

# Factors Affecting Teaching and Learning of Square Roots without Calculators in Secondary Schools: A Case Study of 6 Secondary Schools in Mpika District, Zambia

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## Abstract

This study was undertaken to determine the factors affecting the teaching and learning of square roots of real numbers without the calculators in secondary schools in Mpika District. Three research questions were raised and answered by administering questionnaires. Data was collected by means of four different questionnaires administered to twenty one teachers of mathematics, six Heads of Mathematics Department and one hundred and seventy four pupils drawn from six secondary schools in Mpika District. The five points Likert rating scale was used in determining the factors. Data was analyzed using simple frequency counts, percentages and analysis of variance. The results of the study showed that lack of content knowledge among the teachers, lack of educational materials and library facilities were the factors responsible for not teaching the topic without the calculators. For the pupils, it was due to lack of appropriate mathematics text books, not being taught by the teachers and failure to be given at the beginning of the term the course outline of the mathematics topics to be taught in the term. Some strategies have been adopted to enhance the teaching and learning of square roots without the calculators and a number of recommendations have been made to the government and other stakeholders involved in the provision of education.

**Keywords:** Teaching, learning, square roots, secondary schools, algorithm, calculator

## 1 Introduction

Mathematics is one of the core subjects that is being taught to all the pupils in secondary schools in Zambia because of the role which it plays in both the intellectual human development and economic development of the nation (Ministry of Education, 1996). Many people have come to believe that the mathematical skills that one acquires are important for the disciplining of the mind, developing of logical and critical reasoning, analytical and problem solving skills to a high degree (Dora, 1986). By learning topics like squares and square roots of real numbers without using the calculators, pupils can acquire the deduction, calculation and comparison skills which can enable them to deal with future challenges successfully. The concern by the public and the government over the poor performance in mathematics is an indication that interventions in teaching of mathematics must be designed so that performance in mathematics could be improved because mathematics aims at developing clear mathematical thinking and expression in a learner and also developing ability to recognize problems and to solve them with related mathematical knowledge and skills (MESVTEE, 2012).

### 1.1 Statement of the problem

In Zambia today, many senior pupils and school leavers are unable to find the square root of a number which is not a perfect square without a calculator. This failure is an indication that in secondary schools, there is little or no teaching and learning of finding the square roots of real numbers which are not perfect squares. This failure has made the pupils not to acquire the calculation, deducting and comparison skills, a situation which has continued to contribute to poor performance in mathematics by most of the pupils in the School Certificate Examinations especially in paper one where candidatures are not allowed to use the calculators as shown by the chief examiner's reports of 2008 and 2012. In 2008, in paper one, poorly done was 53.8% while in paper two, it was 39.7% and in 2012, poorly done in paper one was 46.9% while in paper two, it was 45.5%. Hence the need to carry out the study on the teaching and learning of finding the square roots of real numbers without using the calculators

### 1.2 Objectives of the study

The objectives of the study were;

- (a) To determine the factors that make teaching of finding the square roots of real numbers a problem in secondary schools in Zambia.
- (b) To find out why no learning by pupils to find the square roots of real numbers which are not perfect squares without the calculators in secondary schools in Zambia.
- (c) To examine the strategies that can remedy the difficulty in the teaching and learning of finding the square roots of real numbers which are not perfect in secondary schools in Zambia without using the calculators.

### *1.3 Research Questions*

The following research questions were generated to guide this study;

- (a) What factors were responsible for not teaching how to find the square roots of real numbers which are not perfect squares without the calculators in secondary schools in Zambia?
- (b) What were the factors affecting the learning of finding the square roots of real numbers without the calculators?
- (c) What strategies could be adopted to enable teachers to teach the finding of the square roots of real numbers without the calculators and enable pupils to learn finding the square roots of real numbers which are not perfect squares without the calculators in secondary schools in Zambia?

## **2. The Literature review**

### *2.1 Pupil opportunity to learn and learning activities*

The opportunity to learn mathematics effectively is dependent upon a wide range of factors, but among the most important are those which are related to activities and practices within the classroom. The pupils must learn how to critically analyze mathematical problems and produce effective solutions. This requires them to learn, how to make sense of complex mathematics concepts and how to think mathematically. Therefore, the mathematical tasks in which pupils engage should facilitate and support students' conceptual understanding of mathematics, fostering deep connections among mathematical ideas (Hiebert & Carpenter, 1992).

### *2.2 Pedagogical content knowledge*

Shulman (1987) argued that in addition to general pedagogical knowledge and content knowledge, teachers needed to make a link between the two. These are subject matter knowledge and knowledge of classroom organization and management, which are called pedagogical knowledge. Teachers need to have skills and knowledge to apply their philosophy of teaching and instructional decisions. The logic herein is that teachers who possess strong mathematical knowledge at a greater depth and span are more likely to foster students' ability to reason, conjecture, and problem-solve, while also being able to more accurately diagnose and address students' mathematical misconceptions and computational dysfunctions (Kilpatrick, Swafford & Findell, 2001).

## **3 Methodology**

The research design for the study was descriptive survey in which data were collected using a combination of quantitative and qualitative research methods. This entailed the use of a survey technique in which questionnaires were given to 6 Heads of the Mathematics Department, 21 teachers of mathematics and 174 pupils from six secondary schools in Mpika District, Zambia to complete them. Data was analyzed qualitatively by using the constant comparatively method (Glaser and Strauss, 1967) that involved a continual process of comparing pieces of data and identifying similarities and differences between them for generating patterns or categories from the data. The SPSS was also used in the analysis of the data that was collected. Interpretation of data included the use of percentages, frequencies and means.

## 4 Results

### 4.1 Responses of teachers on factors affecting the teaching of square roots

**Table 1 Teachers responses on factors affecting the teaching of square roots.**

Statements	Strongly agree	Agree	Uncertain	Disagree	Strongly disagree	Mean
1. At the college or university, I did not learn how to find a Square root of a real number which is not a perfect square without a calculator.	9	8	1	3	0	4.10
2. I cannot calculate square root of a real number which is Not a perfect square without a calculator.	10	3	1	4	3	3.62
3. The only method I can use to find the square root of a real number which is not a perfect square is using a calculator.	3	8	1	6	3	3.10
4. I have not been teaching finding the square root of a real number which is not perfect without using a calculator.	11	6	0	0	4	3.95
5. The department does not have enough recommended text books for teaching square roots of real numbers which are not perfect squares.	13	7	0	1	0	4.52
6. When I was at the secondary school, I did not learn to find the square roots of real numbers which are not perfect squares without a calculator.	7	7	0	5	2	3.57
7. The process of finding the square root of a real number which is no perfect square is difficult.	0	11	3	4	3	3.05
8. Most parents do not support their children buying them recommended mathematics text books.	6	9	1	5	0	3.76
9. There is inadequate funding for buying mathematics text Books every year from the government.	8	11	0	1	1	4.14
10. The school does not have a library with useful books for teaching square roots of real numbers without the calculators.	9	11	0	1	0	4.33

#### 4.1 Analysis of variance

The null hypothesis ( $H_0$ ); There is little teaching of finding the square roots of real numbers without the calculators in secondary schools.

Level of significance = .05

*Table 2 .Analysis of variance on factors affecting teaching of square roots without calculators*

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	18.572	9	2.064	.800	.625
Residual	28.381	11	2.580		
Total	46.952	20			

From the table 2, the sig value is .625 which is greater than .05. This implies that there is no significant difference among the nine independent variables. The calculated value of  $F_{(9,11)} = .800$  is less than the table value of  $F = 2.90$ . We therefore accept the null hypothesis that there is little teaching of finding the square roots of real numbers without the calculators in secondary schools in Zambia.

Table 3. Estimate of relative contribution of factors to teaching of square roots without the calculators.

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1(Constant)	3.904	4.940		.790	.446
Did not learn square roots at University/college	-.132	.316	-.118	-.418	.684
I cannot calculate square roots without calculator	-.376	.334	-.392	-1.127	.284
Only method using calculator	-.613	.367	-.458	-1.668	.123
Department does not have enough text books	.185	.404	.129	.458	.656
Did not learn square roots at secondary school	.343	.356	.321	.962	.356
Process of finding square roots difficult	-.331	.376	-.251	-.881	.397
Most parents do not support their children	.007	.379	.006	.019	.985
There is inadequate funding from government	.074	.412	.049	.181	.860
Library does not have useful books	.592	.620	.282	.955	.360

From Table 3, out of the nine variables, the teacher knowing only one method of using the calculator made the greatest contribution ( $\beta = -.458$ ). This implies that knowing only one method of finding the square root has negative effect on the academic performance of pupils on finding square roots without the calculators or learning how to find square roots of real numbers at secondary school which leads to poor academic performance among pupils. The more methods teachers know the more effective they would become in teaching square roots without calculators and the better the academic performance by the pupils. The second in the rank of contribution is the teacher not finding the square root without the calculator ( $\beta = -.392$ ) Third in the rank of contribution is the teacher not to have learnt square roots without calculators while at secondary school ( $\beta = .321$ ) and the fourth is library not having useful text books on square roots without the calculators ( $\beta = -.282$ ). The fifth and sixth in order of decreasing magnitude of contribution are the process of finding the square roots without the calculators is difficulty ( $\beta = -.251$ ) and the Mathematics Department not having enough text books on square roots without the calculators ( $\beta = .125$ ). The seventh in contribution is the teachers not learning how to find the square roots without the calculators while at the university/colleges ( $\beta = -.118$ ). The eighth and ninth are the inadequate funding from the government ( $\beta = .049$ ) and most parents not supporting their children in terms of buying the mathematics text books ( $\beta = .006$ ).

The data collected revealed that there was little learning of finding the square roots of real numbers without the calculators because of the following factors:

- (a) Pupils were not being taught by the teachers.
- (b) Pupils lack mathematics text books on finding the square roots of real numbers without using the calculators.
- (c) Pupils were not given course outline of mathematics topics at the beginning of each term.

### 5 Discussion of the results or findings

The findings of the study revealed that teachers were lacking content knowledge on square roots because when they were at the secondary school and at the university, they did not learn how to find square roots without using the calculators. As a result of lack of content knowledge, teachers have not been teaching the topic because for the teacher to teach effectively he needs content knowledge. This implies that the training institutions must ensure that teachers are trained in the key components of effective mathematics instruction which are: Subject-matter knowledge in mathematics (or the teacher's knowledge of the content being taught), mathematics topics

for student mastery and knowledge about how to teach effectively mathematics (or the teacher's knowledge and use of effective instructional strategies in teaching mathematics). Teaching is effective if the teacher is knowledgeable about the content of the subject he is teaching and also if he has the pedagogical knowledge of the subject, failure to which teaching would not be a success. Teaching is complex because it involves a learner who is a human being and it is for this reason that the teacher needs to have in-depth knowledge of mathematics subject matter (Shulman, 1987).

When teaching mathematics, the teacher should form an adequate model of the pupils' ways of viewing an idea and then construct a tentative path on which pupils may move to construct a mathematical idea in line with accepted knowledge (Lyn, 2002). If the teacher lacks the content knowledge, then he can not engage the pupils into the process of mathematical thinking instead he would engage them into what is called the product of mathematical thought (Skemp, 1971).

For teaching and learning to be enjoyable and effective, learning materials should be available. Learning is strengthened when there are enough reference materials such as textbooks, exercise books, teaching aids and enough class rooms (Mutai, 2006). In the absence of enough educational materials, it is impossible to have interactive teaching because pupils if they have to participate fully and actively they need to have background information and they can only have this information if educational materials are available.

The result of the study revealed that pupils had not been learning square roots because they were not being taught by the teachers as revealed in the teacher questionnaire and they did not have mathematics text books that explained how to find the square roots without the calculator. School libraries where they existed lacked useful mathematics text books and many pupils acknowledged that they were not given course outline of mathematics topics which they were suppose to cover in a term and as a result they did not know the topics they were going to learn.

## 6 Strategies to enhance teaching of square numbers and roots

### 6.1 Definition of a perfect square number and its relationship with odd numbers

Let us look at perfect square numbers from **odd numbers** point of view.

If  $S = \{ 1, 3, 5, 7, 9, 11, 13, \dots \}$ , then S is a set of odd numbers.

Let us again look at the sum of the consecutive odd numbers starting with 1.

$$\begin{aligned}
 1 &= 1=1^2 \\
 1 + 3 &= 4 = 2^2 \\
 1 + 3 + 5 &= 9 = 3^2 \\
 1 + 3 + 5 + 7 &= 16 = 4^2 \\
 1 + 3 + 5 + 7 + 9 &= 25 = 5^2 \\
 &\dots\dots\dots \\
 &\dots\dots\dots \\
 1 + 3 + 5 + 7 + 9 + \dots\dots\dots + (2n - 1) &= n^2
 \end{aligned}$$

From the above examples, we can conclude that a perfect square number is the sum of the first  $n$  consecutive odd numbers starting with 1.

### 6.2 Methods that can be used to find square root without calculators

The example is according to Claude and Samuel (1952):

(i) *Claude and Samuel's Method*

Example 1. Find  $\sqrt{522\ 729}$

The trial divisor		52'27'29 ( <b>723</b> ← root
		- 49
The true divisor	142	327
		- 284
2 <sup>nd</sup> true divisor	1443	4329
		- 4329

Explanation: First, separate the number into periods of two figures each, beginning at the right, and placing a mark between them. The number of periods thus formed is equal the number of figures in the root.

Find the largest perfect square which is equal to, or less than the left-hand period 52. This perfect square is 49. Write it under 52; and put its square root, 7, to the right as the first figure of the root. Now subtract 49 from 52 and bring down the next period, 27, and unite with the remainder 3, thus obtaining 327.

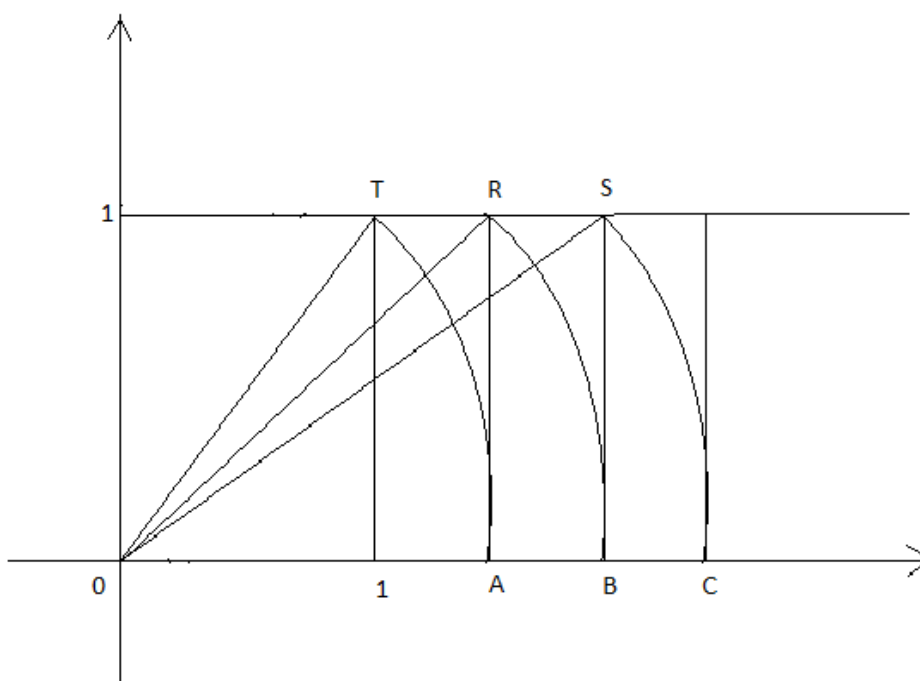
Take twice 7, the first figure of the root, and write it to the left of 327. Find how many times this 14 is contained in 32, which is 2, for the second figure of the root. Place this figure 2 in the root and also to the right of 14, making 142. Now multiply 142 by 2 and write the product 284 under 327. Subtract 284 from 327 and bring down and unite the next period 29 with the remainder 43, thus obtaining 4329. Next multiply 72 by 2 and write it

at the left of 4329 as the second trial divisor. Find how many times 144 is contained in 432, which is 3, for the third figure of the root. Place this figure 3 in the root and also at the right of 144, making 1443, the second true divisor. Multiply 1443 by 3 and write the product under 4329. This gives no remainder. Therefore, 723 is the exact square root of 522 729 ; that is the exact square root of 522 729 ; that is  $723 \times 723 = 522\,729$ .

(ii) *Geometric Interpretations of Square Roots Method*

By constructing the semicircle, square and rectangles in a single diagram, the square root of any real number greater than 1 can be found. To find the square root of 2, draw the square of length 1 unit as in the diagram below, take the diagonal OT of the square to be the radius and the origin being the point where the axes have met. Draw the arc to pass through the square and where it cuts the horizontal axis is the square root of 2, which is OA. To find the square root of 3, draw a vertical line from A to meet a horizontal line from 1 on the vertical axis and let this point be R. Take OR as radius and draw an arc to cut the horizontal axis at B. The length OB is the square root of 3. To find the square root of 4, draw a vertical line from B to meet a horizontal line from 1 on the vertical axis and let the point where the two lines meet be S. Join O and S and let OS be radius with centre O draw the arc to meet the horizontal axis, where it meets the axis is the square root of 4. With the same process, you can find the square roots of any real number without the use of the calculators.

Below is a single diagram showing how to find the square roots of 2, 3 and 4.



$$OA = \sqrt{2} = 1.4$$

$$OB = \sqrt{3} = 1.7$$

$$OC = \sqrt{4} = 2$$

**7.0 Conclusion**

It was found that there was little teaching of squares and square roots of real numbers without the calculators because teachers lacked content knowledge as a result of not learning the topic when they were at school and at the university or college, most of the teachers could only find the square root using the calculator, lack of educational materials in schools, failure by most of the parents to support their children in terms of books and inadequate funding from the government for buying teaching and learning materials. There is need for the government to train more mathematics teachers and deploy them in schools so that the teacher-ratio is reduced and at the same time make teachers be effective because they will have small loads which will enable them to do research. Teaching of squares and square roots would help pupils see the relationship between algebra and geometry and also acquire skills in deduction, comparison, calculation and develop the abilities in logical reasoning, thinking and analysis of problems.

## 8 Recommendations

Based on the findings, discussions and conclusions herewith contained in the study, the following summative recommendations are made to the government and all stakeholders in the education sector:

- (a) Government should buy education materials such as text books according to the syllabus and deploy more mathematics teachers in schools.
- (b) The Heads of Mathematics Department should ensure that teachers should know more than one method of finding the square root of real numbers.
- (c) The Ministry of Education, Science, Vocational Training and Early Education should buy books and other facilities for the school libraries and build libraries at schools where there are no libraries.
- (d) The Heads of the Mathematics Department should ensure that pupils are engaged in mathematical thinking and not in mathematical thoughts by the teachers

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