

The Teaching Candidates' Views About the Impact of Origami-Supported Geometry Teaching on Comprehending the Description, Axiom and Theorem

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Abstract

This study has been made in the purpose of revealing the elementary mathematics teaching candidates' views about the impact of origami-supported geometry teaching on comprehending the description, axiom and theorem. The study group consists of 52 mathematics teaching candidate. A questionnaire which consists of the open-ended questions has been used in the study which has been made with the use of study case design which is one of the qualitative research methods. The impact of origami-supported teaching on understanding the geometry has been interrogated in the questions which have been asked. According to the findings which have been obtained, the teaching candidates have shared the impact of origami-supported geometry teaching on the description, axiom and theorem as they have explained through their own experiences. They have mentioned on these sharings especially that the relation between the description and concept, the relation between the axiom and theorem and the relation between the description and theorem are correctly stated mathematically. As a result of the research, it can be said that the use of origami-supported teaching facilitate to understand its axiomatic structure especially in order that Euclides geometry is comprehended, it can support to interrogate the geometry knowledge correctly and as its result, it can positively affect on the Vam Heile geometry thinking levels.

Keywords: Origami, geometry teaching, description-axiom-theorem, teaching candidates' views

Introduction

The mathematics arised from the necessity and it has developed and come in accordance with the necessity up to today. The necessity on the Euclides geometry has gradually decreased as the computer which is among three great inventions of mathematics has entered into our life, and it has become nearly binin-utilizable in the daily life. This situation has causes as the days pass that there are the problems to understand and interpret the geometry. Furthermore, as the mathematics is based on the Euclides geometry, it makes difficult to learn the mathematics that the geometry isn't understood. In another saying, the success of mathematics is mainly based on understanding the Euclides geometry. The understanding on its geometry means to comprehend how the description, axiom and theorems are made as an axiom within deductional view. According to NTCM, the learning without understanding has been a problem since 1930's and the mathematics learning requires the understanding and the practice of concepts. The learning by understanding facilitates the next learnings. If the students correlates a meaning between the new knowledge and the available knowledge, the mathematics gains meaning and it becomes easy to recall and practice. The well-connected and concept-based views are suitable for being used in the new situations (Skemp,76 akt: NTCM,2000).

When the literature is reviewed, it stands out that all of the students, teachers and teaching candidates have problem to understand the geometry (Ahuja, 1996; Kay, 1987; Mayberry, 1983; Napitupulu, 2001; Viglietti, 2011). Many teaching methods which are suitable for the constructivist approach related to remove these problems have been evaluated. A teaching way which has included in the studies and attracted attention in the geometry teaching especially in recent time gets a support from the origami. The main reason to use the origami, which is known as the art of folding paper, in the geometry teaching is that the folding meets the each phases' geometric concept and terms, and it instantiates the intangible concepts. This aspect of origami has included in the reason to be preferred in teaching the geometry concepts (Brady, 2008; Cipoletti & Wilson, 2004; DeYoung, 2009; Haga, 2008; Kanade, 1980, Krier, 2007; Olson, 1975; Sze, 2005; Wares, 2016; Yin, 2009). The concepts in the studies are limited with the spatial thinking and teaching the planary patterns, and there isn't sufficient study related to whether the origami will be affective in comprehending the main principals of geometry. In this sense, a study which searches whether the origami-supported geometry teaching has an impact on comprehending the geometry's main principals such as the description, axiom and theorem has come up with this study. Euclides geometry is known with learning its main principals. Because the Euclides geometry is an interwoven pattern of the concepts. It is important to know that the origami-supported teaching will be effective to get the geometry's main principals comprehended in terms that the origami strengthens its utility in the geometry education. In this sense, it has been aimed in this study what the impact the origami-supported teaching has on comprehending the geometry's main principals such as the description, axiom and theorem. The results which will be obtained from the study support that the origami can be effective in teaching the Euclides

geometry, it is considered that it can create the learning atmosphere that the teaching candidates can interrogate their own knowledge.

Method

The Research Design

The study case design of the quantitative research methods has been implemented in this study. A survey which has included the open-ended questions has been implemented as the data has been collected, the collected data has been analyzed, the descriptive finding and results related the reviewed situation have been revealed.

The Study Group

The research sampling consists of 52 teaching candidates who received their training at the last grade in The Department of Elementary Mathematics Teaching in a state university in Turkey in the spring term of 2016-2017 education-training year. The reason why the teaching candidates who received their training at the last grade were chosen for the sampling is that they received previously the lesson of origami and they know to fold the paper. Thus, the selection of purposeful sampling has been made in the study.

The Data Collection Tool and The Data Analysis

In the study, the origami activities were made with the teaching candidates of the research for 14 weeks. Firstly, the origami models which were determined by the researcher were folded together in the classroom. Secondly, the problems which require a proof were given to the teaching candidates through the models which have been folded. These problems were determined in a way of obliging to use one or two of the description, axiom and theorem which are the main principals in the Euclides geometry. Then, a chance was given for the teaching candidates firstly to solve the problems and to present them in written in the classroom. These written documents were collected by the researcher at the end and each of the activities were discussed after they were reviewed in the classroom as the brain storming was made. They were emphasized until the full agreement was gotten. A scaling tool which consists of three open-ended questions was used at the end of term in order to reveal the teaching candidates' views about how they got benefit from the origami-supported geometry teaching. In the research, it was fictionalised whether the origami-supported teaching contributed to the Euclides geometry knowledge, if it contributed to it, what kind of a contribution it was, and what should be understood when the origami-supported geometry teaching is mentioned. So it was focused on whether they had a consignment to the description, axiom and theorem knowledges, if they had the consignment, what the reasons were for it. The activities which were implemented for 14 weeks have been presented at Table 1.

Table 1: The activities which were implemented

Weeks	Activity topic	Weeks	Activity topic
1	Modeling point and line with origami	8	Different Geometrical shapes from paper: circle.
2	Euclides axioms and origami	9	Area, paper folding to confirm the formula for area of a triangle
3	Different Geometrical lines from paper	10	Theorems on triangles in origami
4	Bisecting an angle in a square	11	Pythagoras theorem
5	Trisecting an angle in a square	12	Different Geometrical 3D from paper: Pyramid, octahedron, cube
6	Different Geometrical shapes from paper: square, right angle triangle, equilateral triangle, isosceles triangle, isosceles triangle.	13	Different Geometrical 3D from paper: Dodecahedron, icosahedron
7	Different Geometrical shapes from paper: rhombus, paralelogram, trapezium	14	Preparatory folding and euclides geometry

The data which was obtained from the open-ended questions was analyzed with the descriptive analysis method. The data which is obtained in the descriptive analysis is summarized by the themes which are previously determined and they are interpreted and then, presented to the readers. Moreover, the direct quotations were included in order to reflect the persons' views effectively in the descriptive analysis (Miles & Huberman, 1994). Thus, the direct quotations which were made through the answers that the elementary mathematics teaching candidates gave for the questions as their names were hidden and coded as "M₁, M₂...M_n" were included.

Findings

The research findings were obtained from the answers that the mathematics teaching candidates gave for the open-ended question in the data collection tool after the 14-weeks training. The obtained findings were reviewed on the base of sub-problems as the geometry's main principals were based. The answers which were obtained from the findings were reflected onto the tables as they were collected with their common properties. The frequency values in these tables are not on the base of persons but on the combined answers. For example, the

views of two teaching candidates who answered “ the origami makes a form especially to instantiate the intangible theorems in the geometry” and it instantiates to understand the theorem to teach the geometry knowledge with the use of origami” were stated with a single item after they are combined. Furthermore, if a teacher has an opinion for the answers related to the different items, these answers increased one unit the relevant item’s frequency value. For example, if a teacher had answered the question as “ It makes the learning much more permanent to prove a theorem or axiom”, the theorem and axiom in the answer were considered as two different items at the frequency table. Thus, the frequency values of these items were increased as one unit. So the total frequency is different from 52 which is the number of persons in the study group as the frequency values at the tables state the number of item which is obtained from the answers but not from the number of persons.

The findings which were obtained from the research were evaluated under three main topics such as the description knowledge, axiom knowledge and theorem knowledge.

The Description Knowledge

The description knowledge which is one of the geometry’s main principals is the primary knowledge in the geometry. The frequency values related to the answers of research’s teaching candidates which explained the impact of origami-supported teaching on the description knowledge in the written documents have been presented at Table2.

Table 2: The mathematics teaching candidates’ views about the origami-supported geometry teaching’s impact on the description knowledge

Items	f
The impact related to the statement of description	17
The impact related to the determination of concepts which constitute the description	5
The impact related to comprehend the relation between the description and concept	2

17 teaching candidates who participated in the research emphasized that the origami-supported teaching is effective to state the descriptions belonging to the geometry. There are the views of O₃₈ and O₄₅ ;

O₃₈: “ *When I want to give a name for the pattern that I make it, I need to control its properties. It became an effective tool for me to state the description.* ”

O₄₅: “*I have been able to identify the concepts better such as the triangle, bisectrix, square and line segment.*”

5 teaching candidates who participated into the research mentioned the origami-supported teaching’s impact related to determine the concepts of description. The candidate with the code of O₁₆ interpreted as the following;

O₁₆ : “*While I was making the description, I understood that I need not to ignore its main knowledge.*”

Two teaching candidates mentioned the origami-supported geometry teaching’s impact related to comprehend the relation between the description and concept. The candidate with the code of O₄₁ was stated as the following;

O₄₁ : “*I understood the patterns in a better way. Because I described many things through the concepts.*”

The Axiom Knowledge

An axiom which is one of the geometry’ remain principals and that its trueness is accepted by everyone is the primary propositions in the proof of theorem and problem in a sense. When the documents belonging to the mathematics teaching candidates are reviewed, the frequency values related the answers which explain the origami-supported teaching for the axiom knowledge have been given at Table3.

Table 3: The mathematics teaching candidates’ views about the origami-supported geometry teaching’s impact on the axiom knowledge

Items	f
The impact related to recognize the axioms’ necessity	30
The impact related to use the axiom correctly	26
The impact related to understand the difference between the axiom and theorem.	25

30 teaching candidates who participated into the research gave their opinion which explained that they recognized why the axiom is necessary in the geometry teaching. The views of teaching candidates with the code of O₁₄ and O₃₀ have included as the following;

O₁₄ : “ *I couldn’t tell that the angles of triangle is 45-90-45 without using the axioms.* ”

O₃₀ : “*The axioms haven’ t attracted my attention my attention so much.*”

26 of the teaching candidates mentioned that the origami-supported teaching’s impact on using the axioms

correctly. The views of candidates with the codes of O_{12} and O_{21} have been explained as the following;

O_{12} : “ I had superposed two things and I had said that it was an equation before I received the course of origami. I recognized with the lesson that I should use the knowledge, if the knowledge, which will be reached in order that it equates, is available. Otherwise, the last resort should be the axiom. This is definitely something like philosophizing.”

O_{21} : “ I used too many axioms which I was proving by folding the paper. However, I saw that I didn’ t use all of the hypothesis when I finished the proof. I understood that the proof was incorrect. The origami provided me to understand the mathematics. I was difficult to use the axioms. I have interpreted the mathematics in a better way now.”

25 teaching candidates who participated into the research emphasized on the written documents that they understood the difference between the theorem and axiom. The quotations from the students’ views have been included as the following;

O_3 : “ For example, let’s cut out the rectangle into two triangles. If the equation of these triangles is proved with the superposition, it is a meaningless one. It requires to be proved with the direct proof. The students proves only with the use of theorems in the direct proof.”

O_{18} : “ I have learnt to reach the new knowledge and rules on the basis of a few main knowledge, the hypothesis and axiom.”

The Theorem Knowledge

The theorem’s proof which is one of the geometry’ Samim principals ends as the conclusion is gotten on the basis of hypothesis. When the documents belonging to the mathematics teaching candidates are reviewed, the frequency values related to the comments which explain the impact of origami-supported teaching on the theorem knowledge have been included at Table4.

Table 4: The mathematics teaching candidates’ views about the impact of origami-supported teaching on the theorem knowledge

Items	f
The impact related to the statement of theorem	40
The impact related to understand and comprehend the theorem	38
The impact related to the determination of knowledge which will be used in proving the theorem	26

40 teaching candidates who participated into the research mentioned about the impact of origami-supported geometry teaching related to the statement of theorem. The candidate with the code of O_{52} mentioned as the following;

O_{52} : “ I know $a^2+b^2=c^2$ but I couldnt explain it. Then, I was able to state it when it became tangible with the origami.”

Although the candidate stated the pythagorean relation as an algebraic, he/she had difficulty in stating the theorem “ the square’s space which is established on a right angle’s right edges is equal with the square’s space which is established on the hypotenuse.” 38 teaching candidates who participated in the research mentioned about the impact related to understand and comprehend the theorem. The candidate with the code of O_6 mentioned as the following;

O_6 : “ The origami indicated where the descriptions and formulas come from and how they occur. The greatest advantage is that it taught us to make a proof. It provided us to understand the theorem.”

26 teaching candidates who participated in the research mentioned about the origami-supported geometry teaching’s impact related go the determination of knowledge which will be used to prove the theorem. The candidate with the code of O_2 explained as the following;

O_2 : “ The proof is made within the framework of certain rules, the rules are certain. There is too much knowledge that we will not use on the paper. We receive these rules correctly.”

Conclusion and Discussion

Fictional fact of origami, which is known as an art of folding the paper, on the geometry knowledge is one of the most important reasons for it is preferred in teaching the geometry. The geometry consists of an axiomatic structure which is based on the description, axiom and theorem which are known as the geometry’s main principals. It is evaluated only with the geometry’ s impact on comprehending the description, axiom and theorem that it can be effective in teaching. The insufficiency of searches which review the geometry’ s contribution to the description, axiom and theorem, as the geometry’s main principals, is seen as a deficiency in the literature. In the study, it has been aimed to reveal the mathematics teaching candidates’ views on the impact of origami-supported geometry related to comprehend the description, axiom and theorem.

As a result of the research, the most important factor which attracts the attention is that nearly all the

candidates who participated in the research mentioned in their views that the origami-supported teaching has the positive impacts related to comprehend the description, axiom and theorem

As a result of the research, it attracts attention that the mathematics teaching candidates recognized that they have the problem to state the description and theorem correctly. There are many studies in the literature related to the problems to state the description and theorem correctly, the results are parallel with this research (Jones, 1997; Knuth, 2002; Varghese, 2009; Ward, 2004). It is an important chance for each of the teaching candidates to solve their problem that they recognize this situation with the origami. As its result, it has attracted attention that the origami is effective to make a mathematical sentence and so the teaching candidates get the learning atmosphere that they can critically consider the mathematical language that they use it.

Weber (2001) emphasized in his study that the students couldn't prove anything and recognize even whether the proofs are valid or not. In this study, the teaching candidates who tried to prove the theorems that were given as they added a new hypothesis by ignoring a part of the information which was given in the hypothesis recognized with the origami-supported training as a result of the research that the proof can not be made in this way. It provided them to review the proof which was fictionalised by them that their proof and their knowledge on the paper were in contradiction with each other.

Another important finding which was obtained by the research result is that they comprehended the difference between the axiom and theorem. The teaching candidates who compared these concepts with the origami-supported training were able to perceive why the axioms are necessary. It led them to use the axioms correctly that they saw that the available mathematics ignores the knowledge when the axioms are used too much.

Consequently, it was concluded according to the research's results that the geometry will contribute positively to the comprehension of axiomatic structure with the origami-supported teaching. Moreover, the desired level can be gotten as the teaching candidates with the low Van Hiele geometry thinking level are supported with the origami-supported teaching. Thus, it can be supported to grow up the teachers who transfer the knowledge correctly and who can interrogate their own knowledge.

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