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Age- And Gender-Related Change in Mathematical Reasoning Ability and Some Educational Suggestions

Assist. Prof. Dr. Emrullah ERDEM
Adıyaman University, Faculty of Education, Adıyaman, Turkey

Prof. Dr. Yasin SOYLU
Atatürk University, Kazım Karabekir Faculty of Education, Erzurum, Turkey

Abstract
Does the mathematical reasoning ability develop with increase in age? How is mathematical reasoning ability differing according to gender? The current study is trying to find answers to these two questions. The study using cross-sectional design, was conducted with 409 (8th, 9th and 10th grade) students attending to middle school and high school in different provinces of Turkey from different socio-economic environments. Mathematical Reasoning Test (MRT) was used for the data collection. Independent groups t-test was applied in order to analyse the relationship between mathematical reasoning and gender, and additionally ANOVA test was used to determine the differences between grade levels. The analysis shows that as the age increases mathematical reasoning develops and male students perform significantly better than female students in mathematical reasoning. It is very important to (a) take encouraging steps to ensure that women are interested in mathematics instead of discouraging attitudes in society, (b) expose students to higher-level problems in open-ended format without answer options over grades in order to improve their mathematical reasoning.
Keywords: Mathematical reasoning ability, age, gender, middle and high school students

Introduction

The society and researchers claims that “Mathematics is a male domain”. This belief continues to be accepted in modern societies by both genders (Van de Walle, Karp and Bay-Williams, 2010). Is this only a saying or is there any truth in it? Compared to men, are the women considered to be doing less mathematics? Is it just a perception of women or do they actually do less mathematics? After the development of mathematical reasoning according to gender is investigated, the reasons will be discussed and certain suggestions will be made. On the other hand, when you hear the expression such as “I couldn’t do mathematics in the previous years but now I can understand and solve mathematics problems”, you may say *it also happened to me.* Is there really a development in mathematical reasoning as the years past? Are we doing mathematics better as we age? It is expected that thinking capabilities of individuals increase with the age. For example, it is expected that a 15-year-old student will perform better than a 10-year-old student and a 10-year-old student will perform better than a 5-year-old child in mathematics. Therefore, in order to grasp the meaning of these expressions, it is important to scientifically analyse how mathematical reasoning develops with the increasing age.

Age and Gender

Effect of gender and age on mathematics has been a debate among researchers and academicians for years. Researches show that differences can occur more frequently according to gender and increasing age (Fennema and Sherman, 1977; 1978). Sumpter (2016b) indicated that mathematical reasoning of female students are different from mathematical reasoning of male students. It has been argued that there are other factors than physiological reasons, such as class and ethnicity that causes a difference in mathematical success (Brandell and Staberg, 2008; Walkerdine, 1998; Yates, 1997).

Despite the fact that female students work harder and they are more eager to learn mathematics compared to male students (Brandell and Staberg, 2008), researchers, who have conducted studies about the students with different grades, point out that mathematics is a male domain (Bander and Betz, 1981; Brandell, Leder and Nyström, 2007; Brandell and Staberg, 2008; Mendick, 2005; Sumpter, 2012). It is seen that male students like mathematics and regard mathematics as an important part of their future and therefore are more successful in mathematics. On the other hand, the thought “female students should study harder in mathematics” leads to the thinking that the male students are more prone to mathematics than female students (Brandell and Staberg, 2008). Tiedemann (2000; 2002) revealed that teachers perceive male students as more talented in mathematics than female students. Jussim and Eccles (1992) found that the success in middle school mathematics depends on hard work for female students and on intelligence for male students. Another research states that the mathematical success of male students depend on spatial (three dimensional) abilities, while success of female students depend on oral abilities (Klein, Adi-Japha and Hakak-Benizri, 2010). Imitation reasoning is effective on success of female students in mathematics and female students tend to use standard methods in mathematical reasoning (Sumpter, 2016a). Similarly, while female students learn standard algorithms to be successful in mathematics, male students can think creatively with their ability that comes from birth

(Leslie, Cimpian, Meyer and Freeland, 2015).

Wolbers and Hegarty (2010) claim that gender factor creates a difference in terms of learning strategies used by students in mathematical reasoning. For example, male students tend to use metric calculation and focus on the main aspects of the subjects, while female students prefer to use classical and familiar strategies (Dabbs, Chang, Strong and Milun, 1998; Ruggiero, Sergi and Iachini, 2008; Wolbers and Hegarty, 2010). Female students tend to use the strategies they have learned from their teachers while male students develop different strategies and think more abstract (Fennema, Carpenter, Jacops, Franke and Levi, 1998). In the calculations that requires addition and subtraction, female students calculate by using their fingers while male students are doing mental computation (Carr and Davis, 2001). Male students include many probabilities in their thinking and therefore, they try to use different strategies (Sumpter, 2016a). This assertion is expressed by a participant teacher in Sumpter's (2016a) research as "male students push all the buttons on the calculator and think that this will help them".

Why Mathematical Reasoning?

Mathematical reality can be understood with the help of reasoning that is a basis of all the rules and operations of mathematics (Umay and Kaf, 2005). Ball and Bass (2003) point out that mathematical reasoning has the following tasks: (1) Mathematical knowledge can be reconstructed when it is learned conceptually by using reasoning; (2) Reasoning enables revealing and discovering new mathematical thoughts; (3) Mathematical propositions can be verified and proven with reasoning; (4) Reasoning helps the students for generalization of special conditions; (5) Overarching of mathematical concepts and operations can be made with reasoning.

Mathematical reasoning is a common activity, which involves induction, deduction, association, and inference methods, as well as how learners interact with each other to solve the problems (Yackel and Hanna, 2003). Reasoning is defined as a task which is far above the thinking process and the work of thinking thoroughly about all aspects of the problem, event or situation and thus reaching a logical conclusion (Erdem, 2011). The importance and role of mathematical reasoning on learning is emphasized in national curriculum (MNE, 2013) and international reforms (NCTM, 1989; 2000) and researches (Diezmann and English, 2001; English, 1998; Erdem, 2011; 2015; 2016; Erdem ve Gürbüz, 2015; Lithner, 2008; Umay, 2003). It is stated that (1) there is a relationship between mathematics learning and reasoning, (2) effective solutions can be found if a person uses reasoning to solve a problem and so (3) they can make better association (Diezmann and English, 2001; Pellerin, 2012).

It can be said that the notion of mathematical reasoning is expressed with the existence of "if ... than…", "because ..." statements. Mason (2001) stated that the reasoning uses the structure "if ...than", to find out hypotheses and learn how to reach conclusions, and to justify ideas for effective reasoning and to place ideas on solid ground. It is important for a person to justify what s/he thinks to reveal mathematical reasoning. In other words, it is necessary to use *"for this reason ...,"* ,*"because ..."*, *"causes ..."* statements for justification (Erdem, 2015). Thus, mathematical justification enables one to (1) think independently, (2) analyse and explain the ideas of experts, (3) learn the reasonable thinkings without depending on others (Mason, 2001).

Mathematical reasoning in Turkish curriculum (MNE, 2013) is seen as a process of acquiring new knowledge using the tools specific to mathematics (symbols, definitions, relations etc.) and thinking techniques (inductive, deductive, comparison, generalizing etc.). Similar properties of mathematical reasoning can be found in curriculum of other countries. For example, reasoning can be defined as processes with thinking, analysing, proving, assessing, deducting, justifying and generalizing in Australian curriculum (Australian Curriculum Assessment and Reporting Authority [ACARA] (2013). The common ground of researches and curriculums is that mathematical reasoning requires using upper thinking and finding correct solutions or results at the end of the reasoning.

Research on the effects of age and gender on mathematics

There is no coherence between the results of studies that investigated the effects of gender and age on mathematics. For example, Fennema and Sherman (1978) investigated the role of gender difference on mathematical success in the research with 1320 middle school and high school students. They found that the differences in terms of gender can not be generalized and gender related difference is becoming more apparent as the age increases. Armstrong (1985) concluded that male students are more successful than female students between 8th grade and 12th grade which is a period that should be carefully observed. Hyde, Fennema and Lamon (1990) studied the effects of gender on mathematical performance in meta-analysis study, they found that male students perform better than female students in high school and university although there is not a significant effect in elementary school and middle school level. As a result of longitudinal analysis conducted on students between 8th grade and 12th grade (with 2 years' gap), Fan, Chen and Matsumoto (1997) stated that when a comparison is made in the same class and between 8th, 10th and 12th grades, male students have higher mathematical success rates than female students. Mendick (2005) found out that participants (students aged between 16- and 19-years-old) inscribed mathematics as masculine, and concluded that it is more difficult for girls and women to feel talented at and comfortable with mathematics and so to choose it and to do well at it.

Altiparmak and Ozis (2005) made a study to investigate mathematical proving and mathematical reasoning in different age levels. The results are as the following: (1) classification, matching, comparing and ordering concepts are fundamental concepts to develop reasoning in pre-school period and these concepts help in transition to logical thinking; (2) elementary school is in concrete thinking period; (3) middle school period is where student can form assumption on generalizations and evaluate those assumptions; (4) during high school period, abstract thinking is developed and this period is important for induction and deduction. Brandell and Staberg (2008) found that mathematics is a male domain and as the age increases (15-17 years old), views about gender are becoming stronger. Liu and Wilson (2009) showed that male students performed better than female students in 2000 and 2003 PISA results (15 years old) and this result is consistent in both years. Klein et al. (2010) has not found a significant difference in mathematical success of male and female students in preschool. They also found that spatial reasoning in male's mathematical performance and verbal skills in female's mathematical success were effective. DeLay et al. (2015) found that mathematical reasoning develops the age increases. Scheiber, Reynolds, Hajovsky and Kaufman (2015) found that there was no difference between male and female students in mathematics in their research with participants aged 6-21 years, however female students were better at reading and writing. Sumpter's (2016a) study suggested that high school mathematics teachers thought that female students chose standard methods and they use imitation method, while male students were trying to use more and various strategies. Sumpter (2016b) made a survey of high school mathematics teachers to determine how mathematical reasoning changes according to gender and which types of mathematical reasoning female and male students use. Accordingly, male students are more likely to use algorithmic reasoning and female students are more likely to use familiar algorithmic reasoning.

Current Study

Why are we investigating mathematical reasoning in terms of gender? Although female students work harder to improve their ability in mathematical reasoning, there is still a difference between male and female students in terms of development (Brandell and Staberg, 2008). The reasons why female students have lower rates of success in mathematical reasoning compared to male students will be discussed in this study. Implications of this study can prevent stating discriminatory opinions on mathematical reasoning and gender relations (i.e. Mathematics is a male domain) and putting obstacles on the development of female students. The perception that men are more successful in mathematics can lead girls to acquire an identity in this direction and lead to learned helplessness in mathematics. If the mathematical success differs biologically with gender, a mathematical teaching method can be developed in such a way that it will be beneficial to the specific student group. In addition, if the change of mathematical reasoning ability depends on age, the level of mathematical problems, occupations or activities that are directed to the students can be determined according to age groups.

Method

Research Design

The present study is a descriptive research because it aims to determine and compare an existing situation. In such educational research, the events and situations are examined in detail and attempted to describe what they are (Cohen, Manion and Morrison, 2000). This research is also a cross-sectional study as it points out the change of mathematical reasoning according to age and gender

Participants

This study was conducted on 409 (8th, 9th and 10th grade) students attending to middle school and high school in different provinces of Turkey with different socio-economic backgrounds. Attention was drawn to the heterogeneous groups in terms of mathematical success by looking at the mathematics grades of the previous semesters of the participants in each school. Demographic profile of the participants is illustrated in Table 1.

Table 1. Class level and gender distribution of participants

Class Level	**Male N(%)**	**Female N(%)**	**Total (N)**
8th Grade	74 *(52%)*	67 *(48%)*	141
9th Grade	73 *(49%)*	77 *(51%)*	150
10th Grade	57 *(48%)*	61 *(52%)*	118
Total	204 ***(50%)***	205 ***(50%)***	**409**

The reason for this research to start from 8th grades is that some test items that measure mathematical reasoning contain only eighth grade subjects in MNE (2013). For example, a question on the test is as following:

> *In a computer game, a game is played between horses numbered as 7, 9, 12. In this game two traditional dice will be rolled and sum of the upper side of the dice will be taken. If the sum is the same as the number of the horse, the horse will move one step further. The player who reaches the final point earliest wins the game. Which horse number will be more advantageous to win the game?*
>
> *A) 7 B) 9 C) 12 D) Equal.*

This question requires probability knowledge, and probability subject is taught in the 8th grade for the first time in Turkish schools. 11th and 12th grade students were not included in the study with the concern that the test might be insufficient for those students to explain their mathematical reasoning. Students were given codes such as S1, S2, S3...

Data Collection

Mathematical Reasoning Test (MRT) with 33 multiple choice questions developed by Erdem (2016) was used for collecting the data. After the pilot study that consists of 90 students equally distributed to each class level, item analysis has been conducted and six questions have been excluded from MRT since they have item total correlations lower than 30. After this analysis, MRT has the final form with 27 questions (see Appendix-1 for sample questions). To determine whether or not the questions in MRT require mathematical reasoning, two mathematics teachers and two mathematics academicians are consulted as experts. In addition, two students who are not included in the real study are selected and questions that are hard to understand and have biases are corrected after the interviews. According to this pre-study, 50 minutes is determined as appropriate for test period. Additionally, MRT's Kuder-Richardson-20 (KR-20) coefficients are calculated as .816 for 8th grades, .820 for 9th grades and .769 for 10th grades. These values show that test is reliable for all the grades.

Data Analysis

In order to analyse student responses, statistical package program is used (Statistical Package for Social Sciences-SPSS 22.0). Answers of MRT questions are analysed by assigning 1 to every correct answer and 0 to every wrong answer or unanswered questions. Possible lowest score in MRT is 0 and possible highest score in MRT is 27. Total point and general mean of each student is calculated. If the total MRT points are between 0-9 mathematical reasoning is low, between 9-18 mathematical reasoning is medium and between 18-27 mathematical reasoning is high. To determine the normality, Kolmogorov-Smirnov test is used. The data show normal distribution (p= .094). Independent groups t-test is used to determine how mathematical reasoning changes according to gender. ANOVA test is used to determine the difference between grades; Scheffe test of Post Hoc tests are used to determine between which groups did the difference occur.

Findings

Statistical results of mathematical reasoning according to gender are illustrated in Table 2 and according to class level are illustrated in Table 4 and Table 5. Mathematical reasoning levels of all students are given in Table 3.

Table 2 Group statistics according to gender

	Gender	**N**	**Mean**	**Standard Deviation**	**Sig. (2-tailed)**
Mathematical Reasoning	Female	205	14.1463	5.01743	,000
	Male	204	16.7696	4.82000	

As indicated in Table 2, mean for female students' mathematical reasoning is found as 14.14 and for male students' mathematical reasoning is found as 16.76. Significant difference between mathematical reasoning of male and female students were found (p =. 000) and this difference is in favour of male students (as seen from the mean values). In other words, mathematical reasoning of male students are significantly higher than mathematical reasoning of female students.

Table 3 Mathematical reasoning level of participants

Class Level	**Mathematical Reasoning Level**					
	0-9 point, Low		9-18 point, Medium		18-27 point, High	
	Frequency	Percentage (%)	Frequency	Percentage (%)	Frequency	Percentage (%)
8th Grade	32	23	83	59	26	18
9th Grade	24	16	77	51	49	33
10th Grade	4	3	69	58	45	38
Total	**60**	**15**	**229**	**55**	**120**	**30**

Data in Table 3 indicates that 15% of participants have low mathematical reasoning, 55% of participants have medium mathematical reasoning and 30% of participants have high mathematical reasoning. According to these results, it can be said that most of the participants have medium level mathematical reasoning. On the other hand, the number of participants from 8th grades that have low mathematical reasoning is high, where the number of participants from 10th grade that have low mathematical reasoning are low. 9th grade group has the maximum number of participants in high level. To assert a clear comment about mathematical reasoning, individual mean values of each class level should be investigated.

Table 4. Group statistics according to class level

Class Level		**N**	**Mean**
Mathematical Reasoning	8th Grade	141	13.93
	9th Grade	150	15.51
	10th Grade	118	17.19
	Total	**409**	**15.54**

As it can be seen from Table 4, mean values of 8th grade students are determined as 13.93, 9th grade students are determined as 15.51 and 10th grade students are determined as 17.19. This result shows that 10th grade students have the highest mathematical reasoning mean value. It is possible to say that the lowest value of mathematical reasoning is among 8th grade students. This finding shows that as the grade level increases, mathematical reasoning also increases. Yet, it is not possible to say whether there is a significant difference between groups or not. Significance between groups are tested using ANOVA test.

Table 5 ANOVA results

	Sum of squares	df	Mean square	F	Sig.
Between groups	682.997	2	341.499	14.044	,000
In group	9872.416	406	24.316		
Total	10555.413	408			

The results on Table 5 shows that there is a significant difference between groups according to grade levels (p = .000). Yet, it cannot be understood which groups have significant differences. The groups that have significance difference can be determined by using Scheffe test.

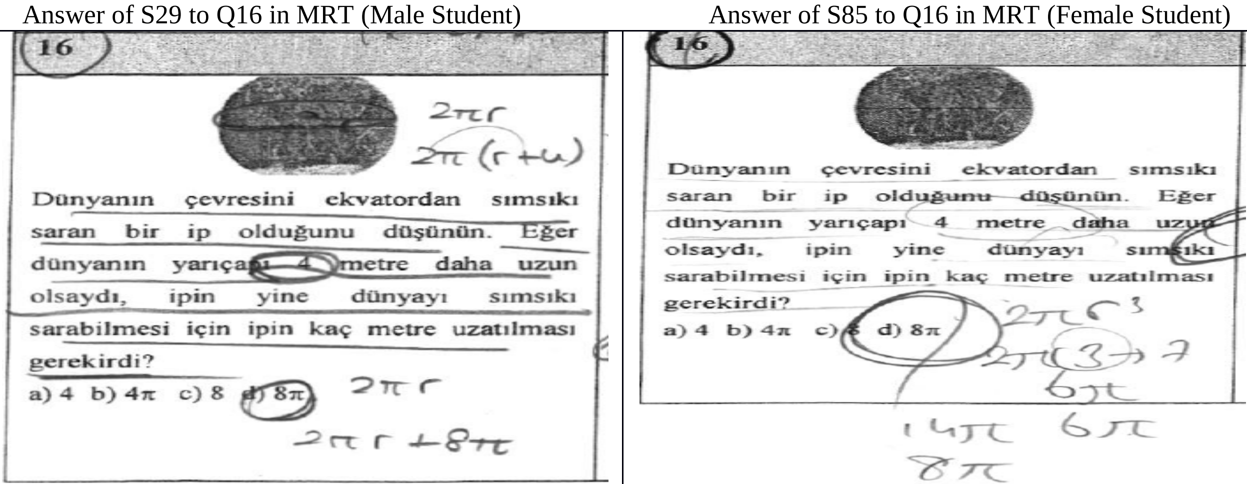
Table 6 Multiple comparison according to class levels (Scheffe)

(I) Grade	(J) Grade	Mean difference (I-J)	Standard Deviation	Sig.	95% Confidence Interval	
8. Grade	9. Grade	-1,57(*)	,57842	,025	-2.9982	-,1561
	10. Grade	-3,25(*)	,61524	,000	-4.7703	-1.7472
9. Grade	8. Grade	1,57(*)	,57842	,025	,1561	2.9982
	10. Grade	-1,68(*)	,60678	,022	-3.1723	-,1908
10. Grade	8. Grade	3,25(*)	,61524	,000	1.7472	4.7703
	9. Grade	1,68(*)	,60678	,022	,1908	3.1723

* Mean differences in meaningful in .05 level.

As presented in Table 6, 10th grade students have significant levels of mathematical reasoning than 9th grade students and 9th grade students have significant levels of mathematical reasoning than 8th grade students. It is possible to say that mathematical reasoning is improving as the class level increases. When answers to MRT questions are compared qualitatively, male students have significantly higher values compared to female students and as the age increases the mathematical reasoning also develops. Below answers of some students to MRT questions are directly transferred and interpreted by comparison. The differences between male and female students are not the same in every participant and on every level. It is seen that female students are better in mathematical reasoning in certain questions compared to male students. On the other hand, students have problems in questions 24, 27 and 16 respectively and these questions have the minimum answering rate. Sample student answers to these questions are given to state differences. For example, student sample answers in 10th grade level consist of question 24 which is considered as the hardest question. Answers of 10th grade students to this question is illustrated, which reveals that they have better mathematical reasoning. Similarly, the answers of 8th grade students are selected from question 16 where they have least problems.

Answer of S29 to Q16 in MRT (Male Student)	Answer of S85 to Q16 in MRT (Female Student)



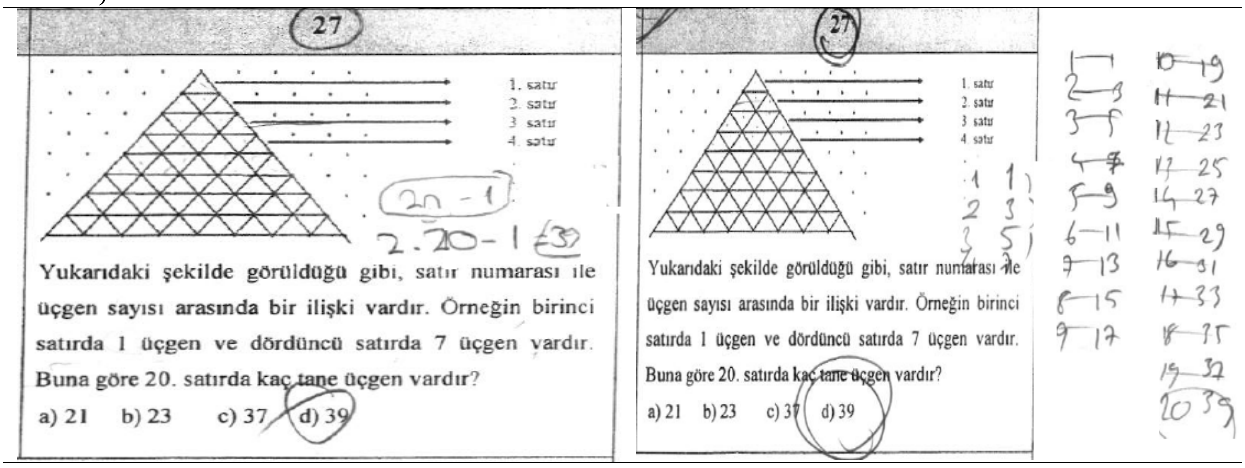
a)	b)

Figure 1. Answers of two 8th graders to Q16 in MRT

Students are expected to do the following reasoning in Q16: The length of a rope that surrounds the Earth is equal to $2\pi r$ as it has circular shape (When the radius of Earth is considered as r). If the radius is 4 meters longer, the new radius of Earth is $r + 4$. In the current situation, the required length of rope to surround the Earth from equator corresponds to perimeter of a circle with $2\pi r$. From here, the new perimeter is $2\pi(r+4) = 2\pi r + 8\pi$ and the rope should be extended 8π. In Figure 1(a) when the answer of male student with S29 code is investigated, it is seen that the student has made the expected reasoning. The student used correct mathematical statements corresponding to verbal problem and correctly executed multiplication and addition and distribution of multiplication on addition with natural numbers. S29 could calculate perimeter of circle using $2\pi r$ formula and found the new perimeter length when the length of rope is extended. The student's MRT point is calculated as 19. This corresponds to *"high"* level (18.00-27.00).

In Figure 1(b) when the answer of female student with S85 code is investigated to the same question, it is seen that the student has made the expected reasoning with special numbers. The student considered the length in first condition as $r = 3$ and calculated accordingly. In the first situation, the perimeter of Earth from equator is calculated as $2\pi 3 = 6\pi$ and when it is 4 meters longer in the second situation the calculation is made as $2\pi 7 = 14\pi$. After these calculations, the length difference is $14\pi - 6\pi = 8\pi$ and the rope should be extended 8π. It does not mean that the student does not have mathematical reasoning as she thinks the radius as a special number. Yet as S29 used mathematical expressions to reach the correct answer shows that he has better reasoning than S85. S85's MRT point is calculated as 15. This corresponds to *"middle"* level (9.00-18.00).

Answer of S127 to Q27 in MRT (Male Student) Answer of S198 to Q27 in MRT (Female Student)



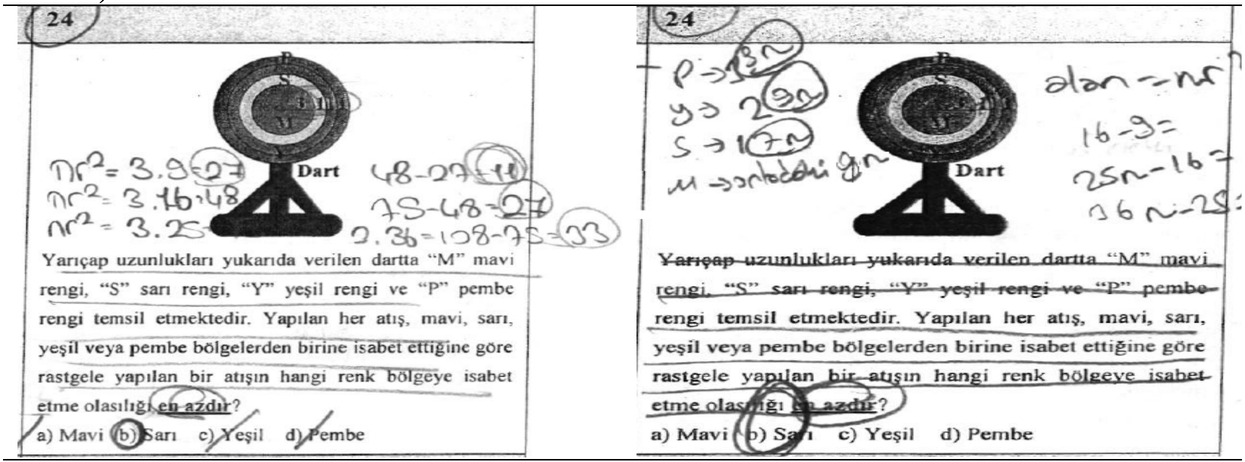
a) b)

Figure 2. Answers of two 9th graders to Q27 in MRT

The students are expected to do the following reasoning in Q27: This question requires to determine the rule of a pattern; triangle number in every line is 2 times 1 minus of the line number. Students should realize the pattern and reach 2x20-1=39 triangles at line 20. In Figure 2 (a) when the answer of male student with S127 code is investigated, it is seen that the student has made the expected reasoning. Student thought the line number as "n" in the pattern and found the answer as "2n-1". After the student found the rule of the pattern he calculated 2x20-1 in the line 20 and found that there must be 39 triangles. The student's MRT point is calculated as 23. This corresponds to *"high"* level (18.00-27.00).

In Figure 2(b) when the answer of female student with S198 code is investigated in the same question, this student reached the correct answer using a different pattern. At first couple of lines student matched the triangles in each line as 1-1; 2-3; 3-5; 4-7. She saw that there is 2 triangles difference between consecutive lines. She continued this pattern and found that line 20 has 39 triangles. The student found a rule between triangle numbers in consecutive lines rather than finding a rule between line number and triangle number. Although this rule is not wrong, if the line number in the pattern is high, for example 100 lines, it will get harder to reach the correct answer. In the same question, S127 discovered a short and practical pattern rule such as "the number of triangles in a line is 2 times 1 minus than the line number." S198's MRT point is calculated as 17. This corresponds to *"middle"* level (9.00-18.00).

Answer of S12 to Q24 in MRT (Male Student)	Answer of S201 to Q24 in MRT (Female Student)



a) b)

Figure 3. Answers of two 10th graders to Q24 in MRT

The students are expected to do the following reasoning in Q24: Which colour area has smaller area, the probability to hit that area is lower. The radius of Blue region which is circular is = $\pi 3^2$= $\pi 3^2$=$9\pi.$ Area of circular Yellow areas is calculated by subtracting Blue circular area with 4-unit radius as $\pi 3^2$=9π. Area of Yellow region is subtracted from Blue region with 4-unit radius and calculated as $\pi 4^2 - 9\pi = 16\pi - 9\pi = 7\pi.$ Area of circular Green areas is calculated by subtracting circular area with 5 unit radius from circular area with 4 unit radius as $\pi 5^2 - \pi 4^2 = 25\pi - 16\pi = 9\pi.$ Area of circular Pink areas is calculated by subtracting circular area with 6 unit radius from circular area with 5 unit radius as $\pi 6^2 - \pi 5^2 = 36\pi - 25\pi = 11\pi.$ After all the areas are calculated, Yellow area has the smallest coverage therefore the probability of hitting this area is the lowest. In Figure 3(a) when the answer of male student with S12 code is investigated, it is seen that the student has made the expected reasoning. It can be said that the student has the knowledge that the areas with the smallest coverage has the lowest probability to be hit. Student discovered that blue area is circular and calculated the area using the formula (A= πr^2). He also discovered that Yellow, Green and Pink areas are circular and therefore area of each circle is the difference of two circles between them. He correctly calculated all the areas and determined that Yellow areas is the smallest. The reason of why this student takes $\pi = 3$,14 may be that he desired the results to be integers. Via this way, it can be said that the student will compare the areas easily. The student's MRT point is calculated as 26. This corresponds to *"high"* level (18.00-27.00).

In Figure 3(b) when the answer of female student with S201 code is investigated to the same question, it is seen that the student has made the expected reasoning with small mistakes. The student discovered that Yellow, Green and Pink areas are circular and therefore area of each circle is the difference of two circles between them. The student calculated the areas of circular shapes except Pink area. The student used the correct calculation to find the area of Pink region but when doing the $36\pi - 25\pi$ subtraction operation, she found $36\pi - 25\pi$ subtraction operation 9π instead of 11π . This mistake does not mean that the student does not have poor mathematical reasoning. This implication is confirmed with MRT point of the student. The student's MRT point is calculated as 24. This corresponds to *"high"* level (18.00-27.00).

Discussion and Conclusion

The results of the present study have two different contributions to mathematics education literature: (1) Mathematical reasoning develops as age increases; (2) Mathematical reasoning of male students is significantly better when compared to female students.

As the age increases, the brain thinks on higher level, and thus, the development of the individual's reasoning is expected. In fact, knowledge is acquired by placing the experience in schemes in the mind. Effective use of information makes it possible to make accurate and excessive correlations between these schemes. As the individual experiences increase, more schemata are formed in the brain, and current schemes are matured and changed and are being rearranged. For example, the schemes that a five-year-old child possesses can vary greatly from those of a ten-year-old child. With the current research, this situation has been revealed using scientific methods. In other words, mathematical reasoning, which requires the use of higher-order thinking processes, has been found to develop with age. This result is supported by Altiparmak and Oziş (2005) and DeLay et al. (2015) with evidences supplied by mathematical reasoning with the increase in age.

One of the most important goals of mathematics teaching is to develop reasoning, which is not only a mathematical but also a basic skill, which helps to obtain logical answers in response to questions (Altiparmak and Ozis, 2005). Mathematical reasoning has a structure that can be continuously improved through cognitive, social, and even affective learning. Mathematical reasoning is an individual culture that is formed by many factors such as the knowledge of the person, the point of view of the world, the experience (Erdem, 2015). It is stated that reasoning is the individual and is a property that can be improved (Umay, 2003). Therefore, it is assumed that individuals may have different levels of mathematical reasoning according to their lifestyle and learning levels. These explanations and conclusions support the conclusion "mathematical reasoning is developing with age" in the current research.

Because of the comparative analysis of the students' solutions, it has been seen that as the age increases, the students generally perform shorter, correct solutions and generalizations. As a matter of fact, it is pointed out that generalization is one of the basic indicators of mathematical reasoning (Ball and Bass, 2003; Herbert, Vale, Bragg, Loong and Widjaja, 2015; Kaput, 1999; Mason, Stephens and Watson, 2009; Stylianides, 2010). For example, in Q20 in MRT, a *"105-page book is required to be numbered starting at page 1. How many numbers will be used at the end of this process?* is answered by a 8th grade student (S14) by writing "1, 2, 3,...,105" separate and calculation how many numbers are written in each page. In the same question a 10th grade student (S72) used the shorter and more practical way of "9 numbers between 1-9; 90x2=180 numbers between 10-99 and 6x3=18 numbers between 100-105." In Q27 in MRT, a 9th grade student with S127 code thought the line number of pattern rule as "n" and found the answer by using "2n-1". After the student found the rule of the pattern he calculated 2x20-1 in the line 20 and found that there must be 39 triangles. In the same question a 8th grade with S338 code wrote 1-1; 2-3; 3-5; 4-7; 5-9; 6-11; 7-13; 8-15 as seen in the question (until eight line) and counted the triangles in each line instead of calculating. This solution shows that the student cannot generalize to this question. On the other hand, while older students do not provide solutions to questions they cannot reason, it is seen that younger students tend to offer a solution although they are wrong. It can be said that younger learners have less mathematical experience and therefore are not afraid of doing mathematics and / or making mistakes. In fact, researches (Baloğlu and Koçak, 2006; Randolph, 1997, Mutodi and Ngirande, 2014) have shown that mathematics anxiety increases with age.

The other main result of the study is that mathematical reasoning of male students is significantly better than that of female students. These results are supported by Armstrong (1985), Brandell and Staberg (2008), Fan et al. (1997), Liu and Wilson (2009). On the other hand, it is possible to encounter studies that reveal that gender does not make any significant difference in mathematical performance (Fennema and Sherman, 1978, Klein et al., 2010, Scheiber et al., 2015). It can be said that several factors are effective in improving the mathematical reasoning of male students: (1) *Differences in preferred strategies in mathematical reasoning.* For example, while female students use strategies they learn from their teachers, male students think more abstract by developing different strategies (Fennema et al, 1998). In the calculations that required addition and subtraction female students calculate using their fingers where male students are doing mental calculation (Carr and Davis, 2001). According to Sumpter (2016a), male students think with more probabilities therefore, they try to use more strategies (Sumpter, 2016a). (2) *Physiological differences*, for example, while female students learn standard algorithms to be successful in mathematics, male students can thing creatively with the endowment (Leslie et al., 2015). A research found that the success in middle school mathematics depends on hard work for female students and depends on intelligence for male students (Jussim and Eccles, 1992). Another research states that the mathematical success of male students depends on spatial (three dimensional) abilities and success of female students depends on oral abilities (Klein et al., 2010). The results of Brandell and Staberg (2008)'s research have revealed that female students find mathematics more boring and difficult. (3) *Different meanings attached to gender in society.* Researches conducted on students with different grade levels point out that mathematics is a male domain (Bander and Betz, 1981; Brandell et al., 2007; Brandell and Staberg, 2008; Mendick, 2005; Sumpter, 2012). Because male students like mathematics and regard mathematics as an important part for their future and the thought that female students should be work harder on mathematics resulted the male students to be seen more prone to mathematics (Brandell and Staberg, 2008). Sometimes teachers seem to have this belief. For example, teachers see male students as having better mathematical reasoning than female students (Tiedmann, 2000; 2002). (4) *Other factors.* It has been argued that there are other factors such as class and ethnicity that causes a difference in mathematical success (Brandell and Staberg, 2008; Walkerdine, 1998; Yates, 1997).

It is very important to take encouraging steps to ensure that women are interested in mathematics instead of the existing discouraging thoughts of 'mathematics of women' in society, both in school and in daily life and in family settings. Piatek-Jimenez (2015), a female academician in mathematics education, has made the following suggestions to identify the factors that influence female university students in mathematics at mathematics and to determine the role of gender in shaping career plans:

> It is important for parents, teachers, professors, and advisor to continue to encourage and support women in their interest in mathematics. Ideally, this encouragement would begin at a young age, but ought to be continued throughout their educational careers. Therefore, in addition to individual encouragement, the development and existence of programs designed to encourage and support women to pursue mathematics likely will continue to change the climate of the field (p.45).

Another issue that needs to be discussed in this research is that students try to reach the true conclusion by using random choices of answer options when they can not reason. This conclusion confirms the researches suggesting that evaluating and developing mathematical reasoning requires the use of open-ended problems without answer options (Erdem, 2015; Erdem and Gürbüz, 2015; Frederiksen, 1984; Henningsen and Stein, 1997; Lannin, 2004). It is stated that open-ended questions provide more advantages in terms of the breadth of conceptual domain, operational and methodological characteristics, than multiple-choice questions (Henningsen and Stein, 1997). It can be said that thanks to the complex, open-ended problems that the answer choices do not involve, the students will try to offer a solution instead of focusing on the options and will be in more judgment.

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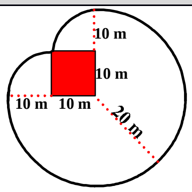
Appendix-1. Some Questions in Mathematical Reasoning Test (MRT)

Q2



As seen above, a sheep is tied to one side of a square-shaped garden (10 m×10 m) with a rope of 20 meters. When the rope is stretched, how many square meters is the area at most in which the sheep can graze?
a) 250π b) 350π c) 450π d) 500π

Q4

A store that sells its products with 100% interest makes a 20% discount to the students. Therefore, what is the percentage rate of interest which this store gets from the students?
a) 20 b) 40 c) 50 d) 60

Q5

There are 4 dice the surfaces of which are signed as (11 5555), (66 2222), (22 4444) and (33 5555). The front surface numbers of which 2 dice are most likely to be 7 in total when they are thrown together?
a) (11 5555) and (22 4444)
b) (66 2222) and (33 5555)
c) (11 5555) and (66 2222)
d) (22 4444) and (33 5555)

Q8

As a rule of a game which is played with a coin, when the coin shows tail two balls are lost, when the coin shows the head 1 ball is gained. Which one cannot be the number of balls of a child who starts to play with 40 balls at the end of the game?
a) 36 b) 39 c) 42 d) 45

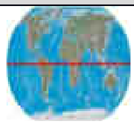
Q13

Of all the money in my pocket, some of them are 5 TL except for two, some of them are 10 TL except for two, and some of them are 20 TL except for two. How much money is there in my pocket?
a) 35 b) 70 c) 105 d) 140

Q14

50 marble balls will be put into 5 boxes without any of the boxes left empty. How many marble balls can be in a box maximum?
a) 10 b) 45 c) 46 d) 50

Q16



Imagine that the world's equator is surrounded tightly by a rope. If the radius of the world were 4 cm longer, then how many meters was the rope supposed to be extended so that it surrounded the world the same way?
a) 4 b) 4π c) 8 d) 8π

Q18

In a group of 25 people, every day of the week is the birthday of at least 1 person. How many people **at most** may have the same birthday?
A)7 B) 8 C) 19 D) 25

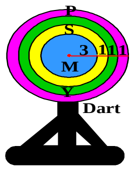
Q20

A book of 105 pages is wanted to be enumerated starting with number 1. When the process is finished, how many numbers will have been used?
a) 105 b) 106 c) 207 d) 211

Q23

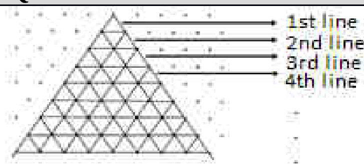
Ahmet paid 235 TL in total for the books each of which costed 5 TL and 10 TL. Given that, how many books did Ahmet buy at **least**?
a) 23 b) 24 c) 45 d) 46

Q24



In the dart radius of which are given above "M" represents blue, "S" represents yellow, "Y" represents green, and "P" represents pink. Given that each shot falls to any of the colored areas, which colored area is less likely to be exposed to a random shot?
a) Blue b) Yellow c) Green
d) Pink

Q27



As seen in the figure above, there is a relationship between line number and the number of triangle. For example, in the first line, there is one triangle and in the fourth line, there are 7 triangles. Given that, how many triangles are there in the 20th line?
a) 21 b) 23 c) 37 d) 39