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Mathematics Teachers' Ability to Investigate Students' Thinking Processes About Some Algebraic Concepts

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Abstract
The study investigated Mathematics teachers' ability to evaluate students' thinking process about some algebraic concepts. The study involved 156 Mathematics teachers randomly selected from some Public Secondary Schools in Bauchi State, Nigeria. Open ended questionnaire was used to collect data from the participants. The teachers were required to respond in detail to the questions asked. The research design was a qualitative one and expository method was used in the analysis of the data collected. It was revealed that, though the teachers showed some success in understanding the students' errors and misconceptions, great majority of them could not assess students' thinking process. The study also revealed that most of the teachers were unable to ask competent questions that could help in evaluating the students' thinking process. Another important finding was the difficulty the teachers themselves had in understanding the problem situations, and hence asked irrelevant questions or make irrelevant statements. It was suggested that training workshops be organized for the teachers to improve their subject matter knowledge about the concepts studied.
Keywords: Ability, Thinking process, Algebraic concepts

1. Introduction

The mathematics teacher is a necessary component in the reform of teaching and learning mathematics. The possession of various structures of knowledge by teachers indicates the kind of teachers they might be. And talking about teachers' knowledge brings to mind the concept of pedagogical content knowledge of teachers. Tanisli and Kose (2013) asked, "What should teachers know?" They noted that the answer can be explained through the concept of pedagogical content knowledge. According to them, pedagogical content knowledge is defined as "the ways of representing and formulating the subject that makes it comprehensive to others". They maintained that pedagogical content knowledge is a kind of knowledge that reveals a teacher's meaningful and effective ways of teaching. Chick and Baker (2005) quoted Graeber as saying "an understanding of common students' misconceptions, and effective strategies to help students avoid them, are important aspects of mathematics pedagogical content knowledge".

Pedagogical content knowledge is the subject-matter knowledge that teachers have and it consists of many components. According to Shulman (1987) pedagogical content knowledge has seven components, among which include subject-matter knowledge, knowledge of students, knowledge of curriculum and knowledge of educational purposes. Tamir (1988) categorized pedagogical content knowledge into four components: knowledge of understanding students; knowledge of teaching methods, knowledge of measurement and evaluation and knowledge of curriculum. Grossman (1990) also identified four components of pedagogical content knowledge as: knowledge of strategies and representations for teaching particular topics; knowledge of students' understanding, conceptions and misconceptions of these topics; knowledge and beliefs about the purposes of teaching particular topics; and knowledge of curriculum materials available for teaching. In the same vein, Fennema and Franke (1992) categorized mathematics teachers' pedagogical content knowledge under four categories, and one of which is knowledge of students.

Shulman (1986) and Park and Oliver (2008) placed knowledge of students in the centre of pedagogical content knowledge, and it is viewed as one of the key components. Knowledge of students is described as a teacher's knowledge of student's procedural and conceptual knowledge, students' thinking processes, learning styles, difficulties and misconceptions in the process of learning a subject.

Several scholars in mathematics education (e.g. Carpenter, Fennema, Peterson & Carey, 1988; An, Kulm & Wu, 2004; Chick & Baker, 2005; Chick, Baker, Pham & Cheng, 2006) have conducted studies about in-service and pre-service teachers' knowledge of students in many subject areas. The findings of these studies indicated that mathematics teachers and pre-service mathematics teachers have incomplete and inadequate knowledge of students in general. Carpenter, Fennema, Peterson and Carey (1988) pointed out that teachers' knowledge of students is necessary for effective teaching. It goes without saying then that mathematics teachers' knowledge of students is important in teacher education.

A study conducted by Tanisli and Kose (2013) revealed that students' background knowledge, the concepts which they have difficulty in understanding and their misconceptions are different from teachers' expectations and predictions about them. Similarly, in a study carried out by Asquith, Stephens, Knuth and Alibali (2007), the findings revealed that mathematics teachers' knowledge of students about the concept of equal sign and variable was different from the students' possible errors. They found out that the participating teachers had difficulty in identifying and predicting students' possible misconceptions and errors about equal sign and variable. Tanisli and Kose (2013) noted that some studies about pre-service mathematics teachers' ability to predict the errors and misconceptions of primary school pupils in relation to algebraic expressions and manipulations indicated that the respondents generally made predictions about only one kind of errors and misconceptions. They said the pre-service mathematics teachers predicted errors and misconceptions which students didn't have.

According to Boz (2002) and Stephens (2008), teachers' predictions and expectations of students' thinking processes fail to identify the ideas and errors behind students' answers, and also fail to explain the sources of students' misconceptions. The studies also maintained that mathematics teachers could not come up with good recommendations that would help eliminate students' errors. Stephens (2008) pointed out that pre-service mathematics teachers had limited knowledge of algebraic concepts as a part of subject-matter knowledge. Boz (2002) noted in his study that though the pre-service mathematics teachers knew the rules for the letter symbols, they could not demonstrate this same success in explaining the reasons for these rules.

Mathematics teachers' ability to probe students' thinking processes is important to effective teaching. Mathematics teachers are expected to understand what students know and what they need to know. This can be done effectively by the questioning skills of the teacher. This is because the quality of the questions asked by teachers plays an important role in identifying students' difficulties. Moyer and Milewicz (2002) said teachers who are able to ask skillful questions can also analyze the depth of their students' thought better. If a teacher does not have an adequate understanding of the subject-matter, he/she will not be able to ask competent questions that would reveal students' errors and misconceptions.

This study sought to examine mathematics teachers' ability to investigate students' errors and misconceptions in some algebraic concepts. The problem therefore is whether secondary mathematics teachers are capable of evaluating students' thinking processes and predicting students' misconceptions in some algebraic concepts.

2. Methodology

2.1 Research Design

The qualitative research approach was employed in this study. Qualitative approach emphasizes the process rather than the product, and the study focused on mathematics teachers' ability to identify, discuss and predict students' errors and misconceptions.

2.2 Participants

The participants to the study were mathematics teachers randomly selected from public secondary schools in Bauchi State, Nigeria. The mathematics teachers who took part in the study were 156, and were of varying qualifications and years of experience.

2.3 Instrument for the study

The instrument for the study was an open-ended questionnaire adopted from Tanisli and Kose (2013), with changes in the names used in the problem statements only. The questionnaire consisted of three open-ended items designed to evaluate mathematics teachers' ability to identify, discuss and predict students' errors and misconceptions with regard to some algebraic concepts. The teachers were requested to answer the questions in detail. The questionnaire is shown in Figure 1.

Question 1: The question "*John is 4 cm taller than Joan. If Joan is n cm tall, how tall is John?"* is being discussed in class. The dialogue among three students is given below: *Adams: John's height is 4n,* *James: No. John's height is 104 cm.,* *Sarah: I think John's height is x+4.* What kind of questions may be asked to each of these students to help them understand their errors?
Question 2: In the question "*In the expression 4n+7, what does the symbol n represent?"* Musa gives the following answer *"n does not mean anything here because there is no symbol "=" in the expression. For example, in an expression such as 4n+7=11, n=1".* Discuss the student's idea.
Question 3: "*a) 4x-1=0 b) x+10=47 c)* $\frac{x}{2}+3=5$ *d) -3x+6=2x+16"* What kind of incorrect answers may be given to the questions above by your students? Try and predict.

Figure 1: Teaching Mathematics Survey

3. Results and Discussion

3.1 Results

The mathematics teachers' ability to identify students' errors, discuss students' thinking process and predict students' errors was examined in this study. And the findings of the study are presented as follows:

3.1.1 Teachers' ability to identify students' errors about the concept of variable

The ability of teachers to ask skilful questions in order to help students recognize their errors and misconceptions is a key component in the teaching and learning of mathematics. Mathematics teachers were presented with hypothetical students' solutions with regard to the concept of variable, and were required to ask questions that would help the students recognize their mistakes. The problem situation and the incorrect responses of the students are presented ("John is 4 cm taller than Joan. If Joan is n cm tall, how tall is John?" Adams: John's height is 4n, James: No. John's height is 104 cm, Sarah: I think John's height is x+4.). The teachers were asked to identify the kind of questions they would ask each student to help them understand their errors and misconceptions. The study revealed that majority of the questions the teachers asked were instructional questions. An instructional question is teaching a student instead of assessing the student's knowledge about the concepts (Tanisli & Kose, 2013).

Examples of the questions the teachers asked to Adams who said that John's height was 4n include: "Do you know the difference between multiplication and addition? Aren't you required to add?" "Is John 4 times taller than Joan?" and "If john is 4n, it implies John is 4 times taller than Joan". The questions the teachers asked James who said that John's height was 104 cm-"If John is 104 cm, does it mean Joan is 100 cm? but we're told that Joan is n cm." and the questions the teachers asked to Sarah who said that John's height was x+4- "Do you not know that n is representing the height of Joan and not x?" and "x is not mentioned in the question, therefore, is your x representing n?" The teachers emphasized their own thinking processes by revealing the answers instead of assessing the students' thinking process in this type of questions.

The study also revealed that a few of the questions asked by the teachers can be categorized as investigative questions. Examples of these questions include: "What does 4n mean to you?", "How do you know Joan's height is 100 cm?", "How did you get 104 cm?" and "What does x stand for?" Regrettably, some of the questions were either irrelevant or inadequate. Examples of such questions are: "What unit of length is used to measure the height of John?" and "Is x the same as n? if so, x and n are unknown".

3.1.2 Teachers' ability to discuss students' thinking process

The question asked to evaluate teachers' ability to discuss students' thinking process about the concept of variable was: "What does n represent in the expression 4n+7?" The teachers' responses to the students' thinking processes revealed their inability to recognize students' incorrect thinking process. Examples of teachers' statements such as, "The student's idea is reasonable, in the sense that n should have a physical quantity or a definition", "The student may be right since 4n+7 is not equated to zero or any number. This means the value of n cannot be found since the equation is not equated to anything", and "4n+7 is not equated to anything, therefore, it means nothing or is undefined" indicate that mathematics teachers were not able to recognize students' incorrect thinking process, and so could not discuss students' errors and misconceptions. Majority of the teachers who participated in this study were unable to analyze students' thinking process.

Another finding of the study is that a few of the teachers made statements that revealed their recognition of students' incorrect thinking process about the question in which the variable n in the expression 4n+7 was asked. Examples of teachers' statements about students' thinking process such as, "Musa did not understand the difference between an expression and an equation. He thinks that the unknown n in the expression 4n+7 cannot be found since the expression is not equal to any number. Therefore, it means nothing to him" and "He does not know the difference between an equation and an expression" indicate that the teachers recognized the student's errors and misconceptions in the interpretation of the variable n in the expression 4n+7.

It was also found that some teachers had difficulties themselves understanding the problems. This is evident from the following questions asked by some of the teachers: "Do you know how to multiply numbers in algebraic form?" and "Do you not know that unknown quantities are given in algebraic form?" This clearly showed that such teachers could not understand the problems.

3.1.3 Teachers' ability to predict students' knowledge about the concept of equation

In order to evaluate teachers' ability to predict students' errors and misconceptions about equations, the teachers were asked to predict students' solutions to the equations: 4x-1=0, x+10=47, $\frac{x}{2}+3=5$ and -3x+6=2x+16. Studies have documented certain common types of errors and misconceptions that students make (e.g. Moyer & Milewicz, 2002 & Hall,2002).

In this study the teachers predicted various types of errors and misconceptions that students make, some of which are documented in the literature. The teachers' predictions are presented in Table 1.

In the first equation, *4x-1=0,* it is documented in the literature that students generally make the errors or misconceptions of *4x=1, x=1-4, x=-3*, which is called "The Other Inverse Error". In this study 49% of the teachers predicted these errors or misconceptions. Majority of the teachers predicted that their students would make the error of $x=-\frac{1}{4}$, which Kieran referred to as "Switching Addends Error". Other errors and misconceptions predicted by the teachers were the "Omission of Error": x=0, and "Absence of Structure Error": x=1. Also 16% predicted the error: x=4, as against 17% of the participants in Tanisli and Kose's study. There were other predictions which have not been documented in the literature.

In the second equation, x+10=47, 66% of the teachers predicted the common errors and misconceptions reported in the literature (Table 1). Also the errors x=47 and x=-37 were each predicted by 33% of the teachers.

In the third equation, $\frac{x}{2}+3=5$, 66% of the teachers predicted the common errors and misconceptions documented in the literature (see Table 1). According to Kieran (1992), most students tend to reach x+3=10, x=7, because they do not take the symmetry of the equation into consideration when multiplying both sides by two. Also, 33% predicted x=4, and this error was predicted by 36% of the participants in Tanisli and Kose's study.

The fourth equation given to the teachers was -3x+6=2x+16,and only 16% predicted the errors mentioned in the literature (see Table 1). The error: $x=\frac{22}{5}$ was predicted by 33% of the teachers, and this error was predicted by just 14% of the participants in Tanisli and Kose (2013). Many other errors analyzed in this study have not been reported in previous studies.

Table 1: Errors and misconceptions predicted by the teachers.

Equation	Common errors and misconceptions in the literature	Number of teachers who predicted	Other predicted errors or misconceptions	Number of teachers who predicted
4x -1=0	4x=1, x +4=1 x=1 - 4, x=-3	77 (49.34%)	x=-1/4 x=5 x=0 x=4 x=1	130(83%) 26(16.6%) 77(49.3%) 26(16.6%) 26(16.6%)
x + 10=47	x=47 +10, x=57	104(66.6)	x=4.7 10x=47 x=37	52(33.3%) 26(16.6%) 52(33.3%)
$\frac{x}{2}+3=5$	x + 3=10, x=7	104(66.6%)	x=4 x=1 x=5.5 x=14 x=10/3 x=13	52(33.3%) 20(12.82%) 32(20.51%) 26(16.66%) 29(18.59%) 23(14.24%)
-3x + 6=2x + 16	x+ 6=16, x=10	26(16.6%)	-3x + 2x=16 + 6 (x+16)/2=5 x=-1 x=2 x=22/5	20(12.82%) 23(14.74%) 30(19.23%) 31(19.87%) 52(33.3%)

3.2. Discussion

The importance of teachers' ability to identify, discuss and predict students' errors, misconceptions and thinking process cannot be overemphasized, if learning is to be effective and meaningful in mathematics. An important finding in this study is the success of the teachers in understanding students' thinking process with respect to variable and equations. This is in line with the findings of Boz (2004), Stephens (2006), Asquith, Stephens, Knuth and Alibali (2007) and Tanisli and Kose (2012).

Another key finding of the study is the inability of most of the teachers to evaluate the students' thinking process through effective questioning. The teachers generally asked instructional questions instead of investigative questions in their effort to identify students' errors and to evaluate their thinking process. This finding agrees with the findings by Moyer and Milewicz (2002) and Tanisli and Kose (2013). According to Moyer and Milewicz (2002) teachers who are able to ask skilful questions can also analyze the depth of their students' thought better. This finding therefore implies that such teachers will not be able to teach mathematics effectively, as they are not able to evaluate the thinking process of their students in depth.

The study also revealed that more than half of the teachers were successful in predicting the common errors and misconceptions reported in the literature with regards to the second and third equations (see Table 1). In Tanisli and Kose's study, more than half of the participants predicted the common errors reported in the literature with respect to the second equation, while only 20% predicted the common errors in the literature with respect to the third equation. On the other hand, only 16% of the teachers predicted the common errors and misconceptions reported in the literature with respect to the fourth equation in this study, while 36% of the participants in Tanisli and Kose' study predicted these common errors and misconceptions. Furthermore, some other errors and misconceptions predicted by the teachers in this study were not predicted in previous studies (see Table 1).

Another important finding of the study was that the teachers themselves had difficulties in understanding the problem situations and also the students' solutions. For instance, in response to the problem situation involving 4n+7, the teachers made statements such as: *The student's idea is reasonable, in the sense that n should have a physical quantity or should have a definition* and *The student may be right since 4n+7 is not equated to zero or any number.* This is implies that the teachers matched the variable n with only a particular number, and hence had difficulty understanding the various uses of the variable concept. Similarly, some of the teachers' questions were irrelevant. Examples, with respect to the question concerning the height of John and Joan, questions such as: *What unit of length is used to measure John's height?, Is x the same as n? If so, x and n are unknown*. This is a revelation of the teachers' failure to understand the problem. And it shows the teachers' need for improvement in their subject-matter knowledge.

4. Conclusion

The inability of the teachers to discuss students' thinking process is regrettable, as this is an indicator to problem in the teaching and learning of mathematics. There can be no effective teaching when teachers lack the necessary knowledge required of them. Generally, the teachers displayed inadequate skills of asking questions that are investigative in nature. Instructional questions, the type which the teachers asked most, cannot identify students' errors and misconceptions or assess students' thinking process. The need for teachers to ask competent questions that would help in identifying students' errors and misconceptions is necessary. The failure of the teachers themselves to have difficulty in understanding the problem, even though the problems were elementary in nature, calls for serious concern. There is therefore, the need for training workshops and seminars to help improve the subject-matter knowledge of the teachers, as this would improve their art of questioning too.

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