

Teaching Permutation and Combination Using Play-way Method

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Abstract

Mathematics from ages have proved itself to be a fearful subject probably because of its high demand of great thinking capability. As if that is not enough, the poor presentation and unfriendly attitude of some teachers worsen the situation such that mathematics continues to attract the interest of very few people and the percentage of females among them is very insignificant. Majority of people studying mathematics resulted to it because they could not get admitted to the course of their choice. Permutation and combination is one of the topics in Mathematics that pose problems to students. In this paper, we explain how permutation and combination could be taught using play-way approach among other methods that could be used.

Keywords: Teaching, permutation, combination.

Introduction

We define effectiveness of teaching as the ability of the students to be able to successfully carry out activities on the treated subject matter. Note that what counts as effective depends on a variety of factors, including the particular learning goals of interest. To achieve this, a lot of things must be put into consideration. This includes: the use of relevant teaching aids, appropriate teaching methods, the use of Student centered approach, and a step by step (from known to unknown) approach in the delivery of prepared teaching procedure. It should be noted that no matter how effective a teaching method could be, if the technical know how of handling the method is lacking on the part of the teacher, the method may still prove not effective. So it is necessary that the teacher study how to handle a particular method he has chosen to employ. Moreover, it is believed by many that play-way method of teaching is only useful or applicable in the primary or kindergarten level of Education, Just as Fredrich Froebel advocated guided play as the best way for a child to learn. I observed that students performed better in the concept of permutation and combination when I adopted the play-way method. This confirmed the result of a study done at the University of Texas [1], that people remember: 10 percent of what they read; 20 percent of what they hear; 30 percent of what they see; 50 percent of what they see and hear; 70 percent of what they say; and 90 percent of what they do and say. It was also surprising to some group of secondary mathematics teachers at a workshop organized for them by the mathematics department Obafemi Awolowo University, Ile-Ife, Nigeria, when I demonstrated to them how some concepts in mathematics can be taught using play-way method. Play-way method involves act of seeing, hearing, saying and doing, which makes it easier to remember. So it could be a way of achieving effective teaching of certain concepts in mathematics. Permutation is an aspect of mathematics that may be confusing if not properly presented to the students. In this paper, we wish to demonstrate how play-way method could be used to teach the concept of permutation and combination in Secondary schools and even in tertiary institution. To teach a concept in mathematics and even other areas of learning, adequate preparation must be made. The preparations include: The concept to be taught, the objectives to be achieved, the category of students to be taught, the choice of instructional materials, as well as a step by step of presenting the concept in order to achieve the set objectives.

Teaching Procedure

Objectives

At the end of the lesson, the students should be able to:

- (1) Define permutation, combination, and fundamental counting principle.
- (2) Work exercises on permutation.
- (3) Work exercises on combination
- (4) Differentiate between permutation and combination questions.

Entry behaviour or previous knowledge: It is expected that the students have learnt the concept of factorial, so the teacher needs to test their entry knowledge by asking them some questions on the concept of factorial after which the teacher gives the definitions of the following:

Permutation

The number of ways that n elements can be arranged in order, is called a permutation of the elements.

Note: In permutation, every different ordering counts as a distinct permutation. For instance, the ordering (a, b, c, d, e) is distinct from (a, b, c, d, e) , etc. that is, order of arrangement matters in permutation.

Combination

The number of ways objects could be combined.
In combination, every different ordering does NOT count as a distinct combination. For instance, the committee $\{a, b, c\}$ is the same as the committee $\{c, a, b\}$ etc. that is order does not matter in combination.

Fundamental counting principle

If one event can occur in m ways and a second event can occur in n ways, then both events can occur in mxn ways, provided the outcome of the first event does not influence the outcome of the second event. This can be stated in another way as: If there are m roots of getting to a point B from a point A , and there are n roots of getting to a point C from B , then there are mxn roots of getting to the point C from A via B . For example, If there are 2 roots of getting to a point B from a point A , and there are 3 roots of getting to a point C from B , then there are $2 \times 3 = 6$ roots of getting to the point C from A via B . Now, we start by giving some examples on permutation.

Example 1

Suppose $n = 5$ people (a, b, c, d, e), are to occupy five chairs, for a meeting. How many such distinct sittings (permutations) are possible? The teacher asks the students to arrange 5 chairs and asked five of them to come for the acting. He poses the question to them, how many choices of sitting does the first person to come has. The first person has 5 choices of sitting and when he finally makes a choice of the chair on which he sits on, there will be 4 chairs left, the choice of sitting of the first person, which is 5 is written down. How many choices of sitting does the second person to come has? The second person has 4 choices of sitting and when he finally makes a choice of the chair on which he sits on, there will be 3 chairs left, the choice of sitting of the second person, which is 4 is written down. In the same way, the third, the fourth, and the fifth person to come will have 3; 2; 1 choice of sitting. Therefore, the number of arrangement of $n = 5$ people on five chairs is given by: $5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$ ways, using the fundamental counting principle. This can be translated as arranging 5 objects taking 5 at a time. Permutation of n objects taking n at a time is done in $n!$ ways

Example 2

Suppose this time we have 3 people to attend the meeting and there are five chairs at the venue of the meeting in how many ways can they sit ('permutation')? Following the procedure in the example 1, the 1st, 2nd and the 3rd person to come will have 5; 4; 3 choices of sitting. Therefore, the number of arrangement of $n = 3$ people on five chairs is given by: $5 \times 4 \times 3 = \frac{5!}{2!} = 60$ ways, using the fundamental counting principle. This can be translated as arranging 5 objects taking 3 at a time. Permutation of n objects taking r at a time is done in $\frac{n!}{(n-r)!}$ ways.

Example 3

How many license plates can be made if there are 3 numbers and 3 letters if
(i) Letter or number could be repeated?
(ii) No letter or number may be repeated?
(iii) The first number must not be zero and no letter or number may be repeated.

Solution

There are 10 numbers and 26 letters.

(i) Any of the 10 numbers could be chosen to be the first, so there 10 choices of the first number and since repetition is allowed that number could be placed among the remaining 9, giving the choice of the second number to be 10 and in the same way, the choice of the third number is 10 as well. Following the same procedure, the choice of the first second and the third letter will be 26; 26; 26. Therefore, the number of plates to be made using the counting principle is $10 \times 10 \times 10 \times 26 \times 26 \times 26$

(ii) In this case, there is no repetition; thus, when a choice of number of letter is made it cannot be placed back. Therefore, the students are to be guided to discover that the number of plates to be made is given by $10 \times 9 \times 8 \times 26 \times 25 \times 24$

(iii) In this case, the first number cannot be zero, so zero is removed from the numbers and there will be 9 choices of the first number, after the choice of the first number, the zero can now be put back, so that there will be 9; 8 choices of the second and the third numbers respectively. Thus, the number of plates to be made in this case is given by $9 \times 9 \times 8 \times 26 \times 25 \times 24$

Let us now give example on Combination. Remember that order does not matter in combination.

Combination

Example

Suppose that instead of arrangement (permutations), we wish to form committees (combinations) of $k = 3$ people from the original $n = 5$: How many such distinct committees are possible?

Solution

This time, the reasoning is a little subtler. From the previous calculation, we know that of permutations of $k = 3$ from $n = 5$ is equal to $\frac{5!}{2!} = 60$. But now, all the ordered permutations of any three people will collapse into one single unordered combination because they are all the same (and there are $3! = 6$ of them), e.g., a, b, c, as illustrated. So combinations of $k = 3$ from $n = 5$ is equal to $\frac{5!}{2!}$ divided by $3!$, i.e. $\frac{5!}{2!3!} = 10$

A set of r objects chosen from a set of n objects is called a combination of r objects chosen from n objects. The number of different combinations of r objects that may be chosen from n given objects is given by $C(n, r) = \frac{n!}{(n-r)!r!}$

Where $C(n, r)$ represents the number of different unordered sets of r objects that could be chosen from a set of n objects and this could be read as n combination r .

Exercise:

State whether each of the following questions is a permutation or combination problem and then answer the question.

- (i) You have 6 books to arrange on a shelf; in how many different ways can you arrange all the books?
- (ii) If you have 5 people, how many different photographs can you take with only 4 people in the picture?
Peter has seven compact discs that John would like to borrow for a party. He has agreed to let him take four of them. In how many different ways could John make his choice?
- (iii) You have 6 people in your club, how many ways can you choose President, Vice-President, and Secretary?
- (iv) There are 5 colors to choose from, how many unique 3 color fags can be made?
- (v) You need to pick three people to be in a committee. There are 7 people to pick from, how many different committees can you make?
- (vi) A four man bowling team is chosen from 5 people, how many different teams are there?
- (vii) A four man bowling team is chosen from 5 people, how many different teams are there?
- (viii) If you have five clean shirts and are going to pack two of them to go on a weekend trip, how many possibilities are there for the two that you select?
- (ix) A class consists of 14 boys and 17 girls. Four students from the class are to be selected to go on a trip.
 - (a) How many different possibilities are there for the 4 students selected to make the trip?
 - (b) If it has been decided that 2 boys and 2 girls will make the trip, then in how many 6 different ways could the 4 students be selected?

Conclusion

The teacher could allow students to come to the board one at a time to solve some of the problem, or allow them to solve some in their note book right there in the class while he or she moves round to check what they are doing and make corrections when necessary, while the rest should be a take home assignment.

Note: As many questions as possible could be given, so that the students have variety of experience of the application of permutation and combination.

References

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