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The Impact of Problem Solving Approach on Students' Performance in Mathematical Induction: A Case of Mukuba University

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Abstract
This study was conducted to explore the impact of problem-solving approach on academic performance of students in mathematical induction at Mukuba University. The design of the study was pre-test post-test control group quasi-experimental design. The first year students enrolled in 2017 studying mathematics constituted the population of this study because mathematical induction is a topic that is taught in a first year foundation mathematics course. The sample was constituted by 40 students who were randomly selected from the population. The Sample was divided into two groups of the same size (I.e. experimental and a control). The treatment of the planned problem-solving approach is Polya's heuristic steps of the problem-solving approach which was developed in 1945.A pretest was used to establish the equivalence and homogeneity of the two groups in aptitude whereas a posttest was used to assess the impact of the intervention on students' performance on mathematical induction. An independent t-test was used to analyze the data at an alpha level of 0.05, which revealed that both the experimental and control groups were equivalent in mathematical competence on the basis of the pre -test ; control group $(\mathrm{M} = 45.40, \mathrm{SD} = 15.371)$; and experimental group $(\mathrm{M} = 45.20, \mathrm{SD} = 14.163)$; $\mathrm{t}(38) = -0.043$ *and* $p - value = 0.966$. The experimental group outscored the control group significantly on the post-test; control group $(\mathrm{M} = 49.00, \mathrm{SD} = 17.162)$ and experimental group $(\mathrm{M} = 61.40, \mathrm{SD} = 7.486)$; $\mathrm{t}(38) = 2.962$ *and* $p - value = 0.005$. Therefore, problem solving was found to have a positive impact on students' performance in mathematical induction.
Keywords: Problem solving, performance, perceptions, first year students

1.0 Introduction

Mathematics can naturally help generate creativity; reasoning and can create an enjoyable environment. Mathematics can also play a significant role in representing, communicating and predicting events. Mathematics is the foundation of science and technology and the functional role of mathematics to science and technology is critical, that no area of science and technology escapes its application (Padmavathy and Mareesh, 2013).

In Zambia, current education reforms are calling for a secondary school curriculum that Provides clear mathematical thinking and expression in the learner, develop the learners' mathematical knowledge and skills, enrich the learners' understanding of mathematical concepts in order to facilitate further study of the discipline, build up an appreciation of mathematical concepts so that the learner can apply these for problem solving in everyday life (Curriculum Development Centre, 2013).

However, besides its importance it is observed that mathematics is one of the most poorly achieved, widely hated and abysmally understood subject in most institutions of learning and Mukuba University is not an exception. For instance, in 2016 academic year examination MAT 110 (mathematics foundation course) had the lowest pass percentage of 37%. Analyzing the performance on each topic of the course reviewed that only 21% of the students attempted the question on mathematical induction and only 9% of them got the question correct.

Evidence of poor performance in mathematics by students in most learning institutions highlight the facts that the most desired technological, scientific and business application for mathematics cannot be sustained. This makes it paramount to seek for an approach for teaching mathematics that aims at improving its understanding and performance by students practically (Okigbo & Osuafor, 2008).

It is therefore argued that the poor performances could be attributed to pedagogical approaches used to teach mathematics. There is a need to consider remedial approaches of teaching quite different from the routine ones. Problem solving is one such a remedial approach that enhances the students‟ active role in the lessons.

A problem-solving approach involves teaching mathematics topics through problem-solving contexts and enquiry-oriented environments that are characterized by the teacher "helping learners construct a deeper understanding of mathematical ideas and processes by engaging them in doing mathematics such as creating, conjecturing, exploring, testing, and verifying" (Lester et al., 1994, p.154). Through problem solving method learners develops problem solving skills. Problem solving is the most useful skill a student can take with them when they leave university. It is problematic to allow students to graduate with first class degrees who cannot handle unfamiliar problems (Rowlett, 2011).

It is against this background that this study was carried out with the aim of assessing the impact of problem-solving approach on students' academic performance in mathematical induction.

1.1 Problem Statement

Mathematical induction is one of the topics that is taught to first year students in their foundation mathematics course at Mukuba University and many other higher institutions in Zambia.

Past Literature shows that most students find it difficult to understand the topic. This is why Kolmos (2007) stated that documenting the difficulties students face in mathematical induction is a common theme in much literature. Furthermore, the contributions of the previous research on mathematical induction inform our specific understanding of some of the difficulties that students encounter in terms of procedural and conceptual knowledge, and mathematical resources required when doing mathematical induction (Brumfiel et al, 2009).

It seems to be generally agreed by college mathematics facilitators that our students do not perform well in this concept of mathematical induction and Mukuba University is not an exception. In the 2016 academic year examination at Mukuba University, MAT 110 (mathematics foundation course) had the lowest pass percentage of 37%. A further analysis by topic tends to suggest that students' poor grasp of the concept of mathematical induction was one of the contributing factors to this quality landscape. For instance, it was revealed that 21% of the students attempted the question on mathematical induction and only 9% of them got the question correct. In the 2015 academic year examinations, of the 28% of the students who attempted the question on mathematical induction only 11% managed to get it correct. Further, in the 2014 academic year examinations, and of the 19% who attempted the question on mathematical induction only 7% managed to answer the question correctly.

This recurring poor performance of learners in this topic calls for concerted efforts by mathematics facilitators in institutions of learning to adopt teaching and learning approaches that will help to improve learners' performance. This persistence in poor performance of learners may mean that the real source and solution to the problem has not been systematically established. This study contends that a possible solution to the problem may lie in the teaching of mathematics using an appropriate teaching approach. The teaching approaches used over the years seem to be addressing low order thinking skills hence the low achievement levels exhibited. The higher order thinking skills may have not been developed through these teaching approaches. For example, (Yahya & Zaman, 2008; Yahya, 2011) attribute students‟ abysmal performance in mathematics to the quality of mathematics teaching.

It was therefore the aim of this study to investigate whether a problem solving approach will be an appropriate alternative to teaching mathematical induction so as to enhance students' performance.

1.2 Objectives of the study

This research was guided by the following research objectives;

Main objective: To find out the impact of the problem solving approach on student's performance in mathematical induction**.**

Specific objectives:

- To find out the challenges students face in mathematical induction.
- To determine if there is any significant difference in students' performance when taught using a problem solving approach and when taught using conventional methods (Lecture method).
- To find out learner's perception towards mathematical induction before and after learning using the problem solving approach.

The above objectives translated into the following research questions

- What are the challenges that students face in mathematical induction?
- What is the impact of problem solving approach on learner's performance in mathematical induction?
- What are the learners' perceptions towards mathematical induction before and after the problem solving approach?

1.3 Theoretical framework

This study draws its theoretical framework from constructivism theory which view learning as a search for meaning. Constructivists believe that knowledge is constructed by the learner and the learner develops understanding through experience and reflection. Constructivism is founded on the belief that learners bring experiences and prior knowledge to the classroom. Learners must connect new information to this background knowledge in order to make sense of it. Learning experiences in constructivist settings are characterized by active engagement, inquiry, problem solving and the teacher is expected to be a facilitator who encourages learners to questions and formulate their own ideas and conclusions.

For instance, Bruner's theory of spiral curriculum is very useful in this study. It refers to the idea of revisiting basic ideas over and over, building upon them and elaborating to the level of full understanding and mastery. This will enable learners to revisit the material that they might have learnt and forgotten. Brunner's spiral curriculum adds that; "each time a learner revisits the material learnt in the past, meaningful learning occurs". He believes that learning is a cumulative process and therefore requires previous learning to be frequently revisited; His learning theory describes three stages of knowing: enactive (action-based), iconic (image-based), and symbolic (language-based). Bruner suggests that when children learn mathematical concepts, they need to go through these stages - from concrete objects to pictorial images and then to abstract symbols (Bruner, 1966 p. 11).

Levy Vygotsky (1896) also advocated for social constructivism. Social constructivism is considered as an extension of the traditional focus on individual learning to addressing collaborative and social dimensions of learning. Social constructivists posit that knowledge is constructed when individuals engage socially in talk and activity about shared problems or tasks (Jones, 1996).

This study was motivated by the need for learners to have the 21st century skills of which problem solving is one of them. Students must also learn the essential skills for success in today's world, such as critical thinking, problem solving, communication and collaboration. When a school builds on this foundation, students are more engaged in the learning process and graduate better prepared to thrive in today's global economy.

This study is also anchored on the need to have sustainable education which can be achieved through sustainable teaching pedagogies. Pedagogies associated with Educational Sustainable Development stimulate pupils to ask questions, analyze, think critically and make decisions. Such pedagogies move from teacher-centered to student centered lessons and from rote memorization to participatory Learning. Educational Sustainable Development pedagogies are often problem/issue based. Educational Sustainable Development pedagogies encourage critical thinking, social critique, and analyses of local contexts which the problem solving approach embraces.

2.0 Methodology

2.1Research design

A research design is a plan or strategy which moves from the underlying philosophical assumptions to specifying the selection of respondents, the data gathering techniques to be used and the data analysis to be done (Maree, 2007). This study is all about the impact of the problem solving approach on learner's performance in mathematical induction. The study employed a quasi-experimental research design which took the form of a pre-test and post-test. This research design was appropriate in the sense that the study was about assessing the impact of problem solving on the learner's performance in mathematical induction. Besides that, the impact of problem solving approach was assessed by comparing the performance of the experimental group, where this method was used and the control group that were taught using lecture method.

2.2 Target population

The study was carried out on the Mukuba university first year students who were doing their mathematics foundation course (MAT 110) in Kitwe district.

2.3 Sample and Sampling Procedure

The sample size constituted 40 first year students comprising of 25 males and 15 females. The first year students were purposively selected because they were the ones taking the mathematics foundation course from which mathematical induction belongs. Systematic random sampling was also used. A list of all the first year students was obtained from which the first participant was selected at random and the rest were selected following the sampling interval.

2.4 Research instruments and data collection

Questionnaires and Mathematics Performance Tests were used to collect information from the subjects. The pre-test and post-test scores were used to compare the groups taught using the problem solving approach and those taught conventionally. Participants' quality of answers given in the pre-test and post-test were also used to check on their challenges in mathematical induction.

The questionnaires were used to collect their perceptions and partly their mode of argument in order to know their challenges.

Both Primary and secondary sources of data were used in this study. Primary data was collected through self-administered questionnaires, tests administered and through observations. Secondary data was collected from different literature related to the topic under study.

2.5 Data analysis techniques

The data was analyzed both qualitatively and quantitatively. Data entry and analysis was carried out using the Statistical Package for Social Science (SPSS) version 20. The pre-test and post-test scores were analyzed using an independent samples t- test statistics to determine if the treatment had effects on the groups.

The data were presented in the forms of tables and figures for easy interpretations. Descriptives such as the mean and standard deviations were used to assess students' perceptions towards mathematical induction. Items with a mean score below 2.4 were considered negative, items with a mean score of (2.5–3.4) were considered neutral, while items with a mean score of (3.5 -5.0) were considered positive.

3.0 Results of the study
3.1 PRE-TEST
A pre-test is a test given prior the intervention. The pre-test allowed to assess whether the groups were equivalent in knowledge towards mathematical induction before the treatment was given to the experimental group.

Table 1 shows the pre-test mean performance between the control and experimental group (N=40)

Group	Sample size	Mean	Std. Deviation	Std. Error
Experimental	20	45.20	14.163	3.167
Control	20	45.40	15.371	3.437

Table1 shows that the mean for the experimental group was 45.20. The mean for the control group was 45.40. The standard deviation for the experimental group was 14.163, whereas that for the control group was 15.371. The number of participants in the experimental group was 20 whereas in the control group were also 20. Table 1further indicates that the control group ($mean = 45.40$) performed better than the experimental group ($mean = 45.20$). This shows that there is a difference between the two groups. However, there was need to establish if the difference was statistically significant by conducting an independent $t-test$.

Table 2 shows Pre-test Independent samples test

Levene's Test for Equality of Variances		t-test for Equality of Means		
F	Sig.	t	df	Sig. (2-tailed)
.034	.855	-.043	38	.966

From the t-test for equality of means it can concluded that the difference between the control and experimental group is not significant ($p-value = 0.966 > \alpha = 0.05, t = -0.43$). Hence, the two groups were equivalent.

3.2 POST- TEST

Table 3 shows the post-test mean performance between the control and experimental group (N=40)

Group	Sample size	Mean	Std. Deviation	Std. Error
Experimental	20	61.40	7.486	1.674
Control	20	49.00	17.162	3.837

Table 3 shows that the mean for the experimental group is 61.40 and the mean for the control group is 49.00. The standard deviation for the experimental group is 7.486, whereas that for the control group the standard deviation is 17.162. The number of participants in the experimental group was 20 whereas in the control group were also 20. Table 3 further indicates that the experimental group ($mean = 61.40$) performed better than the control group ($mean = 49.00$). Looking further at the standard deviations, the experimental group has a smaller standard deviation of (Std deviation=7.486) as compared to the standard deviation of the control group

(Std deviation=17.162) indicating that there was consistence in the marks obtained by the experimental group.

This shows that there is a difference in performance between the two groups. However, there was need to establish if the difference in performance was statistically significant by conducting an independent $t-test$.

Table 4 shows Post-test Independent samples test

Levene's Test for Equality of Variances		t-test for Equality of Means		
F	Sig.	t	df	Sig. (2-tailed)
18.948	.000	2.962	38	.005

From the t-test for equality of means it can concluded that the difference between the control and experimental group is statistically significant($p-value = 0.005 < \alpha = 0.05, t = 2.962$). Hence, there is a statistically significance difference between the experimental and control group.

3.3 STUDENTS' PERCEPTIONS TOWARDS MATHEMATICAL INDUCTION
Perceptions were measured through a questionnaire that had 20 statements which were rated on the following

scale.

1= strongly disagree, 2= Disagree, 3=Not Sure 4=Agree, 5= strongly agree

Perceptions were collected at two different points that is; before and after intervention using a questionnaire.

3.3.1Students' perceptions before the intervention

Table 5 descriptive statistics on student's perceptions towards mathematical induction (Experimental group $n = 20$) before intervention

Item	Mean	Std. Deviation	Skewness	Kurtosis
1. Following rules is important for learning MI	3.7	1.1	-1.4	1.5
2. Learning mathematical induction means finding correct answers	2.5	1.1	0.9	0.3
3. It is important to understand why a method works than to learn rules by heart in MI	3.0	1.3	-0.6	-1.1
4. The best way to learn MI is to see an example of the correct method for solution	4.1	0.7	-1.1	3.7
5. If you persist you will be good at learning mathematical induction	3.9	1.0	-1.2	1.9
6. Those who get the right answer in MI understand it better	3.6	1.1	-0.1	-1.2
7. MI should be learned as a set of algorithms and rules that cover all possibilities	3.0	1.2	-0.5	-0.5
8. Mathematical induction learning is for the gifted	1.4	0.7	1.5	1.2
9. Learning rules and methods by rote are important in mathematical induction	3.2	1.0	-1.1	0.3
10. Learning formal aspects of mathematical induction as early as possible are important	3.5	1.0	-0.9	0.8
11. Learners should learn mathematical induction in groups	2.9	1.0	0.7	-0.9
12. A teacher-centred is the best way to teach learners to solve MI problems.	3.6	1.0	-1.0	1.1
13. Learners should ask questions during mathematical induction lessons	3.2	1.0	-0.3	-0.7
14. A student-centred is effective to actively involve learners in the MI learning process	3.5	1.1	-0.5	-0.2
15. Learners should often be confronted with novel problems to solve.	3.2	1.4	-0.8	-1.1
16. Learners should be given notes to copy when learning mathematical induction.	3.6	1.0	-0.3	-1.0
17. The teacher should stimulate learners to learn mathematical induction on their own.	2.7	1.3	0.4	-0.9
18. Learners should learn MI by working with other learners using worksheets.	3.5	1.0	-0.5	0.8
19. Cooperative work in groups is good for efficient learning of mathematical induction.	4.1	1.0	-1.0	0.3
20. Learners should discover for themselves the desired knowledge in the learning of MI	3.0	1.1	-2.1	2.8

Table 5 suggests that students' perceptions towards mathematical induction ranged from 'not sure' to 'agree' since the mean value in most of the cases is either approximately 3 (not sure) or approximately 4 (agree) in a few cases. Hence, this indicates that student' perceptions towards mathematical induction before the treatment was neither positive nor negative.

3.3.1 Students' perceptions after the intervention

Table 6 descriptive statistics on student's perceptions towards mathematical induction (Experimental group $n = 20$) after intervention

Item	Mean	Std. Deviation	Skewness	Kurtosis
1. Following rules is important for learning MI	4.4	.9	-1.1	1.2
2. Learning mathematical induction means finding correct answers	3.1	1.1	0.4	-1.3
3. It is important to understand why a method works than to learn rules by heart in MI	4.4	1.0	-0.8	-0.3
4. The best way to learn MI is to see an example of the correct method for solution	3.9	.9	-0.7	0.2
5. If you persist you will be good at learning mathematical induction	4.7	.9	-1.2	0.5
6. Those who get the right answer in MI understand it better	4.6	1.5	0.0	-1.5
7. MI should be learned as a set of algorithms and rules that cover all possibilities	3.1	.9	0.1	1.2
8. Mathematical induction learning is for the gifted	1.8	1.2	1.6	1.5
9. Learning rules and methods by rote are important in mathematical induction	3.7	1.2	-0.6	-0.2
10. Learning formal aspects of mathematical induction as early as possible are important	4.5	.9	-0.5	-0.4
11. Learners should learn mathematical induction in groups	4.7	1.2	-0.1	-0.9
12. A teacher-centred is the best way to teach learners to solve MI problems.	3.1	1.3	-0.1	-1.0
13. Learners should ask questions during mathematical induction lessons	4.6	.8	-0.9	-0.8
14. A student-centred is effective to actively involve learners in the MI learning process	4.4	1.0	-0.8	-0.2
15. Learners should often be confronted with novel problems to solve.	3.3	.9	-0.1	-1.0
16. Learners should be given notes to copy when learning mathematical induction.	4.8	1.0	-0.6	-0.5
17. The teacher should stimulate learners to learn mathematical induction on their own.	4.2	1.1	-0.4	-1.3
18. Learners should learn MI by working with other learners using worksheets.	3.8	1.1	-1.2	1.2
19. Cooperative work in groups is good for efficient learning of mathematical induction.	4.4	1.1	-1.1	0.1
20. Learners should discover for themselves the desired knowledge in the learning of MI	3.8	1.1	-0.3	0.3

Table 6 suggests that students' perceptions towards mathematical induction ranged from 'not sure' to 'strongly agree' since the mean value in each case is either approximately 3 (not sure), 4 (agree) or 5(strongly agree) in some cases. Hence, this indicates that student' perceptions towards mathematical induction after the treatment was positive.

3.4 CHALLENGES STUDENTS FACED IN MATHEMATICAL INDUCTION

This study focused on the impact of problem solving on students' performance in mathematical induction. One of the research questions was to determine the challenges students face in mathematical induction. The challenges were observed through proof analysis and proof writing. This was done in the pre-test and post- test administered to the students and part of proof analysis was done in the questionnaire developed.

Table 7 shows the distribution of challenges students faced in mathematical induction.

Descriptions of challenges	N	Percentage
• The statement to be proved could not be represented by some notation say $p(n)$	4	10
• Students could not verify that the result is true for the appropriate initial value when Basis case is not n=1.	2	5
• Unable to clearly show that the induction step $p(k) \Rightarrow p(k+1)$	19	47.5
• A statement of the result $p(k+1)$ that is going to be deduced before the main algebraic part of the proof in the induction step begins could not be shown.	8	20
• Algebra in inductive step must include enough working to show clearly that the expected result has in fact been properly derived.	3	7.5
• Students could not give a clear conclusion to round off the whole proof.	4	10
N= number of students out of 40	**40**	**100%**

The result in table 7shows the challenges students faced in mathematical induction in terms of percentages. The table shows that 10 % of the students had challenges with notations for the statement to be proved. 5% of the students had challenges verifying the appropriate initial value especially when it is not $n = 1$.

47.5% of the students had challenges with inductive reasoning while 27.5% could not show sufficient work showing all the necessary steps. Lastly, 10% of the students could not give a clear conclusion to round off the whole proof.

Clearly, table 7 indicates that most of the students had challenges with inductive reasoning followed by those who could not show sufficient work showing all the necessary steps representing 47.5% and 27.5% respectively.

4.0 Discussion of findings

This chapter is guided by the following three research questions.

- What are the challenges that students face in mathematical induction?
- What is the impact of the problem solving approach on student's performance in mathematical induction?
- What are students' perceptions towards mathematical induction before and after the problem solving approach?

4.1Challenges faced by students in mathematical induction

The findings of the study showed that 10 % of the students had challenges with using appropriate notations for the statement to be proved. 5% of the students had challenges with verifying the appropriate initial value especially when it is not $n = 1$.

47.5% of the students had challenges with inductive reasoning while 27.5% could not show sufficient work showing all the necessary steps. Lastly, 10% of the students could not give a clear conclusion to round off the whole proof.

It is evident from these findings that students indeed had various challenging in the learning of mathematical induction. For instance, majority of them representing 47.5% had challenges with inductive reasoning, particularly showing that $p(k) \Rightarrow p(k+1)$. Another fraction of the students representing 27.5% were unable to convincingly show sufficient work to clarify their solutions. This meant that students did not have a solid background of the subject hence they could not show all their work to justify their solutions.

Some of them also representing 10% were unable to give a clear conclusion to round off the whole proof. This was due to students' lack of enough understanding of the topic. Students could not show were a particular solution to a problem is ending hence, failing to give a clear conclusion to a question.

These findings agrees with what is documented in the available literature (Baker, 2008; Dubinsky & Lewin, 2007 and chow 2003) where it has been shown that most of the students had procedural and conceptual challenges in mathematical induction. Baker (2008) went on further to highlight more challenges students face in mathematical induction that are critical to performance as follows;

(a) knowledge of mathematical symbols and content, (b) the ability to identify the steps in a proof, and (c) the need for and connection between the base case and the inductive step of a proof.

Five aspects were suggestive of being important causal factors:

(i) Insufficient formal mathematics backgrounds.

(ii) The use of a work-backwards or work-forwards strategy.

(iii) Inability to recognize substantial elements in examples of proof.
(iv) Flawed generalizations that acted as prototypes for evaluating other proofs.
(v) Attempts to use every day reasoning and to generate informal rules of inference.

4.2 Impact of problem solving on students' performance

An independent t-test was conducted to establish whether the difference in performance between the control and experimental group was significant. From the t-test for equality of means, $p - value = 0.005$ was compared with the alpha level $\alpha = 0.05$. It was therefore concluded that the difference between the control and experimental group was statistically significant($p - value = 0.005 < \alpha = 0.05, t = 2.962$).

In this study, it has been found that problem solving is a more effective method of instruction for teaching and learning of mathematical induction as compared to conventional (lecture) methods. The results suggest that incorporating problem solving in mathematical induction lessons enhances learners' performance. In addition, problem solving makes learners critical thinkers, encourages collaborative and cooperative learning, and promotes student to student to learning.

These findings are in line with what other researchers have done. For example, (Perveen, 2010; Dannawi, 2013; Seligman, 2007; Wanjala, 2014; Ali, 2010; and Bostic 2011) found that problem solving is a more effective method of teaching and has an impact on academic performance.

Further, the present researcher's view is in support of Wanjala (2014) who stated that problem solving method is learner-centered and hence it enhances learner's team work, peer interactions and raises their learning interests.

4.3 Students' perceptions towards mathematical induction

Perceptions were collected at two different points that is; before and after intervention using a questionnaire.
Perceptions that were collected before the treatment indicated that students' perceptions towards mathematical induction ranged from 'not sure' to 'agree' since the mean value in most of the cases is either approximately 3 (not sure) or approximately 4 (agree) in a few cases. Hence, this indicates that student' perceptions towards mathematical induction before the treatment were neither positive nor negative. This implies that their perceptions before the treatment were neutral.

Perceptions that were collected after the treatment indicated that students' perceptions towards mathematical induction ranged from 'not sure' to 'strongly agree' since the mean value in each case is either approximately 3 (not sure), 4 (agree) or 5(strongly agree) in some cases. Hence, this indicates that student' perceptions towards mathematical induction after the treatment were positive.

This change in perception can be attributed to the treatment (problem solving) used. Students indicated in the questionnaire that; learners should learn mathematical induction in groups; learners should ask questions during mathematical induction lessons, learners should be given notes to copy when learning mathematical induction, if you persist and practice enough, you will be good at learning mathematical induction.

These findings are in line with Akhter et al., (2015) who stated that participants had high levels of perceptions after being taught with the problem solving method in the classroom. These findings have significant implications in the teaching and learning of mathematics in institutions of learning. Mathematics practitioners must be able to look at their pedagogical approaches and see which one enhance academic performance and hence change their perception towards mathematics topics that are perceived to be difficult. This study reviewed that problem solving is one such a method that enhances academic performance and improves perceptions.

5. Conclusion

This study investigated the impact of problem solving on students' performance in mathematical induction. Based on the findings of the study, the following conclusions were drawn;

Firstly, there exists a significant difference in the performance of students taught through problem solving approach and conventional (lecture) method. Students taught through problem solving performed better than those taught by conventional (lecture) method. The difference between the achievement levels can be attributed to the problem solving approach; otherwise both groups had equal basic knowledge of mathematical induction as shown in the pre-test results.

Secondly, the study also investigated students' perceptions towards mathematical induction before and after the treatment (problem solving). The findings showed that students' perceptions towards mathematical induction before the treatment were neutral, that is their perceptions were neither positive nor negative. However, after the treatment their perceptions were positive. Therefore, the difference in perceptions can also be attributed to the treatment (problem solving) used.

Thirdly, the study also investigated the challenges students face in mathematical induction. Based on the findings, students had more challenges in deductive and inductive reasoning. Inductive reasoning moves from specific instances into a generalized conclusion, while deductive reasoning moves from generalized principles that are known to be true to a true and specific conclusion (Godfrey& Hodgson, 2010). Other challenges that students had were; (i) the statement to be proved could not be represented by some notation say $p(n)$ (ii) students could not verify that the result is true for an appropriate initial value especially when basis case is not n=1) (iii) students could not make a clear conclusion to round off the whole proof.

This study has shown that problem solving is an effective approach of teaching therefore, mathematics educators should be able to consider using this approach in the teaching of mathematics in order to enhance the quality of our education.

5.1 Recommendations

Based on the findings of the study, the following recommendations were made.

- The study showed that problem solving is a more effective method of instruction for teaching and learning of mathematical induction as compared to conventional (lecture) methods. Therefore, teachers of mathematics should consider using problem solving approach to improve the academic performance of students.
- The Zambian government should consider equipping institutions of learning with problem solving oriented teaching and learning materials.
- Various institutions of learning should also consider training programs, seminars and workshops for its members in order to create awareness about the problem solving method.

5.2 Recommendations for further research

- In this study, only first year students from one university in one district constituted the sample, hence, a similar study may be done with a larger sample which would enhance the validity and reliability of the conclusion reached.
- This study was conducted at higher learning institution (Mukuba University), a similar study can also be conducted at a lower learning institution like at a secondary school or primary school in order to verify the effectiveness of the problem solving method.

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We also wish to state that this manuscript is a summarized report of a study that was conducted at a Zambian institution of learning called Mukuba University. The source of data is a master's thesis that was conducted in partial fulfilment of the award of a Master of Science in Mathematics Education at The Copperbelt University in Zambia.

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