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Predicting Eight Grade Students' Equation Solving Performances via Concepts of Variable and Equality

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Abstract
This study focused on how two algebraic concepts- equality and variable- predicted 8th grade students' equation solving performance. In this study, predictive design as a correlational research design was used. Randomly selected 407 eight-grade students who were from the central districts of a city in the central region of Turkey participated in the study. Multiple regression analysis was used to answer research questions. The results of the study showed that equality and variable concepts statistically significantly predicted students' equation solving performance. The standardized regression weights (β) for the variable and equality concepts predicting students' equation solving performance were 0.46 and 0.17, respectively.
Keywords: equation solving, variable, equality

1. Introduction

In recent years, the impetus and development during the increment of the knowledge in general also affected the increment of mathematics specifically; however it is not possible for an individual to acquire and catch up with every progress in general. The common goal of most of the countries in the world is to nurture individuals who can attain necessary knowledge and produce solutions to the problems with problem solving systematic and attain necessary knowledge in order to solve problems that is a result of rapid changes. For that purpose, problem-solving skills were emphasized in the curriculum. Problem solving skills in mathematics was listed as one of the four foundational skills after the amendment of the curriculum in Turkey (MEB, 2013).

Algebra is one of the essential subjects in mathematics playing an important role in the development of problem solving skills. Algebra is defined as a generalization of arithmetic (Katz, 2007) and allows individuals to solve specific type of problems (Usiskin, 1988). This definition shows that algebra is a critical tool for problem solving. Problem solving is the primary goal of today's mathematics classrooms; therefore, students need to understand algebra to be a well-equipped problem solver. However, algebra has been a difficult subject for many students and still it is (Hail, 2000).

Two algebraic ideas - variable and equality- are important concepts and Knuth, Alibali, McNeil, Weinberg, and Stephens (2005) indicated that these two are the key concepts for algebraic reasoning. Equality concept and its symbol are the bases of algebraic understanding and are used frequently in most of the levels of school mathematics (Alibali, Knuth, Hattikudur, McNeil & Stephens, 2007). These two key concepts also explain the reason of why learning algebra is difficulty. In other words, if students do not understand equality and variable conceptually, students will experience difficulty in learning algebra.

The concept of variable is a symbol showing us our usage of algebraic mathematical language (Osborne & Wilson, 1992). Understanding the concept of variable is the foundation of transition from arithmetic to algebra and it is necessary in order to use advanced level mathematics meaningfully (Schoenfeld & Arcavi, 1988). Variable can be used in several meanings. These are:

- Variable as an unknown (the meaning of variable in the equation: 2x+1=3)
- Variable as range (i.e., codomain) (the meaning of variable in the equation: y=2x+3)
- Variable as generic structure (the meaning of variable in the equation: x+y=y+x) (Baykul, 2004).

Different and several meanings of variable can be the explanation of why students have difficulties in understanding the concept. As Dominguez (2001) stated, students' difficulty in understanding the concept of variable could be caused by the different usage of it. The concept of variable is generally taught as an unknown in schools; therefore, teaching variable concept with this limited usage could be one of the reasons why students are not able to deal with algebraic expression in advanced levels.

Equality, which is an important concept for algebra achievement, is a connection showing that two mathematical expressions have the same value (Falkner, Levi & Carpenter, 1999; Kieran, 1981). The symbol of equality is necessary in order to understand many subjects in mathematics such as algebraic equations in addition to being complicated symbolic concept (McNeil, Grandau, Knuth, Alibali, Stephens, Hattikudur, & Krill, 2006). Students have insufficient understanding of equality because they generally conceive the concept as a result of arithmetical operation rather than mathematical equality (Baroody & Ginsburg, 1983; MacGregor & Stacey, 1997). Most of the students perceive equality as an operational symbol meaning "solve addition" or "write the result" rather than relational symbol showing mathematical equality (McNeil et al. 2006). For example, participants who were students at the age of 12-13 in Kieran's study (1981) perceived equality as calculating on the left side and writing the answer on the right side of the equal sign. Yaman, Toluk-Uçar and Olkun (2003)

indicated that elementary school students perceived equal sign as an operational symbol rather than relational symbol as well.

The usage of equal sign for different purposes such as arithmetic, trigonometry, algebra, and set theory in mathematics (Saenz-Ludlow and Walgomuth, 1998) also explains why elementary school students misunderstand or have misconception about equality. The usage of equal sign in mathematics has four primary categories:

- an answer of an addition (3+4=7)
- quantitative equality (1+3 =2+2)
- an expression that is right for all values of the variable (x+y=y+x)
- an expression that express the new variable (x+y=z) (Freudenthal, 1983 as cited in Warren, 2006)

The concepts of variable and equality are directly related to equation concept and equation concept plays an important role as problem solving strategy while solving verbal problems. Learning how to solve the concept of equation, specifically solving one-variable linear equations, is indicated as one of the essential components of school mathematics (Andrews & Sayers, 2012) and this study focuses on one variable linear equations. Studies about equations and solving equations showed the difficulty of learning and teaching how to solve equations (Li, 2007). Students have difficulty in formulating algebraic equations to represent knowledge given in verbal problems and learning how to obtain a result to even in simple equations with the use of symbols (Stacey & MacGregor, 2000). A great deal of students memorizes operations used in equations and is not aware of why they use these operations. Studies showed that students have misconceptions or misunderstandings about operations used in equations and the rules of solving algebraic equations (Booth and Davenport, 2013). This originates from nonsystematic and strategic mistakes while students use to simplify algebraic expressions and these students cannot objectivize the operations used in equations (such as their resistance to use same operation to the both sides of equal sign and their propensity to comprehend equal sign as a symbol to finalize operation rather than equality) (Huntley, Marcus, Kahan and Miller, 2007).

As indicated above, generally equality and variable concepts' difficulties cause problems for learning algebra. The same problem is valid for the subject of equation, which has an important place in algebra. As a matter of fact, Knuth, Stephens, McNeil, & Alibali (2006) indicated that middle school students' perception about equality plays a critical role in students' equation solving and verbal problem solving achievement. Knuth, Alibali, Hattikudur, McNeil and Stephens (2008) found that middle school student who had a better equal sign comprehension were able to solve linear equations correctly; in addition Osborne and Wilson (1992) stated that equation solving without building foundations about the concept of variable limited students' learning. In algebra, students should be aware of relational meaning of equal sign as "the same as" besides operational meaning as "do something". This is important when students encounter learning how to solve algebraic equations requiring calculation for the both sides of equal sign (e.g. 3x-5=2x+1) (Knuth et al., 2005). Filloy and Rojano (1989) pointed out that equations having variables on both sides are defined as cut point based upon the difficulties of arithmetic operations and solving these types of problems. Equations having variable only in one side of equal sign are called arithmetic equations (e.g. ax+b=c) whereas variables in both sides of equal sign are called algebraic equations.

Misunderstandings or misconceptions about equal sign generally cause difficulties while solving equations as mentioned above, besides that, Osborne and Wilson (1992) claimed that difficulties of understanding variable in an equation are caused by wrong interpretations of equality concept and equal sign. This situation could be seen as the need for a priority of the examination of the equality concept in equation solving more than the variable concept. However questions about how effective variables are in equation solving need to be answered. There is not a lot of quantitative studies focusing on the concepts of variable and equality together and showing to what extent these concepts are effective for students' achievement of equation solving in the literature. Determining the effectiveness of two concepts on students' achievement of solving equation will shed light on mathematics educators and teachers during designing the process of teaching mathematics. For that purpose, the following research questions were answered in this study:

- How does middle school students' knowledge level about the concepts of equality and variable predict their equation-solving performance?
- How does middle school students' knowledge level about the concepts of equality and variable predict their arithmetic equation-solving performance?
- How does middle school students' knowledge level about the concepts of equality and variable predict their algebraic equation-solving performance?

2. Method

2.1 Participants

In this study, there were 407 eight-grade students who were selected from 5 public schools from the central

districts of a city in the central region of Turkey. This study was performed in the first semester of 2016-2017 academic year. Even though students do not acquire the acquisitions about 8th grade algebra in the first term of their academic year, they should have sufficient skills to solve one variable linear equations and know the concepts of equality and variable because of gaining acquisitions given in the 6th (i.e., algebraic expressions) and 7th grades (i.e., equality, equation, and linear equations).

2.2 Research Design

This study focused on the prediction of students' achievement of equation solving by equality and variable concepts; therefore, prediction design as a correlational research design was used. In correlational research, researcher determines the relationship between two or more variables by using statistical tests. Correlational research design has two types: explanatory and prediction. In prediction design, the anticipated outcome, which is a latent variable, is predicted by measured variables serving as predictor variables (Tekbıyık, 2014). Because this study focused on how the concepts of equality and variable predicted students' equation solving performance (achievement), prediction design as a correlational design was used as the research design of the study.

2.3 *Instruments (Data Collection Tools)*

Author developed an instrument (i.e., test) with open-ended questions by using literature about given concepts (Falkner, Levi and Carpenter, 1999; McNeil and Alibali, 2005). The test contains 6, 8 and 10 questions as a total of 24 open-ended questions to evaluate students' performance on equality, variable, and equation solving, respectively.

Test development process:

- Creating test items assessing performance of equality, variable, and one variable linear equation solving (28 items)
- Obtaining field specialists' opinion about the validity of items whether they are able to assess the acquisitions about concepts (content validity)
- Pilot test items
- Item analysis
- Review and obtain the final form of the test

After obtaining expert mathematics educators' opinions in the field about 28 test items whether they were appropriate to assess concepts they should measure, 150 students took the test and test items were analyzed by using Iteman 3.5 item analysis program. As a result of item analysis, the corrected and final form of the test consisted of 24 items. The item difficulty and item discrimination indices are represented in Table 1.

Table 1. Item analysis results of pilot study

Concepts	Item number	p_j	r_{jx}
Variable	1	0.56	0.60
	2	0.47	0.66
	3	0.62	0.54
	4	0.29	0.52
	5	0.79	0.45
	6	0.12	0.40
	7	0.17	0.31
	8	0.35	0.28
	9	0.39	0.39
	10	0.57	0.70
	11	0.38	0.67
	12	0.58	0.54
Equality	13	0.82	0.62
	14	0.71	0.69
	15	0.82	0.62
	16	0.56	0.51
	17	0.53	0.62
	18	0.52	0.60
Equation-solving	19	0.47	0.74
	20	0.32	0.70
	21	0.60	0.75
	22	0.56	0.75
	23	0.54	0.72
	24	0.46	0.65
	25	0.29	0.67
	26	0.27	0.63
	27	0.21	0.56
	28	0.34	0.60

Four items represented in Table 1 were removed from the test (28 item) because 4 items about the concept of variable had item discrimination indices lower than 0.40; therefore, 24 items formed the final test. Because each right answer was coded as 1, each wrong or blank items were coded as 0, KR-20 was used to determine the reliability in terms of internal consistency. The reliability coefficient was found as 0.98 and this was a very high coefficient showing the test reliability is high as well.

2.4. *Data Analysis*

To examine the relationship between students' knowledge level about the concepts of equality and variable and students' equation solving performance, Pearson product-moment correlation coefficient was used. To determine predictive relationship between students' knowledge level about the concepts of equality and variable and students' equation-solving performance, multiple regression analysis was used. Multiple regression analysis is used to examine the relationship between one dependent variable and multiple predictor variables. During analysis procedure some outliers have an effect on regression model and these outliers make the model not to fit to the hypothetical model (Can, 2013). Therefore, scores' Mahalanobis distances were calculated and two outliers were eliminated, so that the data having 407 scores were analyzed. SPSS (18.0) software was used to execute data analysis.

3. Results

3.1. Results of the first research question

Pearson product-moment correlation coefficient was used to determine the relationship between middle school students' knowledge level about the concepts of equality and variable and students' equation-solving performance. The results of the Pearson's *r* correlation coefficient among three variables are represented in Table 2.

Table 2. The correlation between equality, variable, and equation solving performance

	Equality	Variable	Equation-solving
Equality	1		
Variable	0,61**	1	
Equation-solving	0,45**	0,56**	1

** p<.01

The correlation coefficients between eight grade students' knowledge level about the concept of variable and equality and their equation-solving performance were .45 and .56, respectively. These correlation coefficients were statistically significant (p<.01). Both concepts were moderately statistically significantly correlated with students' equation-solving performance. The correlation coefficient between the concept of variable and equation-solving performance was higher than the correlation coefficient between the concept of equality and equation-solving performance.

To determine the predictive relationship between students' knowledge level about the concept of variable and equality and equation-solving performance, students' equation-solving performance was used as dependent variable whereas students' knowledge level about two concepts were used as independent variables; thus multiple regression analysis was used. The results of multiple regression analysis are displayed in Table 3.

Table 3**.** Multiple regression analysis results for equation-solving

Model	R	R^2	F
Equality-Variable	0.58	0.33	99.99**

Variables	Unstandardized β weights	SE	Standardized β weights	*t*
Equality Concept	.33	.102	.17	3.292**
Variable Concept	.88	.099	.46	8.923**

** p<.01

As a result of multiple regression analysis, the relationship between students' knowledge level about the concept of variable and equality and equation-solving performance (R= .58, R^2=.33) was statistically significant (F=99.99, p<.01). The predictors -equality and variable- explained 33% of the total variation of equation-solving performance. In addition to that, in terms of the standardized regression weights, the order of importance of predictors for equation-solving performance were variable (β=.46) and equality (β=.17), respectively. In other words, the concept of variable had higher significant positive regression weight than the equality, indicating students with the knowledge about variable were expected to have higher equation-solving performance after controlling other variable in the model. However, both predictors were significantly effective on students' equation solving performance.

3.2 Results of the second research question

Another critical question in this research is how students' knowledge level about the concepts of equality and variable predicted students' arithmetic equation-solving performance. To determine predictive relationship between students' knowledge level about the concepts of equality and variable and students' arithmetic equation-solving performance, multiple regression analysis was used when latter was dependent variable and formers were independent variables. The results of the Pearson's *r* correlation coefficient among three variables and multiple regression analysis are represented in Table 4 and 5, respectively.

Table 4. The correlation between equality, variable, and arithmetic equation solving performance

	Equality	Variable	Arithmetic Equation-solving
Equality	1		
Variable	0,59**	1	
Arithmetic Equation-solving	0,40**	0,51**	1

** p<.01

The correlation coefficients between eight grade students' knowledge level about the concept of equality and variable and their arithmetic equation-solving performance were .40 and .51, respectively. These correlation coefficients were statistically significant (p<.01). In other words, both concepts were moderately statistically significantly correlated with students' arithmetic equation solving performance while the correlation between variable concept and arithmetic equation solving was higher than the correlation between equality concept and arithmetic equation solving.

To determine the predictive relation between the concepts of equality and variable and arithmetic equation-solving performance, multiple regression analysis was used where the concepts of equality and variable were independent variables and students' arithmetic equation-solving performance was dependent variable. The results for multiple regression analysis are displayed in Table 5.

Table 5. Multiple regression analysis results for arithmetic equation-solving

Model	R	R^2	F
Equality-Variable	0.53	0.28	76.91**

Variables	Unstandardized β weights	SE	Standardized β weights	*t*
Equality Concept	.19	.06	.15	2.80**
Variable Concept	.54	.06	.43	8.11**

** p<.01

The concepts of equality and variable were statistically significant predictors (F=76.91, p<0.01) of students' arithmetic equation solving performance (R=.53, R^2=.28). The predictors -equality and variable- explained 28% of the total variation of students' arithmetic equation-solving performance. The order of strength of predictors in terms of standardized regression weights for arithmetic equation-solving performance were variable (β=.43) and equality (β=.15), respectively. The concept of variable had higher significant positive regression weight than the equality, indicating students with the knowledge about variable were expected to have higher arithmetic equation-solving performance after controlling other variable in the model.

3.3 Results of the third research question

The third research question focused on how students' knowledge level about the concepts of equality and variable predicted students' algebraic equation-solving performance. To answer this question, multiple regression analysis was used when students' knowledge level about the concepts of equality and variable were independent variables and students' algebraic equation-solving performance was dependent variable. The results of correlation between students' algebraic equation-solving performance and independent variables and multiple regression analysis are represented in Table 6 and 7, respectively.

Table 6. The correlation between equality, variable, and algebraic equation solving performance

	Equality	Variable	Algebraic Equation-solving
Equality	1		
Variable	0,59**	1	
Algebraic Equation-solving	0,35**	0,48**	1

** p<.01

The correlation between students' knowledge level about two concepts and students' algebraic equation-solving performance was statistically significant (p=.01) and the correlation coefficients were .35 and .48, respectively. The relationship between correlation between variable concept and algebraic equation solving was higher than the correlation between equality concept and algebraic equation solving according to correlation coefficients.

To investigate the predictive relationship between the concepts of equality and variable and algebraic equation-solving performance, multiple regression analysis was used where the concepts of equality and variable were predictor variables and students' algebraic equation-solving performance was outcome variable. The results of multiple regression analysis are presented in Table 7.

Table 7. Multiple regression analysis results for algebraic equation-solving

Model	R	R^2	F
Equality-Variable	0.49	0.24	63.46**

Variables	Unstandardized β weights	SE	Standardized β weights	*t*
Equality Concept	.08	.04	.10	1.87**
Variable Concept	.36	.04	.42	7.88**

** p<.01

The results in Table 7 showed that the concepts of equality and variable were statistically significant predictors (F=63.46, p<0.01) of students' algebraic equation solving performance (R=.49, R^2=.24). The equality and variable predictors explained 24% of the total variation of students' algebraic equation-solving performance. In regard to standardized regression weights, the order of importance of predictors for algebraic equation-solving performance were variable (β=.42) and equality (β=.10), respectively. The concept of variable had higher significant positive regression weight than the equality, indicating students with the knowledge about variable were expected to have higher algebraic equation-solving performance after controlling other variable in the model.

4. Conclusion

The results of this study focusing on prediction of students' equation-solving performance by students' knowledge about equality and variable concepts showed that both concepts were statistically significant predictor of students' equation-solving performance. In addition to that, one unit of change in the concept of variable created more increase on the outcome than equality concept, when the other predictor held constant in both cases. Thus, the concept of variable had stronger effect on the prediction of students' equation-solving performance than the concept of equality. How well variable and equality concepts predicted students' performances about algebraic equation having variables on both sides of equal sign (e.g. $ax \pm b = cx \pm d$) and arithmetic equation not having variables on both sides of equal sign (e.g. $ax \pm b = c$) was examined. As a result of this examination, both concepts were statistically significant predictor of students' algebraic and arithmetic equation-solving performance. The concept of variable had stronger effect on the prediction of students' algebraic and arithmetic equation-solving performance than the concept of equality. However both independent variables had stronger effects on the prediction of arithmetic equation not having variables on both sides of equal sign in comparison to the prediction of algebraic equation having variables on both sides of equal sign. Understanding the equal sign as a relational symbol plays an important role in students' learning algebraic equation solving such as 3x-5=2x+1 (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005). Within this study's context, the higher explanation of arithmetic equation-solving performance by equality and variable concepts compared to algebraic equation solving performance showed that understanding equality as a relational concept supported the importance of equality in algebraic equation-solving. Moreover, conceptual comprehension of the variable concept is critical for students' algebraic equation-solving performance.

Bernard and Bright (1984) emphasized the essential influence of variable concept in most instances and specifically in equation solving. The prediction of equation solving performance by the concept of variable in this study supported this important influence. Furthermore, Knuth, Stephens, McNeil and Alibali (2006) stated that there is a strong relationship between understanding the role of equal sign and algebraic linear equation solving when test scores were controlled. In this study, the relationship between equality concept and equation-solving performance is moderately positive and supported Knuth and his colleagues' study results (2006).

References

Alibali, M.W., Knuth, E. J., Hattikudur, S., McNeil, N.M. and Stephens, A. C. (2007). A longitudinal examination of middle school students' understanding of the equal sign and equivalent equations. *Mathematics Thinking and Learning*, 9(3), 221-247.

Andrews, P. & Sayers, J. (2012). Teaching linear equations: case studies from Finland, Flanders and Hungary. *The Journal of Mathematical Behaviour*, 31, 476-488.

Baroody, A. J. & Ginsburg, H.P. (1983). The effects of instruction on children's concept of equals. *Elementary School Journal*, 84, 199-212.

Baykul, Y. (2004). *İlköğretimde Matematik Öğretimi (6-8. Sınıflar İçin).* Ankara, Pegem A Yayıncılık.

Bernard, J. E. & Bright, G. W. (1984). Student performance solving linear equations. *International Journal of Mathematics Education in Science and Technology*, 15(4), 399-421.

Booth, J.L. & Davenport, J.L. (2013). The role of problem representation and feature knowledge in algebraic equation-solving. *The Journal of Mathematical Behaviour*, 32, 415-423.

Can, A. (2013). *SPSS ile Bilimsel Araştırma Sürecinde Nicel Veri Analizi*. Ankara: Pegem Akademi.

Dominguez, A. (2001). *College Algebra Students' Understanding of the Concept of Variable*. Unpublished Doctoral Dissertations. Graduate School of Syracuse University.

Filloy, E., & Rojano, T. (1989). Solving equations, the transition from arithmetic to algebra. *For the Learning of Mathematics* , 9 (2), 19-25.

Hail, C. J. (2000). *The Effects of Using Multiple Representations on Students' Knowledge and Perspectives of Basic Algebraic Concepts*". Unpublished Doctoral Dissertations, University of Kentucky, Lexington.

Huntley, M.A., Marcus, R., Kahan, J. & Miller, J.C. (2007). Investigating high-school students' reasoning strategies when they solve linear equations. *Journal of Mathematical Behaviour*, 26, 115-139.

Katz, V. J. (2007). Stages in the history of algebra with implications for teaching. *Educational Studies in Mathematics*, 66, 185-201.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, 12, 317-326.

Knuth, E. J., Alibali, M. W., Hattikudur, S., McNeil, N. M., & Stephens, A. C. (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School*, 13(9), 514-519.

Knuth, E. J., Alibali, M.W., McNeil, N.M., Weinberg, A., & Stephens, A.C. (2005). Middle school students' understanding of core algebraic concepts: equivalence and variable. *ZDM*, 37(1), 68-76.

Knuth, E. J, Stephens, A.C., McNeil, N.M. & Alibali, M.W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education*, 37(4), 297-312.

Li., X. (2007). An Investigation of Secondary School Algabra Teachers' Mathematical Knowledge for Teaching Algebraic Equation Solving". Unpublished Doctoral Dissertation, University of Texas, Austin.

MacGregor, M., & Stacey, K. (1997). Students' understanding of algebraic notation: 11-15. *Educational Studies in Mathematics*, 33, 1–19.

MEB (2013). *Ortaokul matematik dersi 5, 6, 7, 8. sınıflar öğretim programı ve kılavuzu.* Ankara

McNeil, N., &. Alibali, M.W (2005). Why won't you change your mind? Knowledge of operational patterns hinders learning and performance on equations. *Child Development* , 76(4), 883-99.

McNeil, N.M., Grandau, L., Knuth, E.J., Alibali, M.W., Stephens, A.C., Hattikudur, S. & Krill, D.E. (2006). Middle School Students' Umderstanding of the Equal Sign: The Books They Read Can't Help. *Cognition and Instruction*, 24 (3), 367-385.

Osborne, A. & Wilson, P. S. (1992). *Moving to algebraic thought*. In T. R. Post (Ed.) Teaching mathematics in grades K-8: Research-based methods (pp. 421-442). Boston: Allyn and Bacon.

Saenz-Ludlow, A. & Walgamuth, C. (1998). Third graders' interpretations of equality and the equal symbol. *Educational Studies in Mathematics*, 35 (2), 153-187.

Schoenfeld, A. H., & Arcavi. A., (1988). On the meaning of variable. *Mathematics Teacher*, 81(6), 420–427.

Stacey, K., & MacGregor, M. (2000). Learning the algebraic method of solving problems. *Journal of Mathematical Behaviour*, 18(2), 149- 167.

Tekbıyık, A. (2014). *İlişkisel Araştırma Yöntemi*. Mustafa Metin (Ed.), *Kuramdan Uygulamaya Eğitimde Bilimsel Araştırma Yöntemleri* (pp. 99-114). Ankara: Pegem Akademi.

Usiskin, Z. (1988). *Conceptions of school algebra and uses of variables*. In A. F. Coxford, (Eds.). The ideas of algebra, K-12. 1988 Yearbook (pp. 8-19). Reston, VA; National Council of Teachers of Mathematics.

Warren, E. (2006). Comparative mathematical language in the elementary school: a longitudinal study. *Educational Studies in Mathematics*, 62, 169-189.

Yaman, H., Toluk-Uçar, Z, & Olkun S. (2003). İlköğretim öğrencileri eşit işaretini nasıl algılamaktadırlar?. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 24(24)*, 142-151.