

Theory of Distance Relativity of Fractal Dimensions

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Abstract

The velocity of a moving object is different when measured from a stationary frame of reference and on a moving frame of reference (see the famous train experiment and the Michelson-Morley experiment). Because velocity is relative to the frame of reference, so do the concepts of "distance" and "time". Thus, were born the concepts of relativistic mass, relativistic distance, and the notion of time dilation, which practically revolutionized Newton's classical Physics (Muller, General Theory of Relativity, 1958). In this paper, we investigate how the fractal dimension of the same natural geometric object changes relative to the distance from which a picture of the object is taken.

Keywords: Fractal dimension, Distance, Fractal

1. Introduction

The fractal dimension obtained by the box counting method for a given fractal object is defined as the ratio of the logarithm of the number of copies (m) divided by the logarithm of the scale ratio (r):

$$(1) \qquad \lambda = \frac{\log m}{\log r}$$

When this definition is implemented in computer programs, Equation (1) translates to:

$$(1) \quad \lambda = \frac{\log(\text{pixel size})}{\log(\text{no. of pixels})} \quad \text{(Sasake, 2012)}$$

What is clear from these definitions of fractal dimension is that the concept itself is a "relative concept", that is, the same object can have different fractal dimensions relative to the context in which the fractal dimension of an object is measured. Thus, an earthworm in still water will have a different fractal dimension in flowing water (Palmer, (1992). Benoit Mandelbrot (1967) illustrated this relativity of fractal dimension in his book Fractal: The Geometry of Nature. A ball of thread will look like a point (zero dimension) from afar, will consists of threads of dimension one from a closer view, a circular plate (dimension 2) from yet a closer view and again as a point up close. This phenomenon is a simple re-statement of the observation that objects appear less rugged from afar: mountains appear like triangular outlines when viewed several kilometres away.

For the same frame of reference, the fractal dimensions of geometric objects can be compared and analysed. Results of such analyses revealed several important findings in various fields: Kummel et al. (1987) found the food-search pattern of many organisms to be influenced by the fractal dimension of the environment; Palmer et al. (1992) computed the fractal dimensions of several leaves of forest trees and thereby accounted for the carbon sequestration property of the forest itself; seismic wave patterns and inter-event times of earthquake occurrences in Italy were studied by Macchiato et al.(2003). Of more recent use of fractal dimension, Barrera et al. (2013) used fractal analysis in determining authorship of questioned documents in forensic science; Relators (2013) exhibited the fractal dimensions of the patio-temporal distribution of the bombings and violence in Mindanao over a 30-year period.

This situation is very similar to Einstein's explanations of the Theory of Relativity. The velocity of a moving object is different when measured from a stationary frame of reference and on a moving frame of reference (see the famous train experiment and the Michelson-Morley experiment. Because velocity is relative to the frame of reference, so do the concepts of "distance" and "time". Thus, were born the concepts of relativistic mass, relativistic distance, and the notion of time dilation, which practically revolutionized Newton's classical Physics (Muller, General Theory of Relativity, 1958).

In this paper, we investigate how the fractal dimension of the same natural geometric object changes relative to the distance from which a picture of the object is taken. We shall refer to the results as Theory of Distance-Relative fractal dimension.

2. Methodology

Familiar fractal objects in nature were photographed using a mounted platform by a Canon 550-D 18-55mm lens set at ISO (auto): autofocus, auto-white balance with built-in flash. The objects considered as listed below together with their two-dimensional measurements:

Table 1: Fractal objects with their two-dimensional measurements

Object	Length/Major Axis	Width/Minor Axis	Euclidean shape
Rambutan (nephelium lapacceum)	5.2 cm	4.8 cm.	Small ellipse
Bitter gourd (momordica charantia)	35.6 cm	4.1 cm.	Larger ellipse
Cucumber (cucumis sativus)	17.6 cm	4.3 cm	Ellipse
Durian (dorio zibethinus)	28.0cm	18.0 cm	Larger ellipse
Eggplant leaf (solanum melogena)	21.4 cm	14.8 cm	Quadrilateral
Ilang-Ilang leaf (cananga odorata)	23.9 cm	8.6 cm	Quadrilateral
Jackfruit leaf (art carpus heterophylla)	16.9 cm	8.4 cm	Quadrilateral

The pictures were taken on a straight line measured 1', 3', 5', 7', 9', 11',13',15', 17' and 19' from the object mounted on the platform. The first four (4) objects are fruits or vegetables with either spherical or cylindrical shapes while the last three (3) objects are flat leaves from plants. After taking the pictures, the images were processed using the FRAK.OUT software to obtain the corresponding fractal dimensions at each distance. A table such as the one shown below is then constructed for each object.

Table 2. Sample fractal dimension-distance table

Fractal Object: _______ Length: _______ Width: ________

Distance in Feet	Fractal Dimension

From the scatterplot of fractal dimension versus distance, we estimated a distance relative function:

(1) $\lambda : \mathrm{R}^{+} \rightarrow [0,2]$

We posit that the rate at which λ(d) changes depends on the two-dimensional surface area of the fractal object:

(2) $\lambda'(d) = f(A)$

where *A* is the surface area of the fractal object.

Data analysis, results and discussion

Table 3: Distance-fractal dimension relationship for spherical fruits

Distance	Rambutan	Durian
1	1.8835	1.8558
3	1.8391	1.7768
5	1.8757	1.7713
7	1.8562	1.7378
9	1.8743	1.6671
11	1.857	1.6057
13	1.797	1.5996
15	1.8153	1.6525
17	1.8182	1.6151
19	1.7567	1.5316

Figures 1 and 2 show the scatterplot of the fractal dimensions of the fruits at various distances.

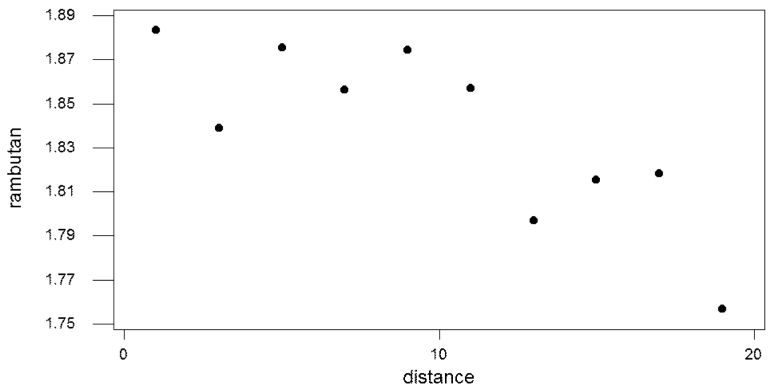

Figure 1**:** Scatterplot of Rambutan (nephelium lapacceum) Fractal Dimension versus distance

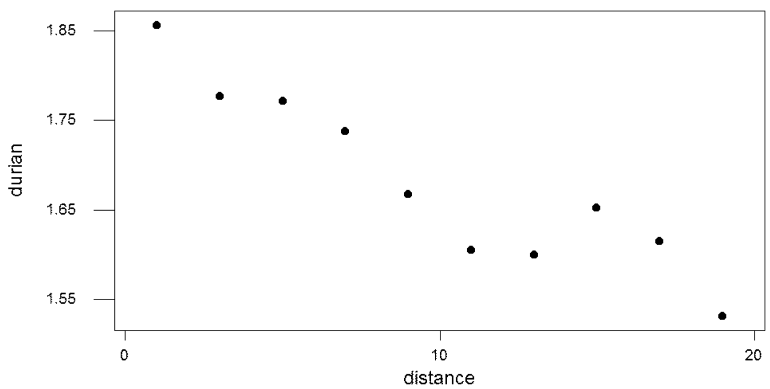

Figure 2. Scatterplot of Durian (dorio zibetinus) fractal dimension versus distance

In both graphs, there is a discernible downward trend in the values of the fractal dimensions as the distance from them increases. We fitted quadratic curves to the scatter of points to obtain:

Table 4. Quadratic regression function for rambutan

The regression equation is rambutan = 1.86 + 0.00255 distance -0.000398 distance squared

Predictor	Coef	SE Coef	T	P
Constant	1.86473	0.02226	83.79	0.000
distance	0.002547	0.005168	0.49	0.637
distance	-0.0003977	0.0002505	-1.59	0.156

S = 0.02302 R-Sq = 74.7% R-Sq(adj) = 67.5%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.0109851	0.0054926	10.36	0.008
Residual Error	7	0.0037107	0.0005301		
Total	9	0.0146958			

Table 5-a Quadratic regression function for durian

The regression equation is durian = 1.87 - 0.0265 distance +0.000550 distance squared

Predictor	Coef	SE Coef	T	P
Constant	1.87334	0.03484	53.78	0.000
distance	-0.026518	0.008089	-3.28	0.014
distance	0.0005502	0.0003921	1.40	0.203

S = 0.03604 R-Sq = 90.0% R-Sq(adj) = 87.2%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.081986	0.040993	31.56	0.000
Residual Error	7	0.009091	0.001299		
Total	9	0.091077			

While the quadratic fits appear to be satisfactory in both cases, we tried another model using the logarithm of the distance as basis. Results revealed, however, that better results are observed only in the case of the fractal dimension for durian fruit. This is reflected in Table 5 below:

Table 5-b Logarithmic regression function for durian

The regression equation is durian = 1.85 - 0.0244 logdistance - 0.0251 logdistsquared

Predictor	Coef	SE Coef	T	P
Constant	1.85346	0.03482	53.24	0.000
logdista	-0.02437	0.04622	-0.53	0.614
logdists	-0.02515	0.01442	-1.74	0.125

S = 0.03575 R-Sq = 90.2% R-Sq(adj) = 87.4%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.082132	0.041066	32.14	0.000
Residual Error	7	0.008944	0.001278		
Total	9	0.091077			

The final models we used were:

$$\lambda(\text{rambutan}) = 1.86 + 0.00255d - 0.000398d^2 \quad (1)$$
$$\lambda(\text{durian}) = 1.85 + 0.0244\log d - 0.0251d^2$$

The vanishing point or the distance at which the shapes become points are:

$$\lambda(\text{rambutan}) = 0 \text{ or } 71.64 \text{ ft.} \quad (1)$$
$$\lambda(\text{durian}) = 0 \text{ or } 3{,}337 \text{ ft.}$$

At roughly 72 feet from the object, the rambutan will be viewed as a point on the plane while at roughly 3,337 ft., the durian fruit will be seen as a point.

Table 6 shows the fractal dimension-distance relationship for the vegetables.

Table 6. Fractal dimension-distance table for vegetables

Distance	Ampalaya	Pipino
1	1.9264	1.9458
3	1.8684	1.9202
5	1.842	1.8767
7	1.8399	1.8632
9	1.8354	1.8688
11	1.8052	1.879
13	1.826	1.8497
15	1.8582	1.8025
17	1.8474	1.8093
19	1.8298	1.814

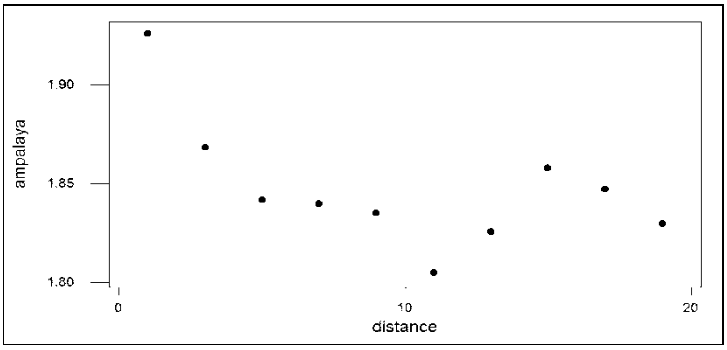

Figure 3. Scatterplot of bitter gourd (momordica charantia) fractal dimension versus distance

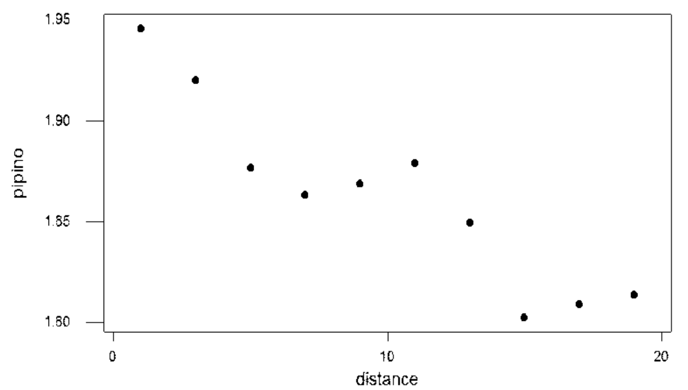

Figure 4. Scatterplot of cucumber (cucumis sativo) fractal dimension versus distance

As in the case of the fractal dimension of fruits, the fractal dimensions of the vegetables appear to be a decreasing function of distance. The regression functions obtained for the bitter gourd (momordica charantia) and the cucumber (cucumis sativo) are shown below.

Table 7. Logarithmic regression function for bittergourd fractal dimension and distance
The regression equation is ampalaya = 1.93 - 0.0814 logdistance - 0.0172 logdistsquared

Predictor	Coef	SE Coef	T	P
Constant	1.92905	0.01496	128.93	0.000
logdista	-0.08138	0.01986	-4.10	0.005
logdists	-0.017227	0.006198	-2.78	0.027

S = 0.01536 R-Sq = 82.7% R-Sq(adj) = 77.8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.0079226	0.0039613	16.79	0.002
Residual Error	7	0.0016518	0.0002360		
Total	9	0.0095744			

Table 8. Logarithmic regression function for cucumber fractal dimension and distance
The regression equation is pipino = 1.94 - 0.0161 logdistance - 0.0101 logdistsquared

Predictor	Coef	SE Coef	T	P
Constant	1.94480	0.01770	109.88	0.000
logdista	-0.01613	0.02350	-0.69	0.515
logdists	-0.010077	0.007332	-1.37	0.212

S = 0.01817 R-Sq = 88.3% R-Sq(adj) = 84.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.0174152	0.0087076	26.37	0.001
Residual Error	7	0.0023117	0.0003302		
Total	9	0.0197268			

The tentative models for the fractal dimension-distance relationship for the vegetable group are:

$$\begin{aligned}\lambda(\text{bitter gourd}) &= 1.93 - 0.0814\log d - 0.0172d^2 \\ \lambda(\text{cucumber}) &= 1.94 - 0.0161\log d - 0.0101d^2\end{aligned} \quad (2)$$

The distances at which the vegetables are viewed as points on the plane are:

$$\begin{aligned}\lambda(\text{bitter gourd}) &= 4854.62 \text{ or } 0 \text{ ft.} \\ \lambda(\text{cucumber}) &= 4224.21 \text{ or } 0 \text{ ft.}\end{aligned} \quad (3)$$

Finally, we considered the three leaf samples. Figures 5,6, and 7 show the scatterplot of the fractal dimensions versus distance.

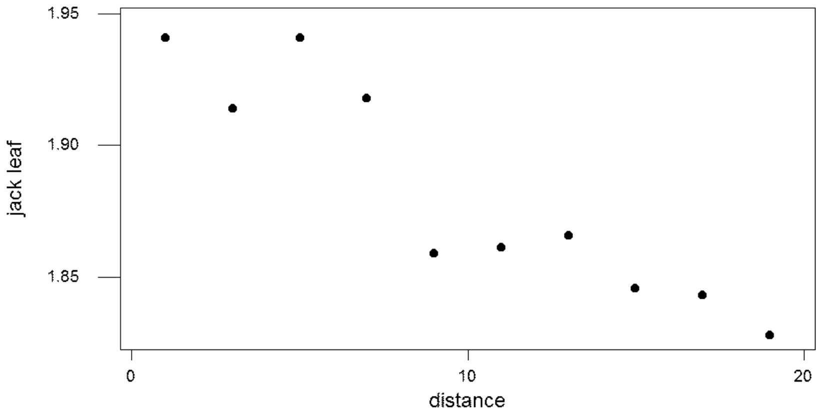

Figure 5. Scatterplot of fractal dimension of jackfruit leaf (Artocarpus heterophylla)

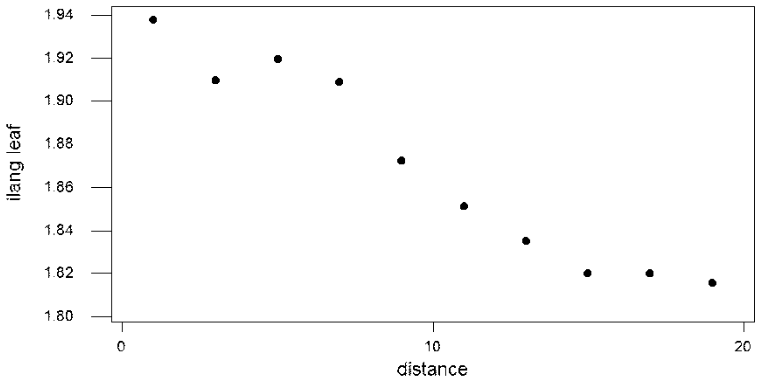

Figure 6. Scatterplot of fractal dimension of ilang-ilang leaf (Cananga odorata)versus distance

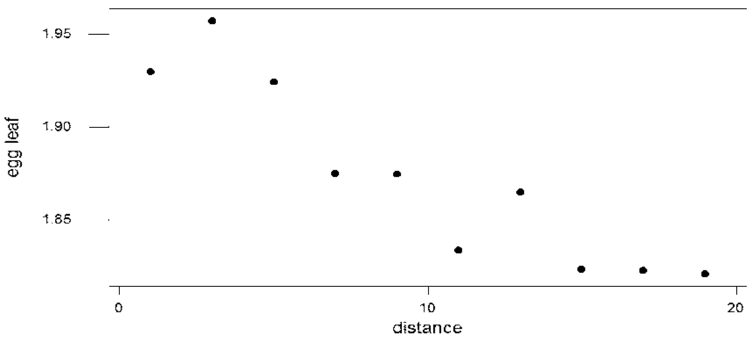

Figure 7. Scatterplot of fractal dimension of eggplant leaf (Solanum melogena) versus distance

Table 9. Logarithmic regression function for the jackfruit leaf fractal dimension and distance
The regression equation is jack leaf = 1.94 + 0.0144 logdistance - 0.0177 logdistsquared

Predictor	Coef	SE Coef	T	P
Constant	1.93871	0.01719	112.76	0.000
logdista	0.01444	0.02283	0.63	0.547
logdists	0.017709	0.007122	-2.49	0.042

S = 0.01765 R-Sq = 86.6% R-Sq(adj) = 82.7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.0140413	0.0070206	22.53	0.001
Residual Error	7	0.0021812	0.0003116		
Total	9	0.0162224			

Table 10. Logarithmic Regression Function for the ilang-ilang leaf fractal dimension and distance
The regression equation is ilang leaf = 1.93 + 0.0168 logdistance - 0.0204 logdistsquared

Predictor	Coef	SE Coef	T	P
Constant	1.93453	0.01240	156.00	0.000
logdista	0.01676	0.01646	1.02	0.342
logdists	-0.020393	0.005137	-3.97	0.005

S = 0.01273 R-Sq = 94.2% R-Sq(adj) = 92.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.0185253	0.0092626	57.14	0.000
Residual Error	7	0.0011347	0.0001621		

Table 11. Logarithmic Regression Function for the eggplant leaf fractal dimension and distance
The regression equation is egg leaf = 1.94 + 0.0218 logdistance - 0.0224 logdistsquared

Predictor	Coef	SE Coef	T	P
Constant	1.93755	0.01824	106.23	0.000
logdista	0.02175	0.02421	0.90	0.399
logdists	-0.022367	0.007556	-2.96	0.021

S = 0.01873 R-Sq = 89.0% R-Sq(adj) = 85.8%
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	0.0198064	0.0099032	28.24	0.000
Residual Error	7	0.0024547	0.0003507		
Total	9	0.0222611			

The fractal-distance functions we found are therefore:

$$\lambda(\text{ilang leaf}) = 1.94 + 0.0144\log d - 0.0177\log d^2$$

$$\lambda(\text{egg leaf}) = 1.93 + 0.0168\log d - 0.0204\log d^2 \quad (4)$$

$$\lambda(\text{jack leaf}) = 1.94 + 0.0218\log d - 0.0224\log d^2$$

The vanishing points are:

$$\lambda(\text{ilang leaf}) = 2551.783 \text{ or } 0 \text{ ft.}$$

$$\lambda(\text{egg leaf}) = 1813.531 \text{ or } 0 \text{ ft.} \quad (5)$$

$$\lambda(\text{jack leaf}) = 1.94 + 0.0218\log d - 0.0224\log d^2$$

Conclusion

From the results of our study we can conclude that “the fractal dimension of any flat geometric object reduces in logarithmic proportion of the distance from the object." While we found out that the mathematical model of the fractal dimension relative to its distance can be expressed by the logarithmic regression function much more has to be done such as determining the relationship of the fractal dimension with respect to the area and volume of such objects.

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