Students' Learning Experiences When using a Dynamic Geometry Software Tool in a Geometry Lesson at Secondary School in Ethiopia

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Abstracts
Students learning experiences were investigated in geometry lesson when using Dynamic Geometry Software (DGS) tool in geometry learning in 25 Ethiopian secondary students. The research data were drawn from the used worksheets, classroom observations, results of pre- and post-test, a questionnaire and interview responses. I used GeoGebra as a DGS tool with research questions relating to the aspects of (1) motivation, (2) interactions and discussions, (3) student-centered learning, (4) conceptual understanding, and (5) Problem _solving strategies. The questions were embedded in an instructional research intervention. The intervention comprised the use of worksheets and applets we developed through GeoGebra. The results showed that the use of a DGS tool with the presence of other factors, such as group work and the use of worksheets brought about certain changes in students 'learning experiences' of the geometrical concepts. Students were well motivated, but discussion and interaction were limited (due to time limitations) and results on students 'conceptual understanding' and problem-solving strategies were only partly satisfactory, but improved during the intervention.
Keywords: Dynamic Geometry Software; GeoGebra; learning experiences; Conceptual Understanding; Problem – Solving Strategies

I.   INTRODUCTION

Within the realm of mathematics, geometry is the study of shapes and space [15] .To bring these objects into classroom, geometry is taught through a variety of representations, such as diagrams, schemes, drawings, and graphs. These representations are a contextual description of geometrical concepts or ideas and may support the process of conceptualization [16] . That is, the use of multiple representations facilitates students' development of geometrical concepts. Traditionally, geometry is taught and learned in a pencil and paper environment. Geometry textbooks at schools provide the above described illustrations. However, sometimes textbook-based illustrations may not be comprehensive, because they lack the visual description of a complete dynamic process needed for the construction of geometrical concepts. This incompleteness arises from the static feature of a textbook medium. In a textbook environment, a dynamic visualization of geometrical figures or shapes is left for an internal (mental) process. To be more explicit, students have to create dynamic geometrical constructions in their minds since a textbook depicts only ideal states of the figures or shapes. Such textbook descriptions do not explicitly picture the construction processes, through which the figures or shapes are idealized. The pencil and paper work only shows the result of the whole construction process, although this process is a product of mental performance.

Hence, creating correct geometrical constructions calls for a spatial imagination, which based on our own observation, turns out to be a hard experience particularly for low-achieving students in Ethiopia. Hence, since it is difficult for low-achieving students to create the required geometrical constructions, they may become demotivated in studying geometry. To supplement the pencil and paper medium in the teaching of geometry, in particular for the lesser motivated students, a new learning environment is proposed by [2] ; [3] ; [9] ;[11] ; and [14] They suggest that the use of technology promotes students' understanding of geometry and therefore, recommend a DGE for geometry teaching and learning. However, it is noted that very little is known about students' use of technology and their concept formation through the use of technology [9].

The central focus in most GDE-related research is on students' investigation of properties of geometrical concepts through the different types of dragging and students' explanations and justifications for geometric conjectures ([1]; [2]; [3]). In line with these studies, this research project will incorporate the use of DGS (GeoGebra) within a geometry class with the purpose of supporting students to develop deeper understanding of geometrical concepts. For this, the applets will be developed under the DGS (GeoGebra), which represent the construction of the intended geometrical concepts.

A.   *Statement of the problem*

Dynamic is understood as the opposite of static. Dynamic connotes motion, action and energy. Dynamic geometry is active, explorative geometry carried out with interactive computer software. It enables to visualize abstract geometrical concepts. Ref. [8] stress that dynamic geometry tools like the Geometer's Sketchpad, the Geometric Inventor, and Cabri offer more opportunities to construct and justify geometrical concepts than the pencil and

paper settings. According to them, a pencil and paper environment has a limited capacity in introducing a geometrical concept with an emphasis on its intrinsic properties. This insufficient feature of a pencil and paper medium causes the tendency in students to construct a limited concept image. Dynamic geometry thus patches up this insufficiency by providing students with the option of generating empirical evidence to progress from particular cases to the general case. In addition, a dynamic geometry medium plays an essential role in developing the proofs of geometrical conjectures [8].

In addition, dynamic geometry is a medium with which students are assumed to alleviate hard psychological experiences that are required for the geometrical constructions and manipulations with pencil and paper. This process is thus facilitated by the dynamic features of the geometry software. Further, the external experience supports the required internal processes needed for theoretical knowledge construction. Therefore, it is assumed that the successful integration of technology into mathematics education has the potential to bring about positive changes in the teaching and learning processes, in particular it combined with student-centered learning activities. In a DGE, students can be invited to develop deeper understanding on geometrical concepts and problem solving strategies informally ([7]; [8]). Also, in a DGE, students can be invited to have mutual communications and interactions [4].

In Ethiopia the mathematics curriculum includes the teaching of geometrical concepts. These are presented in textbooks through axioms, definitions, theorems and proofs. Often, the learning practices consist of the recitation of definitions, formulas and additionally, of solving routine exercises. Students are rarely encouraged to study the processes in which concepts and formulas are derived. Instead, the formulas are memorized with the aim of applying them directly, to solve typical exercises. Within the existing curriculum practice, one cannot expect a good performance from the students on exercises in which they have to justify the validity of given formulas or to solve conceptual problems. It is anticipated that this environment does not offer students the opportunity to deeply understand the introduced geometry concepts and formulas and thus, to solve conceptual problems differing from routine exercises. A conceptual understanding of geometry, that is an understanding based on insight, calls for mental imagination, since the proofs and derivations of formulas are based on the a conceptual understanding of geometry, that is an understanding based on insight, calls for mental imagination, since the proofs and derivations of formulas are based on the flexibility and generalizability of figures or shapes. Textbooks cannot visualize the dynamic nature of geometrical figures on paper. As a consequence, students are forced to mentally investigate the possible properties of geometrical objects without an external way to increase understanding of the related concepts, and therefore students often fail to develop insights into the taught concepts. Thus, internalizing the geometrical representations is a psychological challenge to students in the pencil and paper medium, which makes learning geometry difficult to many students. This problem remains persistent in a learning environment which lacks dynamic features that may facilitate the justification and validation of definitions, axioms and theorems in a perceptive manner.

A possible solution to the above described problem will be proposed by introducing a Dynamic Geometry Environment (DGE) to the teaching and learning of geometry. The Dynamic Geometry Software (DGS) could be incorporated into a regular geometry classroom in order to provide a dynamic as well as visual representation of geometry concepts in a physical sense. In the hands of the students, the DGS play a role of an intellectual tool for studying geometry objects empirically. The general objective of this research will be to explore students' learning experiences when using DGS in order to provide support for learning processes based on the geometry textbook and regular geometry classes. For this, we will design a short intervention, consisting of a series of DGS-based geometry lessons in order to explore the use of a DGS tool in the learning of geometry and its affect on students' learning experiences.

Nevertheless, the successful incorporation of DGS in the learning and teaching of geometry may differ, depending on the social and cultural domain. Therefore, we wanted to research the changes in a geometry class caused by the introduction of DGS under a social and cultural background that differed from previous research.

In this research project the traditional way of the teaching and learning of geometry were supplemented by a new approach, consisting in the use of a DGS tool along with working in small groups. The research goal then will be to explore the extent to which the use of a DGS tool combined with small groups, and how this implies students' learning experiences. It was assumed that a new approach could bring about positive changes to the learning of geometry. Unlike the traditional way of the teaching and learning of geometry, a new approach is believed to provide more offerings in increasing students' participation in whole class discussions, interactions and argumentations on theory construction, and in developing conceptual understanding, and problem solving strategies.

The effective role of the DGS-based lessons on students' learning experiences is assumed to be associated with the aspects of motivation, interactions and discussions, student-centered learning, conceptual understanding and problem solving strategies. The research questions were developed based on these aspects.

Therefore, each of the research questions below represents one of the aspects.
1. To what extent are students motivated to learn geometry with the support of DGS?

2. To what extent does the use of DGS increase students' participation in overall class discussions and interactions with each other and with a teacher?
3. In what ways does DGS provide support for student-centered learning activity in a geometry class?
4. To what extent does DGS amplify the shift from procedural to conceptual oriented understanding of geometry concepts?
5. To what extent does DGS have an effect on strategies developed by students in problems solving?

B.   *Objectives of the study*

The general objective of this research will be to investigate the practical changes that DGS could bring to students' learning experiences in geometry lessons. Our research interest focused on the changes that might occur relating to five aspects; (1) motivation, (2) discussions and interactions, (3) student-centered learning, (4) procedural versus conceptual understanding and (5) problem-solving strategies. Herewith, we are convinced that these aspects are not independent. Rather, they are believed to be interrelated and the consequences of change made in one aspect may affect the other. Hence, it is assumed that the DGS-based learning medium should provide support to each of these aspects.

In order to achieve this, the following specific objectives were formulated:

➢ Investigate to what extent are students motivated to learn geometry with the support of DGS,
➢ Explore the extent in which the use of DGS increase students' participation in overall class discussions and interactions with each other and with a teacher,
➢ Investigate ways DGS provide support for student-centered learning activity in a geometry class,
➢ Explore the extent in which DGS amplify the shift from procedural to conceptual oriented understanding of geometry concepts,
➢ Investigate the effect of DGS have on strategies developed by students in problems solving.

C.   *Significance of the Study*

The important role dynamic geometry tool in geometry is acknowledged by most educators and educationists so that learning geometry in a DGE can offer students opportunities to construct and manipulate geometrical figures and carry out empirical investigations. These activities are almost impossible in a static geometry environment [12].

Ref. [12] view, drawing refers to the material entity, while figuring refers to a theoretical object [8]. Ref. [8] made a clear distinction between drawing and figuring for the following reasons:

1. Some properties of a drawing can be irrelevant. For example, if a rhombus has been drawn as an instance of a parallelogram, then the equality of the sides is irrelevant.
2. The elements of the figure have a variability that is absent in the drawing. For example, a parallelogram has many drawings; some of them are squares, some of them are rhombuses, and some of them are rectangles.
3. A single drawing may represent different figures. For example, a drawing of a square might represent a square, a rectangle, a rhombus, a parallelogram, or a quadrilateral.

Hence, it is not possible to provide an adequate representation for all properties simultaneously in a pencil and paper environment. However, this is an easier task in a DGE.

Moreover, a DGE has a variety of tools that enable students to construct geometrical objects and visualize geometrical conjectures or ideas at a perceptive level. Also, the tools offer flexibility of the objects (the dragging tool). The flexibility of the geometrical construction grants students the opportunity to justify, validate or disprove conjectures or ideas, as well as to build conjectures based on empirical evidence. Thus, DGS is a learning medium which ensures a new learning setting and new interactions, because it includes unique features that support the learning of geometry. A DGS offers tools to manipulate objects in a physical sense, and subsequently, these tools turn into psychological artifacts.

The other significance can also lie on a Vygotskian view which aims at more effective education. A more effective education may be induced by the use of technology, as studied by a number of researchers ([2]; [11]). They conducted research concerning cognitive and psychological effects arising from the use of technology in education. Yet, the consequences of the use of technology in the classrooms may differ depending on the social settings, because of interactive factors relating to cultural, social and motivational features. The aim of the proposed research was to elicit possible practical effects of the use of a DGS tool on geometry learning in terms of improved motivation, active participation, conceptual understanding and problem solving strategies. In this research technology will be used to support the construction of theoretical knowledge. It is assumed within the research that a technology intervention in geometry education would increase students' motivation and support them in constructing concepts and ideas through empirical experience.

II. LITERATURE REVIEW
D. *Dynamic Geometry*

Dynamic is understood as the opposite of static. Dynamic connotes motion, action and energy. Dynamic geometry is active, explorative geometry carried out with interactive computer software. It enables to visualize abstract geometrical concepts. Ref. [8] stress that dynamic geometry tools like the Geometer"s Sketchpad, the Geometric Inventor, and Cabri offer more opportunities to construct and justify geometrical concepts than the pencil and paper settings. According to them, a pencil and paper environment has a limited capacity in introducing a geometrical concept with an emphasis on its intrinsic properties. This insufficient feature of a pencil and paper medium causes the tendency in students to construct a limited concept image. Dynamic geometry thus patches up this insufficiency by providing students with the option of generating empirical evidence to progress from particular cases to the general case. In addition, a dynamic geometry medium plays an essential role in developing the proofs of geometrical conjectures [8]. In the designed activities, usually, proving the validity of geometrical concepts by means of a dynamic geometry tool is realized through dragging the relevant points of the constructed objects towards a situation in which they satisfy predefined conditions. DGS enables the design of such activities in which students explore the relevant properties of the geometrical objects in order to construct a more appropriate concept image [8]. Hence, learning geometry in a DGE can offer students opportunities to construct and manipulate geometrical figures and carry out empirical investigations. These activities are almost impossible in a static geometry environment [12].

Hence, it is not possible to provide an adequate representation for all properties simultaneously in a pencil and paper environment. However, this is an easier task in a DGE. A DGE has a variety of tools that enable students to construct geometrical objects and visualize geometrical conjectures or ideas at a perceptive level. Also, the tools offer flexibility of the objects (the dragging tool). The flexibility of the geometrical construction grants students the opportunity to justify, validate or refute conjectures or ideas, as well as to build conjectures based on empirical evidence. Thus, DGS is a learning medium which ensures a new learning setting and new interactions, because it includes unique features that support the learning of geometry. A DGS offers tools to manipulate objects in a physical sense, and subsequently, these tools turn into psychological artifacts. A number of researchers ([1]; [3]) focused on the dragging modalities (different ways of dragging) provided by the tool. Clearly, this new learning medium provides tangible experiences to learners through physical interactions. This physical support to the emergence of mental processes are in line with a Vygotskian view [17], stating that the use of technology in education has a promising potential in the internalization process [2].

Based on the views discussed at this point, a series of activities was designed for a geometry class in this research project through the dynamic geometry tool GeoGebra in order to explore and construct the geometrical concepts. These activities were based on playing with the appropriate applets designed by me using GeoGebra. The applets were aimed to allow students to investigate the relevant properties of the geometrical objects in order to construct appropriate concept image and procedures. The underlying point to teach the intended geometrical concepts in a DGE was based on facilitating externalization of the representations of the concepts. Usually, such representations being implicitly described in the geometry textbook call for students to use mental performances. However, the dragging on the computer screen can facilitate the externalization of implicit ideas which become visible phenomena that can be shared and discussed [18].

In addition, dynamic geometry is a medium with which students are assumed to alleviate hard psychological experiences that are required for the geometrical constructions and manipulations with pencil and paper. This process is thus facilitated by the dynamic features of the geometry software. Further, the external experience supports the required internal processes needed for theoretical knowledge construction. Therefore, it is assumed that the successful integration of technology into mathematics education has the potential to bring about positive changes in the teaching and learning processes, in particular if combined with student-centered learning activities. In a DGE, students can be invited to develop deeper understanding on geometrical concepts and problem solving strategies informally [7].Also, in a DGE, students can be invited to have mutual communications and interactions [4].

Nevertheless, the successful incorporation of DGS in the learning and teaching of geometry may differ, depending on the social and cultural domain. Therefore, I wanted to research the changes in a geometry class caused by the introduction of DGS under a social and cultural background that differed from earlier research. The general objective of this master research project was to investigate the practical changes that DGS could bring to students" learning experiences in geometry lessons. My research interest focused on the changes that might occur relating to five aspects; (1) motivation, (2) discussions and interactions, (3) student-centered learning, (4) procedural versus conceptual understanding and (5) problem-solving strategies. Herewith, I am convinced that these aspects are not independent. Rather, they are believed to be interrelated and the consequences of change made in one aspect may affect the other. Hence, it was assumed that the DGS-based learning medium should provide support to each of these aspects.

E.   *Motivation*

Motivation was one of the aspects, which is important to students and teachers because of its affects on learning outcomes. Motivation is linked with the emotion which is manifested either in positive (interest, joy) or negative (frustration, anger) emotions depending whether the situation is in line with motivation or not [6]. It was supposed in the research that students would express positive emotions when working with computers in the classes. The value of these positive emotions was also added by employing student-centered group workings. Based on the computer supported student-centered instructions, it was assumed that students would be stimulated to interact with each other for discussing and sharing their ideas.

F.   *Discussions and Interactions*

This aspect was based on the small-group workings that students were invited to employ during the classroom activities within the research. Students in small groups (two and three) were supposed to demonstrate cooperative learning by sharing and discussing their ideas and views. According to the relevant literature review [5]; Working in small groups (pairs) plays an essential role in promoting students" achievements within heterogeneous classes. Accordingly, in the course of the intervention, it was desired to organize small groups of high and low achievers in order to provide the opportunity for them to learn from each other. By working collaboratively, low-achieving students may benefit from communicating with high-achievers. Further, in the course of social interactions based on sharing and discussing the visual phenomena appeared on computer screens, they come to better explore and understand the phenomena. Therefore, it was assumed in the research that computer-based instruction combined with working in pairs would increase both students" interactions with each other and their participation in class discussions. Also, according to [17]; a social environment plays an essential role in the development of individual thought. Therefore, through mediating the emergent phenomena during the classroom activities, students were supposed to promote a level of interactions and communications with each other. These interactions and communications in turn provide the internalization of a social/instructional environment [5].

G.   *Student-centered Learning*

This aspect manifested itself in the computer-based cooperative learning activities, in which the teacher interventions were reduced to a minimum level. It was assumed that students would feel more responsible for their learning. In a DGE it is believed that the use of computer technology provides the basis for the accomplishment of student-centered learning. According to [12]; the incorporation of technology into mathematics education changes the teaching system. All aspects in the classroom, such as the structure of activities and the content to be taught receive new shapes. This applies also to the DGS, acting as a mediator between students and content, and this mediation affects students" learning experience, in particular the interactions and the communication. Furthermore, students interact with the tools of the DGS and their activities result in representations to which they have to react. That is, interacting with DGS students receive feedback on the basis of which they make new interactions. Hence, this interaction-feedback cycle of working was assumed to provide support to student-centered learning activities. Furthermore, according to [4]; it is unlikely to think that the use of technology based student-centered activities alone will enhance performance and collaboration among students. Rather, in order for technological integration in the classroom to be effective, the emphasis on instructional design must be increased. For this, the design of worksheets and applets as instructional materials was applied in the research.

H.   *Conceptual versus Procedural Understanding*

Conceptual understanding was one of the research focuses in this project. According to [10], conceptual understanding is regarded as a key to grasping mathematical concepts and ideas. Conceptual understanding is an important strand in mathematical proficiency development. Students with conceptual understanding come to realize the interconnection between mathematical concepts and representations. Conceptual understanding thereby provides support for students to develop insights into mathematical procedures and ideas and to competently apply them in solving non-routine mathematical problems. Conceptual understanding assists students to acquire better competencies in formulating alternative solution methods and in connecting these methods with each other.

Based on the elaborated definition of conceptual understanding as a mathematical proficiency strand, I assumed that a DGS-based learning medium provides insightful experience for students in learning geometry concepts. Because students develop conceptual understanding of geometry ideas and procedures, they are expected to know in what ways these geometry procedures are deduced and how to apply them in solving geometry problems. Additionally, conceptual understanding, according to [10]; provides support to develop procedural fluency which refers to knowledge of procedures, knowledge of when and how to use them appropriately and competence in performing them accurately and flexibly. Procedural fluency alone, to my own

thinking, is not desirable, nor does it precede the former strand. That is, students without conceptual understanding may get better at performing procedures based on rote memorization in solving routine problems. However, when facing none-routine problems involving strategic skill, such students become at a loss to develop a new appropriate technique in solving them. Hence, conceptual understanding was also supposed to pave a way for developing problem solving strategies or strategic competencies as mentioned in the literature.

I.   *Problem-solving Strategies*

Ref. [10], viewed a strategic competency refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is also known as problem solving. Thus, students should know how to develop a variety of solution strategies as well as which strategies are useful for solving non-routine problems. Consequently, this strand is interconnected with conceptual understanding. In this sense, it was assumed that DGS-based learning might also support students to develop geometry problem solving strategies. Because students learn how geometry concepts and procedures are developed, they are expected to know where geometry procedures may appropriately be used or applied. As well, developing a variety of solution strategies requires that students be able to generalize the representations generated for equally structured problems. This generalizability then becomes a resource for students to develop appropriate strategies in solving specific geometry problems. In sum, I assumed that by learning geometry concepts in a DGE, students develop conceptual understanding of geometry concepts and procedures which in turn become a ground for students to develop appropriate problem solving strategies. This view is also in line with [17]; suggesting that students may be cognitively affected by the new learning resource. Because geometry was presented in visual and dynamic contexts, it was assumed that students were facilitated to develop conceptual understanding underlying the geometry procedures being taught, and accordingly, improve their problem-solving strategies [7].

III.   METHOD

J.   *Research Design*

The research was designed as a classroom intervention, which were supplementing the regular geometry lessons taught by the collaborative teacher. It is aligned with the curriculum topic (area and circumference of circles, arc length, regular polygons,   perimeter, radians and other related facts) and was carried out in the same weeks in which the regular, traditionally taught lessons took place.

K.   *Single - Subject Research Design*

We preferred Single - Subject Research Design for this research from the different form of experimental designs which is appropriate to achieve the intended objectives. Single - subject research designs have a long history in education and continue to grow in popularity. In this era of accountability, they allow practitioners and researchers to examine behavior and interventions in a relatively easy manner. The goal of single - subject research designs is to explain and understand how human behavior functions through systematic study. The historical roots of single - subject research are found in the field of experimental and behavioral psychology. Names like Ivan Pavlov, Edward L. Thorndike, John B. Watson, and B. F. Skinner should come to mind. It is important to note that their work formed the basis for   applied behavioral analysis, which has been associated with a type of true experimental design with a unique twist: the sample size is limited to one participant or a few participants who are treated as one group [13]. In single - subject designs, the participant serves as both the treatment and the control participant. You might be asking, how can one participant serve in both capacities? Good question! The researcher measures participant behavior repeatedly during at   least two different points in time, when a treatment is not present (the control condition) and again when a treatment is present (the treatment condition) [13].

The periods during which the treatment is not present are called   baseline periods, and the periods during which the treatment is given are called treatment periods. Baseline periods are essential to single - subject research.

The baseline period is used to determine the effectiveness of the independent variable (the intervention); therefore careful consideration must go into the collection of baseline data. Baselines allow the researcher to determine the patterns of behavior. Patterns of behavior often include the frequency, duration, and time in between behaviors. Therefore, single - subject researchers use a   repeated measure design. Collection of data points, not only during baseline but during intervention [13].

According to [13]; the goal of a baseline is to establish patterns of behavior to compare to intervention. Therefore, a baseline needs only be long enough to adequately sample this pattern. This judgment depends on the variability of behavior and the patterns of responding.

A critical component of single - subject research is to demonstrate a link or functional relationship between the independent and dependent variable. The researcher hopes to demonstrate that changes in the independent variable will result in changes in the dependent variable. In other words, the dependent variable is

dependent upon the independent variable. In order to establish this functional relationship, the researcher must control for any extraneous or outside variables that could affect performance of the dependent variable. Extraneous variables are threats to the internal validity of a study. Given that single - subject research studies are conducted in complex educational environments, these variables may pose a problem for the researcher. In order to minimize the potential threats to internal validity, the researcher uses a variety of experimental designs [13].

While there are many single - subject designs, the most commonly used designs are A - B - A   designs and multiple - baseline designs. In all single - subject designs, the A phase of the study represents a series of baseline measurements. The B phase of the study involves the measurements that occur during treatment. In addition, all single - subject designs include continuous measurement of behavior throughout all of the phases. So for this research we preferred A - B - A   designs to see the effect of GeoGerba on students learning experience.

The simplest single - subject research design is the A - B - A design. In this design, the researcher obtains baseline data (the first A phase), delivers the treatment (the B phase), and after withdrawing the treatment measures baseline data again (the second A phase). Using the example of time on task and rewards as an example, the researcher would collect data on the number of minutes the participant spends on the task to determine a baseline. The treatment phase or rewards would then be given as the child is engaged in a task.

L.   *Internal and External Validity of Single - Subject Designs*

As with the group experimental designs discussed earlier, single - subject designs are concerned with issues of internal and external validity. Internal validity (the degree to which changes in the participant occur as a result of the treatment) can be ensured in single - subject research if the researcher carefully obtains baseline data: measurements of behavior over a period of time and before the implementation of the treatment.

M.   *Selection of Software Tools*

There are a great number of dynamic software products available which serve the purpose of increasing dynamism of geometry education. For instance, Cabri, WinGeom, Euclide, Cindrella, and GeoGebra all have advantages for geometry education. Some like WinGeom and GeoGebra are free and can easily be downloaded from their official web sites. Despite some common commands like drawing and dragging, the tools have differences. For instance, unlike other tools, GeoGebra provides the option of making interactive applets. Further, because of its controllable dynamic features and free and technical accessibility, GeoGebra 3.0.0.0 is proposed for use in a few regular geometry classes.

N.   *GeoGebra*

For the research GeoGebra 3.0.0.0 the web http://www.geogebra.org/webstart was used. The software is technically available through the internet and can be installed independent of user platform. GeoGebra requires Java 1.4 which can also be downloaded from the web http://java.com/en/download/index.jsp.GeoGebra has a good option of dynamic manipulations with availability of a slider motion tool. This tool enables users to manually manipulate the drawn geometric objects and to monitor interactive changes. The software unites algebra, calculus and geometry subjects. Interchangeably, making algebraic and geometric representations in the same medium is the advantage of this tool. In addition, this software package has an option of applet construction which allows the author to determine the extent of interactivity for users in design time. Constructed applets with dynamic and visual representations thus become explorative sources to students.

O.   *Sampling*

In single - subject designs the sample size is limited to small number of participants who are treated as one group ([13]. Therefore 25 sample students were selected from grade ten. This group was served as both the treatment and the control participant which was based on the design selected. You might be asking, how can one participant serve in both capacities? The researcher measures participant behavior repeatedly during at least two different points in time, when a treatment was not present (the control condition) and again when a treatment was present (the treatment condition) [13]. One cooperative Mathematics teacher was also participated.

P.   *Data Collection Instruments*

Along with the designed instructional materials some research instruments were developed to evaluate the intervention. There were classroom observations through field notes, interviews with the cooperative teacher and with the students, a questionnaire and pre- and post-tests. Therefore the following instruments were used in the data collection: Observation of classroom activities; Interview with the cooperative teacher; Interview with students; Questionnaires; Pre and post tests for students.

Q.   *Method of Data Analysis*

We use qualitative, quantitative and inferential method of data analysis. But the strategy to answer the research questions were more based on making analysis of qualitative data collections from the appropriate data sources. In order to make the process of qualitative analysis easier the findings from the different data sources were first tabulated in the appropriate tables. In the analysis of the findings the relevant data from the different sources were then cross-referenced and appropriately combined in developing the answers to the research questions. Quantitative data analysis; percentage, mean, median were also used to make the analysis more elaborative. Finally paired t-test was used to see the significant effect of the training, researchers used two tests that the students perform before performing the training of the application of DGS and after introducing the application of DGS. The exam performs in two different categories.

1. Students construction skill of geometric figures and
2. Students understanding in theoretical part of geometric lessons before using DGS and   after using DGS.

The analysis of the data from the worksheets, questionnaire, interviews and the classroom observations serves to answer research questions 1, 2, and 3 about the motivation, interactions and discussions, and student-centered learning. In answering research questions 4 and 5 about the conceptual understanding and problem solving strategies, the analysis of the data from the pre- and post-tests, worksheets, and interviews were used.

IV.   RESULT

R.   *Lesson Report*

Observation made from the classroom events in the beginning, the lessons did not meet the expectations towards the effective use of a DGS tool. In the first lesson, the used learning materials (applets, worksheets) did not seem as helpful as they did in the latest. The lesson halfway the intervention concerning the radian was not seem successful. Also, the time allocated for whole class discussions was not sufficient at all. In all, not many students appeared to benefit from the materials in the lessons, although there was by and large increasing benefit toward the end of the intervention. The researchers assume two reasons for this. The first reason may be that learning geometric concepts in a DGS-supported student-centered medium or any related software's was a new experience for the students. That is, they might need a certain period of time to get accustomed to working in such environments. The second reason may be associated with the development of the instructional materials and the activities. That is, the used worksheets and applets could have been designed and developed more coherently had the students'' prior knowledge and experience been known in advance. In addition, the teaching intervention was intentionally limited in order to allow the students to take charge of their own learning. As a teacher, the researcher tried not to intervene much while the students were working with the materials. Whenever the class discussed, the researcher/teacher attempted to intervene in order to stop the students from wasting time on getting nowhere. The researcher interventions were not to reveal the correct answers, but to provide them with some clues to improve their investigations. This was obvious from the discussions during the fourth lesson on the radian. However, the researcher also had to reveal a direct answer in the last lesson in which the students were not able to progress because of a limit which they did not know yet. In this case, the researchers had to show them how to solve it so that they could continue with their investigation.

During the lesson activities, the students were encouraged to discuss their points with each other. At the end of each lesson, whole class discussions were initiated by the researchers/teachers. Indeed, not always was there enough time to extend the whole class discussions because of short lesson periods. For example, the lesson on the radian (the fourth lesson) was not clear to students due to the insufficient time for the discussion. During this discussion, I asked questions to see if the students had a conceptual understanding of it (if they could relate the radian to the degree and arc-length). They stuck with the definition of the radian as they memorized from the textbook. The researcher wanted to know if they could relate the central angle with the arc-length subtended by it. The common mistake was that they all perceived that the radian of an angle is equal to the subtended arc-length. This misconception emerged from the definition based on the unit circle. During the class discussions, based on my question they explored the relevant applet and found out that their perception was not correct.

Furthermore, a whole class discussion initiated at the end of the last lesson on the area of a circle enabled some students to develop conceptual understanding to some extent.

In sum, the effective role of a DGS tool in teaching and learning geometry seemed to be related to the effective role of class discussions and the support of the teacher. Whenever the class discussions were not sufficient, the students did not develop the expected understanding. On the other hand, they seemed to have a better grasp of the concepts when they had enough discussions and arguments.

S.   *Findings from the Pre-And Post-Tests*

Before the intervention, the students were asked to take a pre-test to measure their conceptual ability to solve geometric problems. The presented problems were different than the textbook exercises which usually involve a direct application of procedures. However, the problems developed for the pre-test required the students to

reason before applying a procedure. Intentionally, the pre-test was supposed to provide a support to the assessment of the difference between before and after the intended lessons. Accordingly, a post-test was applied right after the conducted lessons. The problems in the pre-test and post-test were equal, except for the first two problems which only differed in their numbers, but they had an identical structure.

The students were all present on the day of the pre-test. All but one student participated in the post-test. The score of each student on both tests were tabulated in one table in order to facilitate the comparison.

This enabled to assess the changes in the problem-solving strategies of the students and the support of DGS for solving problems. The tables below give the results of some students who have relatively distinctive responses of the pre- and post-tests.

TABLE I: <u>Student 1 score on pre-test and post-test</u>

N<u>O</u>.	Responses of pre-test	Responses of pre-test
1	D=50cm=0.5m $\Rightarrow$ 450:0.5=900 Answer is 900 times	D=40cm=0.4m $\Rightarrow$ 650:0.4=1625 Answer is 1625 times
2	L=85cm and D=5cm , 85:5=17 Answer is 17 times	L=70cm and D=3cm , 70:3=23.3 Answer is 23.3 times
3	No response	No response
4	$S_1=\Pi r^2$=3.14*100=314m^2 $S_2=\Pi r^2$=3.14*400=1256m^2 S= S_2- S_1 = 1256 – 314 = 942m2	No response
5	$$S_6 = \frac{1}{2} * 6 * r^2 * sin\frac{360}{6} = r^2\frac{\sqrt{3}}{2}$$ Answer is $3r^2\frac{\sqrt{3}}{2}$	$$n = 6, S_n = \frac{1}{2} * n * r^2 * sin\frac{360}{n},$$ $$S_6 = \frac{1}{2} * 6 * r^2 * sin\frac{360}{6} = 3r^2\frac{\sqrt{3}}{2}$$

TABLE II: Student 2 score on pre-test and post-test

N<u>O</u>.	Responses of pre-test	Responses of pre-test
1	D=50cm=0.5m $\Rightarrow$ 450:0.5=900 Answer is 900 times	D=40cm=0.4m $\Rightarrow$ 650:0.4=1625 Answer is 1625 times
2	L=85cm and D=5cm , 85:5=17 Answer is 17 times	L=70cm and D=3cm , 70:3=23.3 Answer is 23.3 times
3	No response	P_s =4a, $Dc = \frac{S_c}{\pi}$, $\frac{P_s}{D_c} = \frac{4a}{\frac{S_c}{\pi}} = \frac{4a\pi}{S_c}$
4	D_1=10m, r_1=5m and D_2=20m, r_2=10m $S_1= \pi r^2$=3.14*25=78.5m^2 $S_2= \pi r^2$=3.14*100=314m^2 S= S_2- S_1 = 314 -78.5 = 235.5m^2	D_1=10m, r_1=5m and D_2=20m, r_2=10m S_1=2 πr_1=2*3.14*5=31.4m S_2=2 πr_2=2*3.14*10=62.8m S= S_2- S_1 = 62.8 -31.4 = 31.4m
5	$$S_6 = \frac{1}{2} * 6 * sin\frac{360}{6}$$ $$S_6 = \frac{1}{2} * 6 * r^2 * sin\frac{360}{6} = 3r^2\frac{\sqrt{3}}{2}$$	$$n = 6, S_n = \frac{1}{2} * n * r^2 * sin\frac{360}{n},$$ $$S_6 = \frac{1}{2} * 6 * r^2 * sin\frac{360}{6} = 3r^2\frac{\sqrt{3}}{2}$$ $$S_n = \frac{2\pi}{2\alpha} * r^2 * sin\alpha, \alpha = \frac{360}{n}, n = \frac{2\pi}{\alpha}$$ $$S_n = \pi * r^2 * \frac{sin\alpha}{\alpha}, S_n = \pi r^2$$

TABLE II: Student 3 score on pre-test and post-test

NO.	Responses of pre-test	Responses of pre-test
1	D=50cm=0.5m $\Rightarrow$ 450:0.5=900 Answer is 900 times	D=40cm=0.4m $\Rightarrow$ 650:0.4=1625 Answer is 1625 times
2	L=85cm and D=5cm , 85:5=17 Answer is 17 times	L=70cm and D=3cm , 70:3=23.3 Answer is 23.3 times
3	No response	No response
4	$S_1=\Pi r^2=3.14*100=314m^2$ $S_2=\Pi r^2=3.14*400=1256m^2$ $S= S_2 - S_1 = 1256 -314 = 942m^2$	$D_1=10m, r_1=5m$ and $D_2=20m, r_2=10m$ $S_1=2\Pi r=3.14*5=31.4m$ $S_2=2\Pi r^2=3.14*10=42.8m$ $S= S_2 - S_1 =62.8 - 31.4 = 31.4m$
5	$$S_6 = \frac{1}{2} * 6 * r^2 * sin\frac{360}{6} = 3r^2\frac{\sqrt{3}}{2}$$ Answer is $3r^2\frac{\sqrt{3}}{2}$	$S = \Pi r^2$

T.   *Summary of Students Works on Problems on Pre and Post Test*

Generally, some of the students gave similar responses to the first and second problem of both the pre- and post-test. But there are some significant differences between students in the methods of solution used in both tests. The students were supposed to use the circumference of a circle as an input for solving the problems. Instead, they all performed straightforward arithmetic operations on the given numbers. This points out that the students did not develop a conceptual understanding regarding the circumference of a circle, from the regular lessons by the collaborative teacher and but some of the students improve this ideas using DGS-based lessons. As a consequence, the students did improve managing to solve the problems with respect to the circumference in the post-test. The third problem remains unanswered in the first test. Apparently, the students did not relate this problem to their classroom activities. This also indicates that they have not well understood the basis of π. However, in the class discussions it turned out that they knew about the ratio of a circumference of a circle to its diameter, that produces π from the textbook. Yet, the tests show some improvement, but still they have difficulty in constructing the number themselves. The students' responses to the fourth problem show that they did not conceptually understand the radian, either. This is obvious from the solutions they have developed in pre-test. In the pre-test most of the students confused it with the area. In the post-test they considered the circumference of a circle. Only one student attempted to apply the concept of radian to the correct measurement of the appropriate arc-lengths. In the final problem, however, two students achieved correct results thanks to the appropriate classroom activities. The distinctive results of pre- and post-tests point out that the students have managed to generalize their ideas in order to find out the area of a circle.

In related to test the significant difference of the training the researchers apply the paired t-test based on the student's score out of 15% before and after the training. The paired t-test is used to calculate the difference within each before-and-after pair of test scores and determines the mean of these score changes, and reports whether this mean of the differences is statistically significant. To test this, the researchers use the following hypotheses test:

H_0: μ_d = 0 (students score did not change after they completed the training of DGS)
H_1: $\mu_d \neq 0$ (students score change after they completed the training of DGS, that is either increased or decrease after the training)

Because the researcher recorded before-and-after measurements on each individual student's, all variation that exists between independent members of a sample is absent. Any remaining effect is largely due to the effect of the training related to DGS. If the paired t-test indicates the observed mean difference within pairs is statistically significant, the researchers conclude that the training related to DGS is effective.

Descriptive Statistics: pre test 15_1, post test 15_1
Variable   N   Mean SE Mean TrMean StDev CoefVar Minimum Median Maximum
pre test   25   8.620   0.356   8.630   1.781   20.66   5.000   9.000   12.000
post test   25   10.600   0.419   10.543   2.097   19.78   8.000   11.000   14.500

Paired T-Test and CI: post test, pre test
Paired T for post test - pre test

	N	Mean	StDev	SE Mean
post test	25	10.600	2.097	0.419
pre test	25	8.620	1.781	0.356
Difference	25	1.980	1.229	0.246

95% CI for mean difference: (1.473, 2.487)

T-Test of mean difference = 0 (vs not = 0): T-Value = 8.06 P-Value = 0.000

From the MINITAB 16 output the researcher gives the following interpretation about students score and its training.

The confidence interval for the mean difference between the post test and pre test does not include zero and its confidence interval contains only positive values, which suggests DGS training have a significant difference on students test score. The small p-value ($p = 0.00$) further suggests that the data are inconsistent with H0: $\mu_d = 0$, that is, the pre and post-test do not perform equally. Specifically, students after completing the training (post test   mean = 10.60) performed better than before the training score (pre test mean = 8.62) in terms of giving a correct response on related question. So, the results of the paired procedure led us to believe that the data are not consistent with H0 ($t = 8.06$; $p = 0.000$). And also the box plot shows the hypothesized value (that is H0: $\mu_d = 0$) have a significance difference between student score before and after the training of students score.
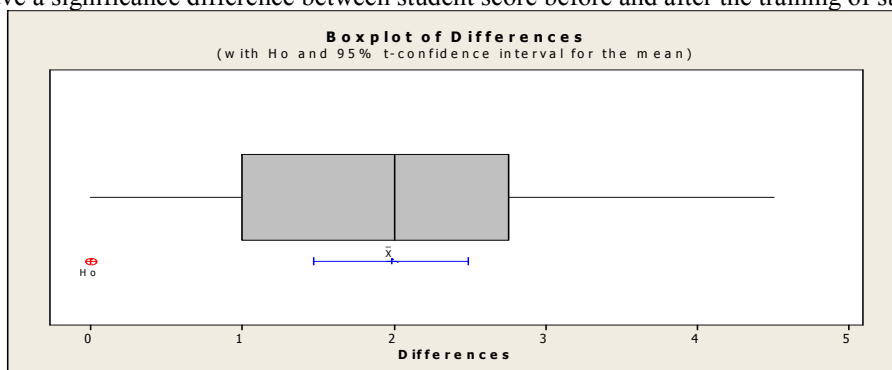

Fig. 1 Difference

U. *Findings from the Questionnaire*

Following the post-test, a questionnaire was administered in order to identify the students' impressions and attitudes towards the implemented lessons. The questionnaire was administered to twenty five students whose responses are tabulated. The quantitative data are provided in Table 4 (in Likert scale). The numbers in Table 4 show the number of responses. One indicates strongly disagree and 5 indicates strongly agree.

V. *Responses Based On the Questionnaire*

The results show that a majority of the students liked studying geometry lessons with the aid of the computer ($\mu=3.8$, $\sigma=0.82$). Only one student is disagreed studying the lesson with the aid of DGS. Most students expressed that they were helped by the computer to learn the taught concepts ($\mu=3.56$, $\sigma=1.01$). A majority of the students think that the textbook does not help a lot to learn geometry concepts and they prefer lessons with computers, not with the textbook ($\mu=3.48$, $\sigma=0.92$). A great number of students were not sure about the effective role of the computer in learning the radian, but from now on, average number of students want to learn all geometry lessons with computers ($\mu=3.32$, $\sigma=1.08$). But, regarding the effective role of the computer in learning geometry, the majority of the students have positive response ($\mu=3.68$, $\sigma=0.83$). Nevertheless, the test results also support this fact. In fact, although the students think they have understood the concept relating to the geometry lesson, the test results as well as the relevant classroom activities also support this.

In the figure also, students answer the question related to students motivation, most of them are liked to learn using the software and also the software support students understanding and they prefer related to the text book.
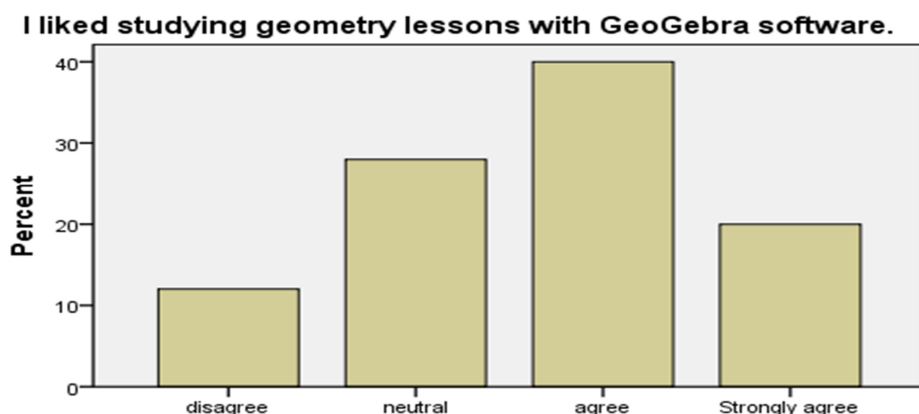
Fig.2: students Interest on lesson with GeoGerba Software

TABLE IV: student's response on Motivation Discussion and interaction after training of DGS

Research Question	Question		Strongly agree	agree	Neutral	disagree	Strongly disagree	Mean	Standard Deviation
Motivation	I liked studying geometry lessons with DGS software.	Freq	5	11	8	1	0	3.8	0.82
		%	20	44	32	4	0		
	The DGS helped me a lot to learn the geometry concepts taught.	Freq	4	11	5	5	0	3.56	1.01
		%	16	44	20	20	0		
	I prefer lessons with computers, not with the textbook.	Freq	1	15	5	3	1	3.48	0.92
		%	4	60	20	12	4		
	From now on, I want to learn all geometry lessons with computers	Freq	3	9	8	3	2	3.32	1.08
		%	12	36	32	12	8		
	Lessons with computers are Interesting to learn geometry.	Freq	4	11	8	2	0	3.68	0.85
		%	16	44	32	8	0		
Discussion and interaction	I interacted with my group mates or the teacher during the lessons.	Freq	7	9	4	3	2	3.64	1.25
		%	28	36	16	12	8		
	I discussed the result of our group work with the other group members.	Freq	6	10	4	5	0	3.68	1.06
		%	24	40	16	20	0		
	I asked questions of the teacher when I did not understand something.	Freq	13	5	7	0	0	4.24	0.87
		%	52	20	28	0	0		
	I interacted with my group mates or the teacher during the lessons.	Freq	10	5	8	2	0	3.92	1.03
		%	40	20	32	8	0		

From the above table 4, student's response related to classroom discussion and interaction, about 64 % students give a response that, I interacted with my group mates or the teacher during the lessons (μ=3.64, σ=1.25) and discussed the result of our group work with the other group members(μ=3.68, σ=1.06). In related to students and teachers interaction during the lesson, about 72% of students asked questions of the teacher when they did not understand something in the lesson this is about the mean student of μ=4.24 with standard deviation σ=0.87 and 60% of them interacted with the group mates or the teacher during the lessons in DGS classroom and also the figure related to students interaction supports almost more than half students interact with their group members and teachers and also ask lesson related questions for teachers.
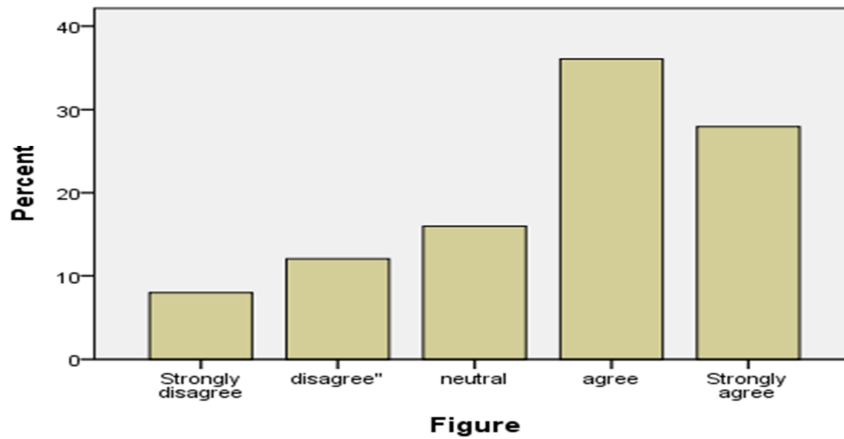
Fig. 3: Students Interaction

In related to student's relationship with teachers and peers, (asking questions, interaction and discussion) students answered they make me active participant in the class and am interested to know everything in the lesson.
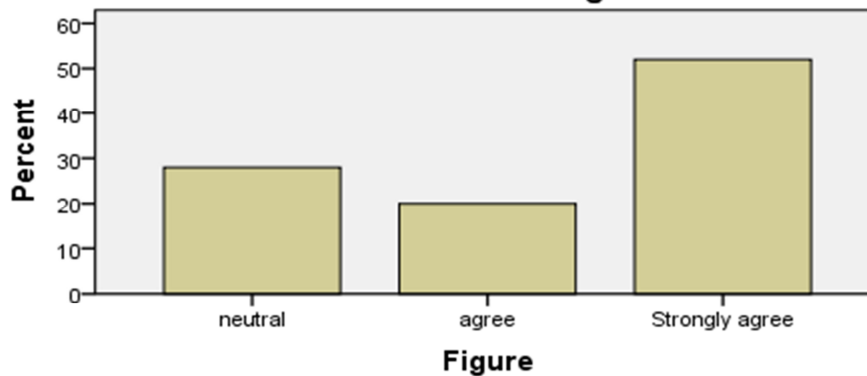


Fig. 4: Students Understanding

TABLE V: Student's response on Student centered Learning Activity after training of DGS

Research Question	Question		Strongly agree	agree	Neutral	disagree	Strongly disagree	Mean	Stand Dev.
Student centered Learning Activity	The lessons with the DGS help me to easily understand the concepts of the area of a regular polygon and the area of a circle independently and with the group members	**Freq**	3	9	4	7	2	3.16	1.21
		%	12	36	16	28	8		
	The lessons with the DGS help me to easily understand the concept of the circumference of a circle independently and with the group members	**Freq**	0	10	10	2	3	3.08	0.99
		%	0	40	40	8	12		
	The lessons with the DGS help me to understand what "radians" are, independently and with the group members	**Freq**	1	11	9	3	1	3.32	0.90
		%	4	44	36	12	4		
	The DGS helps me a lot to learn easily the geometry concepts.	**Freq**	7	8	4	4	2	3.56	1.29
		%	28	32	16	16	8		
	Worksheets in the DGS software helped us a lot to work.	**Freq**	5	8	4	5	3	3.28	1.34
		%	20	32	16	20	12		
	Applets in the DGS help to Students to learn the topics taught	**Freq**	3	10	5	5	2	3.28	1.17
		%	12	40	20	20	8		
	I am interested to explore and show easily the lessons for peers by using DGS	**Freq**	4	11	6	3	1	3.56	1.04
		%	16	44	24	12	4		

According to Table 5, out of 25 students 48% of them are agree and strongly agree that the lesson with DGS help them to easily understand the concepts of the area of a regular polygon and the area of a circle independently and with the group members and 36% of them are not agree this idea, the remaining 16% neutral from both conditions.

In relation to the concept of "radians" the lesson using DGS 48% of them supports the software help us to easily understand about radians but 16% of them not agree this idea and 36% of students remains neutral. On the other hand about 60% of student DGS help us to easily understand the concept of geometry but 24% of them not support these ideas and give the reason on the skill of the computer of students are not that much competent.

In general the lessons with the DGS are much powerful to make students active participant in the class room and apply student centered teaching approach and students cooperate to each other. The figure below also supports this idea.
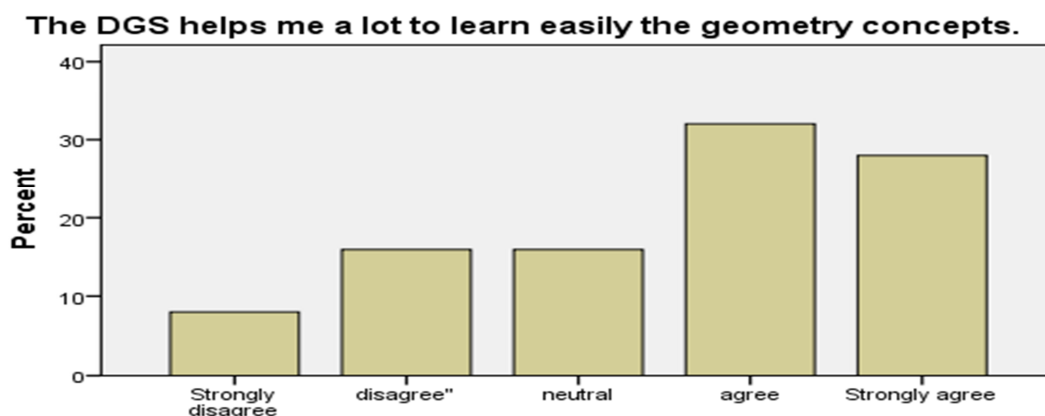
Fig. 5: Concept simplification

The findings from the questionnaire for qualitative responses of students for the questions, which

lesson(s) did you like mostly and dislike?, most students expressed positive remark on the computer based geometry lessons specially the lessons related to arithmetic's (concepts of a circumference and area of a circle) are easily understandable and easy to manipulated. On the other side few students give the comment related to the construction lesson, they need computer skill to control and develop geometric figures and also computer supported lesson make students dependent with the software, nevertheless I like the lesson with DGS and am interested. In addition to application the computer supported learning has also had a positive effect on the students" interactions with each other as well as with the teacher" and increase active participation in the class. In general learning with DGS gives a good opportunity for cooperative learning.

W.   *Findings from the interviews*
1)   *Interview with the students*

At the end of the intervention an interview was conducted with the students. It was made with two selected students' one boy and a girl. The result of the interview indicates that most students liked the geometry lessons with the computer. They think that it helped them to interact with each other in order to discuss and share their ideas. Generally, the applets/work sheets helped them to learn the taught concepts. However, the students think that they were helpless without the guidance of a teacher. Hence, they suggest that the teacher should intervene more. In addition to these, the respondents give answer for the question, which lesson was the best in your opinion applying DGS are, the construction part is more interesting than the others and if you understand well the software makes your life easy. But some of them also comment as the software makes dependent.

The results of the test and classroom activities also show that not all of the students have developed the same degree of understanding regarding the concept of some lessons.

2)   *Interview with the cooperative teacher*

After the interview with the students, an interview was also held with the cooperative teacher. The cooperative teacher did not fully participate in the lessons. For this reason, the interview with him had a limited perspective on the intervention. His answers are only based on the short visits made during the lessons. Also, because he did not know much more about the computer technology, he was not confident in giving justified answers with regard to the role of the computers in students' learning to the level of our expectation . However, his answers indicate that the students appeared motivated when working with the computers and some concepts in geometry can be simplified using computer assistance. They interacted with each other more in order to discuss their views and ideas based on the visual geometrical illustrations. According to him, the applets helped the students to develop insights into the concepts and students also motivated to attend the lessons.

X.   *Discussion*

The aim was to study the students' new learning experiences of the geometrical concepts in the DGS-based learning environment. For this, we focused our attention on the five interrelated aspects; motivation, discussions and interactions, student-centered learning, conceptual understanding and problem-solving strategy. There was an interaction among these aspects, which was presumably triggered by the use of the DGS. The DGS as the central tool of the learning medium was assumed to have provided new learning experiences in the geometry lessons. Unlike earlier research ([1]; [2]; [3]; [12]) on the different types of dragging potentials of the DGS tool in geometry learning, the use of the DGS in this research was primarily based on the use of the pre-designed applets. The applets representing the geometrical concepts restricted the possible dragging interactions. These limited interactions reduced the arbitrary dragging and helped the students to pay attention to the externalization of the representations of the geometrical concepts. The limited interactions did not reduce the supposed efforts to externalize the implicit development of the geometrical concepts.

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