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The Return on Educational Investment in the Face of Depreciation: A New Formulation and Tries to Apply on Individual Data

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Abstract
In this article, we propose to re-examine the relationship between training-returns into studying the factors that influence individual decisions. Previously, it was mainly the expected returns that encourage the individual to rationally make the investment decision.However, this relationship between training and expected earnings is not so obvious. This depends, among other things, on the effect of the depreciation of the stock of acquired human capital, which we propose a new formalization. Let us thus detect a relation which shows that the variation of the net salary is positive as long as the variation of the gross salary is positive. In other words, the net investment will only be positive if the depreciation does not exceed the gross investment.If applicable, the effect of the depreciation will lead to a reduction in the wage gap between skilled and unskilled workers. This result means that even when returns are different, wage convergence may still exist. The results of this study are applicable in terms of budget and forecast of funds intended for training, and to trace the policies of labor market regulation and the problem of unemployment of graduates.
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1. Introduction

Investing in human capital remains one of the main themes of economic policy today in both developed and developing countries.

The consequences of education are disparate, microeconomic and macroeconomic, equally upstream and downstream of productive activity.

The central assumption of all human capital models is that education is recognized as the essential determinant of the structure and evolution of individual incomes. The idea is that individuals, by forgoing an immediate gain that they would perceive if they entered the labor market immediately, hope to increase their productive capacities and the market value associated with it.

In the earliest versions of the human capital model, the worker is assumed to maximize his income and education is considered an investment, just like physical capital or financial capital. The objective of the individual is then to define an optimal path for the accumulation of human capital by controlling his investment in training and taking into account its effects on present and future earnings. Educational strategies are then investment strategies defined by comparing marginal returns on human capital and financial capital.

This trade-off between present and future gains is the basic principle behind the behavior of the individual over his life cycle. And the positive effect of education on wages is arguably one of the most robust empirical results in economics.

The formulation of the earnings function of Mincer (1961) is the starting point of a vast literature devoted to the evaluation of the returns to education, and which remains until now the most solid modeling. The other advances have in turn enabled them to better understand how the present and past environment of an individual affected the returns to education and the differences in the educational choice between individuals, synthesized in particular by Card (2001).

The study of depreciation is particularly one of the most important tracks for better understanding, managing, and optimizing educational investment. Human capital - and its depreciation - is not exclusively about what is produced on the market, it is spread over what produces, what people want and prefer.

The peeling of the depreciation effect of human capital is necessary from a political and social point of view, in the sense that it should guide the directives answering decisive questions, in particular, the optimal duration of education for people, the training standards that should be promoted, the time and duration of post-school training, and the appropriate age to retire.

Our study will be structured as follows. First, we present a review of the literature: Roots, conceptions, and elements of analysis of human capital, then, we extend to the study of the relationship between gain and investment in training with depreciation effect which we Let us present a new formulation, which will be exploited empirically, finally, an estimation of the equations will be advanced.

2. Review of the literature: roots, designs and elements of analysis of human capital

The study of the theme of human capital (C.H) in a competitive market has long been a solid basis for analysis of the economics of education. This study is based essentially on the games of supply and demand. The main aim was to establish the role of education and to assimilate it as a factor generating production. Since then, training is no longer seen as a simple act of consumption, but a profitable investment.

Public and private spending is indeed consciously supported to acquire a stock of productive human capital embodied.

2.1. Origins and basic concepts

The first attempts to defend the investment-education link were adopted by A. Smith (1776), who asserts that training is an investment and that qualification is an asset.

A. Smith states that:

"*A man who has spent a lot of time and work to make himself fit for a profession that requires extraordinary skill and experience can be compared to one of these expensive machines. We must hope that the function for which he is preparing will make him, other than the wages of simple labor, enough to compensate him for all the costs of his education, with at least the ordinary profits of capital of the same value*".

And it is from this reasoning that intermediation is revealed, through which we hope for an income, which gives birth to the first beginnings of the fulfillment of the act of educational investment.

J. S MILL (1848) formulates in the same order as:

"*It can be said without hesitation that the goal of all intellectual training for the mass of people should be to cultivate common sense, to make them capable of making sound and practical judgment of the circumstances surrounding them*".

The originality of MILL's analysis lies in the formulation of the distribution theory, where it explains the differences in wages by the artificial scarcity of certain categories of work, which leaves their valuation on the market above the level balance.

L. Walras (1874) emphasizes the differentiation between the concept of capital (something that can be used more than once) and its service. And it introduces individual aptitudes into the price determination process, and from there replaces the thesis that wages are formed by the play of supply and demand on the labor market, according to marginal productivity of work, which is organically linked to the level of training.

This conception becomes clearer with the analysis of I. Fisher (1911): "man invests in his own person in the hope that the sums thus invested will eventually be reimbursed to him with interest". This directs thinking to the study of values and accumulations, and it is precisely these two concepts (value and accumulation) that will drive most of the studies that come.

However, the most promising study is that presented by G. S. Becker (1964) where he was able to construct a model showing the relationship between earnings, training costs and rates of return.

In this sense, Becker defines the C.H as the set of skills, knowledge, and qualifications which make it possible to improve the future situation of man.

It is therefore considered as an investment in C.H, any act intended to improve future income by bearing a cost. A distinction is then made between two periods: a cost period and a recovery period for the expected benefits which largely depend on the costs already spent.

Becker:

"*Capital depreciates as it can increase thanks to sustained efforts in terms of time and resources, this capital thus formed can fulfill two functions, the first, as an investment, it provides monetary income (producer), the second, it provides psychic income (consumer)*".

So, at equilibrium, the remuneration will depend on these investments which require the mobilization of scarce resources and their yields.

The first attempts to optimize the returns on these investments, as well as the allocation of resources was based on the benefit-cost analysis and the results are presented in terms of equality between costs and discounted marginal returns, its 'therefore establishes an effective rate of return.

Moreover, research advanced by Becker (1961) in the USA has shown the same rate of return on investment in training as in material capital, with the same duration and the same risk.

More recently, analyzes have turned to individual returns, where we can cite J. Mincer (1974) who explained the personal distribution of earnings by the number of years of study.

On the other hand, the depreciation of cognitive skills over the life cycle presents itself as a stumbling block when it comes to examining the profitability of education.

Measures of skill obsolescence are linked to both the cause of obsolescence and how it manifests. It turns out, that the depreciation is slower for more generic skills, knowledge, and personal abilities and it is faster for specific skills.

Despite the relevance of the context of the depreciation of human capital, this factor has been the subject of few empirical studies; this probably comes from the fact that it is not estimable within the framework of the simple model of Mincer (1974). Some authors such as Neuman & Weiss (1995) extended the equation advanced by Mincer to construct a model allowing the estimation of the depreciation rate under certain assumptions. There is also a new trend in the literature, illustrated by other works such as Groot (1998), Arrazola & de Hevia (2004), Arrazola, de Hevia, Risueno & Sanz (2005) and Wu (2007). Their methodology is however very indirect, the estimate of the depreciation resulting from the combination of several parameters in addition to a set of strong assumptions.

The literature that we consulted did not, however, reveal any typology of the consequences of depreciation or the associated models explaining the mutual relationship between the possible effects (Jones, Chonko & Roberts, 2004; Neuman & Weiss, 1995; Van Loo, 2005; Shearer & Steger, 1975; Thijssen, 2005; Lien Laureys, 2014; Brodaty TO, Gary-Bobo RJ and Prieto A, 2014; Zafar Nazarov, Nodir Adilov & Heather LR, 2018).

2.2. Analysis of investment decisions

The decision to invest individually in education is the most influential and decisive investment because it will mark our whole life, but also the most cabalistic, which is why economists have tried to decipher these symbols and limit the ambiguity that surrounds it.

In this regard, we will present and extend the model of Ben Porath, which explains a production function of human capital, and then we seek to determine the equilibrium as well as its implications for given price conditions, under a certain number of assumptions[1].

We start by detecting the investment cost which has two components:

- Direct costs "PD_t" which corresponds to the purchase of goods and services "D" at price "P".
- Indirect costs or opportunity costs which correspond to the shortfall: "$a_0 s_t K_t$" where "a_0" is the remuneration of one unit of capital, and "s_t" the fraction of the stock of CH "K_t" allocated to the production of human capital by the individual rather than at work ($0 \leq s_t \leq 1$).

=> $C_t = a_0 s_t K_t + PD_t$.

Since, the maximum salary is "$E_t = a_0 K_t$", and the available salary will be:

$Y_t = E_t - C_t$ if part of his time is devoted to the production of C.H.

There are therefore two factors that contribute to the production of C.H which are time and other goods and services. These conditions are represented by a production function[2]:

$$Q_t = \beta_0 (s_t K_t)^{\beta_1} \times D_t^{\beta_2}$$

Where "Q_t" is the volume of human capital produced during the "t" period.

β_0, β_1, β_2 are positive parameters that reflect the capacities of the individual as well as the institutional conditions, with $\beta_1 + \beta_2 < 1$.

The change rate of the C.H stock is arranged by the depreciation rate:

=> $\delta K_t/\delta t = Q_t - \gamma K_t$.

The individual finds himself in a situation of a competitive entrepreneur who decides his output according to the prices of his factors, and his production function at prices given by the market. The process can be written as follows:

$Min\ C_t = a_0 s_t K_t + PD_t$ with respect to s_t and D_t with $0 < s_t < 1$.

$S/C\ Q_t = \beta_0 (s_t K_t)^{\beta_1} \times D_t^{\beta_2}$

The equilibrium condition gives:

$a_0 s_t K_t / PD_t = \beta_1 / \beta_2$.

From this relation, we can extract the function of cost (minimum) according to the production "Q_t".

$$C_t = \frac{\beta_1 \beta_2}{\beta_1} a_0 \left(\frac{\beta_1 P}{\beta_2 a_0}\right)^{\frac{\beta_2}{\beta_1+\beta_2}} \left(\frac{Q_t}{\beta_0}\right)^{\frac{1}{\beta_1+\beta_2}}$$

And as a result, we can deduce the marginal cost function:

$$Cm = \frac{\partial C_t}{\partial Q_t} = \frac{a_0}{\beta_0 \beta_1} \left(\frac{\beta_1 P}{\beta_2 a_0}\right)^{\frac{\beta_2}{\beta_1+\beta_2}} \left(\frac{Q_t}{\beta_0}\right)^{\left(\frac{1}{\beta_1+\beta_2}\right)-1}$$

Thus, the marginal cost is increasing according to the production of C.H since:

[1] 1. The stock of human capital (k) is homogeneous and depreciates at a rate « γ ».
2. A loan and a loan may be made at a constant interest rate « r ».
3. The utility function is devoid of the time factor and the stock of human capital.
4. The individual decides first the optimal amount of investment, then the consumption.

[2] All the demonstrations that come are own to the author.

$(1/\beta_1 + \beta_2)-1 > 0$.

The determination of the optimal quantity "Q_t" to be produced goes through the equalization of the "Cm" to "P_t". With "P_t" expresses the maximum price that the individual is willing to pay to acquire an additional unit of CH under the conditions of the depreciation « γ », the economic horizon *(T-t)* and the gross remuneration "a_0", for an interest rate « *r* ».

$$P_t = a_0 \int_0^{T-t} e^{-(r+\gamma)x} dx = \frac{a_0}{r+\gamma}[1 - e^{-(r+\gamma)(T-t)}]$$

With "P_t" " is a decreasing function with the variable time, given the limited lifetime. We are now able to determine the optimal amount of C.H to produce:

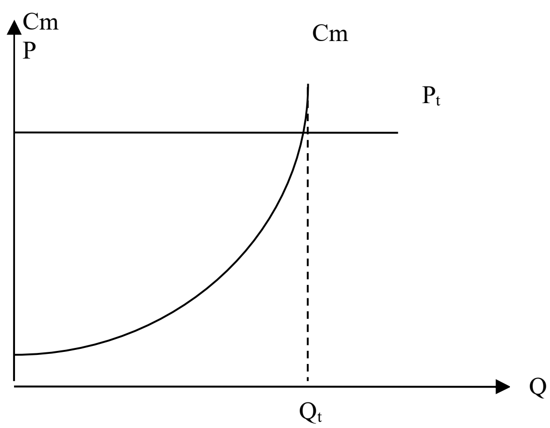
$$Q_t = \beta_0 (\frac{\beta_0 \beta_1}{r+\gamma})^{\frac{\beta_1+\beta_2}{1-\beta_1-\beta_2}} (\frac{\beta_2 a_0}{\beta_1 P})^{\frac{\beta_2}{1-\beta_1-\beta_2}} [1 - e^{-(r+\gamma)(T-t)}]^{\frac{\beta_1+\beta_2}{1-\beta_1-\beta_2}}$$

$$= M[1 - e^{-(r+\gamma)(T-t)}]^N \geq 0$$

(M a constant)

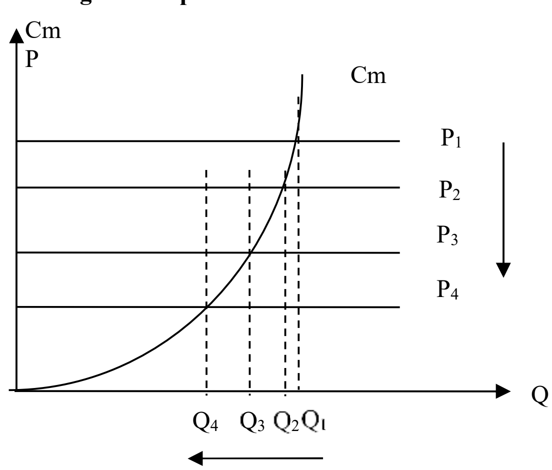
The figure below is a representation of the optimal amount of investment for a single period.

Figure 1. Optimal amount of investment for a period



To analyze the phenomenon for successive periods, it should be noted that the marginal cost (Cm) is independent of time and that the price of the demand "P_t" decreases with time, we will have the following figure:

Figure 2. Optimal Investment Amounts for Multiple Periods



This shows that the optimal amount to invest decreases as one spreads over time, moreover:

$$\frac{\partial Q_t}{\partial t} = MN[1 - e^{-(r+\gamma)(T-t)}]^{N-1}[-(r+\gamma)e^{-(r+\gamma)(T-t)}] \leq 0 \qquad (N \text{ a constant}).$$

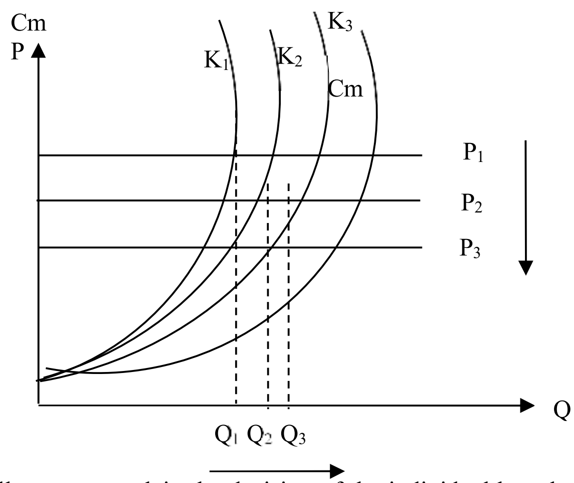
It is now necessary to study the two extreme cases:

- If $s_t = 0$: there is no production of C.H.
- If $s_t = 1$: the individual is totally devoted to the production of C.H, which is compatible with young

people who have not yet entered the labor market where their economic horizon is still wide. Thus, their "P_t" demand prices are high; however their C.H stock still low thus constitutes a constraint, and as this stock increases the constraint relaxes.

Moreover, Becker believes that the investor, in a first phase specializes in the production of C.H when young, so the optimal quantities invested in C.H increase. In a second phase, the time will be shared between work and investment in C.H, the latter is gradually declined throughout of life.

Figure 3. Optimal amount of investment in the specialization period



Thus, the analysis does not only allow us to explain the decision of the individual but also allows us to clearly define the demand function, which is a decreasing function with the interest rate:

$$\frac{\partial Q}{\partial r} = N[1-e^{-(r+\gamma)(T-t)}]^{N-1}[(T-t)e^{-(r+\gamma)(T-t)}]\beta_0(\frac{\beta_1+\beta_2}{1-\beta_1-\beta_2})(\frac{\beta_0\beta_1}{r+\gamma})^{\frac{\beta_1+\beta_2}{1-\beta_1-\beta_2}-1}(-\frac{1}{(r+\gamma)^2}) \leq 0$$

This means that investment in training is just like any other type of investment that is going in the opposite direction with the interest rate and increasing it also penalizes investment in training.

3. Relationship between gain and investment in training with depreciation effect

3.1. Model exposure[1]

To calculate the gross salary « E », account must be taken not only of the gross changes due to the investment but also of the net changes which are due to the depreciation of the $C.H$.

$E_t = E_{t-1} + (C^*_{t-1} - \gamma_{t-1} H_{t-1}) r_{t-1}$ where γ is the depreciation rate.

We know that:

$E_T = r_t H_t$

$$\Rightarrow \quad E_t = E_{t-1} + r_{t-1}(C^*_{t-1} - \gamma_{t-1}\frac{E_{t-1}}{r_{t-1}})$$

$$= E_{t-1} + r_{t-1}C^*_{t-1} - \gamma_{t-1}E_{t-1}$$

Because:

$k_t^* = (C_t^*/E_t)^2$

$$\Rightarrow \quad \frac{E_t}{E_{t-1}} = 1 + r_{t-1}k^*_{t-1} - \gamma_{t-1}$$

$$= 1 + r_{t-1}(k^*_{t-1} - \frac{\gamma_{t-1}}{r_{t-1}})$$

Similarly, we know that:

[1] The formulation presented uses the basic relationships introduced by Becker and Chiswick (1966) and Mincer (1974).

[2] K_t* is the fraction of the gross salary allocated to gross investment.

$$k_t = \frac{C_t}{E_t} = \frac{C_t^* - \gamma_t H_t}{E_t} = \frac{C_t^*}{E_t} - \frac{\gamma_t H_t}{E_t}$$

$$= k_t^* - \frac{\gamma_t H_t}{r_t H_t} = k_t^* - \frac{\gamma_t}{r_t}$$

We will have:

$$\frac{E_t}{E_{t-1}} = 1 + r_{t-1} k_{t-1} \quad \rightarrow \quad LogE_t = LogE_{t-1} + Log(1 + r_{t-1} k_{t-1}^* - \gamma_{t-1})$$

By recurrence:

$$\rightarrow \quad LogE_t = LogE_0 + \sum_0^{t-1} Log(1 + r_t k_t^* - \gamma_t)$$

And with the approximation:

$$\rightarrow \quad Log(1 + r_t k_t^* - \gamma_t) = r_t k_t^* - \gamma_t$$

We can write that: $LogE_t = LogE_0 + \sum_0^{t-1} (r_t k_t^* - \gamma_t) + \mu_t$ [1]

And the net salary: $LogY_t = LogE_t + Log(1 - k_t^*)$

Similarly, we can distinguish between school and professional investments:

$$\rightarrow \quad \begin{aligned} LogE_t &= LogE_0 + \sum_{i=0}^{s} (r_i - \gamma_i) + \sum_{i=m}^{t-1} (r_i k_i^* - \gamma_i) + v_t \\ &= LogE_0 + (r_s - \gamma)s + \sum_{i=m}^{t-1} (r_i k_i^* - \gamma_i) + v_t \end{aligned}$$

Where « *m* » is the moment of entry into working life.

3.2. Implications of the model

We know that the potential salary of an individual is:

$$E_{it} = E_{0i} + \sum_0^{t-1} r_{ij} C_{ij}$$

Following the preceding analysis, we obtain two great results: in the first phase, at the beginning of life, the individual specializes in the production of CH and in a second phase, he divides his time between work and training, but the latter is decreasing with age.

We can also write that: $E_t = E_{t-1} + rC_{t-1}$ since we are interested in one person.

$\rightarrow \Delta E_t = rC_t > 0$

But we know that in the second phase:

$C_{t-1} > C_t$

$\rightarrow C_t - C_{t-1} < 0 \quad \rightarrow \Delta^2 E_t = r\Delta C_t < 0 \rightarrow$ the gross salary profile will be concave.

For the net salary:

$$Y_t = E_t - C_t = E_0 + r \sum_{j=0}^{t-1} C_j - C_t$$

$\rightarrow \Delta Y_t = rC_t + (C_t - C_{t+1}) > 0$ [2] et $\Delta^2 Y_t = r\Delta C_t - \Delta^2 C_t < 0$ [3]

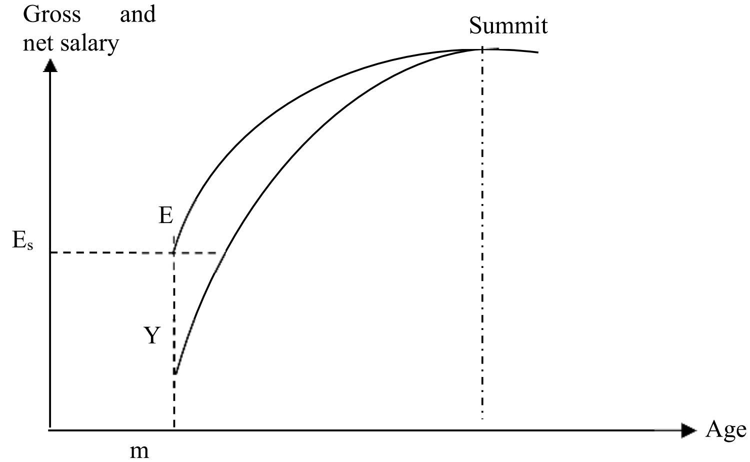
This implies that gross pay is an increasing function of the amount of stock acquired during the domestic phase. But this growth as well, as the net investment, is at a decreasing rate. This should lead to show a concavity of profiles. In addition, a peak will be reached at the end of the investment period. We can, therefore, represent gross and net salary profiles as follows.

[1] This equation can be deduced directly from the previous analyzes.

[2] $\Delta Y_t = \Delta E_t + (C_t - C_{t+1}) > 0$: therefore the slope of the net salary curve is higher than that of the gross wage.

[3] $\Delta^2 C_t$ is supposed to be very small $\rightarrow |r\Delta C_t| > |\Delta^2 C_t|$.

Figure 4: Gross and net salary profiles



The same results will be found if one thinks in terms of profiles of the logarithm of the gross wage and the logarithm of the net salary.

The introduction of the effect of depreciation leads us to reason in terms of net investment (C_t) instead of gross investment (C^*_t) with $C_t = C^*_t - \gamma H_t$ therefore, the evolution salary will depend on the latter.

Indeed, the expression $\Delta E_t = rC_t = r(C^*_t - \gamma H_t)$ shows that gross wage growth is ensured, provided that the gross investment exceeds the loss due, this time to depreciation.

The growth of the wage is not ordered only by the investment, but also by the depreciation, and when "$C^*_t = \gamma H_t$" the gross salary reaches a peak and beyond, it will decrease because we know, d on the one hand, that "C^*_t" " decreases over time, on the other hand, that the depreciation increases with time since it is proportional to the size of the stock. Concerning the net salary profile, we know that:

$Y_t = E_t - C^*_t$

And

$\Delta Y_t = rC_t + (C^*_t - C^*_{t+1}) = r(C^*_t - \gamma H_t) + (C^*_t - C^*_{t+1}) = \Delta E_t + (C^*_t - C^*_{t+1})$.

This is a relationship that shows that the change in net pay is positive as long as the change in gross wage is positive because we know that $(C^*_t - C^*_{t+1}) > 0$In d other words, the net investment is positive, which requires that the depreciation does not exceed the gross investment. First, the gross wage peaks before the net wage, which will peak at a later date, when $\Delta Y_t = 0$, then decline.

Another study seems still necessary; it is that which concerns the comparison of the profiles of the wages between individuals who have distinct behaviors with regard to the investment in CH This study was advanced by J.Mincer (1974) Groot (1998), Neuman & Weis (1995), De Grip et al. (2002), Van Loo (2005), Lien Laureys (2014). We will study its principles, as well as another avenue of analysis that focuses on the depreciation effect.

3.3. Comparison of salary profiles for different investments

We will begin with the analysis of gross wages, where we know that this wage is all the higher as the previous stock of capital will be high but with greater age. In the first stage, it is assumed that the rate of return and depreciation are the same for all individuals, as well as for gross investment after entry into the labor force.

We hope to look for the shape of the curve of two individuals who do not have the same initial investment ($E_2 > E_1$), we will take the age as a base[1].

$\Delta E_1 = rC_1 = r(C^* - \gamma H_1)$

And

$\Delta E_2 = rC_2 = r(C^* - \gamma H_2)$

➔ $\Delta E_2 - \Delta E_1 = r\gamma (H_1 - H_2) < 0$ ➔ $\Delta E_2 < \Delta E_1$

So, we will observe a convergence of wage profiles. In this case, where after-school investments are the same, the effect of the depreciation will lead to the reduction of the wage gap between individuals.

For net wages, we already know that:

$\Delta Y_1 = rC_1 + (C^*_1 - C^*_2)$

$\Delta Y_2 = rC_2 + (C^*_2 - C^*_3)$

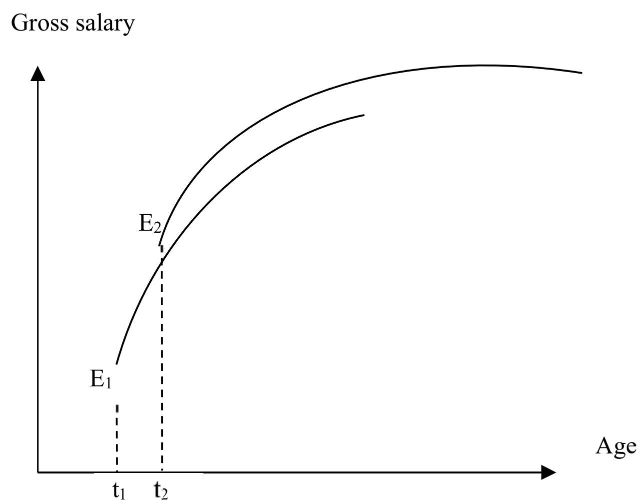
We know that gross investments are the same for individuals:

[1] It was possible to take as a basis the number of years of professional life, which gives the same results.

➔ $\Delta Y_2 - \Delta Y_1 = rC_2 - rC_1$
$= \Delta E_2 - \Delta E_1$
$= r\gamma(H_1 - H_2) < 0.$

This presages that there will also be convergence for net wages, which is ensured by the convergence of gross wages.

Figure 5: Age profiles salary



This convergence does not mean that wages will be equal from a certain point, but it only ensures a reduction in the size of the wage difference. This result seems to be in keeping with the reality, where we notice today that the gap between the wages of the most educated and the less educated tends to decrease.

The problem lies in the level of returns, where it is certain that the returns of investments of the most educated are always higher than that of the least educated. We will consider for this second analysis a rate "r_2" for the first, and "r_1" for the others, with $r_2 = \alpha r_1$ and $\alpha > 1$.

$$\Rightarrow \Delta E_{t2} - \Delta E_{t1} = r_2(C_t^* - \gamma H_{t2}) - r_1(C_t^* - \gamma H_{t1})$$
$$= C_t^*(r_2 - r_1) - \gamma(r_2 H_{t2} - r_1 H_{t1})$$

Therefore, the convergence of wages is again largely controlled by the depreciation; we must ensure "γ" very low to have $\Delta E_{t2} - \Delta E_{t1} > 0$, and therefore the divergence.

➔ $\Delta E_{t2} - \Delta E_{t1} = r_1[C_t^*(\alpha - 1) - \gamma(\alpha H_{t2} - H_{t1})] \succ 0$

If and only if:

$$\gamma \prec \frac{C_t^*(\alpha - 1)}{\alpha H_{t2} - H_{t1}}$$

Even when returns are different, convergence may exist, the same condition is found for net wages.

The same conclusions described above concerning to wage profiles will be true for the log wage profiles by referring, in the analysis, to the fraction "k^*_t".

It should be noted that a correlation can be found between investments during the period of specialization and subsequent investments, that is to say, when entering the working life, which is generally positive[1].

4. Empirical exploitation

The analysis will focus on the survey made by INS Tunisia in 2012, is composed of 63078 people. The average age of individuals is 37.25 years, with a median of 37. The average schooling is 11.48 years, the median is 13. The description of the salary variable shows that the average is 452d the median is 370. The Gini coefficient is 0.317. The experiment is of average 19.89 years, the median equal to 19, and his coefficient Gini equal to 0.35.

4.1. Specification of the equations to be estimated

The objective of this paragraph is to test the gain model which is limited only to school and professional investments, where we will specify two cases:

- The first case without depreciation:

[1] But, that can be as negative or zero.

$$LogE_t = LogE_0 + r_s S + r_p \sum_{i=m}^{t-1} k_i + u_t$$

$$Et\ \ LogY_t = LogE_0 + r_s S + r_p \sum_{i=m}^{t-1} k_i + Log(1-k_t) + v_t$$

- The second case with depreciation:

$$LogE_t = LogE_0 + (r_s - \delta)S + \sum_{i=m}^{t-1}(r_i k_i^* - \delta) + u_t^;$$

$$= LogE_0 + (r_s - \delta)S + r_p \sum_{i=m}^{t-1} k_i^* - \delta t + u_t^;$$

$$Et\ \ LogY_t = LogE_0 + (r_s - \delta)S + r_p \sum_{i=m}^{t-1} k_i^* - \delta t + Log(1-k_i^*) + v_t^;$$

With E_t: the gross salary, Y_t: the net salary, r_s: the average rate of return of educational investments, S: the number of years of schooling, r_p: the average rate of return of professional investments, k_t: the fraction of the salary earmarked for professional investment, δ: the average rate of depreciation of human capital, t: the number of years of working life k_t^* : ,: the fraction of the gross salary allocated to the investment gross, and T: the total length of the period of the net investment.

We will proceed to two regressions which describe the behavior of the individuals concerning the post-school investment the first is linear of the form:

$k_t = k_0 - \frac{k_0}{T} t$, where "$k_0$" is the fraction of the gross salary allocated to the investment in the first period of professional life.

The second one in the form: $k_t = k_0 e^{-\beta t}$, this expression explains the decay of the fraction of the salary allocated to the investment in an exponential way of parameter "β".

We will now introduce these forms into the previous model.

4.2. Specification without depreciation

a/ Returning to the first form, the model will be as follows:

$$LogE_t = LogE_0 + r_s S + r_p \sum_{i=m}^{t-1} (k_0 - \frac{k_0}{T} i) + u_t$$

$$= LogE_0 + r_s S + r_p k_0 t - \frac{r_p k_0}{2T} t^2 + u_t$$

$$Et\ \ LogY_t = LogE_0 + r_s S + r_p k_0 t - \frac{r_p k_0}{2T} t^2 + Log(1 - k_0 + \frac{k_0}{T} t) + v_t$$

Using *Taylor*'s quadratic approximation:

$Log(1 - k_0 + \frac{k_0}{T} t) = -(k_0 + \frac{k_0^2}{2}) + (\frac{k_0}{T} + \frac{k_0^2}{T})t - \frac{k_0^2}{2T^2} t^2$ We will have:

$$LogY_t = LogE_0 - k_0(1 + \frac{k_0}{2}) + r_s S + (r_p k_0 + \frac{k_0}{T} + \frac{k_0^2}{T})t - (\frac{r_p k_0}{2T} + \frac{k_0^2}{2T^2})t^2 + v_t \quad \textbf{(1)}$$

$\Rightarrow$ The logarithm of wages is a parabolic function of time devoted to professional life.

b/ The second form gives the following results:

$$LogE_t = LogE_0 + \frac{r_p k_0}{\beta} + r_s S - \frac{r_p k_0}{\beta} e^{-\beta t} + u_t$$

While the net pay will be of the form:

$LogY_t = LogE_0 + \frac{r_p k_0}{\beta} + r_s S - \frac{r_p k_0}{\beta} e^{-\beta t} + Log(1 - k_0 e^{-\beta t}) + u_t$ Using the same Taylor approximation, we will have:

$$LogY_t = LogE_0 + \frac{r_p k_0}{\beta} + r_s S - (\frac{r_p k_0}{\beta} + k_0)e^{-\beta t} - \frac{k_0^2}{2} e^{-2\beta t} + u_t \qquad \textbf{(2)}$$ (Function of *Gompertz*'s salary).

4.3. Specification with depreciation

If we take into account the depreciation effect the two forms will be as follows:

a/ $k_t^* = k_0^* - \frac{k_0^*}{T} t$

b/ $k_t^* = k_0^* e^{-\beta t}$

The introduction of the first form of investment allows writing:

$$LogE_t = LogE_0 + (r_s - \delta)S + (r_p k_0^* - \delta)t - \frac{r_p k_0^*}{2T^*} t^2 + u_t$$

And

$$LogY_t = LogE_0 - k_0^*(1 + \frac{k_0^*}{2}) + (r_s - \delta)S + (r_p k_0^* + \frac{k_0^*}{T^*} + \frac{k_0^{*2}}{T^*} - \delta)t - (\frac{r_p k_0^*}{2T^*} + \frac{k_0^{*2}}{2T^{*2}})t^2 + v_t \quad \textbf{(3)}$$

The second form of investment gives:

$$LogE_t = LogE_0 + \frac{r_p k_0^*}{\beta} + (r_s - \delta)S - \frac{r_p k_0^*}{\beta} e^{-\beta t} - \delta t + u_t$$ So the net salary will be:

$$LogY_t = LogE_0 + \frac{r_p k_0^*}{\beta} + (r_s - \delta)S - (\frac{r_p k_0^*}{\beta} + k_0^*)e^{-\beta t} - \frac{k_0^{*2}}{2} e^{-2\beta t} - \delta t + u_t \quad \textbf{(4)}$$

We will only use the functions that describe net pay for econometric analysis, namely the functions: ***(1)***, ***(2)***, ***(3)***, ***(4)***.

Models using the linear investment form (***(1)*** and ***(3)***) will be estimated by the equation:

$Log\ (Sal) = \alpha_0 + \alpha_1 EDU + \alpha_2 EXP + \alpha_3 EXP^2 + \varepsilon$

With, *EDU*: the number of years of study, *EXP*: the number of years of professional life, EXP^2: the square of *EXP*.
The coefficients represent in the specification without depreciation:

$\alpha_0 = LogE_0 - k_0(1 + \frac{k_0}{2})$, $\alpha_1 = r_s$, $\alpha_2 = (r_p k_0 + \frac{k_0}{T} + \frac{k_0^2}{T})$, $\alpha_3 = -(\frac{r_p k_0}{2T} + \frac{k_0^2}{2T^2})$

With depreciation we will have:

$\alpha_0 = LogE_0 - k_0^*(1 + \frac{k_0^*}{2})$, $\alpha_1 = r_s - \delta$, $\alpha_2 = (r_p k_0^* + \frac{k_0^*}{T^*} + \frac{k_0^{*2}}{T^*}) - \delta$,

$\alpha_3 = -(\frac{r_p k_0^*}{2T^*} + \frac{k_0^{*2}}{2T^{*2}})$.

Then, models using the exponential investment form (equation ***(2)***) will be estimated by the equation:

$Log\ (Sal) = \beta_0 + \beta_1 EDU + \beta_2 X + \beta_3 X^2 + v$

With $X = e^{-\beta t}$ et $X^2 = e^{-2\beta t}$, the coefficients will be interpreted as follows without depreciation:

$\beta_0 = LogE_0 + \frac{r_p k_0}{\beta}$, $\beta_1 = r_s$, $\beta_2 = -(\frac{r_p k_0}{\beta} + k_0)$, $\beta_3 = -\frac{k_0^2}{2}$.

Equation ***(4)*** will be estimated by the following representation:

$\beta_0 = LogE_0 + \frac{r_p k_0^*}{\beta}$, $\beta_1 = r_s - \delta$, $\beta_2 = -(\frac{r_p k_0^*}{\beta} + k_0^*)$, $\beta_3 = -\frac{k_0^{*2}}{2}$, $\beta_4 = -\delta$.

5. Estimation of equations

5.1. Parabolic shape

The estimation of the model gives:

$$Log(Sal) = 4.433 + \underset{(188.09)}{0.0834}\ EDU + \underset{(82.07)}{0.04109}\ EXP - \underset{(-45.05)}{0.000488}\ EXP^2 + u$$

$R^2 = 39.67\%$. (See Appendix *1*)

The results of the estimations show that all the variables of all the models are significant, and the insertion of the variable "*EXP*" and its square make it possible, on the one hand, to increase the explanatory power of the model (*39.67%*), on the other hand, to correct the average return on educational investment, from *5.98%* to *8.34%* partly eliminating the bias associated with the latter.

According to The model "2" (Appendix *2*), the formula of marginal returns to education is written in the form:

$$\frac{\partial Log(Sal)}{\partial EDU} = -0.0125 + 2(0.0047)EDU$$

This makes it possible to calculate the marginal returns of the different levels of training:

$EDU = 6$ (primary education) we will have r_s (the yield) $= 4.39\%$

$EDU = 9 \Rightarrow r_s = 7.22\%$,

$EDU = 13 \Rightarrow r_s = 10.99\%$,

$EDU = 17 \Rightarrow r_s = 14.75\%$.

The results show that there is an increase in marginal rates of return for all levels (almost two points for each level of training compared to the simple model).

As far as the profitability of professional investments is concerned, it is possible to calculate for different values of "*T*", according to model *2* :

For $T = 20 \Rightarrow k_0 = 43.1\%$ and $r_p = 2.38\%$,

$T = 25 \Rightarrow k_0 = 41.66\%$ and $r_p = 4.19\%$,

$T = 30 \Rightarrow k_0 = 35.33\%$ and $r_p = 7.11\%$.

The coefficient of the interaction variable "*EE*" is positive and significant, which reflects the complementarities between post-school training and formal education.

(See Appendix *3, model 3*).

Calculation of the cumulative duration of professional investments gives:

$M = \sum_{j=0}^{T} k_j = k_0 \frac{T}{2} \Rightarrow M = \frac{T^2}{2}\alpha_2 - T^3\alpha_3$, M is maximum for $T = \frac{\alpha_2}{3\alpha_3} = 28.03 \Rightarrow k_0 = 38.4\% \Rightarrow r_p = 5.76\%$ and $M = 5.38$.

The cumulative duration of investments on the job reaches its maximum for $T=28.03$; its time equivalent is noted for a value of *5.38*. If "*M*" is maximum, the part of the salary used in the professional investors will be *0.384*; the rate of return equivalent to this investment is *5.76%*.

5.2. Estimation of Gompertz's function

$$Log(Sal) = 5.372 + \underset{(194.25)}{0.0836}\,EDU - \underset{(-30.55)}{0.944}\,X - \underset{(-2.71)}{0.0832}\,X^2 + v$$

$R^2 = 39.88\%$. (See Appendix 4, model *4*).

It can be said that the return on schooling and the explanatory power of the model have not changed from the basic model that calls for parabolic regression.

The "β" chosen in this frame is equal to *0.058*.

We can calculate therefore, $k_0 = 40.8\%$ and $r_p = 7.62\%$. Therefore, we can say that this generation has benefited from the investment benefits of previous generations.

Model 5 (*Appendix 5*) which includes the variable "*EXP*" as an explanatory variable, can be used to detect the depreciation coefficient $\delta = 1.27\%$. This coefficient is negative and significant. So the phenomenon of the depreciation of human capital is justified.

The average return of gross education (without depreciation) is then:

$r_s = \beta_1 + \delta = 0.0836 + 0.01279 = 9.64\%$.

6. Conclusion

The purpose of this article is to establish a new relationship between training and wages in the context of human capital.

To make a presentation of the history of birth and development of this training-salary relationship in the context of CH theory was necessary; the study found that this relationship only started to be forged in the mid-fifties. However, one cannot neglect the ancient works which date back to the writing of "the wealth of the nations". It was, therefore, necessary to wait a long time for the conception of training as an investment, and the links between training and differences in wages, to take shape.

The idea was to start from a microeconomic analysis of individual behavior, arriving at an extrapolation which brought out the collective facts.

The internal rate of return to education was the driving force behind this analysis, which necessarily depended on the costs and expected income flows of education. An individual who is supposed to be rational must hope for gains that at least offset the costs so that he expects a return. It is precisely this expected return that stimulates the individual to make the investment decision. But also, the return to education is largely dependent on depreciation.

Depreciation can be seen as a decrease in human capital, human qualities that have not been sustained. In literature, various forms such as knowledge, skills, abilities, ideas are attributed to these human qualities. This can involve a variety of work-related qualities that can become obsolete (Fossum et al, 1986; Kaufman, 1995; Pazy, 1996; Shearer and Steger, 1975; Thijssen, 2005; Van Loo, 2005).

Our study of this individual behavior of investment in the framework of human capital has been able to highlight, on the one hand, that the growth of the gross salary is assured, provided that the gross investment exceeds the loss due to the depreciation, as well as distinguishing a correlation between these two investments, and on the other hand, providing a relationship that shows that the increase in net wages is positive as long as the increase in gross wages is positive, in other words, the net investment is positive provided that the depreciation does not exceed the gross investment.

The study of convergence shows that depreciation can be responsible for reducing the wage gap between individuals with initially different skills.

This allows the construction of a gain function which is ready for empirical analyzes and which reduces the biases which are due to depreciation.

The results of the estimates based on the survey supplied by the INS Tunisia and using the new specification show that there is an increase in marginal rates of return for all levels. Likewise, we demonstrate the complementarities between post-school training and formal education.

The cumulative duration of on-the-job investments reaches its maximum for a relatively long period.

Another explanatory variable was included to detect the depreciation coefficient of $\delta = 1.27\%$. This coefficient is negative and significant. So the fact of the depreciation of human capital is justified.

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Appendix

Appendix 1

Model 1: MCO, Using Observations 1-63078

Dependent variable: l_Sal

	Coefficient	Erreur Std	t Student	p. critical	
Const	4,43314	0,00803527	551,7098	<0,00001	***
EDU	0,0834285	0,000443548	188,0934	<0,00001	***
EXP	0,0410992	0,000500767	82,0726	<0,00001	***
sq_EXP	-0,000488668	1,08461e-05	-45,0549	<0,00001	***

Avg. var. dep.	5,942569	Sd. De, var. dep.	0,572421
Sum squares residues	12467,33	Sd. type of regression	0,444592
R2	0,396785	R2 adjusted	0,396757
F(3, 63074)	13829,70	p. critical (F)	0,000000
Log vraisemblance	-38370,87	Akaike Criterion	76749,74
Schwarz criterion	76785,94	Hannan-Quinn	76760,96

Appendix 2

Model 2: MCO, Using Observations 1-63078

Dependent variable: l_Sal

	Coefficient	Erreur Std	t Student	p. critical	
Const	4,77095	0,00961433	496,2335	<0,00001	***
EDU	-0,012513	0,00164856	-7,5903	<0,00001	***
EXP	0,0479056	0,000499846	95,8408	<0,00001	***
sq_EDU	0,00470899	7,8096e-05	60,2974	<0,00001	***
sq_EXP	-0,000669708	1,09655e-05	-61,0741	<0,00001	***

Avg. var. dep.	5,942569	Sd. De, var. dep.	0,572421
Sum squares residues	11787,83	Sd. type of regression	0,432310
R2	0,429662	R2 adjusted	0,429626
F(4, 63073)	11878,94	p. critical (F)	0,000000
Log vraisemblance	-36603,30	Akaike Criterion	73216,61
Schwarz criterion	73261,87	Hannan-Quinn	73230,63

Appendix 3
Model 3: MCO, Using Observations 1-63078
Dependent variable: l_Sal

	Coefficient	Erreur Std	t Student	p. critical	
Const	4,94456	0,0229813	215,1558	<0,00001	***
EDU	-0,0307382	0,0027418	-11,2109	<0,00001	***
EXP	0,0398128	0,00109387	36,3962	<0,00001	***
sq_EDU	0,00516331	9,52726e-05	54,1951	<0,00001	***
sq_EXP	-0,000583683	1,50703e-05	-38,7308	<0,00001	***
EE	0,00040202	4,83411e-05	8,3163	<0,00001	***

Avg. var. dep.	5,942569	Sd. De, var. dep.	0,572421
Sum squares residues	11774,92	Sd. type of regression	0,432077
R2	0,430287	R2 adjusted	0,430241
F(5, 63072)	9527,257	p. critical (F)	0,000000
Log vraisemblance	-36568,74	Akaike Criterion	73149,48
Schwarz criterion	73203,79	Hannan-Quinn	73166,31

Appendix 4
Model 4: MCO, Using Observations 1-63078
Dependent variable: l_Sal

Avg. var. dep.	5,942569	Sd. De, var. dep.	0,572421
Sum squares residues	12425,23	Sd. type of regression	0,443841
R2	0,398822	R2 adjusted	0,398793
F(3, 63074)	13947,78	p. critical (F)	0,000000
Log vraisemblance	-38264,20	Akaike Criterion	76536,40
Schwarz criterion	76572,61	Hannan-Quinn	76547,62

Appendix 5
Model 5: MCO, Using Observations 1-63078
Dependent variable: l_Sal

	Coefficient	Erreur Std	t Student	p. critical	
Const	5,37202	0,00617764	869,5901	<0,00001	***
EDU	0,0836318	0,000430533	194,2517	<0,00001	***
X	-0,94403	0,0308989	-30,5522	<0,00001	***
sq_X	-0,0832399	0,0306422	-2,7165	0,00660	***

Avg. var. dep.	5,942569	Sd. De, var. dep.	0,572421
Sum squares residues	11804,60	Sd. type of regression	0,432621
R2	0,428850	R2 adjusted	0,428805
F(5, 63072)	9471,584	p. critical (F)	0,000000
Log vraisemblance	-36648,14	Akaike Criterion	73308,28
Schwarz criterion	73362,59	Hannan-Quinn	73325,11