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Investigating First Year Undergraduate Students' Difficulties in Constructing of Proofs in Set Theory

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Abstract

This study reports on first year undergraduate students' difficulties in proof construction in the area of set theory. The study adopted Purposive sampling where by a total of 73 respondents were selected of which 67 were first year Mathematics students and 6 were their lecturers. Data was collected using two instruments i.e. a test for students and questionnaire for students and lecturers. The data gathered were analyzed to explore difficulties faced by first year students when constructing proofs in set theory. The findings from the study showed that first year students had difficulties in construction of proofs in set theory. This was from the incorrect answers obtained in the test questions given where 45 students representing 67% failed question one, 35(52%) students failed question two and 40 students representing 60% failed question three. Among the main difficulties faced by first year undergraduate students at Kwame Nkrumah in constructing proofs in set theory were; poor understanding of set concepts, inability to grasp the mathematical language, lack of real understanding of definition, lack of understanding of what can be classified as a proof, incorrect use of notation and students' lack of giving clear conclusion i.e. if $A \subseteq B$ *and* $B \subseteq A$ then $A = B$.

The study further found that the above difficulties were caused by 1) Lack of knowledge of logical reasoning process; 2) Student's limited deductive reasoning abilities; 3) Incorrect use of terms, symbols and signs; 4) Mathematics done at secondary school is not sufficient pre requisite for proof construction at University level; 5) Inadequate material on proofs in elementary set theory; 6) Poor pedagogy by lecturers and 7) tendency by most students to translate the mathematical concepts using their local language which in turn distort the reasoning.

From the findings, it was evident that little guidance is provided to the first year students as they engage in proof construction in set theory hence, the difficulties. Therefore, there is need for lecturers to organise activities according to those which improve thinking and reasoning skill of students in order to overcome the proof problems. Also, lecturers should structure learning environment in such a way that students may discover answers on their own, but should also use creative explanations, demonstrations and effective questioning.

Keywords: difficulties, proof construction, set theory.

1. Introduction

Zambian University Mathematics Curriculum highlights active, meaningful learning through processes related to proofs. Meaning, students are expected to engage in Mathematics actively, learn how to solve problems, share, explain and justify their solutions and ideas and find relations within Mathematics as well as between Mathematics and other subjects.

Much emphasis is put on proof because it is an essential part of Mathematics and therefore of academic Mathematician's daily practices. It is also important for mathematics educators because proof involves reasoning, conviction and communication and helps meaningful learning (Imamoglu & Togrol, 2015). In other words, proof is the basis of mathematics and consequently, it is important for the students to know what constitutes a proof, why proof is needed and how to construct proof to understand the structure of mathematics (Sari, 2016).

Despite proof being obviously an important part of Mathematics, Studies done all over the world, (Güler & Dikici, 2012; Güler, Özdemir, & Dikici, 2012; Güler, 2013; Güler & Dikici, 2014, Yo & Knuth, 2013) indicate that students across all levels have poor understanding of proofs and have difficulties in constructing their own. Considering the importance of proof in Mathematics education, emphasised both by current research and

Mathematics curriculum and the difficulties that students face regarding proofs, this study aims to investigate first year undergraduate students' difficulties in constructing of proof in set theory: A case of Kwame Nkrumah University.

1.1. Problem statement

Proofs are the heart of Mathematics (Rav, 1999, p.6). But the task of proof construction poses a great difficulty for the students at all levels of learning (Mukuka & Shumba, 2016). One of the potential factors contributing to this could be that they lack proper understanding in handling problems involving set theory, the topic which makes mathematics easier to all, as ideas in all the branches of mathematics (arithmetic, algebra, geometry, calculus etc.) can be explained in terms of sets (Shafigul ,2015).

Besides the importance of set theory especially proof in the teaching and learning of mathematics in Zambia, proof in set theory is one of the poorly achieved topic in most institutions of learning at both Colleges of education and universities. Kwame Nkrumah University is not an exception. For instance, in 2015 Promotional examination in Foundation Mathematics, MAT 110 performance analysis of topic by topic, of the 40% students who attempted the question on proving a set theorem question, only 13% got the question correct. While in 2016, of the 37% who attempted the question, only 12% got it right.

Against this background, the researcher would want to investigate the difficulties first year students at Kwame Nkrumah university face in proof construction that lead to this poor performance on this topic.

1.2. Scope of the study

This study was designed to cover the entire first year Mathematics students of 2017 intake at Kwame Nkrumah University. It focused on examining the difficulties faced by first year undergraduate students in proof construction of set theory. The study also explored causes and forms of guidance offered by lecturers to their students as they engaged in proof constructions .The following were the research questions:

(a) What are the main difficulties that first year students face in relation to proof construction in set theory at Kwame Nkrumah University ?
(b) What might be the origin (causes) of such difficulties?
(c) What forms of guidance do Mathematics lecturers offer to first year students as they engage in proof construction at Kwame Nkrumah University?

1.3. Theoretical and methodological frameworks

The theoretical framework underpinning this study advances the view that the quality of this opportunity to learn is, in turn, shaped primarily by the pedagogical content knowledge of the teacher. In the constructivist classroom, the teacher's role is to prompt and facilitate discussion (Hausfather, 2001). Thus, the teacher's main focus should be on guiding students by asking questions that will lead them to develop their own conclusions on the subject. The philosophical foundation or paradigm on which the data were collected and analyzed was the critical theory. ***Though the study is conducted in Zambian and African setting, the results may be relevant cross-culturally and globally, since proof is a universal mathematical competency***.

2. Literature Review

A Survey conducted by Mujib (2015) on 36 third year students as he was trying to analyse students difficulties in constructing mathematical proof on discrete mathematics showed that the ability to read proof and construct a mathematical proof the students was low. The difficulties faced by students in constructing such proof: (1) understanding of mathematical concepts, (2) language and mathematical notation, (3) strategies of proof, and (4) read of proof. In addition, student perception about mathematics and mathematical proof construction affected student proof. Writing about a good proof was another difficulty faced by students.

In research performed by Mukuka & Shumba (2016) on second year student teachers' conception of algebraic proof reported that, out of the 73 second year student teachers at Mukuba University, more than 75% had limited understanding of the nature and purpose of proof.

The literature review has only reported some of the difficulties students face in proof construction without highlighting the roots (causes) of such difficulties. The researcher also identified a research gap on the form of guidance lecturers' offer to students as they engage in proof construction. Thus, this study was conducted to fill the gap.

3. Methodology and Procedures

This study used a descriptive survey design because the study was intended to present the difficulties first year students' face in proof construction in set theory, as they exist. Both qualitative and quantitative approaches were used in this study. Niglas (2004) points out that quantitative and qualitative method can be combined at different

stages of the research process in the study of the same phenomena for the purpose of triangulation and drawing from the strength of both methods since both have their own strengths and weaknesses. This is to enable both methods complement each other so as to allow the researcher to offset their weaknesses and draw on the strengths of both in order to ensure that the results are valid and not a methodological artifact. Also, Brook (2013) and Orodho (2009:120) support the combination of these methods in order to reveal several dimensions of phenomenon, to deal with the shortcomings of each approach and double check the findings by examining them from several different vantage (clear) points.

This research study was conducted at Kwame Nkrumah University on seventy three (73) participants of whom sixty seven (67) were students in first year studying Mathematics at the same institution and six (6) were their lecturers in mathematics courses. The study used purposive sampling technique which assumes that a sample is selected "in a non-random manner, based on member characteristics relevant to the research problems." Kombo and Tromp (2006: 82) state that, "the power of purposive sampling lies in selecting participants who will provide the richest information for in-depth analysis related to the central issue being studied.

Two instruments were used to collect data i.e. test and questionnaire. This study used both quantitative instruments (structured questionnaire) and qualitative instruments (researcher made written test). Bryman (2006) contends that different research instruments are used in order to extend the breadth and range of inquiry. The first was a researcher – made written test comprising three tasks on proof-writing in set theory given to students in order to provide an in-depth test to discover the nature of particular weaknesses, difficulties and conceptions that first year undergraduate students possess in proof construction. The test items were familiar to the students but they were not exclusively of a routine type as those they met in the classroom or textbooks. This was done to avoid the possibility of reproducing memorized arguments. This was done to answer research question (a).

The second instrument used was a questionnaire; one for students and the other for lecturers. The questionnaire for students was used to collect data relating to students' difficulties in proof construction and the causes of those difficulties answering research questions (b), while a questionnaire for lectures answered to the causes of those difficulties answering research questions (b). Also, gathered information on how they rated the difficulties that first year undergraduate students' face in proof construction and the forms of guidance they rendered to help the students taking care of research question (c)

The questionnaires were then distributed to sixty seven first year students doing Mathematics and six lecturers in mathematics department. Students completed the questionnaires and a test in class respectively. Lecturers completed the questionnaire in their respective offices. The researcher personally visited the research site for the administration of the questionnaires.

A mixed method of analysing data was employed. Data from the test was qualitatively analysed in form of document analysis and presented in narrative and direct quotes as suggested by Orodho (2009, 2012) and Brook (2013). While, data from questionnaires were analyzed using frequency counts, percentages and mean scores. Any mean score that was lower than 3.0 was rejected while any mean of 3.0 and above was
accepted i.e.

$$SA = 5, \quad A = 4, \quad U = 3 \quad D = 2 \quad SD = 1$$

$$\frac{5+4+3+2+1}{5} = \frac{15}{5} = 3.0$$

Also, bar charts were used in some cases to give a visual representation that would enable readers to analyse and interpret data more easily than they could simply by looking at numbers.

4. Results and findings

4.1 Research Question One:

What are the main difficulties that first year Mathematics students at Kwame Nkrumah University face in relation to proof construction in set theory?
To answer the above research question, the following test questions were presented to the first year students in order to obtain this study's research data.

1. Prove $(A \setminus B) \cap B = \emptyset$
2. Prove the De Morgan's law , $(A \cup B)^c = A^c \cap B^c$
3. Prove $A \times (B \cap C) = (A \times B) \cap (A \times C)$

The written test and students' answers were analysed deeply by the researcher. The students' test scripts were marked and results based on each question are presented in the table below.

Table 1: Distribution of Respondents answers

Statement	**Q1**	**Q2**	**Q3**
Correctly answered	22 (33%)	32(48%)	27(40%)
Incorrectly answered	45 (67%)	35(52%)	40(60%)
Total	**67 (100%)**	**67 (100%)**	**67(100%)**

Source: Field Survey, 2017

Considering the incorrectly answered questions, it was evident that first year students faced difficulties with proof construction at various stages of proving in their test questions given. The scripts were marked and the following difficulties were observed question by question and their corresponding percentages as categorised in table 2.

Table 2: Distribution of the difficulties first year mathematics students' face in proof construction in set theory

Description of the difficulties faced	Frequency			Percentage %		
-	Q.1	Q.2	Q.3	Q.1	Q.2	Q.3
• Lack of understanding of the definition	18	10	19	40	29	47.5
• Incorrect use of notation	9	4	4	20	11	10
• Inability to grasp set theory language	7	7	8	16	20	20
• Poor understanding of sets concepts	5	6	4	11	17	10
• Students lack of giving clear conclusion i.e. if A$\subseteq B$ *and* B$\subseteq A$, then $A = B$	0	5	3	0	14	7.5
• Lack of understanding of what can be classified as proof	6	3	2	13	09	05
N = Number of students	45	35	40	100%	100%	100%

Source: Field Survey, 2017

From the above, it can been seen that most of the first year students had difficulties with understanding of the definitions representing 40%, 29% and 47% respectively, followed by those who could not grasp the mathematical language represented by 16%, 20% and 20% respectively.

The above study's findings were created and interpreted based on the written answers to questions posed to the students. Below are sampled students' marked answer scripts presented in figures 1, 2 and 3 respectively.

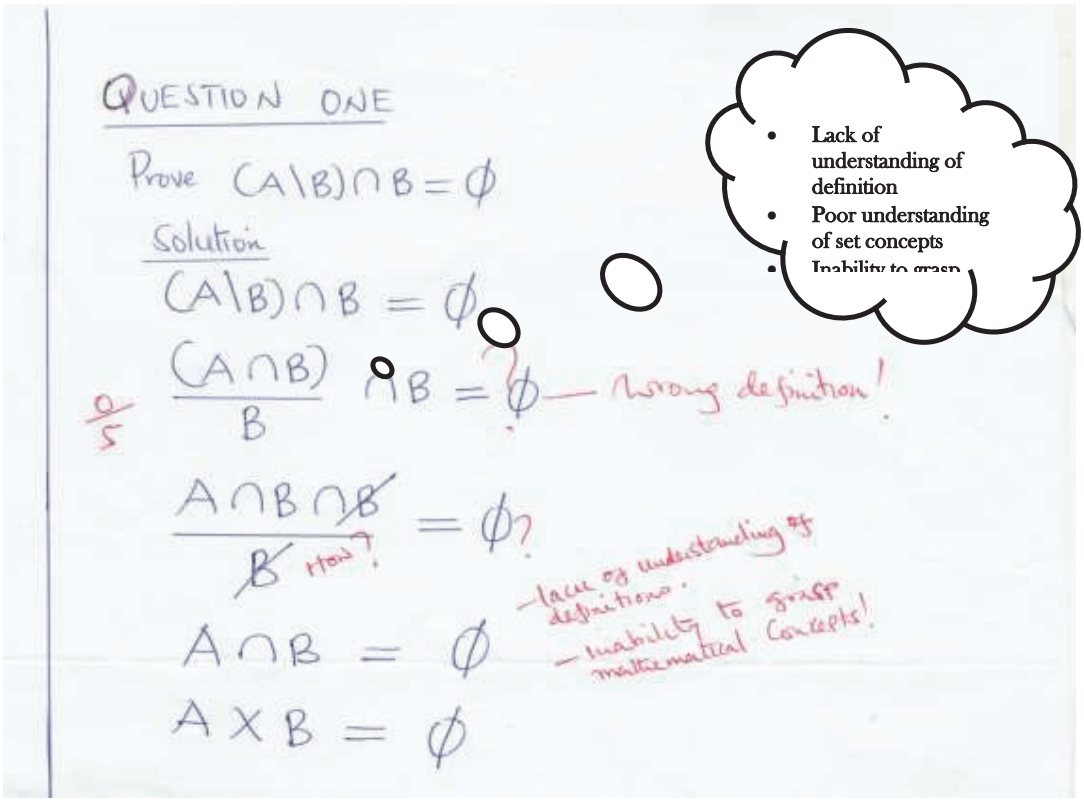

Figure 1: Student A's proof of question no. 1

In the solution of figure 1, students exhibited lack of understanding of definitions and poor understanding of set theory.

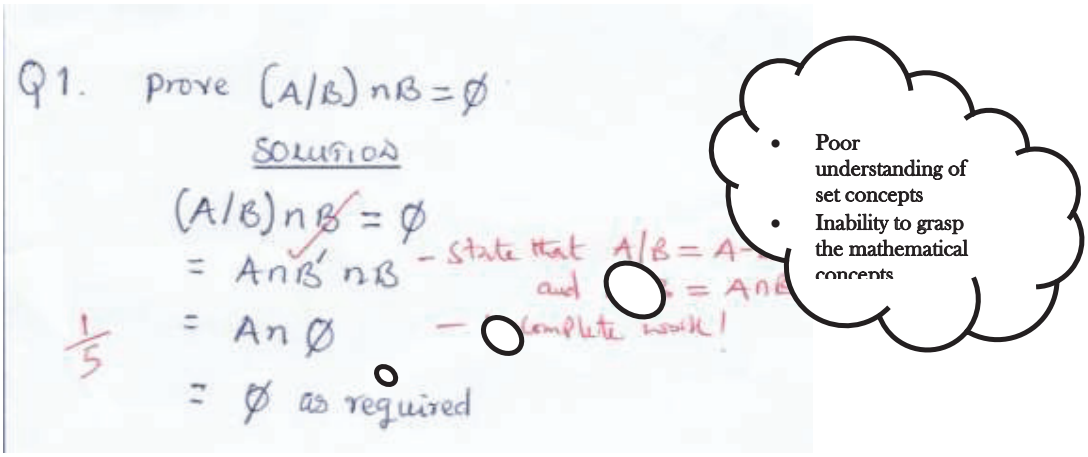

Figure 2: Student B's proof of question no. 2

The presentation in figure 2 shows that students use of incorrect set notation and lack of giving clear conclusion i.e. if $A \subseteq B$ *and* $B \subseteq A$, then $A = B$

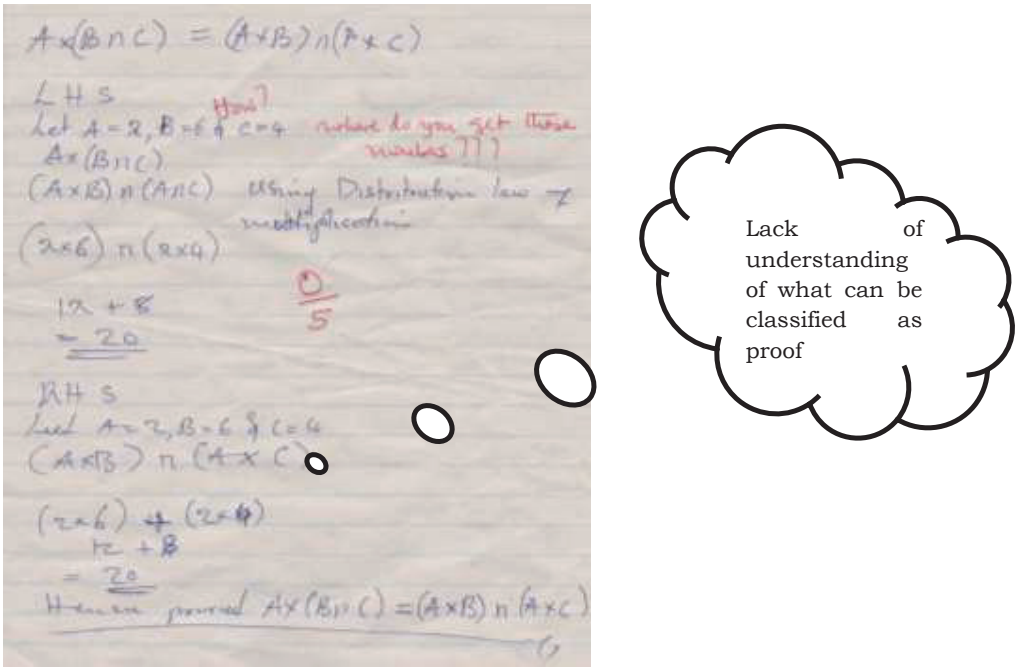

Figure 3: Student C's proof of question no. 3

From demonstration in figure 3, students showed lack of understanding of what can be classified as proof, lack of understanding of the definition and incorrect use of notation.

4.2 Research Questions Two

What might be the origin (causes) of such difficulties?

The answer to the second research question is displayed in Table 3.

Table 3: Distribution of students' responses to questionnaire on the origin (causes) of difficulties first year Mathematics students at Kwame Nkrumah University have in constructing proofs in set theory (n = 67).

S/No	Statement	SA	A	U	D	SD	$\bar{x}$	Decision
a.	Lack of knowledge of logical reasoning process	12	23	04	23	05	3.21	Agree
b.	Student's limited deductive reasoning abilities	12	26	04	16	09	3.23	Agree
c.	Incorrect use of terms, symbols and signs	13	24	04	20	06	3.27	Agree
d.	Inadequate material on proofs of elementary set theory	22	28	04	07	06	3.79	Agree
e.	The mathematics done at secondary school is not sufficient pre requisite for proof construction at university level	34	16	01	11	05	3.94	Agree
	Overall Mean						**3.49**	**Agree**

Source: Field Survey, 2017

The findings from table 3 showed that, lack of knowledge of logical reasoning process mean score stood at 3.21, students limited deductive reasoning abilities mean score was given as 3.23, incorrect use of terms,

symbols and signs gave a mean score of 3.27, mathematics done at secondary school is not sufficient pre-requisite for proof construction at university level had a mean score of 3.79 while inadequate material on proofs in elementary set theory was at a mean score of 3.94 with a Sectional Mean score of 3.49 which indicated the overall acceptance that these were the causes of the difficulties first year students face in proof construction in set theory.

Further, 74.6% of the students indicated that the causes of the difficulties to poor pedagogy by lecturers. They revealed that, lecturers skip steps when explaining the concepts and usually the pace at which they lecture is fast, while 25.4% of the students attributed the causes to intimidations by lecturers. They attribute it to lecturers refusing to allow students to ask questions.

On the same research question, lecturers' responses on the causes of the difficulties first year Mathematics students have in proof construction in the area of set theory were tabulated in table 4.

Table 4: Distribution of students' responses to questionnaire on the Origin (causes) of difficulties that first year Mathematics students at Kwame Nkrumah University have in constructing proofs in set theory (n = 6)

S/No	Statement	SA	A	U	D	SD	$\bar{x}$	Decision
a.	Lack of knowledge of logical reasoning process	04	01	-	01	-	4.33	Agree
b.	Student's limited deductive reasoning abilities	03	03	-	-	-	4.50	Agree
c.	Incorrect use of terms, symbols and signs	03	02	-	01	-	4.17	Agree
d.	Inadequate material on proofs of elementary set theory	05	-	-	01	-	4.50	Agree
e.	The mathematics done at secondary school is not sufficient pre requisite for proof construction at university level	03	01	01	01	-	4.00	Agree
	Overall Mean						**4.30**	**Agree**

Source: Field Survey (2017)

The results expressed above showed that the mean scores of items for the causes of the difficulties first year Mathematics students have in constructing of proofs in set theory were distributed as; lack of knowledge of logical reasoning process stood at 4.33, students limited deductive reasoning abilities was given as 4.50, incorrect use of terms, symbols and signs gave 4.17, mathematics done at secondary school is not sufficient pre requisite for proof construction at university level had 4.50 while inadequate material on proofs in set theory had a mean score of 4.00 with a Sectional Mean score of 4.30 which indicated the overall acceptance that these were the causes of the difficulties first year students face in proof construction in set theory.

The result revealed that 13% of the lecturers said that the causes of the difficulties were as a result of tendency for most students to translate the mathematical concepts using the local language which in turn distort the reasoning while the remaining 87% highlighted the low foundation of mathematical proofs.

4.3 Rating the level of difficulties first year undergraduate students face in proof construction in set theory.

Data from the questionnaire on rating the level of difficulties first year Mathematics students at Kwame Nkrumah University face in proof construction in set theory was analysed.

The findings are shown in form of a bar chart in Figure 4.

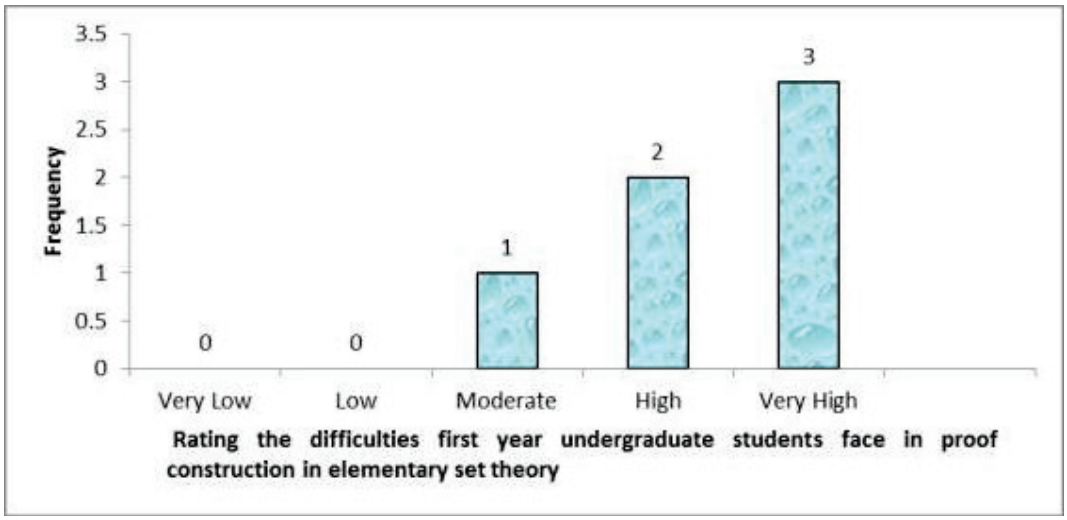

Figure 4: Bar chart of distribution of lecturers' responses to questionnaire on the rating of difficulties first year undergraduate students face in proof construction in set theory.

The findings in figure 4 shows that three (3) lecturer representing 50% rated the difficulties as very high, 2 (33%) said high. The remainder 17% rated the difficulties students face as moderate, while 0 (0%) was recorded for very low and low respectively. There was no doubt, that respondents totally admitted that the level of difficulties first year undergraduate students' face in proof construction in the area of set theory is still high.

4.4 Research Questions Three

What forms of guidance do Mathematics lecturers offer to first year Mathematics students as they engage in proof construction at Kwame Nkrumah University?

On the issue of forms of guidance Kwame Nkrumah University Mathematics lecturers offer to first year Mathematics students as they engage in proof construction, when the six (6) lecturers were asked to give any three (3) forms of guidance of their choice, they render to their students on proof construction in elementary set theory, the result of the eighteen (18) responses expected, only sixteen responses were given as follows; two (2) indicated that they were giving individual remedial work, all the six (6) said they conduct tutorials, three (3) said they provided reference books and five (5) said they encouraged collaboration amongst the students e.g. forming WhatsApp groups. But when asked how often they guide the first year undergraduate students in proof construction in the area of set theory, figure 5 is used to display the result of the statistical analysis of their responses.

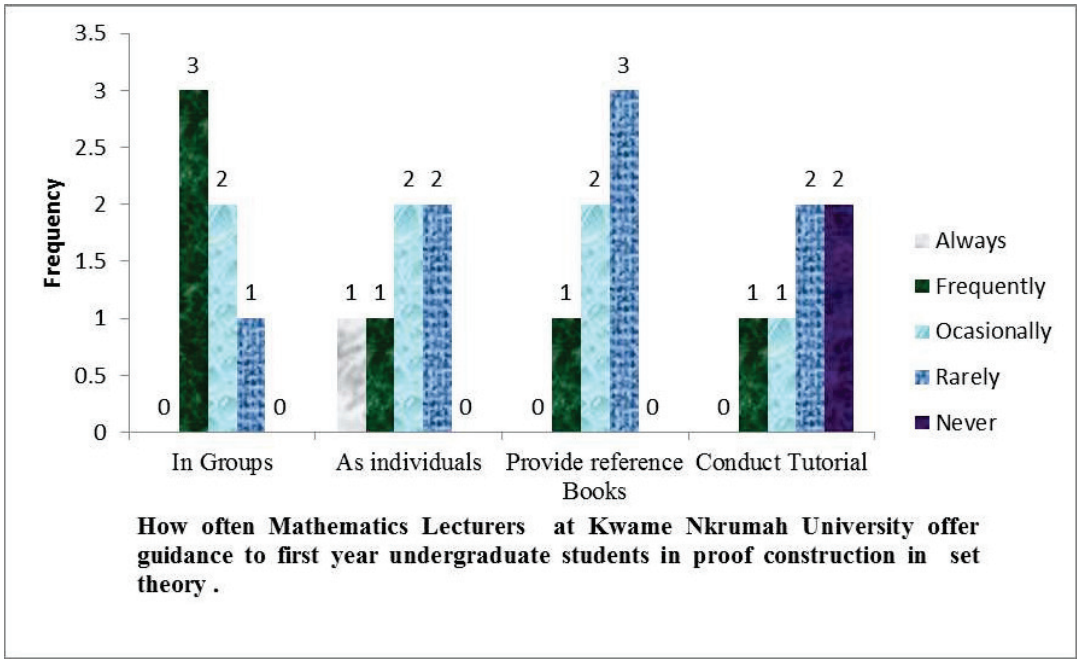

Figure 5: Bar chart showing the distribution of lecturers' responses on how often they guide students as they engage in proof construction in set theory.

From figure 5 above, we see that higher bars are on rarely and never when it comes to 21st century strategies i.e. guiding them as individuals, providing reference books and conducting tutorials. While teaching in groups is seen as the common method as always and frequently has higher bars.

5. Discussions and implications

5.1 Difficulties first year students face in relation to proof construction in set theory

Overall findings from the study showed that first year students had difficulties in construction of proofs in set theory. This was seen from the incorrect answers obtained in the test questions given where 45 students representing 67% failed question one, 35(52%) students failed question two and 40 students representing 60% failed question three. Among the main difficulties faced by first year Undergraduate students at Kwame Nkrumah in constructing proofs in set theory were;

1. Poor understanding of set concepts
2. Inability to grasp the mathematical language
3. Lack of real understanding of definition
4. Student's lack of giving clear conclusion i.e. if $A \subseteq B$ *and* $B \subseteq A$ Then $A = B$
5. Incorrect use of notation

These findings agree with what is documented in the available literature like (Mujib, 2015; Shaker, 2015) where it had shown that most students have procedural and conceptual challenges. Further, the present researcher's view is in support of Mujib(2015) who stated that the main difficulties faced by students in proof construction include; inability to fully conceive the definition of concepts, difficulties in understanding concepts and inability to apply the concepts in problems , real understanding of definitions and use of mathematical notation.

Therefore, the results of this study are consistent with the results of previous studies on other subjects such as Knapp (2005) who concluded that students difficulties in constructing proof fit into two categories. First, students struggle with the logic, language and culture of the proof as determined by the community. Second, students lack the domain specific knowledge, such as definitions, theorems, heuristics and the ability to generate examples.

5.2 The roots (causes) of the difficulties faced by students in proof construction in set theory

The findings on the origin (causes) of the difficulties first year students' face in proof of set theory revealed that students face the difficulties in proof of set theory because of;

a) Lack of knowledge of logical reasoning process

b) Student's limited deductive reasoning abilities
c) Incorrect use of terms, symbols and signs
d) Mathematics done at secondary school is not sufficient pre requisite for proof construction at University level
e) Inadequate material on proofs in elementary set theory
f) Poor pedagogy by lecturers
g) Tendency by most students to translate the mathematical concepts using their local language which in turn distort the reasoning

Baştürk (2010), also reached similar results, and pointed out that the main reasons for difficulties prospective teachers have in mathematical proving is the difference between high school and university mathematics education. Secondly, teacher's negative skills such as frightening students by exaggerating the difficulty of mathematics at University and getting angry with students' questions were cited as the source of difficulty. Thirdly, teacher's insistence in wanting proof of theorems be exactly like they did or dictated in classroom. In short, teachers insist that students should prove theorems like great mathematician as if there was no other way. In addition, the findings reported that students failed to construct proof because they lacked logic and reasoning abilities in problem solving or argument construction.

5.3 Form of guidance lecturers' offer to first year students in proof construction

Though lecturers claimed to have been providing guidance in form of giving reference books to the students, conducting of tutorials, encouraging of collaboration among students and providing individual remedial to students, the findings revealed that very little is done in incorporating the 21st century teaching and learning strategies.

The study further found that first year mathematics student experienced little instruction that focuses on proof in set theory. It was evident from the findings little guidance is provided to the student as they engage in proof in set theory and that the kind of instruction they received marginalize their participation in constructing of proofs in set theory. Hence there is need to raise the instructional challenge to build a classroom where students can be actively engaged in learning about proof as a way to potentially alter a negative, passive stance towards it. Valerio (2012) suggests that a classroom with passive learning environment implicitly reinforce low performance on proof activity in particular and mathematics activities. Therefore, there is need to call for supportive learning environment.

The creation of a supportive learning environment can assist in the development of successful learners in the classroom of proof construction, where students want to learn for the enjoyment of learning, a hub of intrinsic motivation. A supportive environment necessarily involves teachers having high expectations for students' individual learning abilities (Hinde-McLeod & Reynolds, 2007). This entails ensuring learning outcomes fit within a learner's Zone of Proximal Development (ZPD) (Vygotsky, 1978), meaning that teachers need to provide tasks that are challenging yet, through the mediation of quality support, are achievable.

A supportive learning environment is free from discrimination and based on mutual respect, involving the social support of teachers as well as fellow class members (Hinde-McLeod & Reynolds, 2007). It is often found in classrooms that students are hesitant to participate in classroom discussions, for fear of giving incorrect answers and/or being teased by fellow classmates. Lecturers utilising group activities can assist students in the development of social support skills in the classroom (Hinde-McLeod & Reynolds, 2007), through allowing students to understand that it is 'OK' to make mistakes as that is how new learning takes place. The environment is of utmost importance, as it is here that the majority of knowledge is generated and internalised.

6. Conclusion

The results of this study have revealed that most of the first year undergraduate students at Kwame Nkrumah University had difficulties with constructing of proofs in set theory was very low. This could be the reason why first year students in Zambian universities have poor results in Mathematical courses. This can be confirmed by the number of students who fail to proceed to year two. Yopp, 2011, basing his analysis on Bell (1976) and De Villiers (1999) observed that if prospective teachers (students) do not have this wide range of understanding of the functions and purpose of mathematical proof, how likely are they to incorporate them in teaching, learning, and assessment activities? Wu (1996) posited that "logical deduction — proof — is the backbone of mathematics. If we are serious about mathematics education, we should aspire to making every high school student learn what a proof is".

7. Reference

Baştürk, S. (2010). First-year secondary school mathematics students' conceptions mathematical proofs and proving. *Educational Studies*,*36*(3), 283-298. http://dx.doi.org/10.1080/03055690903424964.

Bell, A. (1976). A study of pupils' proof - explanations in mathematical situation. *Educational Studies in Mathematics*, *7*, 23-40. http://dx.doi.org/10.1007/BF00144356

Bryman, A.(2006). Integrating quantitative and qualitative research: How is it done? Qualitative Research, **6**, 97 – 113.

Brook, S.E.(2013).Selecting a sample. Educational Research, 250.

De Villiers, M. (1999). *Rethinking proof with the geometers' sketchpad*. Emeryville, CA: Key Curriculum. http://dx.doi.org/10.1080/00207390120360

Güler, G. (2013). *Investigation of pre-service mathematics teachers' proof processes in the learning domain of algebra* (Unpublished Doctoral Dissertation). University of Atatürk, Erzurum, Turkey.

Güler, G., & Dikici, R. (2012).Secondary pre-service mathematics teachers' views about mathematical proof. *Kastamonu University Journal of Education*, ***20***(2), 571-590.

Güler, G., & Dikici, R. (2014). Examining prospective mathematics teachers' proof processes for algebraic concepts. *International Journal of Mathematical Education in Science and Technology*, ***45***(4), 475-497. http://dx.doi.org/10.1080/0020739X.2013.837528.

Güler, G., Özdemir, E., & Dikici, R. (2012). Pre-service teachers' proving skills using mathematical induction and their views on mathematical proving. *Kastamonu University Journal of Education*, ***20***(1), 219-236.

Guner, P. & Topan, B. (2016). Prospective Elementary Mathematics Teachers' abilities of using Geometric proofs in the Teaching of Triangle. A journal of Science and Mathematics Education, **10**(2), 210 – 242.

Hausfather, S. (2001). Where is the content? The role of content in constructivist education. Educational Horizons, **80** (1), 15-19.

Hinde-McLeod, J. & Reynolds, R. (2007). *Quality Teaching for Quality Learning: Planning through reflection.* South Melbourne: Cengage Learning.

Imamoglu,Y. & Togrol, A.Y. (2015). Proof construction and evaluation practices of prospective Mathematics educators. *European Journal of Sciences and Mathematics Education*, **3** (1),130 – 144.

Knapp, J. (2005). *Learning to prove in order to prove to learn*. Retrieved from http://mathpost.asu.edu/~sjgm/issues/2005_spring/SJGM_knapp.pdf.

Ko,Y. and Knuth, E.K. (2013). Validating proofs and counterexamples across content domains: Practices of importance for Mathematics major, Journal of Mathematics Behaviour, **32**(1), 20 – 35

Kombo, D.K. & Tromp, L.A. (2006). Proposal and Thesis writing: An introduction. Nairobi: Paulines Publication Africa.

Mukuka, A. & Shumba, O. (2016). Zambian University Student Teacher' Conceptions of Algebraic Proofs, Journal of Education & Practice **7**(30).

Mujib, A. (2015). Analysis of students' difficulties in constructing Mathematical proofs on Discrete Mathematics course. A proceeding of International Seminar on Mathematics, Science and Computer Science Education.

Niglas, K. (2004) The Combined Use of Qualitative and Quantitative Methods in Educational Research. Tallinn, Estonia: Tallinn Pedagogical University Dissertation on Social Sciences.

Orodho, J. (2009). Techniques for Data Analysis using Statistical Package for Social Sciences (SPSS) Computer package. Kanezja publisher :Maseno.

Rav, Y. (1999). Why do we prove theorems? *Philosophia Mathematica*, **7**(1), 5-41. http://dx.doi.org/10.1093/philmat/7.1.5.

Sari, M. (2017). Undergraduate students' difficulties with Mathematical proof and proof teaching. Hecettepe University ,Turkey.

Shaker, H. (2015). An exploration of first year university students' constructions of proofs in Elementary Set Theory. (www.wits.ac.za/library).

Valerio, K. (2012).Intrinsic motivaiton in the classroom, *Journal of Student Engagement: Education Matters*, **2**(1), 30-35

Vygotsky, L. (1978*). Mind in Society: The Development of Higher Psychological Processes.* Cambridge, MA: Harvard University Press.

Yopp, D.A. (2011). How some research mathematicians and statisticians use proof in undergraduate mathematics. *Journal of Mathematical Behaviour,* **30**(2011), 115-130.

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