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Algebra is a Dream? Is It a Game?

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Abstract
The purpose of this research is to reveal the perceptions of secondary school mathematics teacher candidates about algebraic thinking through metaphors. The study was conducted with the participation of 160 students on a volunteer basis who study in the department of mathematics teaching in a public university in 2015-2016 education year. In the study, the sentence "algebraic thinking is like …….., because ……." was required to be completed in order to reveal mental structures the students had about algebraic thinking. As a result of the research, it was determined that prospective teachers could produce metaphors related to features such as algebraic thinking and algebra learning. There was categorized such as representation quantities with letters, generalized arithmetic, to analyze the variables in a context, variable, algebraic thinking as patterns, equality and equation. The most used metaphor is in the category of "To analyze the variables in a context". Metaphors help to associate concepts with everyday life. It is therefore advisable to use it in the field of education in order to ensure that the learning is permanent.
Keywords: Metaphors, algebraic thinking, secondary school mathematics teacher candidates

1. Introduction

In today's educational understanding, it is aimed students show the ways to reach information, make comparisons, have creative and critical thinking and link what they have learnt from daily life. In daily life, the necessity to use and understand mathematics is becoming important and constantly increasing. In a changing World, those who understand and do mathematics could shape their future better (MoNE, 2009). Doing mathematics means developing methods for problem solving, realizing whether the results are obtained by carrying out these methods and testing the significance of the results. Mathematics is the science of pattern and order (Van De Walle, Karp and Bay-Williams, 2012). One of the sub-fields of mathematics is algebra. The structure of abstract thinking in which algebra includes provides an important component within the basic information and skills which the individuals would acquire, and it also tends to be a binding rule and a structive element in educational and professional lives (Erbaş, Çetinkaya and Ersoy, 2009). We see that algebra is defined differently in literature. Algebra is the use of letters to represent numbers and quantities (Kieran, 1992). It is the way of a language (Driscoll, 1999) and thinking (Vance, 1998). Based on the definition of Vance, the term of algebraic thinking has emerged. Algebraic thinking is a field which includes in reasoning, symbolic notations and understanding, using and explaining the variables, studying with models to improve mathematical ideas, making transformations between the notations (Kaf, 2007), the notation and structuring of patterns and orders, thinking with generalisations (Hawker and Cowley, 1997), rational thinking, balance, the meaning of variables, the patterns and functions and inductive and deductive reasoning (Greenes and Findell, 1998). Herbert and Brown (1997) defined algebraic thinking and reasoning as words, shapes, tables, graphs and equations in the representation of mathematical knowledge, using mathematical symbols and tools for the analysis of different situations, linking the functional relationships with similar or new situations and performing or interpreting mathematical findings to solve unknowns.

It was noticed from the previous studies that students had difficulty in understanding and falling into misconceptions about algebraic terms (such as equation, equality, variable, algebraic expressions, unknown) (Baki, 1998; Dede and Argün, 2003; Ersoy and Erbas, 1998; Kaput, 1999; Kieran, 1992; MacGregor and Stacey, 1993). The fact that students had difficulty in understanding the topics and falling into misconceptions could be caused by teaching methods. Teaching methods can directly affect mental activities that students may experience. Teachers should teach students in a way that will make them understand algebra and rise the permanence level to the top level (Leitze and Kitt, 2000). In teaching algebra, embodying the abstract terms and creating multiple patterns may increase the understanding and permanence level. Besides, linking it with daily life may help realize conceptual understanding.

One of the methods in linking with daily life is the use of simple analogies. For instance, in teaching negative numbers, using air temperatures below zero is an analogy (Gainsburg 2008). There are also some studies showing the connection between analogy and metaphor (Zwicky, 2010). Analogy or metaphor is beneficial in terms of seeing the complex and well-developed mathematical point of views of the individuals (Schink, Neale, Pugalee and Cifarelli, 2008). Understanding students' belief better will provide information about their acquisition of mathematics and their beliefs which affect the practises (Leder, Pehkonen and Torner, 2002). Metaphors seem like beneficial tools in both the aspect of belief and determining mathematical point of

view.

Metaphors are mental tools used in understanding and explaining abstract and complex phenomenon, mental models which affect people's thoughts about the reality and the world (Saban, Koçbeker and Saban, 2006), the term which can make up social realities by affecting our understanding of the world (Lakoff and Johnson 1999), which guides in revealing unnoticed beliefs and attitudes (Baake 2003) and which helps us array our behaviours (Gross and Hogler 2005). Metaphors are mental models since it is possible to reflect a mental scheme over another by providing relations between two unlike phenomenon (Saban, Koçbeker and Saban, 2006). Therefore, it could be said that metaphors function as a bridge between two phenomena. When someone says "proving is like playing with legos", he means to say that proving can occur when you combine the parts properly. Here, since legos are familiar items with the properties of their formation of new shapes by combining the parts with which people play in their childhood, it is implicated that proving, a phenomenon, can have the same properties.

One of the reasons that metaphors are often used in educational studies is that people can determine the concept traces in their minds with different words which are not relevant to the concepts. Another reason is that metaphors are powerful mental mapping and modelling in structuring and understanding of the individuals' world (Arslan and Bayrakçı, 2006). For each sub-unit in educational field, the perception styles of some masses were studied through metaphors. For instance, the perceptions about the concept of teacher (Aydeş and Akın, 2016; Çocuk, Yokuş and Tanrıseven, 2015; Emerson and Mansvelt, 2015; Koç, 2014; Saban, Koçbeker and Saban, 2006; Yılmaz, Göçen and Yılmaz, 2013)), the concept of student (Saban, 2009), geographical information systems (Akbaş and Gençtürk, 2013), the concept of light (Aygün, Durukan and Hacıoğlu, 2015), the concepts of heat, temperature and energy (Çelik, 2016), physics, physics course and physics teacher (Çetin, 2016) post-graduate education (Bozpolat, 2016), education program (Gültekin, 2013), ideal library (Demir and Güneş, 2016), Sports (Dervent, 2016), technology (Durukan, Hacıoğlu and Usta, 2016), the concept of multigrade classes (İlter, 2015) and smart boards (Aktürk, Mihçi, and Çelik, 2015) and internet (Şahin and Baturay, 2013) were studied through metaphors. Besides, there are also some studies about mathematics. There are some available studies which explore about the beliefs of the prospective teachers of primary school on students, the content of mathematics and school lessons, (Reeder, Utley and Cassel, 2009), perceptions of the students in mathematics department on proving (Cansız Aktaş and Aktaş, 2013), perceptions of gifted and non-gifted middle school students (Bahadır, 2016) and high school students (Horzum and Yıldırım, 2016) on geometry, perceptions of gifted students on mathematics (Arıkan and Ünal, 2015), perceptions of prospective teachers in mathematics on mathematics (Güler, Akgün, Öçal and Doruk, 2012; Güner, 2013a), perceptions of 7^{th} grade students in primary school on mathematics (Bahadır and Özdemir, 2012), perceptions of 12^{th} grade students on mathematics (Güner, 2013b), perceptions of prospectives of class teachers on mathematics (Güveli, İpek, Atasoy and Güveli, 2011), perceptions of bachelors on mathematical problems (Uygun, Gökkurt and Usta, 2016), and beliefs of 9^{th} and 10^{th} grade students on mathematics (Schink et. al., 2008). It was clearly understood that although there are several national and international studies in which perceptions of concepts about teacher, student, mathematics and mathematics problem, no study has been encountered about algebraic thinking which has a great importance in mathematics. With this respect, this study is thought to contribute dramatically to fill the gap in literature.

The purpose of this study is to determine the perceptions of prospective teachers of mathematics in middle school with the help of metaphors. In this respect, some answers were tried to be found for the questions below:

1. What are the metaphors students produced about algebraic thinking?
2. Under which conceptual categories could these metaphors be collected in terms of their common properties?

2. Method

This qualitative study which aims to determine perceptions of prospective teachers about algebraic thinking through metaphors was designed as descriptive. Since it is about perceptions, the data collected through metaphors were subjected to descriptive analysis. Metaphor which is one of the data collecting tools can be used in obtaining powerful and substantial findings (Patton, 2002; Yıldırım and Şimşek, 2008).

2.1 Participants

The study was conducted with the participation of 160 students on a volunteer basis who study in the department of mathematics teaching in a public university in 2015-2016 education year. It was given importance that the participants should come different class levels to assess the study problem in a wide sense. 37 from the first class, 40 from the second, 39 from the third and 44 students from the last class were participated in the study.

2.2 Collecting Data

In the study, the sentence "algebraic thinking is like …….., because ……." was required to be completed in

order to reveal mental structures the students had about algebraic thinking. The documents which the students completed by using their own handwriting constitute the basic data source of the study. In this document, while the word 'like' was given to reveal the connection between the topic of the metaphor and source, the word 'because' was given to provide participants to give reasons and reasonable base for the metaphors they produced. After the prospective teachers were given the form they had to fill in, necessary explanations were made about metaphors. In this respect, the participants were required to write metaphors about the definition of algebraic thinking in accordance with their thoughts and to explain the reasons for the definitions they made. This form was applied to the students nearly for 15 minutes.

2.3. Data analysis

Content analysis method was conducted in the analysis of data obtained from the study. In content analysis, it is aimed to make up specific frameworks by conceptualising obtained raw data and to interpret the meaning by making up codes and categories after the framework is clear (Patton, 2002). In order to obtain data, the stages below determined by Saban (2009) were followed:

The first stage: (naming): after the metaphors obtained from data were listed, the documents including no metaphor or the papers left empty were marked before they were eliminated. These works were conducted by carrying all data into computer.

The second stage (eliminating and increasing): In this stage, the metaphors listed were revised in accordance with justifying between the topic of metaphor and the source of it. Among the metaphors listed, the answers which do not include any metaphor source, those which do not contribute to understand the algebraic thinking and those which include more than one category were determined and excluded.

6 of the answers of the first-class participants, 18 of the second class, 1 of the third class and 20 of the forth class participants in total 57 answers were excluded because of the reasons explained below.

The metaphors which include no source: For instance, in a data as such *"indeed, algebraic thinking varies by interests and natural circumstances. For example, the place where farming is predominantly practised is about productive based and the place where trade is predominantly practised is about money-based or sometimes, the family structure is important. In general theme, imagination and visuals are given."* there is not metaphor source. Besides this, the statements of prospective teachers such as "*algebraic thinking is like finding a more than one unknown operation, because finding the unknown in the formula is the purpose of algebraic thinking.*" were added.

The metaphors that make algebraic thinking to be understood better: For instance, although the statement "*algebraic thinking is like a child, it builds complicated dreams*" seems like a metaphor, it may not be working in interpretation of algebraic thinking. Though the topic (algebraic thinking) and source (the child) of the metaphor is clear, it is agreed that this metaphor is no use in explaining algebraic thinking better (that is, there is no a significant property that could be attributed from the source of the metaphor to the topic of the metaphor). Since it is not possible to explain the stages of algebraic thinking by child metaphor, it was excluded from the data analysis.

The answers including metaphors belonging more than one category: For example, a thought given as such: "*algebraic thinking is like a calculator, because it necessitates fast thinking by practices, comparing the elements from different types, matching and faster operation ability*." falls into three categories: representation of quantities with letter for algebraic thinking, emphasing the term 'variable' and equality.

The third stage (composition and improving category): In this stage, 66 metaphors the students produced and left at the end of the second stage were collected under 6 conceptual categories according to the properties they have. The categories were named to represent the metaphors and the components of the process of algebraic thinking.

The forth stage (validity): in this study, the process of data analysis was broadly analysed to ensure the validity. The conceptual categories which best represent the determined metaphors were explained in detail in the findings by using original quotations. It is important to describe the data analysis broadly and explain how the outcomes are obtained to provide the validity in qualitative researches (Yıldırım and Şimşek, 2008).

The fifth stage (reliability): in this study, first writer's analysis was conducted to provide reliability about the determined metaphors. A form defining the properties of the reached categories was designed. Second writer pointed out whether the given metaphors under the categories were included in that category. Afterwards, the reliability of consensus between the first and second writer were calculated by using the formula (Reliability = consensus / consensus + divergence) of Miles and Huberman (1994). After the operation conducted, %82 consensus was found in the reliability study.

3. Findings

The findings obtained after subjecting the metaphors stated by the participants about the algebraic thinking to the first two of the stages of data analysis are given in Table 1.

Table 1. Metaphors and Frequencies the Participants made up about the Algebraic Thinking

Metaphors	**f**	**Metaphors**	**f**
Proverb	1	Gossip	1
Detail	1	Fruit And Vegetables	2
Riddle	7	Tidying Up The KiPThen	1
Science	1	Absolute Infinity	1
The Source Code Consisting Of Unknowns	1	Ocean	1
To Define Knowledge Mathematically	1	Game	2
Solving A Riddle	1	Knitting	1
A Stationery Seller	1	Going From Part To Whole	1
Riding A Bicycle	1	Pedlar's Trade	1
Empty box	1	Using fingers	1
Soup	3	Plan	1
Dedective	1	Analysis of Problems	1
Sea	1	Detective Film	1
Four Seasons	1	Dream	1
Expressing Feelings	1	Novel Characters	1
Thinking In A Different Language	1	Hide And Seek Game	1
A Football MaPTh	1	Representation of A Pot To A Flower	1
A rainbow	1	Art	1
Sky	1	Dealing with numbers	2
A hammock	1	Eyes of a fly	1
Game with letters	1	Finding solutions for problems	1
Dreaming	1	Abstract thinking	10
The challenges in our life	1	Embodying the abstract	3
A calculator	1	Sudoku	2
To understand everything	1	A Scales	2
As if everything were numbers	1	Inductive	1
Human mind	1	Variety	1
Balls of string	2	3 dimensional thinking	2
Operation	1	Foreign music	1
To Find A Missing Item	7	Puzzle	1
Lego Game	2	Recipe	1
Mathematical Thinking	5	A Road Guidebook	1
Greengrocer	1	A Brain Box	1
Total 103			

When Table 1 was studied, it was detected that from the 103 valid answers 66 different metaphors were obtained. The prevalence of the metaphors ranges from 1 and 10. According to these data, the most prevalent metaphor is the "*abstract thinking*" produced by 10 participants. Subsequently, it was followed by "*riddle*" and "*finding the missing item*" with 7 frequencies, "*mathematical thinking*" with 5, "*thinking an abstract thing*" and "*soup*" metaphors produced by 3 participants, "*balls of string*", "*lego game*", "*fruit and vegetables*", "*dealing with numbers"*, *"sudoku"*, *"a scales*" and "*3 dimensional thinking*" produced by 2 participants. Therefore, it was concluded that various metaphors were produced about algebraic thinking by prospective teachers of mathematics.

Conceptual Categories

In accordance with the third stage of data analysis, the metaphors in Table 1 were compiled and collected under specific categories. While creating the categories, the definition of algebraic thinking and the terms including in it were considered. Accordingly, at the end of the third stage, 6 conceptual categories were made up. The frequencies and percentage of these categories are given in Table 2:

Table 2. The Categories Made Up About Algebraic Thinking

CATEGORIES		f %	Metaphors
1	Generalised arithmetic	6 5,8	Using fingers (1), operation (1), dealing with numbers (2), a stationery seller (1), sudoku (1)
2	To analyse the variables in a context	27 26,2	Foreing music (1), mathematical thinking (5), abstract thinking (10), analysis of problems (1), a brain box (1), ocean (1), dreaming (1), sudoku (1), sea (1), detail (1), 3 dimensional thinking (2), to understand everything (1) to define knowledge mathematically (1)
3	Variable	21 20,4	Science (1), a source code consisting of unknowns (1), thinking in a different language (1), absolute infinity (1), proverb (1), dream (1), empty box (1) hide and seek game (1), variety (1), challenges in our life (1), dedective (1), gossip (1), human mind (1), fruit and vegetables (2) pedlar's trade (1), greengrocer (1), soup (3)
4	Representation quantities with letters	9 8,73	A rainbow (1) as if everything were numbers (1) embodying the abstract (3) novel characters (1) a pot flower (1), expressing feelings (1), game with letters (1)
5	Algebraic thinking as patterns	20 19,4	A game (1), solving a riddle (1), hammock (1), lego game (2), riddle (7), going from part to whole (1), inductive (1), finding a missing item (4), knitting (1), puzzle (1)
6	Equality and equation	20 19,4	Soup (2), eyes of a fly (1), balls of string (2), riding bicycle (1), a scales (2), a road guidebook (1), plan (1), a football match (1), tidying up the kitchen (1), game (1), finding solutions for problems (1) detective film (1), recipe (1), art (1) finding a missing item (3)

When Table 2 was studied, it could be stated that the most used metaphor is in the category of "To analyse the variables in a context" with 27 frequencies. Subsequently, the category "variable" (21) followed. Besides, it was noticed that the metaphors in the categories of "algebraic thinking with patterns" and "equality and equation" were equally distributed. Finally, the metaphors in the categories of "representation quantities with letters" (9) and "generalised arithmetic" (6) followed. It gives clue that their point of view about algebraic thinking was close to mathematics since the metaphors the prospective teachers of mathematics gave in the category of "to analyse the variables in a context" were plenty.

Category 1: Generalised Arithmetic

In this category, 6 (%5,8) of the metaphors accounting for 66 valid metaphors produced by 103 prospective teachers are included. The most prevalent of the metaphors is "dealing with numbers". Other metaphors prospective teachers of mathematics stated are as such: "using fingers", "operation", "a stationery seller" and "sudoku". The expressions in which the prospective teachers stated these metaphors and reasons are as follows:

Algebraic thinking is like using fingers, because it is about operation with numbers. It is about dancing with numbers in the sea of mathematics. (PT2)

Algebraic thinking is like operations, because algebra is like an operation with unknowns. (PT85)

Algebraic thinking is like dealing with numbers, because I think about numbers when it is about algebra, so the field of mathematics other than geometry comes to my mind. (PT35)

Algebraic thinking is like a stationary seller, because you see how much two books and a rubber cost when you label the prices one by one. (PT55)

Algebraic thinking is like four seasons, because it teaches you how to make four operations and it improves your intelligence. (PT64)

Algebraic thinking is like sudoku, because you play games with numbers. (PT120)

When the metaphors were studied, it was noticed that algebraic thinking were considered as a generalisation of making operations with numbers by the prospective teachers. It can also be said that they noticed the numbers had a place in algebraic thinking.

Category 2:to analyse the variables in the context

In this category, there are 12 different metaphors produced by 27 participants. This accounts for 26,2% of the total valid metaphors. The most prominent metaphors are abstract thinking (10) and mathematical thinking (5). It was seen in Table 2 that other metaphors are foreign music (1), problem analysis (1), brain box (1) ocean (1), dreaming (1), sudoku (1), sea, (1) and detail (1). The statements given by prospective teachers of mathematics were explained with direct quotations:

Algebraic thinking is like dreaming, because we embody the topic of algebra in our mind and understand it by using abstract thinking and imagination. (SA76)

Algebraic thinking is like problem solving, because we do reasoning and go from unknown to known. (PT23)

Algebraic thinking is like abstract thinking, because the operation is not made about numbers but unknowns.

(PT79)

Algebraic thinking is like thinking unreal, because it is an abstract term. It is difficult to understand and perceive. (PT82)

Algebraic thinking is like foreign music, because you feel it even if you don't understand the meaning. (PT6)

Algebraic thinking is like sudoku, because it is quite abstract and difficult, if you understand it, it is as if a game. (PT53)

Algebraic thinking is like sea, because it is wide-ranging. (PT103)

Algebraic thinking is like detail, because it is about handling something with overall aspects. (PT107)

Algebraic thinking is like a form of mathematical thinking, because mathematical thinking and algebraic thinking is closely related. (PT5)

When the metaphors were studied, it was detected that high level abilities such as 'to analyse the variables in a context' were more emphasized. When the quotations the prospective teachers of mathematics gave were analysed, it was observed that reaching solutions were suggested by analysing the changes that would happen in a context such as attempt to reach unknown, abstract thinking, problem solving, feeling the existence of something that you cannot see, wide ranging and dealing with details.

Category 3: Variable

In this category, there are 16 different metaphors produced by 21 participants. It accounts for 20,4% of the metaphors accepted as valid. The most prevalent of these metaphors are fruit and vegetables and soup. The others, as seen in Table 2, are science (1), the source code consisting of unknowns (1), thinking in a different language (1), absolute infinity (1), proverb (1), dream (1), empty box (1), hide and seek game (1), variety (1), challenges in our lives (1), detective (1), gossip (1), human mind (1), Pedlar's Trade (1) and greengrocer (1). The expressions obtained from the documents the prospective teachers of mathematics stated for metaphors are seen below.

Algebraic thinking is like science, because education is to learn knowledge in books, but science is to find the unknown. (PT12)

Algebraic thinking is like the source code consisting of unknowns, because it reserves the secrecy system within itself. (PT13)

Algebraic thinking is like thinking in different language, because unknowns are engaged in the work and a mathematical language appears. (PT65)

Algebraic thinking is like absolute infinity, because there is no an end of the value given for the unknown.an unknown could be a rational number or an integer. Therefore, it is absolute infinity.(PT70)

Algebraic thinking is like a proverb, because in algebraic thinking, the symbol has a value but is not known. The proverbs also have value. However, the person who told is not known. (PT72)

Algebraic thinking is like dream, because dreams are abstract. Algebra is also abstract. The things happening in dreams cannot happen in real. But we remember the dream we see and understand and tell about it. (PT77)

Algebraic thinking is like empty boxes, because we find the box according to the operation, that is, we think algebraically. (PT87)

Algebraic thinking is like hide and seek game, because we could think as if finding the missing one. There is something in the end, but it is not certain what it is or where it is. Some certain operations are found at the end of seekings. (PT91)

Algebraic thinking is like variety, because there is any kind of unknown and giving value. (PT131)

Algebraic thinking is like challenges in our lives, because any unknown thing is a challenge in our way. We find the unknowns with our thinking ability and we make our lives easier. (PT143)

Algebraic thinking is like a detective, because detective always finds the unknowns. Likewise, we find the unknowns too. (PT144)

Algebraic thinking is like gossip, because we constantly search the events that we don't know about. In mathematics, we search for the unknown terms. (PT145)

Algebraic thinking is like human mind, because it is full of unknowns. (PT147)

Algebraic thinking is like fruit, because the same fruits function the same things, that is, the same kinds function with the same things. (PT148)

Algebraic thinking is like Pedlar's Trade, because thinking is like a pedlar's job, because for example, they liken Xs or Ys to a known thing. They liken an apple to an X. (PT149) Algebraic thinking is like fruits and vegetables, because it is about operating with the same ones between knowns and unknowns. (PT150)

Algebraic thinking is like a greengrocer, because we make operations by likening unknowns to fruits. (PT151)

Algebraic thinking is like soup, because each ingredient thrown into it is like an algebraic expression and it stirs up trouble. (PT119)

When the statements above were studied, it could be said that prospective teachers of mathematics in middle

school reflected a point of view about the metaphors under the category of variables including in unknown values or changing quantities in general. For the unknown values, some metaphors such as proverb, infinity and gossip could be given as examples. The metaphors which was considered as changing quantities could be exemplified with greengrocer, soup or challenges in our lives.

Category 4: representation of quantities with letters

In this category, there are 7 different metaphors produced by 9 participants. It accounts for 8.73% of the metaphors accepted as valid. The most prevalent of these metaphors is embodying the abstract. The others as seen in Table 2 are rainbow (1), as if everything were numbers (1), novel characters (1), pot flower (1), expressing the feelings (1) and game with letters (1). The written expressions obtained from the documents the prospective teachers of mathematics stated are seen below.

Algebraic thinking is like a rainbow, because it includes all colours (numbers) in it. (PT14)

Algebraic thinking is like as if everything were numbers, because we can easily solve all the things in our lives by turning the problem into simple numbers. (PT27)

Algebraic thinking is like embodying the abstract, because we put expressions such as x and y in place of an object. (PT41)

Algebraic thinking is like novel characters, because while we are reading, we think about the specialties of the characters. Everybody composes a character suitable for themselves. (PT42)

Algebraic thinking is like a floer in a pot, because the unknowns in algebraic expressions, for example x,y,a,b… actually stand for our numbers. (PT96)

Algebraic thinking is like expressing the feeling, because while we are expressing our feelings, we need to symbolize them as in algebra. (PT121)

When these metaphors were studied, it was observed there was a common perception that when letters were used instead of quantities, this would lead to converting an abstract idea into an embodying one. The perception that letters could be used instead of numbers revelaed from the writings of prospective teachers. Besides, it was coincided that there was a thought that letters could be used not only instead of quantities but also things.

Category 5: algebraic thinking as patterns

In this category, there are 10 different metaphors accounting for 19,4% of 103 prospective teachers producing 66 metaphors accepted as valid. The most prevalent metaphors are subsequently riddle (7), finding the missing item (4) and lego game (2). The other metaphors prospective teachers of mathematics produced are game (1), hammock (1), solving a riddle (1), going from part to whole (1), inductive (1), knitting (1) and puzzle (1). The expressions in which the prospective teachers stated about these metaphors and reasons are as follows.

Algebraic thinking is like a game, because the operations look like riddles. (PT101)

Algebraic thinking is like solving a riddle, because it is necessary to find the unknowns. (PT109)

Algebraic thinking is like a hammock, because it consists of ropes connected each other with patterns. After turning ropes a hammock with patterns, only thing that remains is swinging. (PT126)

Algebraic thinking is like a lego game, because when individual lego parts come together, it becomes a new and complex whole. (PT69)

Algebraic thinking is like a riddle, because it is trying to find out an unknown number or it is like adding value to an unknown expression. (PT80)

Algebraic thinking is like a puzzle, because you can reach the whole by connecting the parts close to each other or by finding the ones that have meaningful relations with each other. To find a solution way, taking measures against difficulties. (PT22)

Algebraic thinking is like going from part to whole, because small parts become together and then the whole appears since mathematics is cumulative. (PT84)

Algebraic thinking is like inductive, because we go from parts to whole. (PT43)

Algebraic thinking is like knitting, because however difficult it seems at first glance, it is soon understood how the knits are ordered and knit at second glance. (PT136)

When the metaphors that provide categories of algebraic thinking to be created are studied as patterns, it could be seen that prospective teachers of mathematics created a perception that algebraic thinking is a process of producing generalisation based on patterns. As an example of this perception, hammock, knitting and riddle could be given. However, prospective teachers thought that algebraic thinking includes in the process of inductive thinking. Lego, inductive and going from part to whole could be given as examples for this.

Category 6: equality and equation

In this category, there are 15 different metaphors accounting for 19,6% of 103 prospective teachers producing 66 metaphors accepted as valid. The most prevalent metaphors are subsequently finding the missing item (3), soup (2), balls of string (2) and scales (2). The other metaphors that prospective teachers of mathematics produced are eyes of a fly (1), riding a bicycle (1), a road guidebook (1), plan (1), a football match (1), tidying up the kitchen (1), a game with open rules (1), finding solutions for problems (1), a detective film (1), a recipe (1) and art (1). The expressions in which the prospective teachers stated about these metaphors and reasons are as follows.

Algebraic thinking is like making soup, because we make instant soup according to specific stages. We do the same thing while solving algebraic expressions. (PT99)

Algebraic thinking is like eyes of a fly, because it sees various complexities all at once. It is about balancing. It is to see with multi-dimensional. (PT103)

Algebraic thinking is like balls of string, because students deal with algebraic expressions just as they unrope a string jumbling each other. (PT106)

Algebraic thinking is like riding a bicycle, because you should first learn how to stay in balance, that is algebraic expressions like learning, later pedalling is like using these expressions in questions and finally going forward is like learning more about algebra. There is no end of this road just as mathematics has no end. (PT118)

Algebraic thinking is like scales, because one should notice the equivalent. (PT122)

Algebraic thinking is like a road guidebook, because if we want to reach an outcome, that is getting somewhere, we should discover the shortest and considerable way by using the guidebook. Algebraic thinking is the very same thing. It shows us the way and it makes us understand easier by deepening the meaning. (PT123)

Algebraic thinking is like a plan, because we build an equation in our minds with some certain wishes and then we organize our lives to ensure the accuracy of this equation. (PT52)

Algebraic thinking is like a game with open rules, because the operations to solve the equations are conducted in certain systematic rules. Therefore, algebraic thinking requires a systematic knowledge. (PT25)

Algebraic thinking is like finding a missing item, because we find out situation step by step. That is we first comprehend it through mind, find out the missing item and assess the situation accordingly. What I am saying is that in algebraic thinking, we try to remember the unknown first and think where it could be and then we find out it as a result of various operations and seekings. (PT9)

Algebraic thinking is like finding solutions for problems, because we find X in algebra and we find solutions in regular life. (PT133)

Algebraic thinking is like a detective film, because we constantly look for the unknown and we reach it because of operations.

Algebraic thinking is like a recipe, because the recipe for meal is definite, you put the ingredients. In algebraic thinking, the formula is definite, you put the numbers. (PT153)

Algebraic thinking is like art, because it is made up with the harmony and consistence. (PT159)

It was detected that prospective teachers of mathematics produced metaphors stressing the meaning of equality in the process of algebraic thinking. It was thought that the metaphors; recipe, art, eyes of a fly, riding bicycle and scales might help the symbol of equality to be conceptualized as balance. Prospective teachers of mathematics explained the meaning of algebraic thinking as solving equation and this thought arose from the metaphors that provide this category to be created. For example, from the metaphors such as instant soup, balls of string, a road guidebook, finding solutions for problems and detective film, it was seen that prospective teachers predicated on solution ways of equation.

5. Discussion and Result

When the findings obtained from this study aiming to reveal perceptions of prospective teachers of mathematics about algebraic thinking were studied, it was seen that the first prominent thing is that the metaphors produced consist of a few words rather than a single word or term. For example, the metaphors such as "going from part to whole", "a source code consisting of unknowns", defining knowledge mathematically", "thinking in different languages" and "finding solutions for problems" consist of a few words. It was thought that the reason of this is caused by the fact that the term algebraic thinking given to prospective teachers is an action happening because of a specific process. It could be said that prospective teachers should build a word pattern consisting of a few words since thinking requires a process and since they were required to make up a metaphor about algebraic thinking.

When prospective teachers become teachers in the future, their attitudes and perceptions expected them to transfer to their students about algebraic thinking determine how they will teach these terms (Öztürk, 2007). As is known, metaphors could be used as a powerful searching tool in understanding mental images about a phenomenon people have, in revealing them and in expressing (Bahadır and Özdemir, 2012).

The most prevalent of the metaphor produced by prospective teachers is "*abstract thinking*". Subsequently, "*riddle*" and "*finding the missing item*" repeated by 7 times, "*mathematical thinking*" repeated 5 times, "*embodying an abstract thing*" and "*soup*" produced by 3 participants and the metaphors "*balls of string*", "*lego game*", "*fruit and vegetables*", "*game*", "*dealing with numbers*", "*sudoku*", "*scales*" and "*3-dimensional thinking*" followed. Since the most prevalent metaphor is "abstract thinking" and since this metaphor is included in the category "to analyse the variables in the context" which is one of the important part of algebraic thinking, it was thought that this gave an idea that prospective teachers could keep on their teachings with in-class activities about analysing the relations. The reason why the metaphor "abstract thinking" is so prevalent can be

caused by the fact that prospective teachers regard mathematics as an abstract science. When other metaphors they produced were studied it could be said that they have perceptions about different aspects of algebraic thinking. For example, the metaphor "balls of string" is the one in which prospective teachers showed the perception about algorithmic structure of solving equation.

The results obtained from the categories created in accordance with analysis of data; 5,8% of the metaphors produced by the prospective teachers are in the category of *generalised artihmetics.* These are "dealing with numbers", "using fingers", operation", "a stationary seller", "sudoku" and "four seasons" in subsequence. Since these metaphors are about numbers in general, the category of generalised arithmetics was created. The success of students in algebra depends on deep comprehension of number system. (Seeley and Schielack, 2008). Students cannot easily give up the tendency to think arithmetically which is one of the structures of algebraic thinking (Waren, 2005; Baş, Erbaş and Çetinkaya, 2011). In understanding the rules between operations, it is important to know the patterns between numbers and generalise the operations (Denmana ve Leitlez, 1988). It can be said that the prospective teachers know that generalising numbers is a part of algebraic thinking.

There are 12 (25,4%) different metaphors in the category of "*to analyse the variables in a context*". The prominent metaphors in this category are "abstract thinking" (10) and "mathematical thinking" (5). Also, it was encountered with some metaphors such as foreign music (1), problem analysis (1), brainbox (1), ocean (1), dreaming (1), sudoku (1), sea (1), detail (1), 3 dimensional thinking (1), and understanding everything (1). The reason why the prospective teachers often stated the metaphor of mathematical thinking can be caused by the fact that algebra is one of the sub- fields of mathematics. The statement of the prospective teacher of mathematics "*algebraic thinking is like a kind of mathematical thinking, because mathematical thinking and algebraic thinking is closely related each other*" expresses this idea. In their study on prospective teachers of mathematics, Güler et.al., (2012) searched for their metaphorical perceptions about mathematics. The fact that the prospective teachers produced the metaphor of "abstract thinking" in the category of "view towards mathematics" shows that they regard mathematics as an abstract science. The prevalence of the metaphors in abstract thinking and mathematical thinking matches with the study results of Güler et. al. (2012).

20,6% of the metaphors the prospective teachers produced include in the category of variables. The metaphors in this category are a source code consisting of unknowns (1), thinking in a different language (1), absolute infinity (1), proverb (1), dream (1), empty box (1), hide and seek (1), variety (1), challenges in our lives (1), detective (1), gossip (1), human mind (1), peddlar's trade and greengrocer (1). It was detected that the variable was used in two different meanings: unknown values and changing quantities. It was pointed out that the variable was perceived as unknown values and some difficulties were experienced in perceiving them as changing quantities (Van De Walle, et. al., 2012). It could be said that the metaphors such as proverb, infinity and gossip that were included in unknown values in the category of variable were produced by the prospective teachers. The examples that show that the variable was perceived as changing quantities are greengrocer, soup and challenges in our life. Besides, since understanding the term of variable facilitates the transfer from arithmetic's to algebra, it is of great importance in understanding algebraic expressions in formulas and equations. Students will make generalisations by starting to use variables and thus they will start to use a new language in the expressions in some mathematical situations. (MoNE, 2005). Therefore, the fact that the prospective teachers created metaphors included in the category of variable and that these metaphors were related with two different meanings of the variable shows that they internalized these meanings and they could provide their students to gain different point of views in the future.

In the category of *representation of quantities with letters,* there are 7 different metaphors accounting for 7,8% of the metaphors accepted as valid. The most prevalent of these metaphors is "embodying the abstract". The others are rainbow (1), as if everything were numbers (1), novel characters (1), pot flower (1), expressing the feelings (1) and game with letters (1). When these metaphors were studied, it was noticed that letters were often used instead of quantities and this means there is a common perception that an abstract idea could be embodied. It was also noticed that the prospective teachers used letters instead of not only quantities but also the things.

In the category of *algebraic thinking as patterns,* there are 20 (19,6%) metaphors. The most prevalent metaphors are subsequently riddle (7), finding the missing item (4) and lego game (2). The other metaphors produced are game (1), solving a riddle (1), hammock (1), going from part to whole (1), inductive (1), knitting (1) and puzzle (1). When these metaphors were studied, it could be said prospective teachers of mathematics created a perception that a generalisation should be reached by using patterns. The metaphors that contribute to obtain this result are hammock, knitting or riddle. Additionally, the prospective teachers understood that algebraic thinking includes in inductive thinking process. The examples of metaphor that contribute to obtain this perception are lego, inductive, going from part to whole and puzzle. Generalising the patterns improves the algebraic thinking skills of young children (Akkan and Çakıroğlu, 2012). Kieran (2004) stated that in the development process of algebraic thinking it should be focused not only on quantitative calculations but also on

relationships. Exploring the relations about patterns and making generalisations will help students understand the world around themselves (Akkan and Çakıroğlu, 2012). A student who can make generalisation through patterns can also find opportunity to solve problems by showing mathematical situations and structures and reveal the relations between the quantities. The best introduction to the term variable occurs with the generalisation of patterns and expressing them with letters (English and Warren, 1998). That's why, it is important for students to perceive the relation between patterns.

In the category of *equality and equation,* there are 20 (19,6%) metaphors. The most prevalent metaphors are subsequently "finding the missing item (3), soup (2), balls of string (2) and scales (2)". The others are eyes of a fly (1), riding a bicycle (1), a road guidebook (1), plan (1), a football match (1), tidying up the kitchen (1), a game with open rules (1), finding solutions for problems (1), a detective film (1), a recipe (1) and art (1). It was seen that prospective teachers of mathematics produced metaphors that emphasize the meaning of the symbol of equality in the process of algebraic thinking. The metaphors "recipe, eyes of a fly, art, riding a bicycle and scales" show that the symbol of equality was conceptualized as the balance. The prospective teachers created the metaphor "scales" and this shows they have a point of view that if one side is added a unit, it should be reduced from the other side to keep balance the equality (Van De Walle et.al., 2012). It was understood from the metaphors providing this category to emerge that prospective teachers of mathematics perceived algebraic thinking as solution of equation. For instance, from the expressions of the metaphors such as instant soup, balls of string, road guidebook, finding solutions for problems and detective film, it was understood that the prospective teachers based on the ways of solution of equation. It can be said that conceptualizing the term equality as balance could provide relational understanding for students' problem solving (Van De Walle et.al., 2012). It was thought that when the term equality was interpreted as relational, the solution of equation would also be interpreted as conceptual.

5. Recommendation

When the results obtained from the study were studied, it was noticed that the prospective teachers of mathematics in middle school could produce metaphors about the structure of algebraic thinking. The metaphors produced by the prospective teachers participating in the study can help the structures of algebraic thinking to be taught as conceptual. It was detected that the metaphors the prospective teachers produced are mostly in the category of 'to analyse the variables in a context" and 'variable'. When the categories created were considered, it can be stated that the prospective teachers of mathematics internalized algebraic thinking as conceptual in general since these categories build the frame of algebraic thinking. Therefore, the metaphors the prospective teachers produced can be used as examples in understanding algebraic thinking for students and teachers.

As metaphors are effective tools (Arnett, 1999; Saban, Koçbeker and Saban, 2006) in revealing perception of individuals, the researchers could make similar studies on teachers. Besides, comparative studies which analyse metaphors of teachers and propective teachers about algebraic thinking could be conducted.

References

Akbaş, Y. and Gençtürk, E. (2013). Metaphorical perspective to geographical information system concepts. *Dicle University Journal of Ziya Gökalp Education Faculty*, 21, 179-196

Akkan, Y. and Çakıroğlu, Ü. (2012). Generalization strategies of linear and quadratic pattern: a comparison of 6^{th}-8^{th} grade students. *Education and Science,* 37(165*)*, 104-120.

Aktürk, A. O., Mihçi, S., and Çelik, İ. (2015). Metaphors of high school students about the concept of "interactive whiteboard". *International Journal of Education in Mathematics, Science and Technology*, 3(2), 120-131.

Arıkan, E. E. and Ünal, H. (2015). Gifted students' metaphor images about mathematics. *Educational Research Reviews*, 10(7), 901-906.

Arnett, R. C. (1999). Metaphorical guidance: Administration as building and renovation. *Journal of Educational Administration*, 37, 80-90.

Arslan, M. ve Bayrakçı, M. (2006). Metaforik düşünme ve öğrenme yaklaşımının eğitim-öğretim açısından incelenmesi. *Milli Eğitim Dergisi,* 171, 100-108.

Aydeş, S.S. and Akın, U. (2016). School administrators' metaphorical perceptions about teachers. *Sakarya University Journal of Education,* 6(3), 20-39.,Doi: 10.19126/suje.220173

Aygün M., Durukan Ü.G. ve Hacıoğlu Y. (2015) Fen bilgisi ve ilköğretim matematik öğretmenliği öğrencilerinin 'ışık' kavramıyla ilgili metaforik algıları. *Fen Bilimleri Öğretimi Dergisi ,* 3(2) ,52-64.

Baake, K. (2003). *Metaphor and knowledge: The challenges of writing science.* New York: State University of New York Press.

Bahadır, E., and Özdemir, A. Ş. (2012). 7th grade primary school students' mental images about the concept of mathematic. *International journal of Social science Research,* 1(1), 26-40.

Bahadır, E. (2016). Metaphorical perceptions of geometrical concepts by secondary school students identified as

gifted and identified as non-gifted. *International Online Journal of Educational Sciences,* 8 (1), 118 – 137.DOI: http://dx.doi.org/10.15345/iojes.2016.01.011

Baki, A. (1998, Mayıs). Matematik öğretiminde islemsel ve kavramsal bilginin dengelenmesi. *Atatürk Üniversitesi 40. Kurulu Yıldönümü Matematik Sempozyumu*. Erzurum: Atatürk Üniversitesi.

Baş, S. Erbaş, A.K. and Çetinkaya B. (2011) . Teachers' knowledge about ninth grade students' ways of algebraic thinking. *Education and Science,* 36 (159),41-55.

Bozpolat, E. (2016). Metaphorical perceptions of prospective teachers toward graduate education. *Elementary Education Online*. 15(4): 1114-1130

Cansız Aktaş, M. and Aktaş, D. Y. (2013). Determination of mathematics department students' perceptions about proving through metaphors. *International Online Journal of Educational Sciences,*5 (3), 701-718

Çelik, H. (2016). An examination of cross sectional change in student's metaphorical perceptions towards heat, temperature and energy concepts. *International Journal of Education in Mathematics, Science and Technology*, 4(3) DOI:10.18404/ijemst.86044

Çetin, A. (2016). An analysis of metaphors used by high school students t odescribe physics, physics lesson and physics teacher. *European J of Physics Education,* 7(2), 1309-7202.DOI: 10.20308/ejpe.35860

Çocuk, H.E., Yokuş, G. ve Tanrıseven I. (2015). Pedagojik formasyon öğrencilerinin öğretmenliğe ilişkin özyeterlikve metaforik algıları: Mersin üniversitesi örneği, *Mustafa Kemal Üniversitesi Sosyal Bilimler Enstitüsü Dergisi,*12 (32), 373-387.

Dede, Y. and Argün, Z. (2003). Why do students have difficulty with algebra? *Hacettepe University Journal of Education*, 24, 180-185.

Demir. G. and Güneş, A. (2016). The metaphorical perceptions of the public library administrators on the term of ideal library. *Education Science Society Journal,* 14 (54), 134-151.

Demana, F. and Leitzel, J. (1988). Establishing fundamental concepts through numerical problem solving, In A.F. Coxford (Ed.), The ideas of algebra, K-12, (pp. 61-68), Reston, VA: NCTM.

Dervent, F. (2016). An examination of conceptualization of sport metaphor. *Journal of Education and Training Studies,* 4 (4), 259-268. doi:10.11114/jets.v4i4.1400.

Durukan, Ü.G., Hacıoğlu, Y. and Dönmez Usta, N (2016). Computer education and instructional technology prospective teachers' perceptions of technology. *Journal of Computer and Education Research,* 4(7),24-46.DOI: 10.18009/jcer.15212

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, grades 6-10*. Portsmouth, NH: Heinemann.

Emerson, L. and Mansvelt, J. (2015). Buckets and fire: metaphors in tertiary teaching. *Studies in Higher Education.* 40(10),1872–1888, http://dx.doi.org/10.1080/03075079.2014.886682

English, L. and Warren, E. (1998). Introducing the variable through pattern exploration. *The Mathematics Teacher*, 2, 91, 166-170.

Ersoy, Y. ve Erbas, K. (1998, Kasım). İlköğretim okullarında cebir öğretimi: öğrenmede güçlükler ve öğrenci basarıları. *Cumhuriyetin 75. Yılında İlköğretim I. Ulusal Sempozyumu.* Ankara: Baskent Öğretmenevi.

Erbaş, A.K., Çetinkaya, B. and Ersoy, Y. (2009). Student difficulties and misconceptions in solving simple linear equations. *Education and Science,* 34 (152)

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education,* 11(3), 199–219. doi:10.1007/s10857-007-9070-8.

Güler, G., Akgün, L, Öçal, M.F. and Doruk M. (2012). Pre-service mathematics teachers' metaphors about mathematics concept. *Journal of Research in Education and Teaching.* 1(2), 25-29.

Gültekin, M. (2013). The metaphors that primary education teacher candidates use regarding curriculum. *Education and Science,* 38 (169), 126-141.

Güner, N. (2013a). Pre-servıce teachers' metaphors about mathematıcs. *E-Journal of New World Science Academy-Education Sciences.*8(4), 428-440. http://dx.doi.org/10.12739/NWSA.2013.8.4.1C0597

Güner, N.(2013b). Searching for the exit in amaze? or setting sail for new horizons? Metaphors by twelfth grade students for learning mathematics. *Educational Sciences: Theory & Practice,* 13(3) • 1929-1950. DOI: 10.12738/estp.2013.3.1663

Güveli, E., İpek, A. S., Atasoy, E. and Güveli, H. (2011). Prospective primary teachers' metaphorical perceptions towards mathematics. *Turkish Journal of Computer and Mathematics Education,* 2(2), 140-159.

Gross, M.A., and R. Hogler. 2005. What the shadow knows: Exploring the hidden dimensions of the consumer metaphor in management education. *Journal of Management Education* 29: 3–16.

Greenes, C. and Findell, C. (1998). *Algebra puzzle sand problems (grade 7).* Mountain View, Ca: Creative Publications

Hawker, S. and Cowley, C. (1997). *Oxford dictionary and thesaurus*. Oxford: Oxford University.

Herbert, K. and Brown, R. (1997). Patterns as tools for algebraic reasoning. *Teaching Children Mathematics,* 3(6), 340-344.

Horzum, T.and Yıldırım, G. (2016). High school students' metaphors about geometry. *Mehmet Akif Ersoy University Journal of Education Faculty,* 40, 357-374.DOI: 10.21764/efd.53998

İlter, İ. (2015). Primary school teacher candidates' metaphorical perceptions related to multigrade classes concept. *Elementary Education Online*, 14(4), 1450-1468. doihttp://dx.doi.org/10.17051/io.2015.53507

Kaf, Y. (2007). Matematikte model kullanımının 6. sınıf öğrencilerinin cebir erişilerine etkisi. Yayımlanmamış Yüksek Lisans Tezi, Hacettepe Üniversitesi, Eğitim Bilimleri Enstitüsü, Ankara.

Kaput, J. J. (1999). Teaching and learning a new algebra with understanding. In E. Fennema & T. Romberg (Ed.) *Mathematics Classrooms that Promote Understanding* (pp. 133-155). Mahwah, NJ: Lawrence Erlbaum Associate.

Kieran, C. (1992). The learning and teaching of school algebra. In: Grouws DA (ed.). Handbook *of Research on Mathematics Teaching and Learnin*g. New York: Macmillan Publishing Company.

Kieran, C.(2004). Algebraic thinking in the early grades: what ıs ıt? *The Mathematics Educator*, 8 (1), 139 – 151.

Koç, E.S.(2014). Sınıf öğretmeni adaylarının öğretmen ve öğretmenlik mesleği kavramlarına ilişkin metaforik algıları. *İnönü Üniversitesi Eğitim Fakültesi Dergisi*, 15(1), 47-72. DOI: 10.17679/iuefd.79408

Lakoff, G.and M. Johnson (1999). *Philosophy in theflesh*. New York: Basic Books

Leder, G.,Pehkonen, E.,and Torner, G. (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht:Kluwer.

Leitze, A. R. and Kitt, N. A. (2000). Using homemade algebra tiles to develop algebra and prealgebra concepts. (27.02.2017).

MacGregor, M. and Stacey, K. (1993). Cognitive models underlying students' formulation of simple linear equations. *Journal for Research in Mathematics Education,* 24(3), 217-232.

Miles, M. B. and Huberman, A. M. (1994). *Qualitative data analysis.* ThousandOaks, CA: Sage.

Ministry of National Education [MoNE] (2005). *İlköğretim Matematik Dersi (6-8. Sınıf) Öğretim Programı.* Ankara: MEB.

Ministry of National Education [MoNE] (2009). *İlköğretim Matematik Dersi (6-8. Sınıf) Öğretim Programı ve klavuzu.* Ankara: MEB

Öztürk, Ç. (2007). Sosyal bilgiler, sınıf ve fen bilgisi öğretmen adaylarının "coğrafya" kavramına yönelik metafor durumları. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi(KEFAD),* 8(2), 55-69.

Patton, M. Q. (2002). *Qualitative Research & Evaluation Methods.* ThousandOaks, CA: Sage Publications.

Reeder, S.,Utley, J.and Cassel, D., (2009). Using metaphors as a tool for examining preservice elementary teachers' beliefs about mathematics teaching and learning. *School Science and Mathematics,*109(5), 290-297.

Saban, A., Koçbeker, B. N. and Saban, A. (2006). An investigation of the concept of teacher among prospective teachers through metaphor analysis. *Educational Sciences: Theory & Practice, 6* (2),461-522

Saban, A. (2009). Prospective teachers' mental images about the concept of student. *Türk Eğitim Bilimleri Dergisi* .7(2), 281-326.

Schinck, A. G.,Neale, H. W., Jr., Pugalee, D. K., &Cifarelli, V. V. (2008). Using metaphors to unpack student beliefs about mathematics. *School Science and Mathematics,* 108, 326 - 333.

Seeley, C. & Schielack, J.F. (2008). A look at the development of algebraic thinking in curriculum focal points. *Mathematics Teaching in the Middle School,* 13(5), 266-269.

Şahin, Ş. ve Baturay, M. H. (2013). Ortaöğretim öğrencilerinin internet kavramına ilişkin algılarının değerlendirilmesi: bir metafor analizi çalışması. *Kastamonu Eğitim Dergisi,* 21 (1), 177-192.

Uygun, T., Gökkurt B. ve Usta, N.(2016). Analysis of the Perceptions of the University Students about Mathematics Problem through Metaphor. Bartin University Journal of Faculty of Education, 5(2), 536 – 556. Doi: 10.14686/buefad.v5i2.5000187677

Warren, E. (2005).Young children's ability to generalize the pattern rule for growing patterns .In Chick, H. L. & Vincent, J. L. (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, *Vol.* 4 (pp. 305-312). Melbourne: PME.

Vance, J. H. (1998). Number operations from an algebraic perspective. *Teaching Children Mathematics, 4*, 282-285.

Van de Walle, J.,Karp, K. & Bay-Williams, J. (2012). *İlkokul ve ortaokul matematiği: gelişimsel yaklaşımla öğretim.* (Çev. Ed. Soner Durmuş). Ankara: Nobel Yayıncılık.

Yıldırım, A. ve Şimşek, H. (2008). *Sosyal bilimlerde nitel araştırma yöntemleri*. Ankara: Seçkin Yayıncılık.

Yılmaz, F., Göçen, S. ve Yılmaz, F. (2013) Öğretmen adaylarının öğretmen kavramına ilişkin algıları: bir metaforik çalışma. *Mersin University Journal of the Faculty of Education*, 9 (1), 151-164.

Zwicky, J. (2010). Mathematical analogy and metaphorical insight. *For the Learning of Mathematics*, 30(1), 9-14