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Understand Addition through Modelling and Manipulation of Concrete Materials

Chengyong Poon* Keejiar Yeo Noor Azlan Ahmad Zanzali

Faculty of Education, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia

* E-mail of the corresponding author: cypoon2@live.utm.my

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Abstract

Manipulation of concrete materials was commonly employed to support students with learning difficulties in mathematics using prescriptive pedagogies. However, a behaviourist framework of learning may lead to a lack of conceptual understanding. A case study conducted at a suburban elementary school which involved a teacher and for nine-year-old students to develop student thinking, mathematical modelling was integrated with manipulation of concrete materials in a mathematics remediation classroom. Qualitative data was collected using observation, interview and students' work. The observation revealed that explicit instruction used in modelling activity hampered students in acquiring conceptual understanding and mathematical process skills. When modelling was initiated by the students, they showed improvement in both. Therefore, teachers should guide students with learning difficulties to participate in modelling activity rather than merely follow re-enact procedures.

Keywords: Mathematics learning difficulties, Manipulative, Modelling, Remediation

1. Introduction

Traditional responses from mathematics educators to remedial intervention for students with mathematics learning difficulties focus on the behaviorist framework of learning (Bryant, Bryant, Gersten, Scammacca & Chavez, 2008; Tournaki, 2003; Fuchs & Fuchs, 2001; Mercer and Miller, 1992). By providing explicit instruction, concrete-representation-abstract (CRA) sequence is used to teach struggling students to understand mathematical concepts, operations, and applications. Although the use of CRA sequence with scaffolding was found effective in improving basic skills of number sense, students might still face difficulties in retention of mathematical knowledge and skills over time, together with misconceptions in mathematics (Ketterlin-Geller, Chard & Fien, 2008). Misconceptions of students are likely the consequence of rote-learning as they are given instructions to develop basic arithmetic skills in absence of sense-making (Ma, 1999; Cawley & Parmar, 1992).

During explicit instruction, materials are commonly used to demonstrate procedures for students to re-enact. Over-reliance on prescriptive pedagogies and concrete materials might result in difficulties of students to make sense of mathematics and development of mental strategies (Moscardini, 2009). Use of concrete materials to build mathematical meaning is consistent with a constructivist approach when it is used by students to make sense of problems (Slavin, 2009). Based on observational data of student learning, Moscardini (2009) found that children with moderate learning difficulties were able to use materials in a sense-making way. However, discussions between the author and the participating teachers showed that the teachers used concrete materials to demonstrate procedures for students to practice.

In a study conducted by Ketterline-Geller et al. (2008), the effects of two supplemental interventions on mathematics achievement of low-performing students were examined. Knowing Math intervention was designed to re-teach fundamental math concepts and principles using a conceptual approach to instruction and student think-aloud. In another invention, Extended Core, teachers provide extended time and follow a format based on systematic and explicit instruction of material presented in the core curriculum. The authors found that the students in the Extended Core group performed better than other students but the findings were not significant. Due to the limitations of that study, causal interpretations of results were also not justified as the teaching and learning processes were not studied. The alignment between student characteristics and intervention features was not investigated too.

Explicit instruction might discourage students from sense-making and reasoning, and thus hamper them from gaining conceptual understanding of mathematics. Instead, remedial interventions should be designed to encourage students develop their understanding of the relationships among mathematical ideas, and the connections between mathematics and reality. To obtain an in-depth understanding of their learning and the instruction, the teaching and learning processes in remedial interventions should be investigated.

2. Instructional Approach

We planned instructional activities to help struggling students to understand meaning of addition, and the connection between this number operation and real-life problems. Direct modelling was integrated with manipulation of concrete materials to enable the student make sense and reason.

2.1 Mathematical Knowledge

In the view of constructivist advocates, knowledge should be constructed through active participation in learning activities (Slavin, 2009; O'Donnell, Reeve & Smith, 2007). Students should develop their conceptual understandings to become procedurally proficient as they have more cognitive resources to apply their knowledge and skills. As students are engaged in learning mathematics, some teachers tend to downplay the development of skill proficiency as the development of conceptual understanding is emphasized (Evans, 2007). In teaching and learning mathematics, both procedures and concepts are intertwined and necessary for expertise in mathematics (Reys, Lindquist, Lambdin &Smith, 2007). Students should be guided to make meaningful connections between them in order to let them learn mathematics with understanding. Mastery of procedural knowledge enables skilful application of rules or algorithms while conceptual understanding helps students to link mathematical ideas in networks of connected meanings. Besides, students can incorporate new information into these networks and identify relationships among different pieces of information (Reys et al., 2007). Hence, conceptual understanding requires students actively think about relationships and make connections. Students will learn when to use a procedure, how to do it, apply it in new situations, and judge if the results are reasonable. Learning environments should support students to make explicit links and gain "a balanced connection" (National Council of Teachers of Mathematics, 2000) between conceptual understanding and computational proficiency.

Reys et al. (2007) recommended the use of materials and place value to ensure that students do not just learn algorithmic procedures by rote, but they also learn with understanding. Materials could function as a bridge between real-life problem contexts and the abstract representation of mathematics. Thus, students must manipulate materials personally to construct an understanding of when and how an algorithm works. Algorithm for whole-number operation is based on the concept of place-value (Reys et al., 2007). Students should be guided to link the place-value concept directly with renaming ideas to help them develop algorithms for each operation.

2.2 Modelling

Modelling has its root in constructivism (Confrey, 2007). It might support central connections among disciplines and development of students' thinking. Greer and Verschaffel (2007), and Niss, Blum & Galbraith (2007), suggested that the process of modelling includes identifying the real world aspects, the mathematical domain and the correspondence, carrying out operations within the mathematical domain, interpreting result of those operations, and evaluating conclusions with regard to the real world domain. Niss et al. (2007) and Confrey (2007) recommended the use of mathematical modelling to support learning of mathematics. Simultaneously, learning of mathematics also could be used to develop students' competency in applying mathematics and making sense (Usiskin, 2007).

Usiskin (2007) recommended the use of mathematical modelling of addition in elementary arithmetic to answer counting problems involving small whole numbers before it is applied to situations with large numbers. However, teachers need to consider the language used during modelling and make students recognize the limitations of each model. The purpose is to avoid students applying an incorrect model at a given problem context and situation.

To help struggling students establish connection between conceptual understanding and procedural knowledge of addition, they need to build a part-part-whole schema for numbers (Resnick, 1989; Van de Walle, 2001). Cathcart, Pothier, Vance & Bezuk (2011) suggested the use of join model prior to the number-line model in developing students' meaning for addition. According to Reys et al. (2007) and Cathcart et al. (2011), the number-line model as a semi-concrete model is often a difficult model for children to understand and thus should not be the first model used to represent addition. In this research, participating students were assisted to transfer from a concrete model to a more abstract model such as the number-line model as they already had sufficient understanding of the join model.

2.3 Manipulation of Materials

A wide range of daily life objects could be used to introduce problems to students and as a concrete visual tool to useful mathematics (Alsina, 2007). During direct modelling, concrete materials are used to represent a problem (Cathcart et al., 2011). It assists students to generate understanding of addition when they are required to solve problems using a model of addition.

Manipulative materials could function as a link between a problem context and the abstract representation of mathematical ideas (Cathcart et al., 2011; Reys et al., 2007) as it helps students to recognize those abstract representations through actions upon objects. However, adequate time should be provided for students to manipulate the materials and reflect on the process of manipulation (Thompson, 1991) as this process might increase the cognitive load of students who are not used to hands-on activities (Mayer and Wittrock, 2006). Teachers must be aware that materials themselves carry no actual mathematical information (Moscardini, 2009; Reys et al., 2007; Thompson 1994). Hence, in the process of manipulating materials, students should be consciously encouraged to develop their understanding of the relationships between the materials manipulation process and the abstract representation of mathematical ideas (Moscardini, 2009).

3. Research Purpose

The purpose of this qualitative research was to understand the teaching and learning processes of a mathematics remediation classroom that applied direct modelling of addition through manipulation of materials. In short, this research was carried out to investigate

i) behaviours of a mathematics remediation teacher in using modelling and manipulation of materials to help students generate understanding of addition and solve problems, and

ii) behaviours of students with mathematics learning difficulties in using modelling and manipulation of materials to generate understanding of addition and solve problems.

Generally, this research was attempted to provide an alternative instructional approach to overcome the mathematical difficulties in acquisition of mathematical knowledge and process skills. Hence, behaviours of participants during modelling and manipulation of materials were studied.

4. Method

4.1 Research Design

As this research was carried out to investigate the teaching and learning process in a mathematics remediation classroom, a case study research design (Creswell, 2008; Merriam, 1998) was applied. It enabled us in generating an in-depth understanding the behaviours of the participating teacher and students during the process of modelling and manipulation of materials.

4.2 Setting and Participants

A teacher who was officially appointed as a remediation program teacher at a suburban elementary school was selected for this research. This school was located at a Malay village and hence most of the students were from the village. The teacher was a Malay man from an urban area who was posted to this school. He had six years of experiences in teaching remediation program students. Four students were selected to participate in this research after administration of a screening test and a diagnostic test. These students were all from the village. All of them were in the third year of their schooling and had mastered basic skill of whole number including counting but still needed remedial intervention in improving basic skills of whole number addition. They were familiar with the join model of addition but all of them had no idea of the number-line model. In solving word problems which were explained to them orally, all of them tried to find the answer by simply performing any arithmetic operation on the numbers appeared in each question.

4.3 Collection and Analysis of Data

As we intended to understand the perception and behaviours of the participating teacher and students during the teaching and learning process, we used classroom observation, interview with the participating teacher and students, and the work of students to gather qualitative data. Data collected from classroom observations was recorded in the form of video clips and analyzed using a qualitative approach recommended by Creswell (2008) which involved transcribing, segmenting, coding, creating themes, and inter-relating themes. To obtain an in-depth understanding of the students' work, we compared it with the related video clips of classroom observations and interviews. It might enable us to understand the conditions under which the students produced their work.

4.3.1 Observations of Instructional Activities

In order to understand the teaching and learning process, we recorded each class session using a digital camera. During whole-class interactions, observations were focused on the behavior of the participating teacher and students. Three sessions were recorded and each session was approximately thirty minutes.

4.3.2 Interviews

After implementation of a task, an interview was conducted with every participant individually. Each interview session was approximately fifteen minutes. Interview with the teacher was intended to understand his behavior and perception of the teaching and learning process regarding the use of modelling and manipulation of materials in helping students generate understanding of addition. We also intended to understand his belief regarding his instructional approach during each session. Interviews with the students were intended to understand their behavior during implementation of each instructional activity. We also interviewed the students to find out their perceptions towards their teacher's instructional approaches.

4.3.3 Data Analysis

To understand the overall interactions and behaviors of participants during the teaching and learning process, we coded video and interview transcripts using coding schemes that were prepared based on our literature review as suggested by Miles and Huberman (1994). After all the transcripts were reviewed, the coding schemes were modified to account for the patterns that emerged from the data. Subsequently, the coding schemes were applied to all the transcripts.

After the coding process, we identified the corresponding video clips and work of students so that we could perform conversation analysis on the video clips and student work. Analyses at a micro level were carried out to understand the interactions between the teacher and students, and among the students. We used the analysis process which was used by Belland, Glazewski and Ertmer (2009) to understand the behavior and perception of the participants. During the process, we noted the context in which the teaching and learning process was carried out, participant facial expressions, and gestures that were visible among participants.

Assertions were developed based on themes generated from the analyses process (Belland et al., 2009). In the process, we checked the accuracy of the assertions against interviews, video clips and work of participants. Using member-checking technique, we checked the findings with the teacher if the descriptions and interpretations were accurate and reliable (Merriam, 1998). We also asked him whether the themes were accurate to include.

4.4 Validity and Reliability

Understanding is the core of this investigation. Hence, the criteria for trusting this research would be definitely different from that of the experimental study. Triangulation was used to improve the accuracy of the research findings. Evidences from different individuals, types of data and methods of data collection, could be used to support each other (Creswell, 2008). Hence, observational field notes, interviews and students' work were collected to enable us examine each information and find evidence to support a theme. As suggested by Merriam (1998), we aimed at providing "enough detailed description of the study's context to enable readers to compare the fit with their situations". A qualitative research aims to describe and explain human behaviours instead of confirming laws of those behaviours. Hence, to access the reliability of documents and personal accounts, we applied techniques recommended by Merriam (1998) such as using triangulation and describing the process of data collection, analysis and interpretation.

4.5 Instructional Activities

Three tasks were planned and involved the use of modelling and manipulation of concrete materials. These tasks were carried out in a mathematics remediation classroom and took approximately two hours. Prior to the implementation of every task, the teacher was given a training session so that he would try to minimize the use of explicit and direct instruction by gradually incorporating a student-centred approach. In the first task, the teacher explained a problem-context by manipulating concrete objects. Students were expected to represent the situation with a number sentence. Later, students were required to manipulate concrete objects to represent a situation described orally by their teacher, and write a related number sentence. The second task required these students to create a problem situation themselves. Then they were expected to represent their ideas using manipulation of concrete objects and a number sentence. Finally, each of them would be given an incomplete number sentence. By manipulating concrete objects and describing a problem situation, they should justify the answer to that incomplete number sentence.

5.0 Results

Research findings were presented based on the sequence in which instructional activities was implemented. Focus of Task 1 was teacher-directed modelling. As students were used to explicit instruction, this task was used to help the students understand addition and how modelling of manipulative should be performed. Task 2 involved students working in pairs to create a problem context and manipulate objects. They were expected to engage in discussion and decision-making while performing the modelling and manipulation. In Task 3, every student was required to manipulate objects based on an incomplete number sentence given to each of them. During pair work, we found that the more-able peer always dominated the task and left less or no opportunity for their less-able peer to perform. Hence, this task enabled every student to participate in the sense-making and hands-on activity. Besides, the teacher was allowed to use it as an informative testing tool.

5.1 Teacher-directed Modelling

Mr. Harris explained a context related to the join model of addition. The students were interested with the context of fishing by a river because this was their favourite leisure time activity. He instructed two students to manipulate fish to represent two groups of objects. After that, all the students wrote a number sentence which would represent the action of joining the two groups of fish. Mr. Harris kept giving instruction on what to write so that his students could write according to his instruction.

Next, the teacher explained a problem context based on the number-line model. Nine cards were arranged in a row between a picture of a 'river' and a toy house. Two toy models which represented a boy and a girl were put at the 'river'. By manipulating these objects as shown in Figure 1, Mr. Harris explained a situation and instructed his students to write a related number sentence. The number sentence was written in the form of addition with a missing addend. After that, the students found the answer (missing addend) by counting cards in that setting. To assess Fatimah's understanding, Mr. Harris described another similar situation and manipulated the models. He asked Fatimah to find the answer by counting. Fatimah immediately wrote an incomplete number sentence: 4 + □ = 9. Without saying anything, Fatimah counted the cards between the toy tree and the 'house'. Mr. Harris wanted her to answer orally and thus she answered "five" in a very low voice. According to Mr. Harris, Fatimah disliked speaking to anyone. It was the culture of the village that "working is better than speaking". Fatimah could recognize the addends and sum in this situation. Hence, she was able to represent it with a correct number sentence. Her preferred mode of communication was writing instead of speaking.

Later, Mr. Harris asked his students to represent the situation in the "form of numbers". He described what should be done by asking them to 'draw a big box…plus…empty box…equals to'. Mr. Harris found that Farib had written '4 + □ 9 = 5'. Farib explained that he simply wrote following what he listened from Mr. Harris.

During the implementation of this task, Mr. Harris controlled the teaching and learning process step-by-step. The students were not encouraged to think and communicate mathematical ideas among themselves. He told us that the participating students were excited about the toys. If they were allowed to 'play with the toys', the task would not be completed. In his opinion, a teacher-controlled approach in modelling a real-world situation would be more effective to help his student gain conceptual understanding of the number-line model.

5.2 Creating a Problem Situation and Manipulating Objects

Farib and Hafiz arranged the cards of 'river' and 'house' following the previous example. After that, Farib moved a toy model of a boy across two cards and put it between the second and the third cards. Mr. Harris asked him how far the boy had walked. Surprisingly, Farib answered "three kilometres". Nazrah told Mr. Harris that the answer should be "two" but her teacher did not response. After thinking for a short while, Farib and Hafiz agreed with the answer and continued to move the toy model towards a tree which was put between the fifth and sixth cards. Farib and Hafiz could not explain the situation that they had modelled. Hence, Mr. Harris tried to guide them.

Mr. Harris : How many more kilometres did he walk?

Farib : Four kilometres!

Mr. Harris : Where was he just now?

Nazrah : Two kilometres.

Hafiz put the boy model back to the previous position and Mr. Harris repeated the questions. As Mr. Harris realized that his students had difficulties understanding the number-line model, he pointed to the boy model and the tree again. Hafiz moved the toy model towards the tree and Mr. Harris put a pen to indicate the distance walked by the boy. However, Farib and Nazrah still insisted that the distance should be 'four kilometres'.

As Mr. Harris did not ask Farib to justify his answer, an interview with Farib was conducted after that. Farib explained that he assumed the boy was standing on the third card as shown in Figure 2 although the location of the boy was between the second and the third cards. When Nazrah told the class that the answer was "two kilometres", he thought he should assume that the boy was standing on the second card. After moving the model across another three cards and placing it between the fifth and sixth cards, Farib assumed that the boy was standing on the second card and was moved to the fifth card as shown in Figure 3. Hence, he counted the number of cards by referring to the second, third, fourth, and fifth cards. His experience in the first case had caused his misconceptions in counting the distance and the position in the number-line model. As Mr. Harris did not prompt to understand his thinking but merely emphasizing the correct answer, Farib's misconception caused him to make another mistake after that.

Although Nazrah seemed to understand the number-line model at first but her answer to the second position of that boy also indicated that she had misconception in using this model. In an interview with Nazrah after the activity, she explained that she could see that the boy had been moved across two cards and thus the answer was obviously two kilometres. When the boy was moved from the border of the second and third cards to the border of the fifth and sixth cards, she counted the distance by "two, three, four, and five", and thus there were four cards counted.

5.3 Student Manipulation of Objects Based on incomplete Number Sentence

Nazrah was given a card showing '3 + 3'. She put the toy model of a boy on the first card, followed by the second and third card. Hafiz realized that her demonstration was wrong and thus he said "That's only two." Nazrah took the model and put it on the fourth card. Realizing that Nazrah had difficulty in understanding number-line model, Mr. Harris explained the differences of putting the model between the third and fourth cards, and on the third or fourth card.

When Fatimah was given the question '4 + □ = 7', immediately she used her fingers to start counting. Mr. Harris stopped her and asked her to manipulate the objects. However, she kept quiet and refused to do it. Hence, Mr. Harris manipulated the toy model and guided Fatimah to simply tell the location of the model. After telling the location of 'four kilometres' and 'seven kilometres', she still used mental strategy to find the answer. She explained to us that she was counting her fingers mentally without referring to the manipulation of objects. Obviously, she was used to finding answer to an arithmetic problem using counting technique and without connecting it to real situation.

Hafiz and Farib were also able to manipulate objects and model a situation correctly for the question they each received. However, they had difficulties in explaining the situation and the actions. They told us that they were used to give short answer to questions of their teachers. Justifying solutions and communicating mathematical ideas were difficult for them.

6.0 Discussion

Generally, we found that modelling and manipulation of concrete materials that students were passive in learning in an environment which was teacher-directed. They were passive in the construction of knowledge and mathematical processes. When the students were encouraged in participating in modelling and manipulation of materials, they were prompted to engage in sense-making and hands-on activities. In the teaching and learning process, student misconceptions were identified and thus follow-up actions were enabled. The students were also offered opportunities to practice mathematical process skills such as communication, making connection and reasoning.

6.1 Difficulties in Understanding Number-line Model of Addition

Join model of addition was the first model used in our instructional activity. Cathcart et al. (2011) recommended that this model should be used to introduce mathematical ideas of addition to early grade students as manipulation of countable objects might be easier for students to understand. As a semi-concrete model, number-line model of addition is often introduced after students are familiar with the join model. We found that our participating students often 'changed' the number-line model to join model in making connection between a situation and its related number sentence. For example, although Farib had manipulated the objects in Task 2 correctly, he decided the position and distance of the objects based on a join model. Even if Nazrah had stated the distance correctly, we found out that she also interpreted the number-line model based on the join model when she was asked to model a situation in Task 3. Mr. Harris had used a teacher-demonstration approach to explain the number-line model of addition during Task 1. Students were not required to participate in any hands-on activity or discussion. Therefore, their learning was passive when knowledge was not constructed through concrete experiences. These examples indicate that students must be offered a lot of opportunities to model addition using manipulative rather than a follower in re-enacting procedures which are demonstrated by the teacher. As a concrete-operational learner, these students should be involved in physical and mental experiences in order to construct knowledge meaningfully (Slavin, 2009). Through reasoning and actions which were initiated by the students themselves, they could construct knowledge actively and meaningfully.

Mr. Harris believed that the use of a behaviourist approach in the mathematics remediation classroom would benefit students' learning. His purpose of directing the teaching and learning process during implementation of Task 1 was supported by the advocates of behaviourist approach. For example, when Fatimah was given a problem to assess her understanding but Mr. Harris did not stimulate her to engage in active thinking. Moreover, the question posed was similar to the previous example and explained with manipulation of objects by the teacher. Fatimah only needed to modify the answer of the previous example and found the answer by simply counting cards. According to Joyce, Weil, and Calhoun (2009), student learning time and achievement success rate should be maximized in a behaviourist framework of learning. Regarding the fact that his students were merely following his instruction in performing the procedures rather than making sense of mathematics, Mr. Harris assumed that involving students in hands-on activity without control by teacher would be wasting time. He preferred to maximize their learning time and success rate in performance. As a result, modelling and manipulation of concrete materials were used as a tool to explain mathematical ideas and problem-solving. Instructional activity of Task 1 was teacher-directed and thus the learning was passive. Students were not required to make sense to gain conceptual understanding of addition. The perception and practice of Mr. Harris had produced a passive mode of learning in the remediation classroom. Students received knowledge which was delivered to them in a passive learning environment although instructional activities involved modelling and manipulative.

Apart from the issue of instructional approach, Mr. Harris did not consider the importance of language when he explained the number-line model by manipulating objects. His actions were not described clearly and that led to misunderstanding of his students. He also did not encourage and prompt his students to talk about mathematics. Lack of conversation and discussion during modelling using manipulative might be one of the factors which led to misconceptions of his students regarding the number-line model. All the students understood the number-line model based on their understanding of the join model. Besides, a student misunderstood Mr. Harris's instruction in writing a number sentence and thus wrote a number sentence which does not make sense. This evidence indicates that combination of a teacher-directed instruction and spoken of incomplete sentences could cause misconceptions and misunderstandings. Our findings confirm that language should be taken seriously during the modelling and manipulation process as recommended by Usiskin (2007).

Involving students with learning difficulties is important to help them gain conceptual understanding but feedback and follow-up actions should not be neglected. During the modelling activity in Task 2, a more student-centred approach of instruction was applied. Farib's misconception was identified but there was no follow-up action. Mr. Harris should have prompted Farib and Nazrah to explain their thinking process so that their misconception could be identified and corrected. An interview with Mr. Harris revealed that he considered the modelling activity as a tool to demonstrate meaning of addition and for students with learning difficulties to practice prescribed procedures. Response of Mr. Harris was consistent with findings of Moscardini (2009) where concrete materials were intended as artefacts to help pupils with moderate learning difficulties to practice rehearsed procedures rather than for investigating solutions. Hence, a modelling process could be passive and does not guarantee authentic problem solving or sense-making as suggested by Usiskin (2007). The outcome of using modelling depends on the instructional approach and strategies used by classroom teachers. Misconceptions of Farib and Nazrah could have been corrected earlier if Mr. Harris had taken initiative to prompt them in discussing their mathematical ideas regarding the number-line model. Therefore, in applying a student-centred instruction, it is important to constantly motivate students in sense-making and also support them with feedback and follow-up actions.

6.2 Improving Mathematical Process Skills

Generally, students participated in mathematical processes during the implementation of Task 2 and Task 3. On the contrary, they were passive in learning activities of Task 1 and thus did not practice mathematical process skills. Given a problem context and situations, students were supposed to solve problems during modelling activities and manipulation of objects. They needed to translate a real situation into mathematics and work out mathematically (Niss et al., 2007). Explicit instruction did not offer any opportunity for students to solve problems on their own initiative as work was instructed and controlled by their teacher during the first task. However, as they were guided by their teacher in subsequent tasks, they began to gain some experiences in problem-solving during hands-on activities. The students needed to understand their tasks and plan the modelling in order to link a real-world situation and its related mathematical domain, or vice versa. Next, they would have to carry out the modelling by manipulating objects and connected their work to a number sentence. The last step in the problem solving process, checking solution and connecting it to the real-world situation, were not emphasized. In short, the participating students were involved in solving problems and making connections.

In the process of modelling and manipulating objects, the students were required to plan their actions and thus involved in sense-making. When they were told that they had made mistakes, proactively they took their initiative to reflect on their solution. Consistent with findings from research of Moscardini (2009), our students had shown their ability to perform simple reasoning. They also could represent a real-world situation with a number sentence. Mr. Harris could involve them in representing a real-world situation in a variety of ways such as drawings and other contexts (Reys et al., 2007). Besides, the missing-addend number sentences could be related to subtraction too. They should be encouraged to create another problem context rather than suggesting a similar context to the previous example.

Our findings indicate that participating students were weak in their communication skills. As these students were also having learning difficulties in Malay language and English, they experienced difficulties in reading and writing. Hence, all the problems were explained by their teacher during instructional activities. Hafiz and Farib could not explain their modelling while Nazrah had problem explaining her justification. Fatimah was reluctant to speak and preferred writing her answer. All of them tended to simply gesture or give short answers to the questions of their teacher. In an environment which encourages students to work rather than talk, involving students in mathematical discussions could be challenging. This finding could be related to the effects of culture on learning. Previous studies found that students from Asian culture respond more positively to quiet, private environment (Borich, 2011). Some of our participating students were used to a passive learning environment which did not require them to talk about mathematical ideas and manipulate objects. Motivating them to perform the modelling and manipulation of objects was a challenging task. Yet, all the students were making slight improvement during implementation of Task 2 and Task 3.

A student-centred approach of instruction is needed for the mathematical processes to occur. Instructional activities involving modelling of mathematical ideas and manipulating objects could induce cooperative work and hands-on experiences. It is also critical to motivate students with mathematics learning difficulties using prompts and questions.

7.0 Limitation of the Study

In this study, we only managed to identify four participating students from the early grades of this school who had sufficient understanding and basic skills in whole number but still needed remediation in addition of whole numbers. All the four students were also having difficulties in reading and writing of Malay language but could understand simple Malay language. They could not understand English. Hence, instructional activities were carried out in Malay language and problem contexts were explained orally.

We focused on the mastery of knowledge and skills in Addition of Whole Number as the participating teacher and students needed to follow an official syllabus fixed for the Remediation Program. However, instructional activities and approaches could be decided by the teacher who was officially assigned to this program. Further investigation should be carried out to understand the teaching and learning process of using modelling and manipulative in other basic skills such as subtraction, multiplication, and division.

8.0 Conclusion

Integration of modelling and object manipulation could be used as a tool for teachers to explain mathematical ideas in a teacher-led discussion. However, a student-centred instruction should be applied to improve students' conceptual understanding and thus procedural knowledge. If students are merely required to re-enact procedures demonstrated by teacher in modelling and manipulation of materials, they might result in misconceptions. Hence, teachers must provide prompts and support to encourage students in explaining their understanding and engaging in mathematical processes. Apart from that, language used during instruction should be considered seriously. Other than acquiring conceptual understanding and procedural knowledge, modelling and manipulative could engage students in mathematical processes and thus help them to improve their mathematical process skills. The results provide evidence for the use of an approach which integrates modelling and manipulative in mathematics learning for students with learning difficulties.

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Chengyong Poon is a PhD student in the Faculty of Education, UTM Johor, Malaysia. She has a master degree in mathematics education. Currently, she is a doing her doctorate under a federal scholarship in the field of remedial education and educational psychology. Her research is mainly focusing on teaching and learning of mathematics for students with learning difficulties.

Keejiar Yeo, Assoc. Prof. Dr., is a lecturer in the Faculty of Education, UTM Johor. She has vast experience in teaching particularly in Educational Psychology and Malay Language. Her writings and research are mainly focusing on educational psychology, reading readiness, disabilities, teaching and learning.

Noor Azlan Ahmad Zanzali, Prof. Dr., is a lecturer in the Faculty of Education, UTM Johor, Malaysia. He has vast experience in teaching mathematics education and statistics both at the graduate and undergraduate levels. Currently is heading a research group that looks at issues in the assessment of mathematics skills and understanding.

Notes

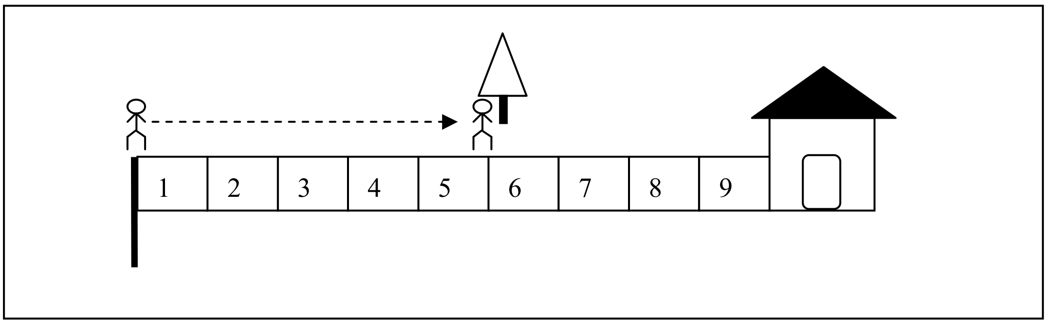

Figure 1. Modelling of a Problem by Mr. Harris

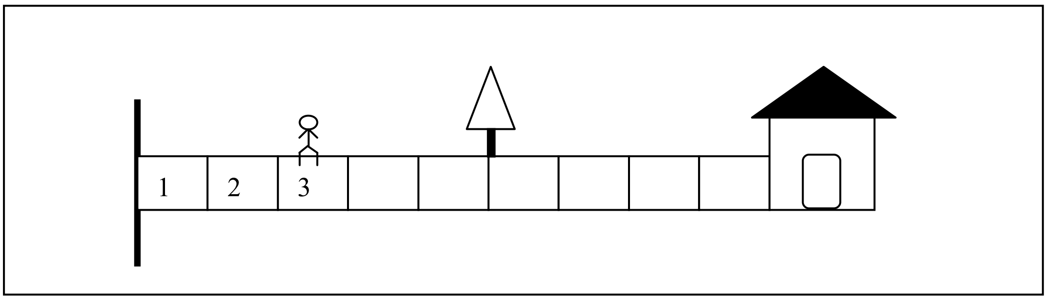

Figure 2. First Position of the Boy as Perceived by Farib

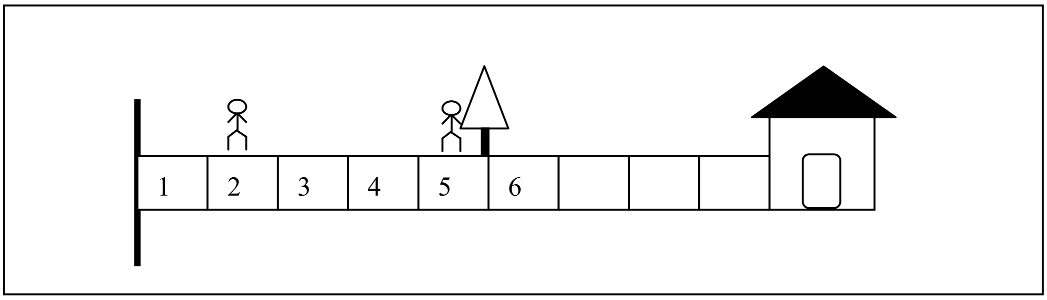

Figure 3. First and Second Positions of the Boy as Perceived by Farib