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Study on Mathematics Teacher Candidates' Ability to Form and Mathematically Define Geometric Surfaces

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Abstract
The aim of this study was to determine how successful elementary school teacher candidates are in forming and mathematically defining the surfaces of spheres, cones, cylinders and circular objects with the aid of various geometric elements. The study group consisted of a total of 40 teacher candidates in their fourth year attending a public university in Turkey during the 2014-2015 academic year. The study was performed using a qualitative study method, and the data were analyzed using descriptive analysis. The study results demonstrated that elementary school teacher candidates were more successful in forming and defining basic surfaces when they worked by interpreting real-life models and adapting them to mathematical contexts.
Keywords: Geometric Surfaces, Mathematics Teacher Candidates, Mathematic Education.

1. Introduction

The word geometry is formed by the combination of the words *geo* and *metry*, and means the "measurement of the earth." As such, geometry occupies an important place in daily life, the understanding of nature, mathematics, and other fields of science. The importance of geometric and spatial thinking can better be understood by the fact that we live in a three-dimensional world. In our daily lives, many problems are solved using spatial thinking and principles. For example, Battista (1990), described that one of the most important factors that affect geometric skills and the ability to solve geometric problems are "spatial abilities." And such spatial thinking abilities or skills can only be acquired through geometry classes. As previously described by Hoover (1996), individuals with a better grasp of spatial and geometric concepts exhibit more advanced understanding of mathematical concepts, and utilize their experiences and learning in geometry to further improve their spatial intuition. The literature describes spatial skills as being directly associated with success/performance in mathematics, and there are numerous studies reporting that students are more successful when they are able to visualize geometric and spatial concepts in their minds (Fennema & Sherman, 1977; Tartre, 1990; Pribyl & Bodner, 1987; Rafi, Samsudin & Ismail, 2006).

Many researchers consider that spatial thinking is, in general terms, an extension of geometric thinking, and that it can be described using mathematical modelling (McClurg, et al., 1997; Christou, et al. 2007). In the broadest terms, mathematical modelling can be defined as the process of expressing the relations between mathematical and non-mathematical events, cases and situations through the use of mathematics, and identifying the mathematical patterns between these events, cases and situations (Vershaffel, Greer & De Corte, 2002).

There are many studies in the literature aiming to form mathematical models based on real-world problems (Ortiz & Dos Santos, 2011; Kaiser & Schwarz, 2006; Sriraman, 2005; Maaß, 2004; Berry & Hauston, 1995). However, there are only a limited number of studies attempting to determine how teacher candidates use mathematical concepts to form new mathematical concepts through mathematical modelling (or by communicating mathematically). For this reason, there is a need for studies exploring the effectiveness with which individuals can form new mathematical concepts by using pre-existing ones. The main theme of this study is the mathematical modelling of what we shall designate throughout this manuscript as "basic surfaces," which include the surfaces of spheres, cones, cylinders and circular objects. The significance of this study rests on the fact that teacher candidates are individuals who, throughout their professional lives, will be teaching mathematical concepts to students within the frame of academic mathematics programs. In this study, our aim was to determine the degree to which elementary school mathematics teacher candidates are able to form surfaces with a given set of geometric elements and then mathematically model them. The results of this study will not only give the teacher candidates the opportunity to critically review their own level of knowledge, but may also serve as a guide for future studies on the subject.

2. Methods

The study was performed using a qualitative study model. The goal of qualitative studies is to obtain in-depth information instead of making generalizations, and to clearly and openly identify information regarding a case or individual.

The study aimed to observe the extent to which teacher candidates are able to mathematically inter-relate the geometric elements used for defining basic surfaces, and mathematically identify these geometric surfaces. To this end, the study was structured according to a case study design, which is a qualitative study method. According to Yin (1998), the case study's "a strategy that is preferred when the emphasis in on the questions 'why?' and 'how?', and the researcher has a very limited window to influence the real-life phenomena that are being studied." The main purpose of cases studies is to carefully and meticulously describe the cases/phenomena being studies, and the restructure it as necessary (Flick, 2009). According to Patton (1990), "Questions beginning with 'why?' are an excellent and logical tool use for demonstrating cause-and-effect relationships. Such 'why?' questions give the opportunity to predict beforehand the consequences of events. Moreover, these questions allow the analytical and deductive evaluation of the phenomena in question, and of how they are experienced, perceived and understood.

2.1. Study Group

The study group consisted of 40 fourth-year elementary school mathematics teacher candidates receiving education at a Turkish public university during the 2014-2015 academic year.

2.2. Data Collection Tool

The study first evaluated the subjects covered within the scope of the Analytical Geometry-I course – which is taken as part of the third-year student program – during the 2013-2014 academic year, and the extent to which the teacher candidates have grasped the course subject. This course involved the analytical evaluation of basic surfaces, and described how basic surfaces should be formed and analytical equations should be obtained within the frame of analytical thinking. As such, the course provided students the ability to analytically and mathematically model the geometric concepts they previously learned during their education. The study also assessed the teacher candidates' understanding of the mathematical definitions of basic surfaces, which were taught to them during the second (spring) semester of their second year. Nearly all of the teacher candidates correctly defined the basic surfaces. Within the scope of this study performed during the 2014-2015 academic year, the teacher candidates were given various geometric elements, and asked to form surfaces with them. They were then asked to define the same surfaces they formed, and to also explain in detail why they formed these shapes/surfaces as such. This allows us to observe and verify the approach under which the surface and its associated definition were formed.

2.3. Data Analysis and Interpretation

For qualitative data analysis purposes, this study employed the descriptive analysis method. The study data were organized according to the themes identified by the study questions (Miles & Huberman, 1994). A pre-questionnaire was administered prior to the origami activity to determine the initial knowledge of the students. In this process, the forms completed by the teacher candidates were first numbered, and then merged under common themes following the detailed evaluation of each form. The common themes that were identified were then tabulated according to their frequency. To increase the reliability of the study, the common themes were also separately evaluated by two academicians specializing in the field. The issues identified by these academicians were solved, and an agreement was reached with regards to the content of the common themes.

3. Results

The surfaces formed by the teacher candidates were divided into four main groups, and each surface was interpreted and evaluated separately. The obtained results are presented below. In addition, we also directly cite below some of the answers provided by the teacher candidates.

Sphere Surface

The teacher candidates were first given a point F fixed in space and a moving point A, and asked to form a surface with the aid of these points. They were then asked to define the surface they just formed. They were expected to form a mathematical relation between the two points to form a sphere surface. An evaluation of the study participants' responses revealed that only 25% of the teacher candidates knew that a fixed point and moving point could jointly form the surface of a sphere. The student (or teacher candidate) with code O_2 answered this study question by saying: *"If we keep the distance between the fixed point F and the moving point A as constant, the movement of point A in space across the planes x, y and z will create a sphere."* Student O_{17}, on the other hand, said *"We can obtain many forms by using the point locations of A. For example, if points F and A remain at an equal distance from each other, we can obtain a sphere."* It can be noted that these students formed the object (i.e. the sphere) by assuming that the two points remained at an equal distance of one another. Considering that the concept of a sphere is one of the first geometric concepts to be taught to teacher candidates, and that that sphere is used in many other courses during their education, the percentage of teacher candidates who were able to properly solve/answer this study question was found to be fairly low.

It was observed that 37.5% of the participating teacher candidates considered that it is not possible to create surfaces or forms with the geometric elements in question (points A and F). The teacher candidate O_{11}

stated that *"If F is a fixed point and A is a moving point, then it won't be possible to form an object with these two points. Because there are no points of intersection between these two points."* Teacher candidate O_{32} said *"These two point will give us the line |FA|; but the length of this line will vary according to the position of point A. For this reason, the two points will not form an object."* An evaluation of teacher candidates who provided such responses indicated that their incorrect answers stemmed largely from a failure to gain adequate geometric thinking skills. The basic problem they encountered in this context included the inability to make geometric inferences based on the provided data; the lack of spatial thinking skills; and the inability to distinguish the properties of fixed and moving points.

Among the teacher candidates, 32.5% described that the geometric elements in question (points A and F) can be used to form a circle. Teacher candidate O_{28} stated that *"We can obtain a circle. With the fixed point F, and an equidistant point A, the two points can draw a circle."* These teacher candidates provided answers that involved a single plane, while the study question specifically required them to think spatially (i.e. at a multi-planar level).

All of the teacher candidates were then asked to define and describe the surfaces they formed. The intention here was to assess the extent to which they could correctly use the data at hand to mathematically define the objects/surfaces they formed. It was observed that only 10% of the study participants were able to provide a correct mathematical definition for their objects; all of these study participants belonged to the group of teacher candidates who described that points A and F would form a sphere. And evaluation of the written answers provided by the teacher candidates showed that 62.5% attempted to provide a mathematical definition by identifying mathematical relationships between the provided and available data. However, the majority failed to established a correct mathematical relationship between the fixed and moving points. Only 20% of the teacher candidates attempted to provide a definition by using modelling; these teacher candidates associated the provided data with real-life models of spheres, and described that this could be used to define the sphere surface. It was noted that interpreting the data based on real-life models allowed them to identify the relevant mathematical relationships more easily.

The teacher candidate O_9 gave the following definition: *"If you plant a pin with a colored head onto a round object, such that the tip of the pin reaches its exact center, the tip of the pin can be considered as forming the fixed point, while the head of the pin will constitute the moving point. Together, they will form a sphere. In this context, a cluster of points lying at an equal distance from a fixed will be defined as a sphere."* It was a significant finding that all of the teacher candidates who provided a correct mathematical definition used such modelling approaches to provide a mathematical definition for the surface was significant. This suggests that it is a more effective approach form and define concepts through models.

Cone Surface

The teacher candidates were asked to form a surface by using a fixed point T, a fixed curve (c), and a moving line d, and to then define the surface they formed. Mathematically speaking, there are an infinite number of lines crossing a point. Among these lines, the ones intersecting with a curve will form the surface of a cone. The shape of the cone surface is determined mainly by the curve. An evaluation of the teacher candidates' responses indicated that 52.5% of them were of the opinion that the given data could not be used to form a surface, since there was no information regarding the geometric shape of curve (c). However, it is a common practice in mathematics courses to take a representative curve (as part of these expression) in order to solve a theorem or problem. These teacher candidates described that they could form a surface only after curve (c) is defined. Teacher candidate O_{23} stated that *"We cannot form a surface, because the curve remains undefined. If a circle was given instead, the lines passing through the center of the circle would form a surface."* It can be noted from this answer that the teacher candidate did not consider the fixed point T as the center of a circle in their attempt to form a surface. Teacher candidates of this category generally approached this problem from the standpoint of a single plane, and hence had difficulties in establishing relationships between the data.

Only 18% of the teacher candidates described that they could form a cone surface with the given data. Teacher candidate O_{31} described that *"The fixed point F and the fixed curve c will be equidistant at every point, and will form the surface of a cone together with the moving line d."*

It was observed during the study that only 15% of the teacher candidates could define the surface they formed as a cone surface when asked. Based on the definitions provided by the teacher candidates, it was determined that 12.5% could correctly describe the surface of a one by using real-life models. Teacher candidate O_5 said: *"The natives of the American continent formed conically-shaped tents (draws a figure). If we assume that the sticks supporting the tent are the moving line, that the ground is the curve of contact, and that the point of intersection of the sticks/lines at the top of the tent represent the fixed point, we will then obtain the surface of a cone. In space, the surface formed by a fixed point F, a fixed curve c and a set of moving lines d constitutes, and is defined as, the surface of a cone."* The teacher candidates who provided such model-based responses further described the characteristics of cone surface by emphasizing that the curve (c) would give the surface its shape; that this curve doesn't necessarily have to be a straight curve; and that the curve does not need to be enclosed as well.

Cylinder Surface
The teacher candidates were then asked to form a surface by using a fixed curve (c) in space, a fixed line d, and a moving line l, and to then define this surface. The teacher candidates were expected to mathematically establish the relation between the fixed line, fixed curve and moving point, and thereby to form a cylinder surface.

It was determined that 27.5% of the study participants formed a surface based on the movements of line l that intersects the curve (c), and which is perpendicular to the fixed line d. Teacher candidate O_{25} said: *"If the moving line intersects with the curve, while remaining parallel with the fixed line, it will form the surface of a cylinder."* It was observed that 57.5% of the teacher candidates could not answer this question correctly.

When the teacher candidates were asked to define the surfaces they formed, only 5% were able to do so correctly. Teacher candidate O_{33} gave the following answer: *"We can think of a circular curtain hanging downwards, with a seam on one of the sides of the curtain representing the fixed line, and the bottom edges of curtain representing a fixed curve. Thus, we can think of all seams, or threads, extending downwards in parallel to the fixed line, and leaning against the fixed curve. The surface formed by a set of lines parallel to a fixed line and leaning against a fixed curve will, by definition, be that of a cylinder surface."* Teacher candidate O_8, on the other hand, gave the following definition: *"This question reminded me of the meridians and parallels of the earth, with line d being like to equator. I can define this surface accordingly…"* While 27.5% of the teacher candidates were able to form cylinder surfaces, only 5% of them were able to define these surfaces correctly, and all of the teacher candidates had reached the correct definition through models.

Circular Surface
In the fourth exercise, the teacher candidates were given a fixed line and a moving curve, and asked once again to form a surface and to define the surface they created. The teacher candidates were expected to form and define a circular surface with the geometric elements. An evaluation of the study participants' written responses showed that only 32.5% were able to establish a mathematical relationship between the given elements to mathematically form a surface. Nearly 4% of the teacher candidates formed a surface by drawing the shape of a vase. On the other hand, 47.5% of the teacher candidates attempted, and failed, to form surfaces by randomly using additional points or lines.

When defining the surfaces, only 2.5% of the teacher candidates used real-life models to form and define circular objects. Teacher candidate O_{39} said: *"When making pottery, the potter uses his thumb to continuously draw a curve and shape the clay. Meanwhile, the potter's wheels keeps turning around its own axis. It can be noted that, while the axis remains fixed, the curve is in constant motion. Let's assume there is a fixed line l and a curve c in space. Revolving the line around this curve will result in a circular surface."* The percentage of teacher candidates who were able to correctly form the surface, as well as the percentage share of teacher candidates who were able to do so based on real-life models, highlighted the importance and role of such models in mathematical modelling.

An overall evaluation of the study results indicated that many of the teacher candidates had little or no understanding that they should use mathematics to describe the relationship between geometric elements, and that they did not pay attention to the level and dimension at which they were expected to solve the problem (i.e. three-dimensionally, as opposed to two-dimensionally or at a plane level). This appears to be the main reason for the difficulties they experienced in forming surfaces. On the other hand, teacher candidates who were able to form the surfaces correctly made use of real-life models that facilitated the task of forming and defining surfaces.

4. Discussion and Recommendations
Three-dimensional objects are included into the mathematics curricula in all countries across the world. The study of three-dimensional objects such as sphere, cones and cylinders are ultimately based on the study of geometric surfaces. For this reason, it is important for teacher candidates to have a good grasp of this fundamental subject which they will be teaching to their own students in the future. At the same, the study of these concepts also helps teacher candidates improve their ability to think and interpret these geometric concepts independently. As such, a thorough understanding of these concepts will enable teacher candidates to transfer geometric knowledge through individualized models suitable to their current teaching environments, rather than using pre-prepared information packages. By evaluating the teacher candidates' ability to form, draw and define basic surfaces, this study aimed to examine teacher candidates' level of understanding of these concepts.

By its very nature, mathematics not only requires a comprehensive understanding of its basic concepts, but also an understanding of how these concepts are inter-related in mathematical terms; for all individuals, learning these relationships is an essential part of mathematics education. The role of the teacher should be to help students grasp mathematical concepts and their characteristics. The students, on the other hand, should question how theorems and concepts can be used and inter-related when solving problems, and draw inferences accordingly. However, in an increasingly modern and globalized world, there are numerous factors that can distract students from focusing on the course subject even in the class environment. For this reason, there is need for an educational approach that would allow individuals to better focus on course materials, and also remind them of the concepts they are learning during the times they spend outside of class. Based on the observations and results of this study, it is possible to state that interpreting mathematics and its concepts based on real-life models (with which individuals can relate to) will contribute significantly to the development of students' mathematical thinking skills.

An evaluation of the literature reveals that there are numerous definitions for mathematical modelling (Galbraith & Catworthy, 1990; De Corte, Verschaffel & Greer, 2000; Chinnappan, 2010). There are also many studies demonstrating that teacher candidates who receive mathematical modelling courses are more successful in solving real-life mathematical problems, and emphasizing the importance of teaching mathematical modelling (Blum & Niss, 1991; Sriraman, 2005; Borromeo-Ferri & Blum, 2009; English, 2006; Taghi Mosleh & Jenaabadi, 2015). Studies also indicate that, when dealing with real-life problems, most students are not able to utilize their mathematical knowledge as effectively as desired (Arcavi, 2002; Busse, 2005; Carpenter et al., 1983). The present study has demonstrated that teacher candidates capable of properly communication mathematical concepts and inter-relations were better able to interpret real-life models, and to adapt these models to a mathematical context. Consequently, it would not be realistic to expect real-life problems to be solved effectively – and for mathematical modelling to be performed accurately – in the absence of proper mathematical communication. For this reason, new mathematical concepts should primarily be taught and interpreted within the scope of already known mathematical concepts, and it is important for fundamental concepts to be adapted to real-life problem by highlighting cause-and-effect relations.

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