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Van Hiele Geometric Thinking Levels of Pre-Service Teachers' of E.P. College of Education, Bimbilla-Ghana

Anas Seidu Salifu* Abdul-Rahaman Yakubu Fuseini Issahaku Ibrahim
Tutors , Department of Mathematics & ICT, E.P. College of Education, P.O. Box 16, Bimbilla-Ghana

Abstract
The purpose of the study was to determine Pre- Service Teachers (PSTs) geometric thinking level in E.P. College of Education, Bimbilla using the Van Hiele's Model. The study population was 473 level 200 PSTs comprising of science 82 (17.3%) and general programme 391 (82.7%). The sample for the study was level 200 General programme PSTs numbering 351(74.2%) of the population. The female sample was 133 (37.89%) while the male was 218 (62.11%) Convenient and simple random sampling techniques were adopted in selecting the college and the general programme PSTs respectively. The research instruments used was the Van Hiele Geometry Test (VHGT). From the results, the lowest score of 3.0, highest score of 16.0 and a modal score of 8.0 were recorded. The mean and standard deviation scores were is 8.79 and 2.49 respectively. The analysis revealed that 131 (37.3%) did not reach any of the VHGT levels. Also, 114 (32.5 %), 73 (20.8%), 28 (8.0%) and 5 (1.4%) attained levels 1, 2 3 and 4 respectively. No PST reached level 5. The correct response percentage rate for the various VHGT levels are 56.5%, 48.9%, 36.2%, 21.7% and 15.7% for levels 1,2,3,4 and 5 respectively. Also, the overall correct response percentage rate for the entire 25 items is 35.8% It was recommended that tutors of colleges of education in Ghana should adopt practical approaches in teaching geometric concepts.
Keywords: Geometry, Pre-Service Teachers (PSTs), Van Hiele Geometry Test (VHGT)

1.0 Introduction

The main aim of the Ghanaian Colleges of Education mathematics curriculum is to help pre- service teachers to think critically and apply mathematical knowledge effectively in their teaching career. It is also aimed at equipping trainees with the requisite knowledge to solve real life problems and make decisions in life (University of Cape Coast (UCC), Institute of Education (2014).

The various levels of geometric thinking play significant roles in PSTs' geometric thinking. These levels of geometric thinking are described in Van Hiele's model as visualization, analysis, informal deduction, deduction and rigor (Battista, 2002; Noraini, 2007). However, the way some mathematics tutors conduct their teaching of geometry in Ghanaian Colleges of Education does not encourage PSTs' thinking process. This is as a result of the fact that they focus only on recognizing and naming geometric shapes and learning to write symbols for simple geometrical concepts (Abdul-Halim & Effandi, 2013).

The role of geometry in the Ghanaian Colleges of Education mathematics curriculum cannot be over emphasized. It has had great impact in peoples' lives emanating from the need of human beings to specify quantities, to measure figures, land and earth and make maps (Sunzuma, Masocha, & Zezekwa, 2013). The study of geometry offers many foundational skills of logic, deductive reasoning, analytical reasoning and problem solving (Armah, Cofie, & Okpoti, 2017). According to Russell (2014) cited in Armah, Cofie and Okpoti (2018) geometry is also linked to many other areas in mathematics such as measurement, algebra, calculus and trigonometry and is used daily by architects, engineers, physicists, land surveyors and many more professionals.

A number of factors account for the difficulties of PSTs in learning geometry. Among other factors are language, visualization abilities and ineffective instruction (Baffoe & Mereku, 2010). These difficulties have been rectified in some western countries and a few countries in Africa who have used the Van Hiele's model effectively to improve performance of students in geometry. It has also helped inform curriculum developers and teachers across various levels of education about the importance of the Van Hiele's model. The Van Hiele's geometric test has also improved the student geometric thinking levels in the countries which adopted it (Van Hiele, 1999; Clements, 2004; Adolphus, 2011; Abdullah and Zakaria, 2013).

Currently research works on assessing students' geometric thinking level in Ghana is not common and because of that there is little or no comprehensive descriptions of Ghanaian students' geometric thinking level that can inform the intervention plan of teachers. Thus this gap has made earlier interventions of enhancing students' geometric thinking to have little effect on students. Also for basic school learners to attain these respective levels of geometric thinking, PSTs who are their prospective teachers need to have attained a level of geometric thinking above these levels in order to assist them by providing appropriate learning experiences. These arguments might be clearly explained by finding the Van Hiele levels of PSTs in geometry since the Van Hiele theory has been a facilitator for much of the renewed interest in geometry.

1-1 Statement of the problem

Improving PSTs' geometric thinking level is one of the basic goals of teaching mathematics in Ghanaian colleges of education. Geometric thinking is important in many scientific, technological and professional subjects (Olkun, Sinoplu, & Deryakulu, 2005). Some studies have pointed to the significant role of the teaching method in developing geometric thinking skills (Gneim, 2012). The criteria of the mathematics curricula of the different educational levels which stemmed from National Council of Teachers of Mathematics for the different stages and the National Council of Teachers of Mathematics in the USA emphasized the importance of having students being able to describe the geometric shapes, analyze their characteristics and make a comparison between the geometric systems (NCTM, 2000).

The scholars paid much attention to the concepts, as learning them is considered as very important goal in all levels of learning, because they provide the learner with organized cognitive structure that could be used to distinguish new examples and interpret new situations related to them. Moreover, some educationists, such as Khalifa (1999), Al-Khateb (2011) and Mashdani (2011) harped on the necessity of taking care of the concepts and focusing on the concept formation process through integration of teaching the math concept starting from the abstract phase that represents the common characteristics that distinguish the concept elements, followed by the generalization phase where the new elements are highlighted till the discrimination phase where the students can distinguish between the concept's elements. Despite the importance of mathematical geometry, learning it faces many difficulties. Ismael (1998) pointed out that geometry is one of the difficult branches of mathematics facing students and this difficulty refers to the lack of geometric concept acquisition. Learning transfer effect is one of the most important issues in education because it is one of the goals of the ministry of education of Ghana and one of the most important goals of the modern methods of teaching.

Developing the right attitudes towards mathematics which is one of the emotional goals that hopefully could be achieved in the educational process is not less important than the skill and cognitive aspects. To achieve the skill and cognitive goals, PSTs should have positive attitudes towards studying mathematics. To develop the right attitude towards geometry, PSTs must know that mathematics as a vital subject develops continuously and they should appreciate the social and civil role of the geometric knowledge and its effect in the nation's progress. The college tutor has a major role in developing positive attitude towards the study of geometry in particular and mathematics at large by presenting it using a method that PSTs can understand. One of the theories that helps greatly and effectively in teaching geometry is Van Hiele model which attracted scholars and educationists attention the world over because it helps effectively in teaching geometry. The Van Hiele model is considered one of the most important models in teaching geometry, and the geometric concepts and thoughts are developed through five phases within an educational program. These phases represent the development of the thinking process in geometry, in addition to the geometric knowledge acquisition. Learning geometry is not easy and many PSTs fail in developing the appropriate understanding of geometric concepts and in acquiring the geometric problem solving skills. Idris (2009) attributed this failure to students' weakness in geometric thinking and teachers' failure to use effective and appropriate teaching methods that can help them overcome their difficulties.

The Van Hiele theory has been used in many developed nations to enhance the teaching and learning of geometry, but in Ghana, there has been little investigation involving the Van Hiele theory. Thus very little work have made use of the Van Hiele's theory to determine the level of geometric thinking of Ghanaian PSTs and also to improve the teaching of geometry. Meanwhile there is every indication that many students in Ghana including PSTs in E.P. College of Education, Bimbilla have serious problems with geometry. For instance 28.8% of PSTs failed while 42.3% got weak passes in the geometry paper in the 2015 academic year for Colleges of Education in Ghana. Also, 23.2% PSTs either failed or obtained a weaker grade of D+ or D in the 2017 academic year end of the semester examination, (Institute of Education, professional board report UCC – Ghana, 2015, and 2017). For this reason, this study seeks to determine PSTs' geometric thinking level in E.P. College of Education, Bimbilla using the VHGT .

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1.2 Purpose of the study

The purpose of the study was to determine PSTs' geometric thinking level in E.P. College of Education, Bimbilla using the Van Hiele's Model. In pursuance of this purpose, this research question below was formulated to guide the study.

1.3 Research Question

- Which stages of Van Hiele levels of geometric thinking do general programme PSTs in E.P. College of Education, Bimbilla attain**?**

1.4 Significance of the study

Overlooking the relevance of this research may not be laudable. The findings of this study are expected to reveal some of the deficiencies that exist in the teaching and learning of mathematics especially geometry and it will also serve as a guide for tutors in colleges of education to vary their methodology to enable PSTs attain higher levels of geometric thinking. Furthermore, the finding of the research will help sharpen most of the PSTs' analytical skills in understanding geometry. It will promote and sustain trainees' interest to learn mathematics, especially geometry as well as motivate them to improve upon their learning. This study will help deepen trainees' understanding of modeling and better equip those who plan curriculum to meet the needs of Colleges of Education students in geometry in particular and mathematics at large.

Finally the research will be of great importance to educational planners especially Ministry of Education, Ghana Education Service, West African Examinations Council, and other beneficiaries of education as well as organizations that do have roles to play in the promotion and development of Mathematics Education in Ghana.

2.0 Theoretical Framework

2.1 The van Hiele Theory

The van Hiele theory of geometrical thinking was developed by Pierre van Hiele and his wife Dina van Hiele-Geldof (Armah, et al, 2017; Armah, et al, 2018; Anas, 2018) out of the frustrations the pair and their students experienced during teaching and learning of geometry. The two were Dutch researchers and mathematics teachers. The theory emanated from their thesis at the University of Utrecht in 1957 (Usiskin, 1982). In the 1960s Soviet Union carried a research based on the theory (Haviger and Vojkůvková, 2015). Also in the 1970s Americans also carried out several studies (Usiskin, 1982 and Senk, 1985) which influence NCTM Standards and Common Core State Standards. Van Hiele Model suggests that geometrical thinking has five closely related stages. The theory has enumerated three parts as (i) the existence of levels, (ii) the properties of the levels, and (iii) the phases of learning from one level to the next level (Vojkuvkova, 2012; Crowley, 1987). Initially van Hiele theory was numbered from 0 to 4. However, (Alex & Mammen, 2016; Howse & Howse, 2015; Siew & Chong, 2014) studies have increased it to level 5. This modification permits a sixth level named pre-recognition for students who will not achieve van Hiele level 1, Mason (1998). The five sequentially and hierarchical separate levels by Van Hiele (1986), are as follows (1) Visual, (2) Analysis, (3) Order, (4) Deduction, and (5) Rigor (Usiskin, 1982; Alex & Mammen, 2016) in ascending order of difficulty.

2.2 Van Hiele Theory Levels

Table 1: Characteristics of van Hiele Geometry Levels

Levels	Characteristics
Level I (Visualization)	Students recognize figures by their appearance. They make decision based on intuition not reasoning.
Level 2 (Analysis)	Students recognize figures by their properties. They can analyze and name properties of figures, but they cannot make relationships between these properties.
Level 3 (Informal deduction)	Students can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning.
Level 4 (Deduction)	Students can construct theorems within an axiomatic system. They know the meaning of necessary and sufficient conditions of a theorem
Level 5 (Rigor)	Students understand the relationship between various systems of geometry. They can compare, analyze and create proofs under different geometric systems.

Source: Karakus & Peker, (2015, p. 339).

2.3 Studies on Van Hiele Theory in Ghana

Anas (2018) study was carried out in five Mathematics/Science Colleges of Educations in the three northern regions of Ghana. The purpose of the study was to investigate Van Hiele levels of geometric thinking among Mathematics PSTs. The population was 412 of which two hundred and ninety- eight (298) were used as the sample. The sample composed of 16.8% female and 83.2% male. The Anas study recorded the following Van Hiele Level 0 (50.3%), Van Hiele Level 1 (23.5%), Van Hiele Levels 2 (14.8%), 9.1% Van Hiele Levels 3, 2.34% Van Hiele Levels 4 and 0 % Van Hiele Level 5. Anas concluded that only 11.44% were eligible to teach basic school mathematics in Ghana. His study again concluded that 88.56% of Mathematics PSTs attained between levels 0 to 2 hence were not eligible to teach mathematics at the basic schools. Anas' (2018) recommendation was that Colleges of Education should adopt Van Hiele phase-based learning of Geometry.

In a study by Armah, et al (2017), whose study focused on Van Hiele geometric thinking of PSTs' in Ghana before going out for teaching practice, a sample was drawn from 4 Colleges of Education and from three (3) regions of Ghana. The sample used were 300 PSTs in the second-year. From the results analyzed, it was clear that 16.33% of the PSTs were at level 0 based on Van Hiele levels. The following were also recorded for the

PSTs' attainment in Van Hiele geometry test levels, 27% for levels 1, 32 % for levels 2, 17.67% for levels 3, 6% for levels 4 and 1% for levels 5. The conclusion based on Van Hiele geometry test levels was that 75.33% of PSTs were lower than their expected future JHS 3 students.

Asemani, et al (2017) research subjects were 200 Ghana's Senior High School final year students selected from three (3) municipalities in the Central Region. The study sample documented 44% males and 56% females. It was clear from the quantitative analysis of students who did not meet any Van Hiele Geometric thinking level was 42.5%. Further analysis revealed that 33% of the final year students attained Van Hiele level 1, while level 2 recorded 22.5%. The rests are 1.5% and 0.5% for levels 3 and 4 respectively. The paper concluded that 43% of Ghana's final year Secondary School students did not attain any Van Hiele Geometric thinking level.

Baffoe & Mereku (2010), reported their study by a sample size of 188 from Winneba Senior High School and Zion Girls schools. The sample students came from the Winneba metropolis. Baffoe & Mereku measured the Van Hiele levels of geometric thinking of Ghana's Senior High School (SHS) 1 students. Their study was conducted when the students were four weeks old in the SHS campus level. The emanated results of the study indicated that Ghana's SHS 1 students were lagging behind on performance as compare to their colleagues from other countries when the Van Hiele geometric thinking test analysis was done. The quantitative results analysis showed that 59%, 11%, 1% of Ghana's SHS 1 students attained Van Hiele levels 1, 2 and 3 respectively.

3.0 Methodology

3.1 Research Design

The descriptive research design was employed to investigate the general programme PSTs geometric reasoning levels and achievement scores using VHGT in E.P. College of Education, Bimbilla-Ghana.

3.2 Population

The study population was four hundred and seventy-three (473) level 200 PSTs. The science / mathematics PSTs were 82 (17.3%) and general programme PSTs were 391(82.7%) of the student population. From the population 325 (68.7%) were male while 148 (31.3 %) were female.

3.3 Sample Size and Sampling procedure

The sample for the study was General programme PSTs numbering Three hundred and fifty-one 351(74.2%) of the population. Out of the sample one hundred and thirty-three 133 (37.89%) were female PSTs while two hundred and eighteen 218 (62.11%) were male PSTs. Convenient and simple random sampling were as adopted in selecting the college and the general programme PSTs respectively.

3.4 Administration of the research Instrument

In order to find answers to the research question for this study, the entire VHGT 25–item multiple choice test developed by Usiskin (1982) under the Cognitive Development and Achievement Secondary School Geometry (CDASSG) special programme was administered to the general programme PSTs to measure their geometric thinking levels in the 2017/2018 academic year, second semester to level 200 PST in the Colleges on 9th June 2018. The duration of the test was 45 minutes. Before administering the test a written permission request was sent to Prof. Zalman Usiskin for the use of his test items. He gave his approval and advice through an email. The VHGT questions are organized chronologically into five (5) subgroups such that it starts from the very cheap to the most difficult. Each subgroup covered the Van Hiele geometric thinking levels. The VHGT is popular and has been used by most researchers notably (Hoffer, 1983; Usiskin, 1982; Mayberry, 1983; Fuys et al 1988; Abdullah & Zakaria, 2013; Halat, 2008; Armah et al, 2017; Anas ,2018; Asemani et al, 2017)) used it to assess students geometric thinking levels.

3.5 Validity and Reliability

Researchers like (Usiskin, 1982; Atebe, & Schafer, 2008; Baffoe & Mereku, 2010; Anas, 2018), have acknowledged the VHGT as been reliable and valid. Two experienced mathematics tutors of the researchers department subjected the VHGT to both face and content validity. The tutors concluded that it met the PSTs' standard and hence cleared it for administering having compared it to the Colleges of Education Geometry course outlines. Reliability of the test items were tested when thirty (30) PSTs were involved in the writing the pilot test. The 30 subjects of the pilot test were not part of the actual study sample. Kuder-Richardson formula 20 methods were utilized to determine the reliability coefficient of the instruments. A reliability coefficient of 0.72 was realized from the pilot analysis which indicates a high degree of reliability of the instrument.

3.6 Nature of the questions

Table 2: Characteristics of van Hiele Geometry Thinking Test

Questions	Levels	Features
1-5	1	It is about visual form. It aims to determine whether the students recognize the shape by looking at the shape of the figure.
6-10	2	It is concerned with the Characteristics of the forms and on the one hand it aims to show that the students do not know the forms and on the other hand they do not know the Characteristics of the forms.
11-15	3	It determines whether students can recognize the relationships between forms. They identify students who respond correctly to questions in this group and have proven that they have knowledge of axioms.
16-20	4	It is the question of reasoning and logical deduction. In these questions, it is determined whether the students are at a level of understanding and writing.
21-25	5	The questions at this level are used to determine whether students can reason in Euclidean and Euclidean geometries.

Source: (Hurma, 2011, p.60)

3.7 Grading Systems and Level Assignment

The 25 items multiple choice test was allocated 1 mark each a correct answer. So the expected maximum mark was 25 while a minimum mark expected was 0. The researcher applied the '3 out of the 5' correct success criterion for the level assignment recommended by Usiskin (1982). A student is said to have mastered a given VHGT level if he/she correctly answered at least '3 out of the 5' items in any of the 5 subgroup correctly. The PSTs were assigned a weighted sum as follows as proposed by Usiskin (1982).

1. 1 mark for attaining the standard on items 1-5 (Level 1, Recognition)
2. 2 marks for attaining the standard on items 6-10 (Level 2, Analysis)
3. 4 marks for attaining the standard on items 11-15 (Level 3, Ordering)
4. 8 marks for attaining the standard on items 16-20 (Level 4, Deduction)
5. 16 marks for attaining the standard on items 21-25 (Level 5, Rigor)

This study adopted the Modified Case because it fits more students consistently than the classical Van Hiele level Usiskin (1982). Usiskin (1982) emphasized that in assigning levels in the Modified Case requires that a student's responses must satisfy property 1 of the levels that is students at level n satisfy the criterion not only at that level but also at all preceding levels. Usiskin reiterated a student who satisfies the criterion at levels 1, 2 and 5 would be assigned the Modified Van Hiele Level 2. Also, student who satisfies the criterion for only level 3 only would not be assigned modified Van Hiele level.

3.8 Data Analysis

Quantitative descriptive data analysis approach was used by making use of Microsoft Excel 2013 and SPSS Version 23. This approach of analysis is consistent with (Usiskin, 1982; MayBerry, 1983; Atebe, 2008; Armah et al; 2017; Asemani et al, 2017; Anas, 2018; Baffoe & Mereku, 2010) whose studies on van Hiele theory of geometry learning also analyzed data quantitatively.

4.0 Results

4.1 Results

The purpose of this study was to use Van Hiele Geometric thinking test to assess and classify level 200 PSTs in E.P. College of Education Bimbilla. The researcher organized the results of the study by employing descriptive statistics and presented it in a bar chart.

Table 3: Descriptive statistics on the total score of the PSTs

Number	Mean	St. Dev	Maximum	Minimum
351	8.79	2.49	16.00	3.00

From table 3, the minimum mark obtained by the PSTs was 3 and a maximum of 16 . The mean and standard deviation of the PSTs score are 8.79 and 2.49 respectively.

Table 4: Total scores obtained by the PSTs in VHGT by Cumulative Frequency.

Score	Number of students (N)	Cumulative (N)	Percentage (%)	Cumulative Percentage (%)
3	2	0.6	0.6	0.6
4	5	1.4	1.4	2.0
5	19	5.4	5.4	7.4
6	43	12.3	12.3	19.7
7	48	13.7	13.7	33.3
8	53	15.1	15.1	48.4
9	51	14.5	14.5	63.0
10	45	12.8	12.8	75.8
11	32	9.1	9.1	84.9
12	**20**	**5.7**	**5.7**	**90.6**
13	20	5.7	5.7	96.3
14	9	2.6	2.6	98.9
15	3	0.9	0.9	99.7
16	1	0.3	0.3	100.0
Total	351	100.0	100.0	

From table 4, it is clear that 298 out of 351 PSTs obtained less than half of the total score. This represents 84.9% of the total number of PSTs who took part in the VHGT. Also, 20 (5.7%) PSTs obtained exactly half the total mark. Also, 33 PSTs representing 9.4% of the total number performed above the half mark of 12. It was amazing to see that the maximum mark scored in the test was16 out of 25, which was obtained by only one PST . A close look at the performance of PSTs was weak.

Table 5: PST subtest 1perfomance on each item in the VHGT level 1

Level	Choice Item	A		B		C		D		E		Blank			
		(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	Total no.	Total %
Level 1	**1**	0	0.0	**277**	**78.9**	5	1.4	64	18.2	1	0.3	4	1.1	351	100
	2	6	1.7	8	2.3	8	2.3	**322**	**91.7**	4	1.1	3	0.9	351	100
	3	131	37.3	3	0.9	**208**	**59.3**	4	1.1	3	0.9	2	0.6	351	100
	4	116	33.0	**142**	**40.5**	24	6.8	15	4.3	51	14.5	3	0.9	351	100
	5	158	45.0	56	16.0	68	19.4	**42**	**12.0**	20	5.7	7	2.0	351	100
TOTAL		411	117.1	486	138.5	313	89.2	447	127.4	79	22.5	19	5.4		

In level 1 of the VHGT, PSTs recorded a very good performance, especially in questions 1, 2, 3 and 4. Questions 1, 2 and 3 percentages are above 50% while question 4 and 5 percentages are below 50% For instance 227 PSTs representing 78.9% of the total number had question 1 correct, while 322 representing 91.7% had question 2 correct. Question 3 was correctly answered by 208 PST representing 59.3% and 142 PSTs representing 40.5% were able to answer question 4 correctly.

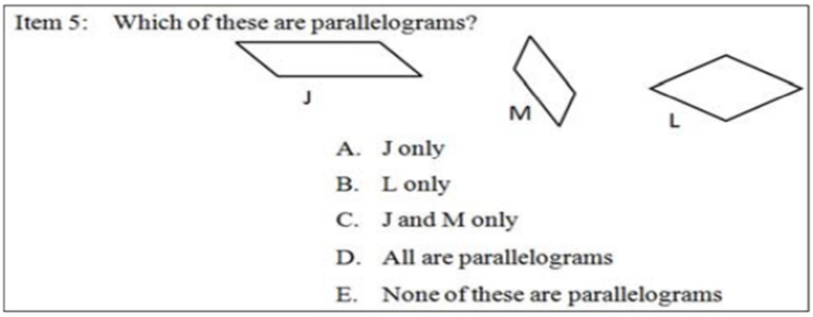


Figure 1: sample item in subtest 1.

Figure 1 is question 5 of the VHGT. The correct option was D and only 42 PSTs representing 12% of the total number had it correct. The reason for the massive inability of the students to get the question right was that they are not aware that all quadrilaterals are parallelograms. The overall correct response rate in subtest 1 of the VHGT level1 is 56.5%.

Table 6: PSTs subtest 2 performance on each item in the VHGT level 2.

Level	Choice Item	A		B		C		D		E		Blank			
		(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	Total no.	Total %
Level 2	6	33	9.4	**136**	**38.7**	145	41.3	35	10.0	0	0.0	2	0.6	351	100
	7	91	25.9	7	2.0	40	11.4	28	8.0	**184**	**52.4**	1	0.3	351	100
	8	**140**	**39.9**	23	6.6	78	22.2	56	16.0	52	14.8	2	0.6	351	100
	9	8	2.3	22	6.3	**289**	**82.3**	10	2.8	19	5.4	3	0.9	351	100
	10	33	9.4	48	13.7	61	17.4	**109**	**31.1**	90	25.6	10	2.8	351	100
		305	86.9	236	67.2	613	174.6	238	67.8	345	98.3	18	5.1		

PSTs performance on questions 7 and 9 is quite good because their correct responses rate were above 50%. . Out of 351 PSTs, 184 representing 52.4% and 289 representing 82.3% respectively responded well to items 7 and 9. Contrary to this, PSTs performance on items 6, 8, and 10 were discouraging since 136 representing 38.7%, 140 representing 39.9% and 109 representing 31.1% respectively had right answers. It also indicates that at least 60% of the PSTs had wrong answers each for items 6, 8, and 10.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.

Which of (A)-(D) is not always true?

A. PRQS will have two pairs of sides of equal length.
B. PRQS will have at least two angles of equal measure.
C. The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.
D. Angles P and Q will have the same measure.
E. All of (A)-(D) are true.

Figure 2: sample item in subtest 2

Figure 2 shows question 10 of the VHGT level 2. This question was poorly answered by PSTs since out of 351, 242 representing 68.9% had wrong answers to this question. This is an indication that PSTs lacked knowledge of properties of a rhombus. The overall correct response rate in subtest 2 of the VHGT level 2 is 48.9%.

Table 7: PSTs subtest 3 performance on each item in the VHGT level 3.

Level	Choice Item	A		B		C		D		E		Blank			
		(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	Total no.	Total %
Level 3	11	133	37.9	86	24.5	**60**	**17.1**	11	3.1	40	11.4	21	6.0	351	100
	12	45	12.8	**223**	**63.5**	25	7.1	4	1.1	40	11.4	14	4.0	351	100
	13	**191**	**54.4**	4	1.1	9	2.6	138	39.3	6	1.7	3	0.9	351	100
	14	**26**	**7.4**	24	6.8	134	38.2	27	7.7	127	36.2	13	3.7	351	100
	15	58	16.5	**136**	**38.7**	28	8.0	36	10.3	85	24.2	8	2.3	351	100
		453	129	473	135	256	73	216	62	298	85	59	17		

Level 3 of the VHGT was meant to test PSTs knowledge in the relationship between different types of figures. PSTs performed woefully at this level. From table 7 it is clear that out of 351 PSTs, 60 representing 17.1%, 223 representing 63.5%, 191 representing 54.4%, 26 representing 7.4%, and 136 representing 38.7% answered items 11, 12, 13, 14 and 15 respectively. The best item answered correctly by PSTs was 12 and the least performance was in item 14.

14. Which is true?

(A) All properties of rectangles are properties of all squares.

(B) All properties of squares are properties of rectangles.

(C) All properties of rectangles are properties of all parallelograms.

(D) All properties of squares are properties of all parallelograms.

(E) None of (A)-(D) is true.

Figure 3: sample item in subtest 3

Figure 3 is question 14 of subtest 3 of the VHGT. From the subtest 3, 26 out the 351 representing 7.4 % had this item right. This performance was abysmal. This performance is as a result of lack of knowledge on the properties of rectangles and squares. They don't know that the two have familiar properties .The overall correct response rate at this level was 36.2%.

Table 8: PST subtest 4 performance on each item in the VHGT level 4.

Level	Choice Item	A		B		C		D		E		Blank			
		(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	Total no.	Total %
Level 4	**16**	59	16.8	39	11.1	**72**	**20.5**	53	15.1	120	34.2	8	2.3	351	100
	17	164	46.7	62	17.7	**64**	**18.2**	35	10.0	18	5.1	8	2.3	351	100
	18	106	30.2	64	18.2	51	14.5	**97**	**27.6**	29	8.3	4	1.1	351	100
	19	210	59.8	34	9.7	64	18.2	**22**	**6.3**	11	3.1	10	2.8	351	100
	20	126	**35.9**	**50**	14.2	94	26.8	48	13.7	29	8.3	4	1.1	351	100
		665	189.5	249	70.9	345	98.3	255	72.6	207	59.0	34	9.7		

Table 8, presents PSTs subtest 4 performance on each item in the VHGT level 4. Subtest 4 is meant to test PSTs ability to exhibit deductive geometric proofs, understanding the role definitions, and theories. PSTs are expected at this level to be able to give reasons for a statement in formal proofs. Table 8 indicates that 72 (20.5%), 64 (18.2%), 97 (27.6%), 22 (6.3%) and 126(35.9%) of PSTs correctly answered items 16, 17, 18, 19, and 20 respectively which was quite discouraging. At this level, PSTs had their best performance in item 20 and worst performance in item 19.

19. In geometry:

(A) Every term can be defined and every true statement can be proved true.

(B) Every term can be defined but it is necessary to assume that certain statements are true.

(C) Some terms must be left undefined but every true statement can be proved true.

(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.

(E) None of (A)-(D) is correct.

Figure 4: sample item in subtest 4

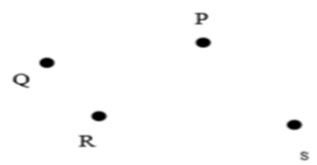
From the analysis, PSTs have difficulties understanding simple deductive geometric proofs, understanding the role of definitions theories, and axiom's. Meanwhile students at SHSs are supposed to be at this level before completion. The overall correct response rate at this level was 21.7 %.

Table 9: PSTs subtest 5 performance on each item in the VHGT level 5.

Level	Choice Item	A		B		C		D		E		Blank			
		(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	(n)	(%)	Total no.	Total %
Level 5	**21**	197	56.1	**27**	**8**	70	19.9	15	4.3	40	11.4	2	0.6	351	100
	22	103	29.3	100	28	64	18.2	38	10.8	**43**	**12.3**	3	0.9	351	100
	23	153	43.6	36	10	34	9.7	**76**	**21.7**	49	14.0	3	0.9	351	100
	24	19	5.4	40	11	46	13.1	181	51.6	**63**	**17.9**	2	0.6	351	100
	25	52	14.8	127	36	90	25.6	**66**	**18.8**	14	4.0	2	0.6	351	100
		524	149.3	330	94	304	86.6	376	107.1	209	59.5	12	3.4		

Table 9 shows PSTs performance in subtest 5.Out of the 351 PSTs 27 (8%), 43 (12.3%), 76 (21.7%), 63 (17.9%) and 66 (18.8%) correctly answered items 21, 22, 23, 24, and 25 respectively. Question 23 was best answered and the worst item answered was 21. The overall correct response rate at this level was 15.7%.

21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R and S, and the lines are {P,Q}, {P,R}, {P,S}, {Q,R}, {Q,S}, and {R,S}.



Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common. The lines {P,Q} and {R,S} are parallel because they have no points in common. From this information, which is correct?

A. {P,R} and {Q,S} intersect. B. {P,R} and {Q,S} are parallel. C. {Q,R} and {R,S} are parallel. D. {P,S} and {Q,R} intersect. E. None of (A)-(D) is correct.

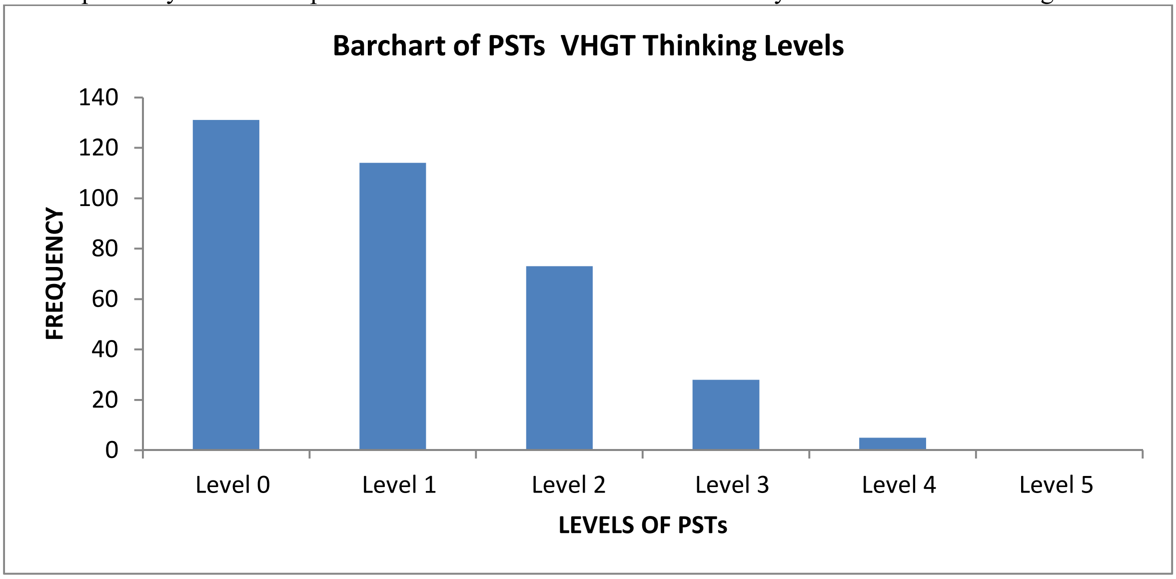
Figure 5: sample item in subtest 5

At this level, PST were expected to work with a variety of axiomatic systems. That is being able to study non-Euclidian geometries by comparing different systems and also seeing geometry in the abstract.

Table 10: PSTs performance on VHGT level attainment.

Level	No level reached	Visualization level 1	Analysis level 2	Order level 3	Deduction level 4	Rigor level 5
No	131	114	73	28	5	0
Percentage	(37.3%)	(32.5 %)	(20.8%)	(8.0%)	(1.4 %)	(0 %)

As shown in table 10, no PSTs reached level 5 and only 5 (1.4%) attained level 4. Also, 131 (37.3%) reached no VHGT. Further analysis indicates that, 114 (32.5 %) , 73 (20.8%) and 28 (8.0%) attained level 1,2 and 3 respectively. Pictorial representation of the various levels attained by the PSTs is shown in figure 6 below.



4.2 Discussion of Results

This study was meant to investigate the Van Hiele Geometric thinking levels of level 200 PSTs of E.P. College of Education, Bimbilla – Ghana. It is clear from the descriptive analysis of the overall performance of the 351 PSTs that the lowest score was 3.0, highest score was 16.0 and a modal score of 8.0 was recorded. The mean score is 8.79 and the standard deviation is 2.49. The correct response rate for the various VHGT levels are 56.5% for questions 1-5, 48.9% for questions 6-10, 36.2% for questions 11-15, 21.7% for questions 16-20 and 15.7% for questions 21-25 for levels 1,2,3,4 and 5 respectively. Also, the overall correct response rate for the entire 25 items is 35.8% For the levels analyses , 298 (84.9%), of the PSTs obtained less than half of the total score of 25 , 33 (9.4%) of the PSTs obtained marks above half of the total score and, 20 (5.7%) PSTs obtained exactly half the total.. This is a clear indication that PSTs performed woefully similar to Anas (2018). It revealed from the results of the VHGT that 114 (32.5%) reached Van Hiele level 1 which is the visualization stage where geometric figures are identified based on their appearance. Level 2 of the VHGTT tested the ability of PSTs to analyze figures geometrically and only 73 (20.8%) reached this level. Apart from that 28 (8.0%) reached level 3, 5 (1.4%) reached level 4 and no student reached level 5. Also 131 (37.3%) of level 200 PSTs in E.P College of Education, Bimbilla, Ghana reached no level of VHGT. This is disturbing because with similar sample within southern Ghana by Armah et al (2017) only 16.33% of the total participant reached no level of the VHGT. This is an indicator that majority of the level 200 PST in E.P College of Education, Bimbilla cannot teach their prospective basic school pupils since the pupils are supposed to be at level 3. These findings are in line with previous students by (Anas , 2018; Halat and Sahin, 2008; Pandisco and knight , 2010; Ndlove, 2014; and Armah, et al, 2017).The findings also reveal that a little above half of level 200 PST in E.P College of Education , Bimbilla were found to be operating at van Hiela level 1,2,3 and 4 Also only a few were at level 3 and 4 which is consistent with other studies by (Anas, 2018; Armah, et al, 2017; Halat , 2008; Erdogan and Durmus, 2009; Halat and Sahin, 2008; Pandisco and Knight 2010) who also recorded very low performance in van Hiele levels 3, 4 and 5 . If van Hiela level 4 is the pre-requisite for students leaving SHS and majority of level 200 PSTs are below this level, then something serious needs to be done about teaching geometry at the SHS. Basic school teachers need to be at level 3 and above if they should function well at the basic school (Crowley, 1987; Usiskin, 1982; and Armah, et al, 2017). Further analysis has proven that 33 out of the 351 level 200 PSTs representing 2.2% of the total number are qualified to teach their prospective basic school students. This is to suggest that all level 200 PSTs who are below the van Hiele level 3 will have extreme difficulty in teaching students at the basic school level. Item by item analysis also revealed that level 200 PSTs could easily visualize and identify properties of plane shapes which is for only level one and two. It also revealed from item to item analysis that level 200 PSTs lacked understanding of the relationship between different geometrical shapes. The PST also lacked the knowledge to create meaningful definitions and give informal arguments to justify their reasoning. Lastly level 200 PST lacked understanding in logical implication and class inclusion. These findings concur with Baffoe and Mereku (2010) and Armah, et al (2017) who observed that Van Hiele level 3 is most difficult for learners because class inclusion such as square being types of rectangles was not understood by PST only a few PST understood geometry reasoning at level 4 of the VGHT. It is discovered from this study that geometry teaching is essentially focused on level 1 and 2 and very little or no emphasis on the advance levels 3, 4 and 5.

5.0 Conclusion
For the geometric thinking levels of the VHGT, 131 (37.3%) of the PSTs did not reach any of the levels. Also, 114 (32.5%), 73 (20.8%), 28 (8.0%), 5 (1.4%) reached levels 1, 2, 3 and 4 respectively. No PST attained level 5. The analysis revealed that the minimum score was 3, maximum 16 , and a mode 8 The mean and standard deviation of the PSTs score are 8.79 and 2.49 respectively. The correct response rate for level 1 was 56.5%, level 2 was 48.9%, level 3 was 36.2%, level 4 was 21.7% and level 5 was 15.7%.in the VHGT. Also, 35.8% was recorded as the overall correct response rate for the entire 25 items. PSTs who scored less than half of the total score was 298 (84.9%), 20 (5.7%) obtained exactly half the total marks of 12. While those who scored above half of the total score was 33 (9.4%) The implication is that only 33 out of the 351 level 200 PSTs representing 2.2% of the sample are qualified to teach their prospective basic school students.

5.1 Recommendations
Based on the findings of this study, it is recommended that

- Colleges of Education geometric syllabus should be revived in terms of content and scope in line with current standards
- Mathematics tutors in Colleges of Education should adopt practical approach to teaching and learning of geometric as a course
- The supervisory body of Colleges of Education, The National Council For Tertiary Education (NCTE) and the Institute of Education, University of Cape Coast (UCC) should organize seminars and workshops for Mathematics tutors in colleges of Education on Van Hiele phase based instruction so that they will be able to incorporate it in their instruction to maximize PST achievement.
- Curriculum developers and writers of Mathematics textbooks should also adopt the Van Hiele phase based instruction.

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