were four resistivities and three thicknesses. The upper and lower limits in the search spaces of the parameters are also listed (Tables 4 and 7 for VES; Tables 5 and 8 for MT).

The population size was set at 100, with a sub-population of 20 each [20 20 20 20 20] and a crossover of 0.5. The simulation terminated the iterations at 50 generations for MT and 200 generations for VES. Since this inversion method is a kind of probabilistic approach using random numbers, it finds models near a global minimum solution. Several inversion runs were executed, where each inversion had the same upper and lower boundary conditions for the resistivity and thickness parameters; these were made so large that they did not require any changes during each inversion process.

Table 3 - Input data for 1D MT for the GA-based Inversion

Genetic Operation (Initial Model)	Resistivity [ohm.m]	
	Thickness [m]	
	Frequency [Hz]	
Noise [%]		
Smoothing Parameter		
Lower Boundary	Resistivity [ohm.m]	
	Thickness [m]	
Upper Boundary	Resistivity [ohm.m]	
	Thickness [m]	
Coefficient for LCI	RANGE	-
Initial Population		0 – 100
Migration Interval		0 – 10
Cross-Over		0 - 0.5
Generation		0 - 50

Table 4 - Input data for VES for the GA-based Inversion

Genetic Operation (Initial Model)	Resistivity [ohm.m]	
	Thickness [m]	
	Frequency [Hz]	
Noise [%]		
Smoothing Parameter		
Lower Boundary	Resistivity [ohm.m]	
	Thickness [m]	
Upper Boundary	Resistivity [ohm.m]	
	Thickness [m]	
Coefficient for LCI	RANGE	-
Initial Population		0 - 200
Migration Interval		0 - 10
Cross-Over		0 - 0.5
Generation		0 - 200

3.0 Result and Discussion

3.1 Application on synthetic data: Simultaneous Inversion of four-layered model

The assumed synthetic earth model has uneven formation resistivity with a corresponding thickness. Its third layer has the highest resistivity and thickness. The conductive layer (layer 2) is located between the high-resistance formations (layers 1 and 3). The search limit required by the GA inversion technique to estimate the model parameters of this formation is presented in Table 5 for both MT and VES.

Table 5: The synthetic earth model

Synthetic Model			VES Search Space		MT Search Space	
Layer Number	Resistivity (ohm-m)	Thickness (m)	Resistivity (ohm-m)	Thickness (m)	Resistivity (ohm-m)	Thickness (m)
1st layer	50	100	0.01-150	0.1 -150	0.01-150	0.1 -200
2nd layer	10	100	0.01-100	0.1- 150	0.01-100	0.1- 200
3rd layer	100	200	0.01-100	0.1-250	0.01-100	0.1-250
4th layer	20	∞	0.01-100		0.01-100	

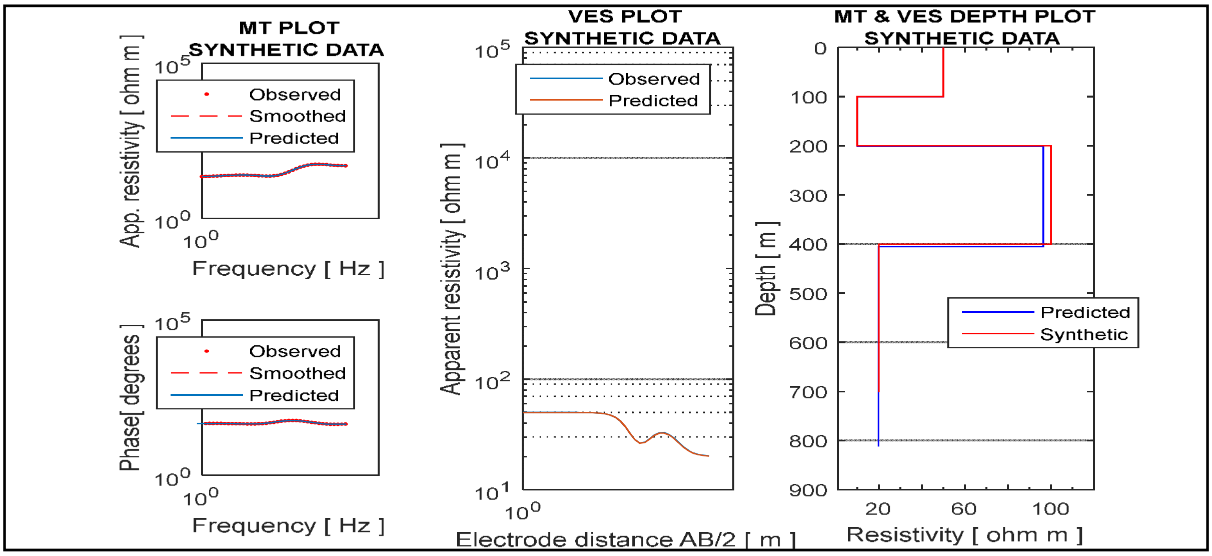


Figure 6 - Fitting of synthetic(experimental) and predicted simultaneous MT&VES model; (a) Top: apparent resistivity as a function of frequency for the synthetic MT model; (a) Bottom: phase as a function of frequency for the synthetic MT model; (b) Middle: apparent resistivity as a function of electrode spacing for synthetic VES model (c) Depth plot of formation thickness as a function of apparent resistivity for the simultaneous MT and VES model; the red line indicates the synthetic data, and the blue line represents the predicted data.

After extensive test runs, a population of 200, a generation of 200 and a cross-over fraction of 0.5 were considered adequate for the 1D simultaneous inversion of synthetic MT and VES data. When the forward modeling code was run, the results produced an apparent resistivity curve with 40 data points dispersed logarithmically. The theoretical apparent resistivity curves, the true and inverted resistivity models, and the observed (experimental) curves, are shown in Figure 6b. For MT, red dotted lines represent the model's curve for the observed apparent resistivity, and a blue solid line represents the model's predicted apparent resistivity, which is produced by the GA inversion (Figure 6a top). Likewise, for VES, the curve for the observed apparent resistivity of the model is illustrated by red solid lines, while the predicted apparent resistivity, which is produced by the GA inversion, is illustrated by a blue solid line (Figure 6b). The depth plot generated by the GA simultaneous inversion of the MT and VES synthetic models is shown in Figure 6c, where the red solid line represents the synthetic model depth plot and the blue solid line represents the predicted model depth plot.

Figure 6 shows that the predicted apparent resistivity curve and the observed one are very similar. For this model, the apparent resistivity root mean square error is 0.31% (Table 6). The optimal subsurface parameters of the predicted model obtained by the GA simultaneous inversion of the MT and VES synthetic models are summarized below (Table 6), with a depth plot in Figure 6c.

Table 6: Predicted model parameters for MT and VES synthetic data.

Synthetic Model			Predicted Model *	
			RMS = 0.31%	
			C.O = 0.8; GEN. = 200; POP.= 200	
Layer Number	Resistivity (ohm-m)	Thickness (m)	Resistivity (ohm-m)	Thickness (m)
1st layer	50	100	50.04	98.2
2nd layer	10	100	10.67	109.2
3rd layer	100	200	104.91	190.99
4th layer	20	∞	20.01	∞

**RMS: Root-mean-square error, C.O: cross over fraction, GEN: Number of generations POP: Number of population (individual models)*

The root-mean-square (RMS) gives an indication of the accuracy of the GA inversion technique. It represents the percentage deviation of the predicted model's apparent value from the measured (experimental) data. Equation (3) was used to quantify the apparent resistivity curve error, referred to as the root-mean-square error. The error recorded for the predicted model was less than 0.31%. It can be seen from Table 6 that the inverted parameters are close enough to the synthetic model, which in turn suggests a reliable estimate of layer parameters by the GA optimization technique.

An error margin of less than 0.31% was recorded in this work when the simultaneous GA inversion technique was implemented on a four-layer synthetic MT and VES data. This is similar to that reported by Hamimu et al

(2015) who applied GA inversion on VES synthetic data and Wijanarko and Grandis (2019) who applied it on MT synthetic data.

Hamimu et al. (2015) applied the GA inversion technique on several VES synthetic earth models, and all the synthetic models reported an error margin less than 5% for each layer, while Wijanarko and Grandis (2019) generated three layered earth models from MT synthetic data. They employed the genetic algorithm to find the model parameters for the 1D MT synthetic data. The root-mean-square error recorded between the inverse model and the synthetic MT data was less than 7%. This shows the applicability of the GA optimization techniques in estimating the model parameters of both VES and MT synthetic data.

3.2 Application of GA inversion on observed (real) Field Data

3.2.1 Standard 1D VES Inversion

The search limit required by the GA inversion technique to estimate the model parameters of the real VES data is presented in Table 7. After several trials, a population size of 200 with 100 generations and a cross-over fraction of 0.5 were considered for the 1D simultaneous inversion of observed (real) VES data. Optimal results were obtained when a 4-layer earth model was used in the modeling of the real VES data. The first, second, and third layers recorded an average resistivity of 10.16 ohm-m, 23.39 ohm-m, and 5.41 ohm-m, respectively, with corresponding average formation thicknesses of 5.18 m, 13.25 m, and 28.18 m. The third layer, which has the largest formation thickness, is also the least resistive (indicating the presence of an aquifer).

Table 7: True model, search space and genetic operators for 1D VES inversion

	Predicted Model					
Number of layers	Search space for GA inversion (Resistivity and thickness boundary)				RMS = 4.31%	
					C.O = 0.5 GEN. = 100 POP. = 200	
	ρ_{min} $(\Omega.m)$	ρ_{max} $\Omega.m)$	h_{min} (m)	h_{max} (m)	ρ $(\Omega.m)$	h (m)
1st layer	0.1	20	0.1	15	10.16	5.18
2nd layer	0.1	15	0.1	20	23.39	13.25
3rd layer	0.1	10	0.1	30	5.41	28.18
4th layer (Half space)	0.1	50			30.00	∞

The apparent resistivity curve produced by the forward modeling code's run consists of 15 logarithmically distributed data points. The observed (field) data and predicted apparent resistivity curves, along with the inverted resistivity model's depth plot, are shown in Figure 7. The curve of the observed apparent resistivity of the model is illustrated by blue solid lines, while the predicted apparent resistivity is illustrated by a red solid line (Figure 7). The predicted model's depth plot is shown by a blue solid line in Figure 7b. It can be seen from this figure that the subsurface layers were clearly delineated by GA inversion of VES field data, with a RMS error margin of 4.31% (Table 7).

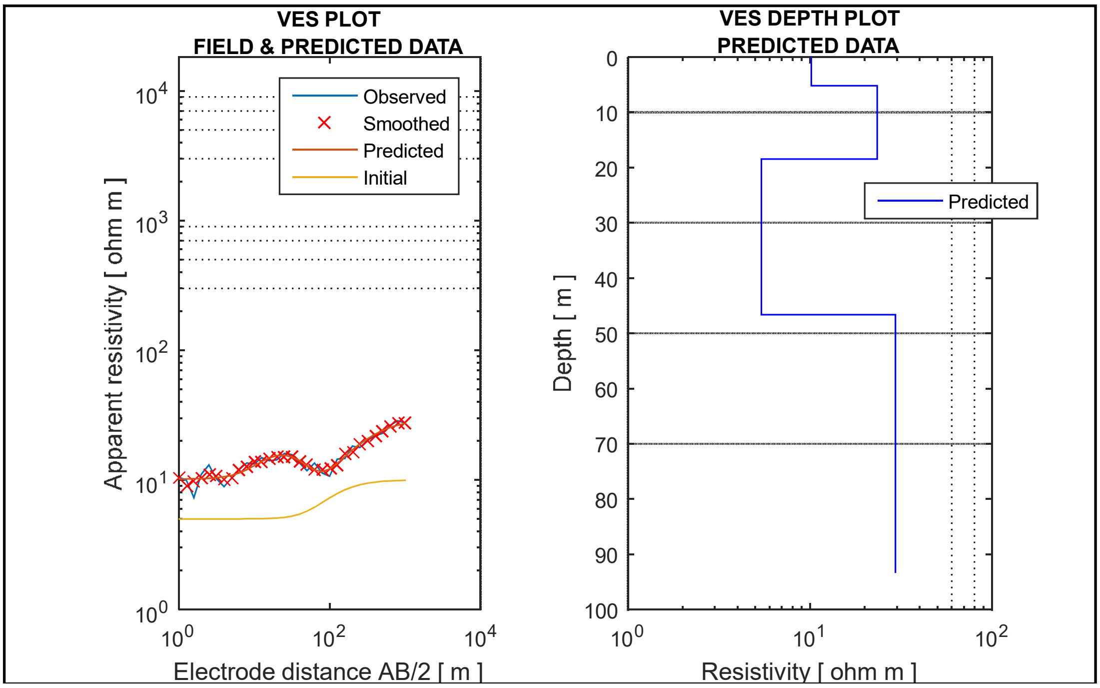


Figure 7 - Fitting of observed and predicted VES 1D model (a) apparent resistivity as a function of electrode spacing for 1D VES data; (b) depth plot of formation thickness as a function of apparent resistivity for 1D VES predicted model.

The GA optimization technique, when applied to other measured VES data, reliably modeled the subsurface layer. Jha M.K. et al. (1998) applied the GA inversion technique to interpret field apparent resistivity data and later correlated the interpretation with available well logs in the same field. They recorded a root-mean-square error of less than 10% for 29 sites out of 38 that were investigated.

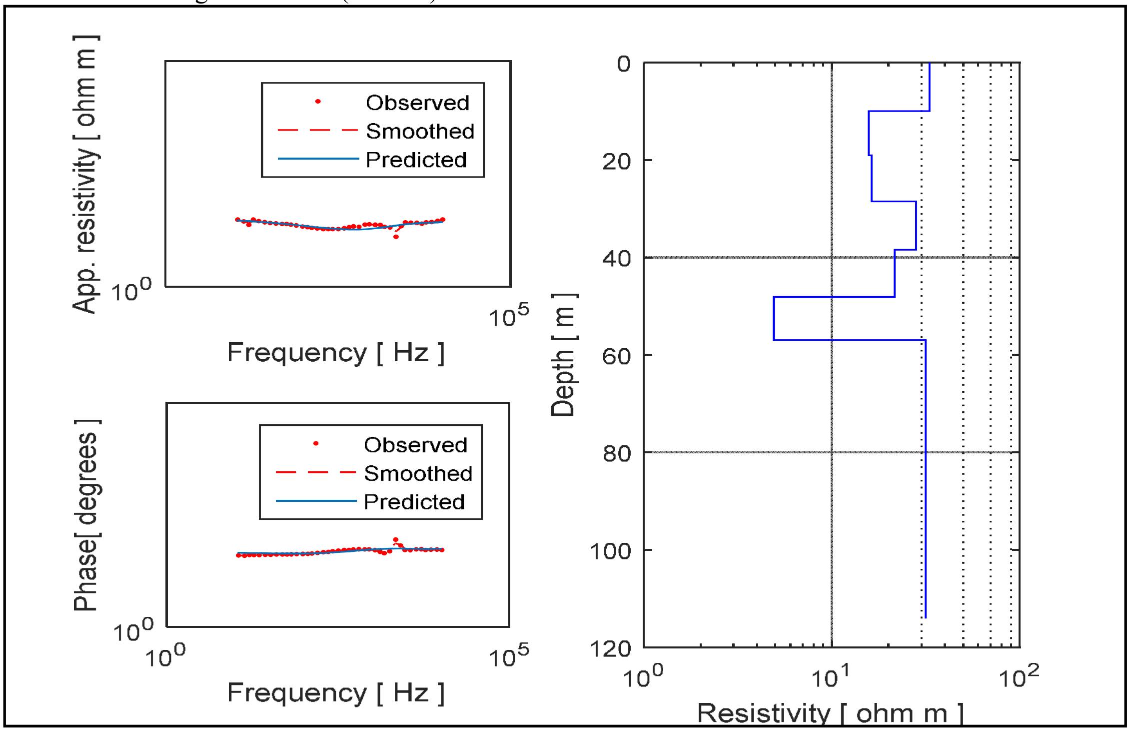
3.2.2 Standard 1D MT Inversion

For a 1D MT model of the subsurface from the real MT measurement, 7 layers were assumed. After several trials, the following GA operator parameter inputs were considered for the inversion of the MT observed data: population size of 200, 100 generations, and a cross-over fraction of 0.5. The average thickness for each layer was 9.4 m, with an average resistivity of 21.7 ohms per meter. A root-mean-square of 18.3% was recorded. It also shows that layer 6 is the least resistive layer with a resistance of 5 ohms per meter and a thickness of 8.9 m (Table 8).

Table 8: Predicted model, search space and genetic operators for 1D MT inversion

	Predicted Model					
Number of layers	Search space for GA inversion (Resistivity and thickness boundary)				RMS = 18.3%	
					C.O = 0.5 GEN. = 100 POP. = 200	
	ρ_{min} $(\Omega.m)$	ρ_{max} $\Omega.m)$	h_{min} (m)	h_{max} (m)	ρ $(\Omega.m)$	h (m)
1st layer	0.1	40	0.1	15	32.00	10.00
2nd layer	0.1	20	0.1	15	16.00	9.00
3rd layer	0.1	20	0.1	15	17.00	11.00
4th layer	0.1	40	0.1	15	29.00	8.00
5th layer	0.1	30	0.1	15	22.00	10.00
6th layer	0.1	10	0.1	15	5.00	8.90
7th layer (Half space)	0.1	40			31.00	∞

The 34 data points that made up the forward modeling code's output for the observed apparent resistivity curve were dispersed logarithmically. The observed (field) data and predicted apparent resistivity curves, along with the inverted resistivity model's depth plot, are shown in Figure 8. The curve of apparent resistivity against frequency shows red dotted lines for the observed apparent resistivity, while the predicted apparent resistivity is illustrated by a blue solid line (Figure 8a (top)). Likewise, the curve of phase against frequency shows the observed apparent resistivity of the model with red dotted lines, while the predicted apparent resistivity is illustrated by a blue solid line (Figure 16a (bottom)). The predicted model depth plot is shown by a blue solid line in Figure 8b. It can be seen from this figure that the subsurface layers were clearly delineated by GA inversion of MT field data, with a RMS error margin of 18.3% (Table 8).



(a) ***(b)***

Figure 8: Fitting of observed (experimental) and theoretical MT 1D model;((a) Top) Apparent resistivity as a function of frequency for the inversion of test MT 1D data; ((a) Bottom) Phase as a function of frequency for the inversion of real 1D MT data. (b) Depth plot of subsurface resistivity as a function of depth (thickness) inverted from 1D MT data.

The GA optimization technique, when applied to the observed and measured data, reliably modeled the subsurface layer. Shi et al. (2000) applied the MGA (Multi-scale Genetic Algorithm) to over 20 magnetotelluric data sets. For a six-layer model, he recorded a root-mean-square error of 16.6%. The identified formation layers correlated well with drilled well-bore data. This further shows the applicability of GA inversion of MT data in delineating the subsurface layers, with a root mean square error (RMSE) less than 20%.

4.0 Conclusion

The stochastic GA-inversion method was encoded in the MATLAB programming language and was successfully tested by applying the simultaneous GA inversion on synthetic MT and VES data to determine the model parameters (thickness and resistivity) of a synthetic earth model. For the synthetic data, it generated model parameters of resistivity and thickness with a root-mean-square error less than 0.31%.

The GA inversion codes were also applied separately to field MT and VES data. For the VES data, a RMSE was less than 5%, and that for MT was less than 20%. For the separate 1D models of VES and MT GA-based inversions, the formation depth models were satisfactorily delineated, giving a four-layer depth model and a seven-layer depth model for VES and MT, respectively.

This indicates the applicability of GA inversion in interpreting measured field data. Though the GA inversion program in this project can inverse both MT and VES data at the same time, it is only capable of accepting one input. This gives credence to the robustness of the simultaneous GA inversion program. Again, the low RMS values recorded for the synthetic and field MT and VES data during the inversion clearly show that simultaneous inversion of the VES and MT data sets by the GA optimization technique is feasible and effective in modeling subsurface layers.

To improve the lateral description of subsurface formation, lateral constraints can be included in the GA inversion technique to further examine the horizontal variation of resistivity in the subsurface. One major limitation of this work was the inability to apply the simultaneous GA inversion technique to both VES and MT data sets measured from the same field (location). It will be prudent for this to be done whenever such data is available.

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