Journal of Education and Practice www.iiste.org
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.9, No.17, 2018

Teaching of Earth Geometry at Secondary School in Zambia

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Abstract
The study explores the challenges associated with teaching Earth Geometry in secondary schools. Selected secondary schools of Mufulira District in Zambia were the focus of the study and a total of 21 Teachers and 97 pupils formed the sample population to whom questionnaires were administered. In addition, a 5-point Likert scale formed part of the research instrument used to investigate the phenomenon. The collected data was analysed using SPSS which allowed for simple frequency counts, percentages, means and analysis of variance. To further understand the phenomenon under study, data collected from the pupils by way of interviews and document reviews was analyzed using the QDA miner 4.0 version software. It was established that pupils faced the following challenges: computation of the shortest distance between points on the same latitude, distinguishing between knots and kilometres per hour, and distinguishing great circles and small circles.
Keywords: learners, teachers, earth geometry, latitudes, knots and kilometer, circles

1. Introduction

Mathematics is an important tool for development and improvement of a person's intellectual competence in logical reasoning, spatial visualization, analysis and abstract thought. Learners' adequate acquisition of knowledge in mathematics, results in developed numeracy, reasoning, thinking and problem solving skills. Geometry a word derived from Greek meaning "Earth measurement" is one of the oldest branches of mathematics. Having arisen in response to practical problems such as those found in surveying; it is concerned with spatial visualization. Researchers have realized that the study of geometry need not be limited to plain geometry focusing on the study of flat surfaces and rigid three-dimensional objects (solid geometry) but should even be extended to the study of most abstract thoughts and images perhaps represented and developed in geometric terms.

Sidhu (2008:331) defines geometry as "a science of space and extent." Geometry is said to deal with the position, shape and size of bodies, although it has nothing to do with the material or physical properties of those bodies (Sidhu, 2008). Geometry is said to comprise of those branches of mathematics that exploit visual intuition to remember theorems, understand proof, inspire conjecture, and perceive reality (Royal Society/JMC 2001). The Royal Society/JMC (2001) report suggests that the aims of teaching geometry can be summarized as follows:

- to develop spatial awareness, geometrical intuition and the ability to visualize;
- to provide a breadth of geometrical experiences in 2 and 3 dimensions;
- to develop knowledge and understanding of and the ability to use geometrical properties and theorems;
- to develop skills of applying geometry through modeling and problem solving in real world contexts;
- to develop useful ICT skills in specifically geometrical contexts;
- to encourage a positive attitude towards mathematics; and
- to develop an awareness of the historical and cultural heritage of geometry in society, and of the contemporary applications of geometry.

Given the above definition of geometry, and a consideration of the aims of teaching geometry, it is possible to see why it should be included in the school mathematics curriculum. The Zambian Curriculum Development Centre (CDC) responded to this call by strengthening the teaching and learning of geometry through the re-introduction of earth geometry in the mathematics curriculum for Zambian secondary schools in 2004. In a study conducted by Tembo (2013), one of the respondents from the CDC indicated that Earth Geometry was re-introduce in the syllabus because of its use in other studies such as navigation.

2. Geometry in Mathematics Curriculum

Because of the diverse roles of geometry, Zambian curriculum developers together with other stakeholders saw it fit to introduce a section of geometry called earth geometry in the secondary school syllabus. According to Volderman (1998); geometry provides a more complete appreciation of the world we live in. For instance, Earth Geometry's focus on spherical geometry is important in the sense that it enables learners gain knowledge of how to locate different places on planet earth. It also enables learners to comprehend that differences in longitudes of different places brings about differences in time. The study of geometry contributes to helping learners develop the skills of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. Geometric representations can equip the learners to make sense of other areas of mathematics such as fractions and multiplication in arithmetic, the relationships between the graphs of functions (of both two and three variables), and graphical representations of data in statistics. Furthermore, it also promotes spatial reasoning which plays an important role in other curriculum areas such as science, geography, art, design and technology. Working with practical equipment can also help develop fine motor skills.

Battista (1999) and Lappan (1993) state that Geometry plays an important role in primary and secondary school mathematics curricula in Malaysia and other countries. According to Jones (2002) geometry provides a culturally and historically rich context within which to do mathematics. Using particular strategies teachers may present geometry in a manner that stimulates learners' curiosity and encourages exploration thus enhancing the learning process and attitudes towards mathematics. When the teacher encourages learners to discuss problems in geometry, articulate their ideals and develop clearly structured arguments to support their intuition, it can lead to enhanced communication skills and recognition of the importance of proofs. The contribution of mathematics to learners' spiritual, moral, social and cultural development can be effectively realized through geometry (Volderman, 1998).

Geometry is a rich source of opportunities for developing notions of proof. Tembo (2013) established the worthiness of emphasizing visual images in teaching geometry, particularly those images which can be manipulated on the computer screen such that learners could be invited to observe and conjecture generalizations. New developments in computing technology mean that the 21st century is an era in which spatial thinking and visualization can exist. The teachings of geometry demands that teachers make it a point to display visual aids to enable learners make interpretations and draw conclusions.

3. Geometry Teaching Methods

The debate surrounding the instruction of geometry was complicated in the middle of the nineteen century as a result of moving the course from college level to the high school level. Although the maturity level of the learners was lowered, there was no organized adjustment in course contents. Since then, numerous committees have addressed the need for adjustment and have offered a wide range of recommendations aimed at reforming geometry instruction in schools. Despite these repeated attempts to change the teaching of geometry in the intervening decades, few fundamental changes have been fused into the widespread practice. Part of this reason has been due to the lack of reform (Allendoefer, 1968). A study by William (1968) however, revealed that Euclid's traditional approach to the subject is considered by many to be a significant part of man's cultural heritage. In the traditional Euclidean geometry, many students experienced difficulty writing proofs and most students were unsuccessful in solving geometrical problems. This was reported in many surveys for example, in Usiskin (1983) and Hoffer (1986).

Jones article of 2002, 'Issues in the teaching and learning of geometry,' also strongly advocated that one of the reform ideas was to base much more of school mathematics on the idea of function and to aim further at mathematics that would lead to calculus and linear algebra. The room for this innovation was made by reformulating all parts of the mathematics curriculum. However, to remove solid geometry and to convert trigonometry components into parts of a course about functions seemed to be the practical effect. The impact of these changes was a reduction in the amount of geometry while at the same time, increasing emphasis on co-ordinate geometry and introducing some elements of transformation geometry and topology. As a result, the amount of geometry taught in the Euclidean fashion probably became much reduced.

Many teachers in schools have tried different methods and programmes to make learners understand geometry, sometimes with success and sometimes not (Chakerian, 1972). The tendency by most teachers to teach geometry by informing learners of the properties associated with planes or solid shapes, and then completing the exercises contributes to poor performance in geometry. Such an approach entails that little attempt is made to encourage thinking and reasoning skills in learners such as those requiring the solving of non-routine problems and deriving proof (Hoyles and Jones, 1998). This approach posed problems to both teachers and learners; and both began to dread geometry. Teachers became frustrated because their poor conceptual understanding led to learner's poor geometry achievement. It could be argued that to teach geometry effectively to learners of any grade, age or ability, it was vital to ensure that learners understood the concepts and the logical steps involved rather than learners solely learning rules. Kalejaiye (2000) states that poor performance in geometry is as a result of teachers not involving learners in their teaching and the adoption of the rote learning style. The manner, in which geometry was taught, appeared not to link class work and real-life situations. The way geometry was taught, there seemed to be no connection between class work and real-life situations. Thus, this may have contributed to poor performance as learners did not see its relevance. In this regard, the programmatic document; the 1989 Curriculum and Evaluation Standards for School Mathematics (NCTM, 2000) recommended that geometric topics be introduced and be applied to real world situations whenever possible. However, this did not imply that immersing learners in real world situations automatically led to geometrical understanding (Schwartz, 2008). It is argued that hands-on activities were a popular way to establish a connection between instruction and real-life. This was so because, the challenge of geometry instruction was to elevate the learners' experience with the real-world objects to the level of Mathematics. Overall, research on the teaching and learning of geometry indicated that physical experience, especially the physical manipulation of shapes was important to all ages, and that a wide variety of geometrical experiences were necessary in order for learners to gain firm understanding of geometry.

In many geometry classrooms today, teachers merely introduced learners to facts about geometry and then drilled them with concepts in deductive reasoning (Mullis, 2000). Learners were seldom given the opportunity to discover and conceptualize geometry on their own. Hoyles and Jones (1998) argue that although the deductive method is central to Mathematics and intimately involved in the development of geometry, providing a meaningful experience for learners at school appeared to be difficult. Research shows that learners fail to see the need to distinguish forms of mathematical reasoning such as explanations, arguments, verifications and proofs (Jones, 2002). An added reason advanced for learners' poor performance was that teaching methods concentrated much on calculations rather than problem solving and proof which encouraged critical thinking (Yeo, 2000).

According to the National Council of Teachers of Mathematics (NCTM, 1989), the major goals of secondary school geometry are the development of mathematical reasoning abilities and the promotion of a deeper awareness of the real world. To help learners achieve this goal, the NCTM suggest that reasoning in relation to shapes should be aided by coordinate and transformation techniques as well as the traditional synthetic techniques such as flip, turn and sliding of an object. In keeping with the attainment of this goal, the Zambia mathematics curriculum advocated a move away from the traditional deductive Euclidean geometry to investigative Earth Geometry.

In Zambia, the secondary educational policy requires learners to undergo five years of secondary school study. During this period, learners are compelled at the second and fifth year of study to sit for an examination. At both the second and fifth stages, mathematics is one of the compulsory examinable subjects for all learners. Nevertheless, following independence in 1964, the education sector in Zambia has experienced several changes which have included changes in the secondary school Mathematics curriculum. The following are some of the curriculum development changes particular to Mathematics since the 1960s:

- In 1964, the Zambian government adopted and adapted the existing British colonial curriculum.
- In the 1970s, a change from traditional mathematics to modern mathematics was made. The change was necessitated by changes that were implemented in Europe in the late 1960s. The introduction of Modern Mathematics brought about new content into school mathematics such as sets, functions, groups as well as new teaching approaches emphasising the discovery methods.
- Change from a two year Junior and a three years Senior Mathematics syllabus to a three year Junior Secondary and two-year senior mathematics syllabus.
- Introduction of two Junior Secondary Mathematics Syllabi for more able and less able learners.
- In the early 1980s, there was a change from a three year Junior and two year Senior Secondary mathematics to a 2 year Junior and 3 year Senior Mathematics Syllabus.
- Introduction of a single mathematics Syllabus for Junior Secondary mathematics.
- A change in Senior Secondary Mathematics Syllabus from Syllabus B and Syllabus C to Syllabus D, around 1983-1984.
- Syllabus D introduced new topics such as linear Programming, Vectors, Probability and Function.
- Removal of Earth Geometry from the Syllabus.
- In 2000-2004, change from Syllabus D to Zambia Senior Secondary Mathematics Syllabus.
- Introduction of Zambian authored Senior Secondary Mathematics text books.
- Re introduction of Earth Geometrical into the Senior Secondary Mathematics Syllabus in the early 2000s.

The introduction of Earth Geometry into the Secondary School Mathematics Syllabus was meant to be taught at senior level. Despite the perceived usefulness of mathematics and geometry in particular, learners' failure rate as regards the topic Earth Geometry in Zambia, has repeatedly been high compared to other topics (MOE, 1996). Most performance reports by the Ministry of General Education (MOE) in Zambia as well as the Examinations Council of Zambia (ECZ) are littered with concerns on the poor performance of learners, especially in the natural sciences of which mathematics is part and parcel. Such concerns are a clear indication of the need to design mitigation interventions in the teaching and learning of mathematics so as to improve the performance of learners in geometry and earth geometry in particular. It is against this background that, the present study focused on identifying challenges that secondary school teachers and learners face in earth geometry so as to chart the way forward as regards the best practices to mitigate the identified challenges. It is predicted that exploring and identifying the challenges will result in interventions that will eventually lead to improved quality of learning mathematics, earth geometry in particular and its improved applications in society.

In essence the research study sought to address the following:

(i) Identify the challenges encountered by secondary school teachers and pupils in Earth Geometry.

(ii) Establish the relationship between teachers' perceptions and pupils' perceptions regarding the identified challenges

(iii) Determine the appropriate measures to undertake in mitigating the identified challenges.

Many applications of earth geometry are found in geography. Geography itself is nothing but a scientific and geometrical description of the earth in its universe. The dimension and magnitude of the earth, its situation and position in the universe, the formation of days and nights, latitudes and longitudes, height above sea level, etc., are some of the numerous learning areas of geography which need the application of mathematics in general and earth geometry in particular (Sidhu, 2008). In this sense, it can be said that earth geometry is a topic in mathematics that acts as a point of contact between mathematics and geography. In this study, earth geometry is referred to as the size, shape and position of two and three dimensional figures. It is concerned with the location of places, the calculation of distances and determination of time differences between places on the Earth's surface.

4. Geometry Learning Theories

According to the Government of Ireland (1999), mathematics encompasses a body of knowledge, skills and procedures that can be used in a rich variety of ways: to describe, illustrate and interpret; to predict, and to explain patterns and relationships in number, algebra, shape and space, measures and data. For this study, a mathematics curriculum has been defined as planned experiences in mathematics offered to learners under the guidance of the school. These experiences are extracted from the syllabus. A syllabus is simply an academic document that communicates course information content and defines expectations, activities and responsibilities which learners are exposed to in the form of learning experiences and are later evaluated or examined on. Therefore, the major goal of the mathematics curriculum is to develop learners' mathematical literacy in an appropriate as well as socially relevant manner.

Geometry provides a culturally and historically rich context within which to do mathematics. There are many interesting, sometimes surprising if not counter-intuitive results in geometry that can stimulate learners' curiosity to know more and understand why. According to Jones (2002), of the range of theoretical work concerned with geometrical ideas, that of Piaget (and colleagues) and of van Hiele's is probably the most well-known. The Piagetian work has two major themes. The first theme is that our mental representation of space is not a perceptual "reading off" of what is around us. Rather, we build up from our mental representation of our world through progressively reorganizing our prior active manipulation of that environment. Secondly, the progressive organization of geometric ideas follows a definite order and this order is more experiential (and possibly more mathematically logical) than it is historical. The first of these Piagetian themes, concerning the process of the formation of spatial representations, remain reasonably well-supported by research.

According to the van Hiele model, learners advance through levels of thought in geometry. Van Hiele characterized these levels as visual, descriptive, abstract/relational, and formal deduction. At the first level, students identify shapes and figures according to their concrete examples. At the second level, learners identify shapes according to their properties; and here a learner might think of a sphere as being a three-dimensional figure. However, one of the major challenges faced by learners in most Zambian secondary schools is that of viewing a sphere as an ordinary plane circle (Tembo, 2004). At the third level, learners can identify relationships between classes of figures (for example, that the earth is spherical in shape) and can discover properties of classes of figures by simple logical deduction. At the fourth level, learners can produce a short sequence of statements to logically justify a conclusion and can understand that deduction is the method of establishing geometric truth. According to Van Hiele's model, progressing from one level to the next is more dependent upon the teaching method applied than on age of the learner. The evidence available suggested that all types of geometric ideas appear to develop overtime, becoming increasingly integrated and synthesized (Hoyles and Jones, 1998). Poor performance in geometry may be as a result of the manner in which it was taught, where logical sequence was not followed and the skipping of the Van Hiele model. This demands that care must be taken when teaching geometry. No doubt, teachers must create striking classroom display, suspend geometrical models and involve pupils in making models, to get them decide on definitions and then to let them explore the logical consequences. According to Ben-Chairn et al (1989) visualizing cross sections of solids is very difficult for learners lacking ample prior concrete experiences with solid objects. Due to the limited geometric experiences learners may not have enough opportunities to develop and exercise their spatial thinking skills for effective geometry learning, hence resulting in poor performance.

Given traditional teaching methods, research suggests that lower secondary learners perform at levels one or two with almost 40% of students completing secondary school below level two. The explanation for this, according to the van Hiele model, is that teachers are asked to teach a curriculum that is at a higher level than the learners. Going by van Hiele's model, it is not possible for learners to bypass a level. Learners are not able to see what the teacher is able to see in a geometric situation and therefore learners do not gain from such teaching. With this background, the development of the questionnaires as the main research instrument was considered. The researchers' vast experience of teaching mathematics at secondary school level led to the assertion that a learner cannot understand earth geometry concepts without understanding concepts such as trigonometric ratios, surface areas and arc lengths; especially those associated with a sphere.

The researchers noted the possibility of several domains existing in the conceptual framework within which challenges faced by secondary school learners and teachers could be understood, however, in this particular study, the challenges were investigated using the following criteria:

- Firstly, questionnaires were administered to learners so as to establish the challenges they encounter understanding earth geometry concepts
- Secondly, questionnaires were distributed to teachers in order to establish and investigate challenges encountered in teaching earth geometry
- Thirdly, both teachers and learners were interviewed to seek further understanding regarding participants' responses to the administered questionnaires.
- Finally, teachers' and pupils' responses were analysed together to answer all the research questions fully.

As pointed out already, the possibility of there being many domains and better criteria for investigating the issue at hand exist, but it was the researchers' belief that the criteria used in this study was robust enough to address theories of geometric thinking, learning, and teaching.

Earth geometry is a vital and essential component of the mathematics curriculum in the sense that it contributes to helping learners to develop the skills of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. Geometric representations can be used to help learners make sense of other areas of mathematics. Spatial reasoning is important in other curriculum areas as well as mathematics: science, geography, art, design and technology. Working with practical equipment can also help develop fine motor skills in the learner. Hence, it was deemed fit to investigate the challenges that secondary school teachers and learners face in earth geometry and identify ways of improving learner achievement in this topic. It is foreseen that understanding these challenges will compel other stake holders to devise mechanisms that will help reduce the learning problems in mathematics as a whole.

It is an open secret that high school geometry with its formal (two-column) proof has been considered difficult and detached from practical life, because of the teaching methods employed. It can be argued that the poor performance in earth geometry by learners at high school was a manifestation of the type of geometry taught in lower grades. The lack of proof in geometry in earlier school years, greatly contributed to poor performance at a higher level. Battista and Clements (2000) state that the situation obtaining was further compounded by the fact that most teachers at primary level did not even teach the 'impoverished' geometry, which meant that most learners reached high school level with a low Van Hiele way of understanding; hence making it difficult for learners to understand the topic. This is because learning geometry requires formal deduction at a typical high level of the geometry taught.

In Africa also, there are studies that have been conducted in order to determine the influence of teachers' content knowledge and their classroom practices on learners 'poor performance in mathematics and in particular Earth geometry. For instance, in South Africa in the late 1990s, it was argued that, Geometry education in South Africa was in a state of turmoil and a great number of learners performed poorly in it at that time (Mogari, 1998). This observation came with the suggestion that there was need to find an alternative approach to the study of shapes and space –related mathematics. Lately, the National Curriculum Statement era in South Africa has tried to address the problem of poor performance in Geometry reflected in Mathematics Senior Certificate results from 2003 to 2008. This was done by pushing Geometry to a paper written optionally by mathematics learners in matric during those years. Furthermore, most teachers did not teach Geometry at all since it was optional for examinations in grade 12. Only 28 percent of matric learners wrote the Geometry paper in 2009 and of those 19.3 percent performed poorly. Moreover, higher institutions were stressed with introduction of the foundation phase where the concepts that were not addressed in school mathematics were taught to aspiring engineers from scratch. This included Geometry. However, the Curriculum Assessment Policy Statement has reinstated the examination of Geometry in senior certificate from 2011 to date. This has posed problems for both teachers and learners in Geometry (Department of Basic Education, 2011).

5. Methodology

Outlined below are the methodologies, research design, data collection methods and instruments used in this study. The purpose of the research design used in this study was to generate valid answers to all the three research questions, and also to map out how the research instruments were to be managed and administered. Sampling procedures are highlighted and the reasons for choosing them given.

Since the study involved the perceptions of grade 12 learners and teachers regarding the challenges encountered in learning and teaching Earth Geometry it was found suitable that the research design be a descriptive survey. This is due to the fact that some open-ended questions were used to explore challenges that the learners and teachers encounter regarding the teaching and learning of earth geometry at senior secondary level. The descriptive survey design was found to be ideal for this study in that it involved gathering of original data for purposes of describing certain challenges, opinions, attitudes, relationships and orientations that are held by learners and teachers on earth geometry at school certificate level. Furthermore, the design allowed the researcher to obtain in-depth information from the 93 learners and 21 teachers at various schools in Mufulira district of Copperbelt province, Zambia.

5.1 Target Population and the Sample

The target population comprised grade12 pupils and teachers in urban secondary schools of Mufulira district. The idea was to target learners eligible to write grade 12 examinations; and teachers of mathematics in selected secondary schools of Mufulira district. The descriptive survey research used both probability and non-probability sampling procedures to collect data. Six schools were selected purposively and stratified sampling method was used to determine the number of pupils and teachers to sample from each school; making up a sample size of 100 pupils and 30 teachers of mathematics. Thereafter, learners were selected systematically from each school and teachers were purposively sampled. Purposive sampling method was used to select teachers so as to ensure that participant teachers were those who had taught earth geometry before. To reduce researcher biasness affecting the sample study triangulation was employed to determine the sample of participants.

5.2 Research Instruments

Questionnaires, focus group discussion, structured interview and content analysis of mathematics textbooks, ECZ's SCE and GCE final past papers were the research instruments of the study.

Questionnaires were administered to 97 grade 12 pupils, 6 Heads of Mathematics Departments and to 21 teachers of mathematics in the six participating secondary schools of the study (see appendix A and B respectively). The focus group interview guide as research instrument served to determine the learners' views regarding how their teachers assist in developing their ability to comprehend the concepts particular to Earth Geometry as well as in establishing other intervening variables affecting the process of learning. Content analysis as data collection instrument aided the review of the Zambia secondary school syllabus books 10, 11 and 12, Secondary school mathematics content. The reviewed content was accessed from the following: a course for grades 10,11-12, Achievers Senior Secondary Mathematics and others; ECZ chief examiner's reports for 2014, 2012 and 2011, ECZ past question papers for SCE/GCE paper 1 and 2 for the years 2015, 2014, 2013, 2012 and 2011 and test papers of the schools involved in the study. The purpose of documentary content review was to compare notes on class practices, text book notes and assessments at grade 12 and also to have an insight into the content of these materials.

Data collected via questionnaires administered to learners as well as to teachers, addressed three research questions. Selected questions appearing in the questionnaire administered to teachers were also included in the questionnaire administered to learners. This was for the purpose of providing information addressing the second research question. The vital sections in the two research questionnaires required both learners and teachers to state their views and perceptions regarding factors responsible for the difficulties or challenges encountered in teaching and learning of earth geometry. The questionnaire tasks were adapted and informed by previous research particularly that conducted by Tembo (2013) which addressed the perceptions of teachers and learners in the teaching and learning of earth geometry. This allowed for comparison of results of prior studies in order to further evaluate their significance. Out of the targeted sample of 100 learners, 93 responded giving a response rate of 93% and 21 out of the target sample of 30 teachers also responded giving a response rate of 70%.

Oral interviews were conducted for a selected number of respondents, a fortnight following the administration of questionnaires. This was done to allow for the further probing of respondents' thoughts concerning factors responsible for difficulties encountered in the teaching and learning of earth geometry and also to clarify insights in written explanations to the questionnaire items. However, only available respondents were interviewed. Data collected was analysed into categories of meaning as informed by previous research with particular emphasis on strategies that could be adapted to enhance better teaching and learning of earth geometry in schools.

It should be noted that before questionnaires' distribution to the intended respondents, a pilot study was conducted on one of the schools. This was done in order to gain assurance of validity and reliability of the data and data instruments. Following the pilot study, the questionnaire data collection instrument was revised and apriori codes were developed in order to obtain the data that would provide answers to the research questions. Adhering to the coding categories employed in previous research when analyzing survey and interview data, also permitted for comparisons with prior studies. This is useful because it allows for a discussion of the significance of results in the context of previous work.

6. Results

The following is a presentation of results providing answers to the three research questions. Data from both written responses and oral responses reveal the challenges that both teachers and pupils encounter in the teaching and learning of earth geometry. For each questionnaire item, a quantitative summary of the written responses has been given. Thereafter, responses from oral interviews provide insights into the answers received. A total of 93 respondents from the five named schools completed the questionnaires. Out of the 93 learners, 53 were male and 40 were female. Twenty-one teachers were also involved in the completion of questionnaires.

To obtain responses to research question one of the study; Item 2 from the learners' questionnaire was designed to solicit responses to particular areas learners perceived to be difficult or found challenges understanding. In order to establish whether teachers were aware of the challenges learners faced in the process of learning earth geometry, a similar item was included in the teachers' questionnaire. **Table 2** below gives a summary of both teachers' and learners' responses. Respondents who faced challenges understanding a particular subtopic ticked "yes" while those encountering no challenges with particular subtopic were asked to indicate "no". Percentages and frequencies of responses for both teachers and learners regarding a particular subtopic are presented in **Table 2.**

Table 2: Perception: Areas of Learners and Teachers Perceived Difficulty

	Pupils (n = 93)		**Teachers (n = 21)**	
	Yes	***No***	***Yes***	***No***
Location of places	17 (19%)	74 (81%)	1 (5%)	20 (95%)
Distance between two points on the same longitude	25 (27%)	66 (73%)	5 (24%)	16 (76%)
Distance between two points on the same latitude	23 (25%)	69 (75%)	11 (52%)	10 (48%)
Calculation of the shortest distance between two points on the same latitude	46 (52%)	42 (48%)	18 (86%)	3 (14%)
Time calculations	38 (41%)	55 (59%)	9 (43%)	13 (57%)
Differences between small circles and great circles	52 (58%)	37 (42%)	12 (57%)	9 (43%)
Difference between speed in knots and speed in km/h	53 (58%)	38 (42%)	12 (57%)	9 (43%)

Results from Table 2 above suggest that learners' perceptions were somehow related to the teachers' perceptions. For instance, **81%**of the learners perceive understanding the concept of locating places on the earth's surface as easy while **95%** of the teachers share the view of learners. Comparably, **58%** of the learners' responses indicate the perceived difficulty as regards understanding the difference between speed in knots and speed in kilometers per hour, whereas 57% of the teachers' responses indicate agreement with learners' views.

Table 3: Learners' Reasons for Perceived Difficulties Understanding Earth Geometry

Perceived reasons	**Responses (n = 93)**						**Mean response**
	SA	***A***	***NS***	***D***	***SD***	***Missing***	
1. Teachers do not take time to explain the concepts because they always want to complete the syllabus on time	26 (28%)	21 (22.6%)	10 (10.8%)	17 (18.3%)	18 (19.4%)	1 (1.1%)	**2.67**
2. It is difficult to recall appropriate formulae during tests/exams	23 (24.7%)	30 (32.3%)	8 (8.6%)	20 (21.5%)	9 (9.7%)	3 (3.2%)	**2.82**
3. The topic has no connection/relevance to real life situations	7 (7.5%)	6 (6.5%)	17 (18.3%)	17 (18.3%)	39 (41.9%)	7 (7.5%)	**1.72**
4. Lack of text books that contain information on earth geometry	12 (12.9%)	18 (19.4%)	12 (12.9%)	19 (20.4%)	26 (28.0%)	6 (6.5%)	**2.21**
5. Some pupils have a negative attitude towards mathematics as a whole	51 (54.8%)	30 (32.3%)	5 (5.4%)	3 (3.2%)	3 (3.2%)	1 (1.1%)	**3.48**
6. The approaches/methods of teaching used were not appropriate	13 (14.0%)	15 (16.1%)	12 (12.9%)	27 (29.0%)	21 (22.6%)	5 (5.4%)	**2.26**
7. The language used by the teachers was too abstract	6 (6.5%)	12 (12.9%)	10 (10.8%)	23 (24.7%)	40 (43.0%)	2 (2.2%)	**1.80**
8. No teaching aids were used	11 (11.8%)	13 (14.0%)	16 (17.2%)	16 (17.2%)	34 (36.6%)	3 (3.2%)	**2.01**

Table 3 above illustrates attributed reasons (except for items 3 and 7) responsible for learners' difficulties in understanding Earth Geometry.

Table 4 below demonstrates Teachers' perceived reasons for the difficulties faced in understanding Earth Geometry. However, learners rejected Item 3 as not being among the reasons for the difficulties encountered in comprehending earth geometry. This indicates that learners and teachers appreciate the relevance of earth geometry to real life situations and both respondents concur as regards the reasons responsible for the difficulties encountered in learning.

Table 4: Teachers' perceived reasons for the difficulties faced in understanding Earth Geometry

Perceived reasons	Responses (n = 21)						Mean response
	SA	***A***	***NS***	***D***	***SD***	***Missing***	
Pupils' lack of pre-requisite knowledge	8 (38.1%)	8 (38.1%)	2 (9.5%)	2 (9.5%)	1 (4.8%)	Nil	**2.90**
It is difficult to recall appropriate formulae during tests/exams	4 (19.0%)	12 (57.1%)	3 (14.3%)	0 (0%)	2 (9.5%)	Nil	**2.57**
The topic has no connection/relevance to real life situations	1 (4.8%)	2 (9.5%)	2 (9.5%)	6 (28.6%)	8 (38.1%)	Nil	**1.43**
Lack of text books that contain information on earth geometry	Nil	12 (57.1%)	1 (4.8%)	6 (28.6%)	1 (4.8%)	1 (4.8%)	**2.67**
Some pupils have a negative attitude towards mathematics as a whole	3 (14.3%)	12 (57.1%)	1 (4.8%)	3 (14.3%)	1 (4.8%)	1 (4.8%)	**2.75**
No orientation workshops on how to teach earth geometry	4 (19.0%)	12 (57.1%)	3 (14.3%)	1 (4.8%)	Nil	1 (4.8%)	**2.7**
Pupils' failure to visualise the sphere in three dimensions	4 (19.0%)	10 (47.6%)	3 (14.3%)	3 (14.3%)	Nil	1 (4.8%)	**2.6**

Both teachers' and learners' responses indicated that some of the identified challenges led to:

- Learners' failure to answer questions on earth geometry
- Learners' poor performance in earth geometry resulting in low achievement rate in mathematics as a whole
- Learners' lack of concentration on the topic
- Learners' negative attitude towards mathematics on the whole
- A number of teachers to shun teaching the topic

7. Measures to Mitigate Teaching/Learning Challenges in Earth Geometry

Respondents advanced the following as measure to mitigate the identified challenges in reaction to research question three. The following are measures advanced by teachers:

(i) Enhancing continuous professional development (CPDs) meetings in schools
(ii) Introducing basic concepts of the topic early enough (most of them suggested that the basic concepts regarding earth geometry should be introduced at junior secondary school level.
(iii) Enhancing the use of teaching aids
(iv) Before earth geometry is taught, teachers should make sure that pupils have the necessary pre-requisite knowledge.
(v) Making text books that contain information on earth geometry available to pupils
(vi) The topic must be incorporated in mathematics teaching methodology courses at college/university levels.
(vii) Teachers should try to talk about the place of the topic in mathematics as a whole and the relevance to real life.

The above advanced mitigation measures were coded and with the help of qualitative data analysis software, QDA miner version 4.0, percentages for each advanced measure was computed. The findings were that of the respondents' advanced measures namely; (i), (iii) and (iv) accounted for more than 20%. Mitigation measure (i) accounted for 30% of the responses while both the third and fourth advanced mitigation measures each accounted for 23% of the total number of responses. The remaining mitigation measures advanced by teachers accounted for less than 20%.

On the other hand, a number of learners' advanced measures to mitigating the learning challenges encountered in earth geometry were in line with the measures advances by teachers. For example; the view that earth geometry should be introduced at junior secondary level came out strongly from learners as compared to teachers especially that it accounted for 23% of the total responses compared to 11% of the responses from teachers. Other mitigating measures learners expressed included the following:

(i) Availability of materials such as text books
(ii) Teachers to motivate the pupils as they teach and not to discourage them
(iii) Uses of approaches that can make learners comprehend– teacher's explanation should be clear.
(iv) The ministry to employ qualified teachers

Emphasis on teachers motivating the learners accounted for about 20% of the responses. The rest of the mitigation measures received less than 20% support from learners.

In summing up the results and findings of the study, results show that a number of responses advanced by teachers are similar to those advanced by learners. This is an indication that teachers are aware of the challenges learners face as they are taught earth geometry. It can be seen that areas needing enhanced attention during the process of learning earth geometry include calculation of the shortest distance between points on the same latitude, distinguishing between knots and kilometers per hour as well as the differences between great circles and small circles. More than 56% of the pupils indicated that the above-mentioned sections of the topic were very difficult. Furthermore, it was notable that more than 55% of teachers' responses acknowledged learners frequently faced challenges in understanding the said sections of the topic.

8. Discussion

With regard to the first research question: *What challenges do secondary school teachers and pupils encounter in Earth Geometry?* Results presented in Table 1 indicate that there is no section of earth geometry that was easy for all participants in the study, for at least one learner and one teacher at the most acknowledged for each section the concept as being difficult to understand. However, the areas that were endorsed by the majority of both teachers and learners (More than 50% of the responses) were (i) calculation of the shortest distance between two points on the same latitude (ii) distinguishing great circles from smaller circles on the earth's surface and (iii) distinguishing speed in knots and speed in kilometers per hour.

Furthermore, when both categories of study participants were presented with a variety of factors perceived to be responsible for the challenges they faced, yet again, both teachers and learners were in agreement in the manner they responded to this questionnaire item. It would appear both teachers and learners were in agreement on nearly all the proposed factors attributed to difficulties encountered. For learners, the most contributing factors to the difficulties in understanding earth geometry were (i) teachers' inadequacy in explaining earth geometry concepts, (ii) the topic had too many formulas to memorize (iii) lack of text books containing information regarding earth geometry, (iv) negative attitude towards mathematics as a whole, (v) wrong approaches and methods of teaching by teachers and (vi) inadequate teaching and learning aids on earth geometry. Teachers' responses were similar to those of learners and this was demonstrated by teachers indicating the similar difficulties to those indicated by learners. In addition, teachers also indicated that learners' failure to visualise three dimensional objects and inadequate prerequisite knowledge also contributed to poor performance in earth geometry.

The second research question: *To what extent do teachers and pupils agree on the identified challenges?*

The degree of agreement between teachers and learners on items presented to them was measured using Spearman's rank correlation coefficient. The computed correlation coefficient was 0.657, indicating a strong association in the manner both teachers and pupils responded to the perceived reasons that were presented to them. It is salient to note that teachers and learners are in strong agreement as regards the poor foundation of learners, lack of hard work by learners as well as lack of instructional or teaching aids as being among the major problems associated with teaching and learning of earth geometry. These findings are in line with the findings of a research undertaken in Nigeria at Rivers State University of Science and Technology (Telima, 2011) which investigated problems of teaching and learning geometry in secondary schools.

The third research question interrogated the measures to be undertaken in order to mitigate identified challenges. In this regard, both teachers and learners advanced a number of measures that could be undertaken to mitigate the challenges associated with the teaching and learning of earth geometry. Notable ones among the submissions from both teachers and learners include; (i) enhancing CPDs in schools, (ii) enhancing the use of teaching and learning aids so as to enable learners to visualise three dimensional objects and (iii) ensuring that learners are well equipped with the prerequisite knowledge before introducing them to earth geometry.

Results presented in Table 2, in line with the responses of both categories of respondents imply strong agreement as regards the questionnaire items that relate to sections of earth geometry perceived to be challenging to learners. For instance, 81% of the learners acknowledged that it was easy to understand the concept of locating places on the earth's surface and 95% of the teachers were also of comparable view.

In totality, 58% of the learners' responses concur that it was difficult to understand the difference between speed in knots and speed in kilometers per hour while 57% of the teachers' responses were equally of the same view. Although the target population of the present study differs from the previous ones, there is an indication of a slight improvement in the sense that some areas that were perceived to be difficult previously, were not perceived as such in the present study. For instance, Tembo (2013) established that among the challenges learners faced were: difficulties in sketching the points, calculating the distance between two points along the great circle and also the difficulties in finding the angles of latitude and longitude. The present study indicates that very few learners had challenges with sketching or locating points on the earth's surface as well as finding the distance between points on the same longitude or latitude.

The most cited reason for learner's poor grasp of the topic was attributed to learners' negative attitude towards mathematics. The other reason was attributed to teachers' failure to explain concepts clearly and teacher's failure to inspire learners. Lack of teaching aids made it difficult for the pupils to visualize the spherical nature of earth in three dimensions. The study further revealed that many learners struggle when it comes to spatial visualization. The typical belief is that one is either born with spatial sense or not. But this belief is unfounded. What has been established is that rich experiences with shapes and spatial relationships when provided consistently over time can and do develop spatial sense. Without geometric sense, pupils cannot grow in their spatial sense or geometrical reasoning (Martin and Strutchens, 2000).

Results presented in Table 4 illustrate that teachers are in agreement that difficulties learners face understanding earth geometry could be attributed to learners' inadequate pre-requisite knowledge. This view accounted for 76.2% of the responses, while learners' failure to recall appropriate formulae during tests or exams accounted for 76.1% of the responses. Learners' negative attitude towards mathematics as a whole accounted for 71.4%, while failure by schools to hold orientation workshops such as CPDs on how to teach earth geometry accounted for 76.2% of the responses and; learners' failure to visualise the sphere in three dimensions accounting for 66.6%.

According to the ECZ mathematics examiners' report (ECZ, 2004), the language of geometry more so the comprehension of geometry terms, plays a key role in learning and understanding of geometric concepts (Clements, 2001). It has been established that when learners failed to grasp the concepts, they resorted to memorization and only to later complain that the topic had too many formulas to memorise. Strutchens (2001), counsels that geometry learnt by memorizing geometric properties rather than by exploring and discovering the underlying properties was limited, superficial and short-lived. Based on the results and findings of the study, it is evident that the problem of poor grasp of earth geometry still persists despite the noble efforts that have been made by mathematics educators and other stakeholders. It is clear that the effects of the identified challenges lead to pupils' poor performance in earth geometry in particular and mathematics as a whole.

A topic such as earth geometry plays a critical role in navigation as well as technological development; therefore, the issue of adequate physical facilities for instruction cannot be over emphasised. Physical facilities such as models help learners to grasp the concepts in earth geometry which usually is taught in an abstract manner. Teachers of mathematics have, in most cases drawn the sphere on a two-dimensional chalk board which learners fail to visualize in three dimensions. Earth geometry is a special case of spherical geometry. When distances measure a boat or aircraft travelling between any 2 places on the Earth, straight line distances are not used since what is needed is to go around the curve of the Earth from one place to another. When it comes to planning approaches to teaching and learning earth geometry, it is important to ensure that provision in the early years of secondary school encourages students to develop an enthusiasm for the subject by providing opportunities to investigate spatial ideas and solve real life problems. There is also a need to ensure that there is good understanding of the basic concepts and language of earth geometry in order to provide foundations for future work and to enable learners consider geometrical problems and communicate ideas. Furthermore, Jones (2002) contends that human beings live on a solid planet in a 3D world and, as much of man's experience is through visual stimulus, this means that the ability to interpret visual information is fundamental to human existence. To develop an understanding of how spatial phenomena are related and to apply that understanding with confidence to solve problems and make sense of novel situations, earth geometry has to be part of the educational experience of all learners.

In addition, the aspects and considerations that the research participants submitted in the present study also tend to make earth geometry a demanding topic to teach well. Teaching earth geometry well in this case, will have to involve knowing how to recognize interesting geometrical problems and theorems; appreciating the history and cultural context of earth geometry, and understanding the many and varied uses to which earth geometry is put. It means appreciating what a full and rich geometry education can offer to learners when the mathematics curriculum is often dominated by other considerations (the demands of numeracy and algebra in particular). It means being able to put over all these things to learners in a way that is stimulating and engaging, and leading to understanding, and success in mathematics assessments.

For teacher educators in colleges and universities, incorporating an aspect of how geometry can be taught effectively should be one of the priority areas. This should be packaged together with issues to do with curriculum and instruction for mathematics education. School managers and subject associations such as the Zambia Association for Mathematics Education (ZAME), should consider this as a matter of agency to send serving mathematics teachers for training and seminars or for CPD meetings for effective teaching of mathematics and geometry in particular in our secondary schools. Geddes and Fortunato (1993), claimed that quality of instruction was one of the greatest influences of the learners' acquisition of geometry knowledge. Strutchens (2001) advised that instruction in geometry should emphasize hands-on explorations, developing geometric thinking and reasoning, making conjectures and carrying out geometric projects.

The present research only focused on earth geometry alone. However, future research can focus on geometry as a whole and attempt to establish the relationship between earth geometry and other branches of mathematics such as algebra. Furthermore, this study depended heavily on the views and perceptions of learners and teachers which reduced the degree to which the findings may be generalised. Future research can go an extra mile to establish an intervention leading to curbing the learning and teaching problems that are currently being exhibited.

9. Conclusion and Recommendations

This study has established the challenges faced by learners undergoing Earth Geometry instruction. From both teachers' and learner's point of view, challenges encountered were (i) poor grasp of earth geometry concepts (ii) inability to visualize objects in three dimensions. The challenges faced by learners (as perceived by them) were caused by poor teacher instruction from teachers of mathematics; learners were also of the view that there were too many formulas to memorise; no teaching aids were used; lack of text books in schools and generally negative attitude of learners towards mathematics as a whole. For teachers, no CPDs were organised to orient them on how best the topic could be taught. In this respect, it is concluded that not all people think about geometric ideas in the same manner. Certainly, people are not all alike, but all are capable of growing and developing the ability to think and reason in geometric contexts. The research of two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof provided insight into the differences in geometric thinking and how the differences came to be.

The results of this study echo those that have been carried out in other parts of the world meaning that geometry is one of the most important areas in mathematics. It is visual, intuitive, creative, and demanding. Therefore, mathematics educators should use their imagination and tap into that of their learners. There is need for teachers to create striking classroom displays, suspend geometrical models from the ceiling of their classrooms, involve their learners in making things and imagining things, get them to decide on definitions and then explore the logical consequences.

The research recommends the following to the Ministry of General Education and stake holders:

- For teacher educators in colleges and universities, it should be one of their priority areas to incorporate an aspect of how geometry can be taught effectively. It should be packaged together with issues to do with curriculum and instruction for mathematics education.
- School managers and subject associations such as the Zambia Association for Mathematics Education (ZAME), should as a matter of urgency consider sending serving mathematics teachers for training and seminars or CPD meetings for effective teaching of mathematics and geometry in particular. Therefore, strengthening the school CPD management team to introduce/improve lesson study culture at schools.
- The quality of instruction was one of the greatest influences of the learners' acquisition of geometry knowledge. In this sense, it is recommended that teachers make use of team teaching to achieve these goals. Hence, improving quality of lesson study at schools in terms of enhancing active/subjective learning of learners especially in mathematics lessons.
- Capacity building of mathematics teachers on the improvement of their pedagogical content knowledge (PCK) through the practice of lesson study.

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