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The Teaching and Learning of Integer Operations: The Case of Number Rule and Conventional Method

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Abstract
This study looked at how to employ the Number Rule to help JHS students perform better when learning about integer addition and subtraction. To ascertain the impact of the Number Rule as an instructional aid in the teaching of addition and subtraction of integers on the performance of Junior High School students in mathematics, a mixed-method approach (mostly quasi-experimental and use of interviews) was used. It was simple to choose two schools. Both classes remained unaltered, with one serving as the control group (34 students) and the other (37 students) as the experimental group. The control group received instruction using the conventional approach, while the experimental group received instruction utilizing the Number Rule. With the help of a teacher-made achievement test with fifteen (15) essay-style questions, both groups were pre- and post-tested. According to the study's findings, pupils who were taught using the Number Rule approach did better on the post-test than those who were taught using the Conventional method. Additionally, the Number Rule method makes the lessons more time-efficient, simple to learn, engaging, and practical. As a result, it was suggested that teachers employ the number rule to teach mathematical concepts, particularly the addition and subtraction of integers. The study is significant to teachers and teacher trainees as the result depicts the effectiveness to the use of manipulative (number rule) in teaching and learning of integer additions and subtraction.
Keywords: Addition of Integers, Convention Method, Integers, Integer Operations, Manipulatives, Number Line, Number Rule, Operations on Integers, Subtraction of Integers, Whole Numbers
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Introduction
Giving students the chance to actively control specific components of the phenomenological world is one of the pedagogical strategies used in mathematics teaching (Heddens & Speer, 2008; Izydorczak, 2003; Moreno, 2005; NCTM, 2000; Olson, 1988). This method depends on carefully constructing the phenomena that best illustrate the mathematical idea being explained. In essence, these phenomena act as real-world examples of mathematical ideas and are referred to as modeling those ideas in mathematics education. "Manipulatives" are educational tools created especially for these active manipulations. This fundamental notion of manipulatives has been expanded to include computer-based manipulatives or "virtual manipulatives" with the development of digital technology (Schackow, 2007; Tversky & Morrison, 2002). According to Sobel and Mallet (1972), the set of integers is made up of whole numbers and their integer counterparts. Integers were viewed by Steel (1973) as whole numbers having direction, including up-down, east-west, below-above, loss-gain, and decrease-increase. Skyeem (1966), on the other hand, defined integers as the collection of natural numbers. According to Ellerby (1966), integers travel forward and backward. Integers are positive and negative whole numbers, including zero, according to Keedy and Bettinger (1981). According to Kwakye and colleagues, integers can also be called “Signed Numbers” (Kwakye, et al., 2022). On the number line, the negative numbers are displayed to the left of zero. Positive and negative numbers, as well as zero, are considered integers. The number line is a useful graphical tool used by Kauffmann (1989) to illustrate the idea of integers. To him, every whole number that is not zero can be related to its opposite, which is to the left of zero; for example, 1 can be related to -1, and so on. The set of integers, he said, is the set of whole numbers and their negative analogues. Integers are defined as the universal set for counting numbers, whole numbers, and negative numbers, for instance...-4,-3,-2,-1,0,1,2,3,4... which is denoted by Z, according to Abbiw, Adjei, Adu-Gyamfi, Amissah, Awuah, Dogbe, Eshun, Folson, Hutchful, and Minta (1994), Ghana Senior Secondary School Mathematics Book 1 pg.

Middle school children should be exposed to integers via the stock market, population growth or decline, and driving speed, according to Sheffield and Cruikshank (2005). They recommend a card game that students might play to get practice adding integers. The positive integers are represented by the black cards from a standard deck of cards (excluding the face cards), and the negative integers are represented by the red cards. Each player receives five cards that are dealt face down. Each player takes his hand of cards. The sums on the cards in each player's hand are added. The player takes one card from the player on his left after each turn. The objective is to acquire the highest absolute score possible. When a player scores 50 or more, the game is over.

The authors come to the conclusion that in order for students to become proficient in integer addition and subtraction, they must study problems that make sense to them and collaborate with their classmates to find solutions.

Students learnt about integers in the Algebra Project (Moses, Kamii, Swap, & Howard, 1989) by simulating a journey. The tour was expanded upon using four units. Students learned about direction, displacement, and equivalent in the first unit. Students developed a new definition of subtraction in the second unit by contrasting the ends of pairs of displacements. Students learned about relative coordinate systems in the third unit and connected them to integers as displacements with magnitude and direction. The concept of integer addition was connected by the pupils to combining displacements in the last unit. Dienes (2000) talked about integers in six steps. He thinks about a dance circumstance in one of the early stages. Students learnt about integers in the Algebra Project (Moses, Kamii, Swap, & Howard, 1989) by simulating a journey. The tour was expanded upon using four units. Students learned about direction, displacement, and equivalent in the first unit. Students developed a new definition of subtraction in the second unit by contrasting the ends of pairs of displacements. Students learned about relative coordinate systems in the third unit and connected them to integers as displacements with magnitude and direction. The concept of integer addition was connected by the pupils to combining displacements in the last unit. Dienes (2000) talked about integers in six steps. He thinks about a dance circumstance in one of the early stages.

In conclusion, the number line model and the neutralization model are the two sorts of models that are utilized to introduce integers. The neutralization model has four reportedly advantageous features. First, it is consistent with other academic disciplines. For instance, positive and negative charges are drawn to one another in science and balance one another out. Similar to the two-color counters where a black counter balances a red counter, this does the same thing. The second benefit comes from the mathematical soundness of representing integers as ordered pairs. In that one can determine if there are more positives or negatives, using two-color counters is similar to using ordered pairs as a representation. Third, the student's intuitive grasp of numbers is stimulated by the neutralization. Fourth, cardinality, which tends to grow before ordinality, is highlighted by the neutralization model (Wilkins, 1996). Students who use two-color counters can focus on the activities at hand rather than fretting about the placement of the integers on the number line.

The neutralization model's inability to simulate all varieties of integer multiplication and division issues is one of its main drawbacks. Additionally, operations involving rational numbers cannot be included in the application of the neutralization model. There are various benefits and drawbacks to the number line concept. Because the numbers on the right are larger than the numbers on the left, it is possible to discern order within the set of integers. The number line can be used to model the operations of addition, subtraction, multiplication, and division. Additionally, rational numbers can be used with number line operations. Uncertainty about how subtraction works is one of the number line model's drawbacks. Finding a missing addend was one approach studied for modeling subtraction, while moving backward on the number line was another. Students may find this confusing because, before learning about integers, they linked subtraction with "taking away," but with a number line they cannot take anything away. Some pupils find it challenging to figure out what they are counting on the number line. Instead, then counting the spaces that correspond to the units, some students count the hash marks designating the integers (Carr & Katterns, 1984).

Models Used to Understand Integers

There are numerous studies utilizing two models that are used to comprehend integers and the operations that may be done on them. The number line and neutralization models are these. The researcher will go over studies that employed these two sorts of models as well as variants of these two types in this part. The two types of models are equilibrium models and number line models, according to Janvier (1983). Subway stations that need to determine direction and weather issues that need to determine temperature are two examples of when the number line model is used. Since pupils are already aware with the concept of opposites represented by "east" and "west" and "above" and "below," these models enable one to apply more specific meaning to the numbers on the number line. Voting, dancing, and the game of scoring and forfeits are a few more examples of uses of the neutralization model that have been studied in the literature. Peled (1991) investigated two distinct number line conceptualizations. One type included a continuous number line with numbers arranged in ascending order, from smaller to larger. The second kind of number line had a divided number line that broke apart at zero. Efforts were made either in the direction of or away from zero. The pupil would choose the amount required to reach zero and proceed from there.

The neutralization model appears to be a superior fit for comprehending integers and integer operations, despite the fact that the number line is the model that most kids use when they are first introduced to integers (Davidson, 1987). Children comprehend cardinality, which deals with quantity or number of things, before ordinality, which is concerned with position (Davidson, 1987). Therefore, it would seem that more scholars ought to look into the neutralization model. A neutralization model has been used in a lot of investigations in recent years. The neutralizing method has been applied to hot air balloons, dance, attitude, and voting, among other things. These themes are well-known to the students, giving them the opportunity to apply prior knowledge to a new subject. Liebeck (1990) used the game "scores and forfeits" to introduce integers to third and fourth grade students. Each participant in this game had a maximum of five black counters—referred to as "scores"—and five red counters—referred to as "forfeits." Each student took turns drawing a card and used their red and black counters to follow the instructions on the card. One card said, "You won a treasure hunt, 3 scores," for instance. The student would lay out three black counters in this scenario. If the student's card showed that they owed money, they would set out the appropriate number of red counters. Students noted how far along they were at the conclusion of each turn. The use of numerical phrases to represent a move was encouraged among the students. When one player accumulated five red or five black counters, the game was ended. Students immediately understood that placing two red counters out had the same impact as taking two black counters away because they only had five of each sort of counter. Students were able to learn and remember how to add and subtract integers because they were able to relate everyday circumstances to positive and negative integers. It is simpler for pupils to associate scenarios in the game with situations involving positive and negative integers when the model is related to experiences, they are familiar with.

Integer knowledge was divided into four levels by Peled (1991). Simply extending the number line to include negative integers constitutes the first level. According to the neutralization model, the learner is introduced to negative integer amounts at this first stage, which are often unfavorable amounts. Two counting numbers are subtracted at the second level. When the subtrahend is greater than the minuend, the outcome is a negative integer. At this level, the same process as when counting numbers is utilized, but this time, one can go beyond zero and have a negative result. According to the neutralization approach, pupils are able to subtract a larger whole number from a smaller one at this level by taking what they can away and figuring out how much is still missing to produce a negative result. The third level makes use of the fact that, in some situations, adding and subtracting with integers and counting numbers are equivalent. For instance, adding 4 and 7 and adding a minus sign results in - 4 + (- 7). Consider the equation - 5 - (- 3) as 5 - 3 with a minus sign appended for a related example. When considering the neutralization model, the kind of issue only pertains to circumstances in which there are sufficient examples of one type of integer to do the subtraction. This procedure is merely a "take away" extension of subtraction. Level four consists mathematical problems like 4 + (- 7) and 4 -, which do not have the same outcome as counting numbers (– 7). If one is considering the neutralization model, he must decide if there are more positives or more negatives for the addition operation. In order to subtract as many items as the addend (subtrahend) specifies, the learner must figure out how to acquire enough of the type of integer indicated by the sum (minuend).

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In conclusion, the number line model and the neutralization model are the two sorts of models that are utilized to introduce integers. The neutralization model has four reportedly advantageous features. First, it is consistent with other academic disciplines. For instance, positive and negative charges are drawn to one another in science and balance one another out. Similar to the two-color counters where a black counter balances a red counter, this does the same thing. The second benefit comes from the mathematical soundness of representing integers as ordered pairs. In that one can determine if there are more positives or negatives, using two-color counters is similar to using ordered pairs as a representation. Third, the student's intuitive grasp of numbers is stimulated by the neutralization. Fourth, cardinality, which tends to grow before ordinality, is highlighted by the neutralization model (Wilkins, 1996). Students who use two-color counters can focus on the activities at hand rather than fretting about the placement of the integers on the number line. The neutralization model's inability to simulate all varieties of integer multiplication and division issues is one of its main drawbacks. Additionally, operations involving rational numbers cannot be included in the application of the neutralization model.

There are various benefits and drawbacks to the number line concept. Because the numbers on the right are larger than the numbers on the left, it is possible to discern order within the set of integers. The number line can be used to model the operations of addition, subtraction, multiplication, and division. Additionally, rational numbers can be used with number line operations. Uncertainty about how subtraction works is one of the number line model's drawbacks. Finding a missing addend was one approach studied for modeling subtraction, while moving backward on the number line was another. Students may find this confusing because, before learning about integers, they linked subtraction with "taking away," but with a number line they cannot take anything away. Some pupils find it challenging to figure out what they are counting on the number line. Instead, then counting the spaces that correspond to the units, some students count the hash marks designating the integers (Carr & Katterns, 1984).

Some Common Mistakes Made When Adding and Subtracting Integers

When adding and subtracting integers, pupils might make a variety of mistakes. The learner subtracts the number with the smaller magnitude from the one with the larger magnitude in a technique known as symmetric subtraction (Davis, McKnight, Parker, & Elrick, 1979). When teaching youngsters how to subtract, a teacher could inadvertently instruct them that they should always take the smaller number away from the larger number or that the problem is impossible to solve. Symmetric subtraction was shown to explain roughly 50% of all the responses given by the 28 fifth graders who participated in the study by Davis, McKnight, Parker, and Elrick (1979).

The type of error that results from the use of first-grade frames is another type of error (Davis, McKnight, Parker, & Elrick, 1979). Children are frequently instructed that addition is a binary process requiring two numbers. They typically move to the spot directly above if they only have one number in the left-hand most spots without regrouping so that they have two numbers to add. For instance, if a youngster is instructed to add 217 + 42 + 26 in column form, they will understand that they must add two numbers together. They add the two to what they already had for a total in the tens place after noticing the two in the hundredth place. The learner would receive 105 in this case, which is less than one of the addends. Dealing with integers can make this considerably trickier. The sequential interpretation of symbol strings is a third mistake that students make, and it has to do with how they approach reading. They carry out the calculations as they move from left to right. Because the 5+2 was provided to them at the left-hand side of the issue, a youngster who made this type of blunder would add the 5+2 and append a negative sign, yielding -7 for the example -5+2.

Finally, it is important to address students' misconceptions about numbers in a way that enables them to construct and improve concepts based on prior knowledge and experiences. Rathmell (1980) investigated student reactions to NAEP (National Assessment of Educational Progress) example problems in which they had to match a number line with positive and negative numbers that modeled addition or subtraction to a symbolic presentation of the problem. It was discovered that for addition issues, 45% of students aged 9 and 39% of students aged 13 added the numbers that were located at the ends of the arrow. Only 14% of 9-year-olds and 33% of 13-year-olds could correctly match the drawing with the subtraction problem when it came to addition. This result indicates that a lot of kids don't know how to utilize a number line as a model for addition or subtraction (Rathmell, 1980).

In conclusion, there are various errors that students make when adding and subtracting integers. Some students use the same rules as they did for whole number addition and subtraction to the addition and subtraction of integers. Others approach the issue from the left to the right while adding the sign to the left of the initial integer. When a topic is initially given to a class, students are less likely to make mistakes because they take their time to apply what they have learned. Due to extensive expertise with the method by the time students reach the expert stage, they also make less mistakes at this stage. The majority of mistakes made by students occur during the middle stages (Anderson, 1983).

Statement of the problem

According to research, kids have trouble conceptually and logically grasping the idea of addition and subtraction of integers (Bolyard, 2005; Ferguson, 1993; Lytle, 1992; Shore, 2005; Wilkins, 1996). Instead, they have acquired techniques that will direct them toward the right solutions to issues. Many pupils struggle with basic algebra because they don't grasp how numbers work. "Achieving algebraic competence in the solving of problems and equations requires the extension of the numerical domain from natural numbers to integers throughout the process by which twelve- to thirteen-year-old kids acquire algebraic language" (Gallardo, 2002, p. 171). Negative integers are used by algebra students as coefficients, constants, and problem solutions. Other branches of advanced mathematics cannot be conceptually grasped without a solid algebraic foundation. In BECE Mathematics 2 as of April 2009, the Chief Examiner for the West African Examination Council (WAEC) indicated that pupils struggle with integer operations, making it difficult for them to connect such operations to the addition and subtraction of column vectors. In order to educate and learn integer operations, a comparison between the use of the Number Rule and the conventional method was investigated.

Research questions

i. How do students view the use of number rule in the teaching of addition and subtraction of integers?
ii. What is the effect of number rule on students' achievement of addition and subtraction of integers?

Methodology

The study's used mixed-methods research design. Both the guided interview method and the quasi-experimental design were used.

Research design

The study's research design was based on a non-randomized pre-test, post-test control group that was quasi experimental in nature. This was done in an effort to demonstrate the impact of two different teaching strategies for mathematics. Although the independent variable is manipulated and a few controls are included to increase the internal validity of the results, this design prevents individuals from being randomized to treatment conditions at random. Because the study involves manipulating an independent variable and working with pre-existing groups of students rather than selecting students at random for treatment, a quasi-experimental design was adopted. It used two learning groups: a control group and an experimental group (students taught the number rule) (students taught without the number rule but with the Conventional method). The experimental group consisted of the students assigned to receive instruction using manipulative materials (the number rule), whereas the control group consisted of the students instructed using standard conventions of teaching, such as writing on the board.

Given that randomization disrupts the school's teaching schedule, it may be challenging to randomly assign subjects to treatment in studies conducted in educational settings. Classes in junior high exist as whole units, and school officials typically forbid their dismantling and reconstitution for research (Fraenkel & Wallen, 2000). Some of the sources of internal validity are also under the control of the quasi-experimental research. These circumstances made quasi-experimental research more appealing because it offers some control without interfering with the school's curriculum. However, because the study is conducted in a natural environment, it might be broadly applicable to other environments with a similar environment. The design is used to explain how manipulative objects impact students' math achievement during instruction.

Population

All first-year junior high school pupils in Ghana's Central Region were the target audience. A group of first-year students from Mpeasem AME Zion JHS and Jacob Willson Sey JHS, both in Cape Coast Metropolis of the Central Region, made up the accessible population. The approximate 10-kilometer distance between the two schools was deemed sufficient to exclude any interaction between the research participants' students. Students in their first year of JHS were taken into consideration since, in addition to studying a significant portion of the Primary and JHS curriculum, they were exempt from having to immediately prepare for the final Basic Education Certificate Examination (BECE). Because of their proximity and accessibility to JHS 1 pupils, the schools were chosen.

Sampling and sampling procedures

Two schools in the Cape Coast Metropolis were chosen using a convenience sample method. This was the case since the sample was easily accessible and representativeness was not a concern. 71 JHS 1 students from two junior high schools made up the study's sample. The addition and subtraction of integers were taught in one of the schools using the number rule as the experimental group, and the identical subject was taught in the other school using no number rule as the control group. To prevent bias, the schools were randomly divided into experimental and control groups. Both groups had similar socioeconomic backgrounds, diversity in their upbringing, and a range of abilities, including high, average, and low. The regional grading of the Basic Education Certificate Examination revealed that the two schools likewise shared nearly identical features in terms of academic performance (BECE). The students ranged in age from 13 to 14 on average. 37 students from the treatment group out of the total 71 in the sample underwent the lessons using the instructional materials (Number Rule) for three weeks. In contrast, the control group of 34 students received conventional instruction without the use of instructional resources (number rule). For each group, Fraenkel and Wallen (2000) advise using at least 30 participants. Therefore, this amount was suitable for the investigation.

Instrumentation

In this study, a teacher-made achievement test and a questionnaire were both employed as instruments. A set of tasks known as an accomplishment exam is used to evaluate a person's proficiency in subjects including reading, writing, arithmetic, and the sciences. The level of instruction for which a pupil is prepared is frequently determined by a student's performance on achievement tests. High accomplishment levels typically signal readiness for higher teaching and mastery of grade-level content. Low accomplishment scores may signal a need for improvement. This now also serves as a means of evaluating the pupils' level of skill. Better teaching techniques should result in pupils learning more in school, achieving higher success levels, and being more proficient overall. Two parallel sets of achievement tests, each with fifteen short-answer questions, were utilized to collect the data. The achievement tests were created by the researcher with assistance from the supervisors. The pre-test was one, and the post-test was the other. The purpose of the pre-test was to ascertain the participants' baseline knowledge, their math proficiency for placement, and the difference in proficiency at the conclusion of the program. While the post-test covered the integer topic from JHS year one, the pre-tests included integer topics from the primary 6 mathematics curriculum and textbook. On their answer sheet, students were obliged to carefully explain how they had answered each question. On May 3, 2013, the post-test instrument had a pilot test at Swedru International School in the Agona West municipality. Regarding age, socioeconomic and educational background, and academic performance, the school shared similarities with the two schools included in the primary study. One learner was seated at a time in the test's well-organized classroom, where they were also told of the test's requirements. They were instructed to fill out free answer papers with the necessary data. With the help of the mathematics teacher, they received the question papers, and at precisely 9:00 am, they got to work as invigilators went about to ensure independent work. The majority of pupils finished before the end of the 45th minute, and only a few finished in exactly one hour. The test items received over 20 points in scoring, which was translated to 100%.

Validity and reliability

My supervisors and a few JHS mathematics teachers critically reviewed the test item selection to make sure the instrument's content validity. The test items were examined, and any that required revision was done. Additionally, a mark awarding scheme was created, scored out of 20, and converted to 100% using the solutions to each of the exam items. The JHS One mathematics curriculum and textbook's topic of addition and subtraction of integers was covered in the achievement test. The test items were pilot tested at Swedru International School in Agona West Municipality, which shared the same characteristics as the two schools used in the main study, to determine the reliability of the instrument after it was constructed. This was done to check the elements' clarity and to determine the instrument's internal coherence. 38 responders who were JHS 1 students and were not a part of the study sample took the test. The dependability coefficient, which was based on the spearman-Brown formula's split-half approach. The result was 0.896. Using SPSS, the Pearson moment correlation coefficient was calculated and found to be 0.812. According to Litwin (1995), a correlation of 0.70 or above could be considered generally acceptable as having good dependability.

Data collection procedure

The head of the department of science and mathematics education provided an introduction letter, and copies of the letter were sent to each head of the institution that was visited. The Headteachers and the concerned mathematics instructors were able to provide the assistance and cooperation that was required thanks to this introductory letter. Following the approval of the request, the heads of the institutions at the chosen schools were called and personally informed of the significance of the study. This built the necessary rapport with the potential schools and enlisted their assistance. On May 13, 2013, the experimental school administered the pre-test before allocating the teaching times and starting the investigation (Mpeasem JHS). One learner was seated at a time in the test rooms, which were thoughtfully set up so that everyone was aware of the test's requirements. They were given answer sheets and asked to fill them out with the necessary data. With the help of the mathematics teacher, they received the question papers, and at precisely 9:00 am, they got to work as invigilators went about to ensure independent work. The majority of pupils finished by the 50th minute, while only a few finished in exactly one hour. The next day, May 14, 2013, the same test was administered in the control school (Jacob Willson Sey JHS) under the same testing circumstances as in the experimental school. The test items received over 20 points in scoring, which was translated to 100%. For each lesson, preparation was done in advance before it was taught. This involved creating detailed lesson plans for each lesson. For the lessons involving experimental groups, Ross and Kurtz's (1993) recommendations for creating lessons incorporating mathematical manipulatives were utilized. They advised teachers to make sure that:

a. Manipulative materials have been chosen to support the lesson's objectives.
b. Significant lesson plans have been made to familiarize students with the manipulative materials and corresponding classroom procedures.
c. The lesson involves the active participation of each student in the classroom.
d. The lesson plan includes procedures for evaluation that reflect an emphasis on the development of reasoning skills. (p. 256).

In all six lessons, the experimental group's students received instruction using manipulatives. The following subtopics were covered in the lesson plan: addition of positive integers, addition of negative integers, subtraction of positive integers, subtraction of negative integers, addition and subtraction of positive and negative integers, addition and subtraction of positive and negative integers, application of integers to real-world situations. We spent time explaining how to use the manipulative items before each lesson. They were divided into two-person groups, with each group having a leader and a secretary. The group exercises that students were to complete in each lesson were explained to them in the beginning. The Number Rule and other resources pertinent to the subject matter were provided to group leaders. Every class included a generic exercise that all groups completed and reported their results on. I walked around to see how each group was operating and to offer support as needed. For a class discussion and general conclusions to be reached, certain chosen groups were required to submit their solution. Students were given additional examples to practice with in their books as well as instructions on how to answer issues based on the subtopic that was covered.

It was shown that students were enthusiastic about using the resources, exchanging ideas with their peers, and socializing freely. In addition to being able to answer application questions, students were able to apply the fundamental ideas they had learned in the study of related disciplines. For instance, after learning how to add two negative numbers, students were able to extend the concept by understanding that the sum of two negative integers will always be negative. Some of the students were taken aback by their ability to understand things that they had previously assumed could only be learned through memorization. The majority of students argued that manipulative materials should be used to structure math lectures because they help students establish concepts that they would never forget. The majority then went on to make up their own number rule, which they applied to their schoolwork. Manipulative resources, according to Sutton and Krueger (2002), have the additional benefit of engaging students and raising their interest in and love of mathematics. Since sustained interest in mathematics leads to improved mathematical competence, students who have access to manipulative materials report being more motivated to study mathematics. According to Seefeldt and Wasik (2006), teachers should give students the chance to engage with assignments that have open-ended aims rather than predetermined outcomes. These activities give the kids the ability to investigate their own inquiries and produce a range of responses. "These encounters teach kids to consider the world from many perspectives and teach them that there are several approaches to problem-solving. Mathematicians often use the technique of multiple solution generation (Seefeldt & Wasik, 2006, p. 250).

In contrast, other physical difficulties I encountered included a lack of area in the classroom for group activities and a lack of time for more practice examples. Three weeks of instruction without the number rule in any of the six lessons were given to the control group. Therefore, the majority of the topics were introduced using the questions and answer method based on the pertinent prior knowledge. The presentation of the lesson, which was dependent upon the subtopic being discussed, came next. When necessary, number line graphics were used to illustrate the concept of integer addition and subtraction, and the formulae were written out on the whiteboard. Under each lesson, the students were guided through a variety of instances. Following some rehearsed instances, the students were then invited to the board to present their solutions to the class as a whole. A summary of the lesson and an assignment were provided. It was noted that the lesson favored the proficient students and ignored the unique learning styles of the others. The majority of the tasks and assignments that were given following each class reflected this. Since most of them memorized the formulas without actually understanding them, they were unable to apply the concept they had learned to solve difficulties. They applied formulae incorrectly in several circumstances, leading to incorrect solutions; the formulae were many and difficult to understand. Following some rehearsed instances, the students were then invited to the board to present their solutions to the class as a whole. A summary of the lesson and an assignment were provided. It was noted that the lesson favored the proficient students and ignored the unique learning styles of the others. The majority of the tasks and assignments that were given following each class reflected this. Since most of them memorized the formulas without actually understanding them, they were unable to apply the concept they had learned to solve difficulties. They applied formulae incorrectly in several circumstances, leading to incorrect solutions; the formulae were many and difficult to understand.

Data analysis

The hypotheses were tested using the non-parametric Mann-Whitney U test for 2 independent samples at a 5% level of significance. The non-parametric test for independent-samples t-test method compares the means of two distinct groups of cases that are selected through convenient or intentional sampling. Again, the data acquired contradicted the independent t test's presumptions, so the Mann-Whitney U test was applied. For instance, the data's distribution was not random.

Additionally, the effectiveness of the treatment in the experimental group was examined using the paired-Samples t-test. Using the Paired-Samples t-test method, you can compare the means of two different variables for one group. To determine whether the average deviates from 0, the process computes the differences between the values of the two variables for each occurrence. The idea that there is no difference between the pre-test and post-test was tested using this approach. Statistical Package for Solutions and Services (SPSS) Version 16 was used to analyze the research topics. At the 0.05 alpha levels, the following analyses were performed using SPSS version 16:

a. On the post-test, the Cronbach's alpha reliability test was run.
b. To determine if there was a statistically significant difference between the pre-test scores of the students who used the number rule (the experimental group) and the students who did not use the number rule, the Mann-Whitney U test for two independent samples was utilized (control group).
c. The research question "What is the effect of number rule on students' grasp of addition and subtraction of integers?" was analyzed using the paired-Samples t test.
d. The description of the students' opinions on applying the number rule was analyzed both subjectively and quantitatively by calculating the proportion of each rating given on the questionnaire that was given to the experimental group. How do students perceive the employment of the number rule in the teaching of integer addition and subtraction? is the research issue that this was done to address.

For both the pre-test and the post-test, the script for each student was coded with the same number, and the total marks earned for each question were recorded next to each question. Excel was used to record the raw scores, and SPSS version 16 was used to import the Excel data. Since the results for each question were not uniform, the scores for each question were scaled into a base of 10 for further processing analysis (some questions carry more marks than the other). Every unanswered question received a score of zero to guarantee that it was included in the analysis.

Results and Discussions

Table 1. Scores in Achievement Test by taught with Number Rule and Conventional Method

	Number Rule Method		Conventional Method	
Marks	Pre-test *F* (f %)	Post-test *F* (f %)	Pre-Test *F* (f %)	Post-Test *F* (f %)
$1-10$	−	−	1 (2.9%)	1(2.9%)
$11-20$	−	−	−	−
$21-30$	7 (18.9%)	2 (5.4%)	4 (11.8%)	3 (8.8%)
$31-40$	2 (5.4%)	3 (8.1%)	15 (44.1%)	6 (17.6%)
$41-50$	16 (43.2%)	4 (10.8%)	7 (20.5%)	17 (50%)
$51-60$	11 (29.7%)	9 (24.3%)	5 (14.7%)	6 (17.6%)
$61-70$	1 (2.7%)	11(29.7%)	2 (5.9%)	−
$71-80$	−	5 (13.5%)	−	1 (2.9%)
$81-90$	−	3 (8.1%)	−	−
$91-100$	−	−	−	−
Total	37 (100%)	37 (100%)	34 (100%)	34 (100%)

F-Frequency

Discussion of the Achievement Test Scores in the Study

Table 1 displays the distribution of pre- and post-test results together with the associated percentages for students who were taught the number rule and those who were not. According to the distribution of the students who were taught the number rule, 7 (18.9%) had a pre-test score between (21-30), but only 2 (5.4%) had a post-test score between (21-30), and only 1 (2.7%) had a pre-test score between (61-70), but 3 (8.1%) had a post-test score between (81-90). The distribution in the students taught with Conventional method showed that 1 (2.9%) had the score $(1-10)$ in the pre-test and again $1(2.9\%)$ had scores $(1-10)$ in the post-test and $2(5.4\%)$ had the score $(61-70)$ in the pre-test but $1(2.9\%)$ had the score $(71-80)$ in the post-test. This demonstrates that students who were taught using the number rule often achieved greater post-test scores than those who were taught using the conventional approach. Once more, it has been commonly noticed that both student groups improved on their pre- and post-test results.

According to the data distribution in Table 1, both groups' pre-test performance was marginally below average. This can be because the test items were challenging or because both groups might have forgotten what they learned in Primary 6. The post-test results revealed that the students who had been taught using the number rule had somewhat outperformed those who had been taught using the conventional technique. This may be explained by the fact that the majority of students taught using the conventional method utilized the wrong calculation procedure. Additionally, the post-test was a little more challenging than the pre-test. Additionally, it could be because students who were taught using the conventional method did not have the depth of understanding necessary to transfer and apply their mathematical knowledge to novel contexts and solve issues. The ability to develop functional understanding and to become autonomous learners and thinkers is a key argument for hands-on learning. Therefore, the intervention was to blame for the increase in learning outcomes for the students who were taught the number rule.

Table 2. Comparing of Pre-Test Scores of Students Taught with Number Rule and Conventional Method

	N	Mean	SD
Number Rule Students	37	43.78	14.11
Conventional Method Students	34	41.18	12.67

Table 2 compares the means and standard deviations of the students' pre-test results. Table 2 demonstrates that there was a 2.60 difference between the two means. Also slightly different was the standard deviation. Therefore, to determine whether the mean difference of 2.60 was significant, the Mann-U Whitney's test was performed for two independent samples. The Mann-Whitney U test details are presented in Table 3.

Table 3: Comparing Students Taught with Number Rule and the Conventional Method

	Students Pre- test Scores
Mann-Whitney U	482.000
Wilcoxon W	1077.000
Z	−1.716
Asymp. Sig. (2-tailed)	.086

The Mann-Whitney U 2-independent-sample test was used to compare the mean pre-test scores of students who had been taught using the number rule and those who had been taught using the conventional approach. The results are shown in Table 3. The test resulted in $p>0.05$ ($p=0.086$), indicating that. According to statistical analysis, there was no significant difference between the mean scores of the students using the number rule ($M=43.78$, $SD=14.11$) and the students using the conventional technique ($M=41.18$, $SD=12.67$) on the pre-test; $Z=-1.716$, $p=0.086$. According to the statistical analysis, there was no discernible difference between the pre-test performance of the two groups of pupils. Since there was no statistically significant difference between the groups, the practical difference of 2.60 between them may be the result of chance. In conclusion, before the intervention, the math proficiency of the two groups was equal. This suggested that the groups employed in the study shared similar traits at the start of the investigation.

Analysis of Post Test Scores

"There is no significant difference in the mean achievement of learners taught addition and subtraction of integers using number rule and learners taught without number rule," was the null hypothesis that was put to the test. This was in opposition to the alternative premise, which said that "there is a significant difference in the mean achievement of learners taught addition and subtraction of integers using number rule and learners taught integers without number rule." The results from the post-test data are shown in Table 4.

Table 4: Comparing of the Post-Test Scores of Number Rule and Conventional Method

	N	Mean	Std. Deviation
Number Rule Students	37	61.76	15.51
Conventional Method Students	34	46.47	10.41

Table 4 shows that there was a mean difference of 15.29 between the post-test scores of the students who were taught using the number rule and those who were taught using the conventional approach. The Mann-Whitney U test for 2 independent scores was conducted to see whether there is a statistically significant difference in means, as shown in Table 5 below.

Table 5: Comparing Students Taught with Number Rule and the Conventional Method

	Students Pre-test Scores
Mann-Whitney U	264.000
Wilcoxon W	825.000
Z	−4.107
Asymp.Sig. (2-tailed)	.000

The results of the Mann-Whitney U test for two independent samples are shown in Table 5. This test was used to compare the mean post-test scores of students who used the number rule with students who used the conventional technique. Since the test result was p 0.05 ($P = 0.000$), this was found. According to statistical analysis, the students who were taught using the number rule had significantly higher mean scores than those who were taught using the conventional technique (M=46.47, SD=10.41; Z=-4.107, p=0.000). The findings demonstrated that, with a mean difference of 15.29 between the two groups of students, those who had been taught using the number rule had performed better on the post-test than those who had been taught using the conventional technique. The size of the mean difference was significant (eta squared = 0.277). The employment of number rules in the addition and subtraction of integers during instruction and learning considerably enhanced students' performance, even if the difference in achievement was huge.

The study literature has repeatedly come to the conclusion that pupils who were taught using manipulative materials outperformed those who were not. The findings were in line with those of studies by Moyer (2001), Munger (2007), Sowell (1989), Witzel, Mercer, and Miller (2003), which found that students who were taught mathematics using manipulative materials by instructors who were knowledgeable about and committed to the theory of manipulative use outperformed their peers in terms of test scores. This is due to the fact that in each of the experiments listed above, students who learned number rules using manipulative materials performed mathematically substantially better on the post-test than students who received conventional instruction. The current study's findings about the impact of employing number rules were in agreement with those of Mustafa al-Absi and Nofal's (2010) study, which found that the use of manipulative materials in mathematics instruction and learning has an impact on students' mathematical accomplishment. What impact does the use of the number rule have on students' comprehension of integer addition and subtraction? was the study issue that was being looked at.

Table 6. Paired Sample T-Test Comparing the Pre-Test Scores and Post-Test Scores of the Students Taught with Number Rule

	N	M	SD	df	t	Sig. (p)
Pre-test	37	43.784	14.113			
Post-test	37	61.757	15.511	36	−6.5386	0.000

Effect of Treatment

A paired-samples t-test using data from Table 6 was used to assess how manipulative materials affected students' achievement. From the pre-test (M=43.784, SD=14.113) to the post-test (M=61.757, SD 15.510), t=-6.793, p=0.000, i. e. P0.05, it was found that the learners' achievement had increased statistically significantly. The substantial effect size shown by the eta squared statistics (0.482) suggested that manipulating materials had a beneficial impact on students' achievement.

Table 7. Paired Sample T-Test Comparing the Pre-Test Scores and Post-Test Scores of the Students Taught with Conventional Method

	N	M	SD	df	t	Sig. (p)
Pre-test	34	41.180	12.667			
Post-test	34	46.470	12.922	33	−2.485	0.018

Effect of Treatment

A paired-samples t-test was performed using data from Table 7 to assess the impact of the conventional method on student achievement. This showed that the learners' achievement increased statistically significantly from the pre-test (M = 41.180, SD =12.667) to the post-test (M = 46.470, SD = 12.922), t = -2.485, p=0.018, i. e. P 0.05. The moderate effect size revealed by the eta squared statistics (0.157), however, suggested that the conventional approach had little to no impact on students' achievement as compared to those who were taught using manipulatives.

Students' Experiences in Using Number Rule to Learn Addition and Subtraction of Integers

In order to respond to the research question, "How do students view the employment of number rule in the teaching and learning of addition and subtraction of integers? ", data gathered through questionnaires were employed. The questionnaire's ten questions centered on the experiences and perceptions of the students about the usage of lessons based on number rules. According to the responders, applying the number rule to an integer problem makes the lesson simple to comprehend, more applicable, engaging, and time-efficient. They added that the number rule fosters collaboration, improves output, and deepens comprehension. Table 8 displays the students' ratings of the use of the number rule in relation to the subjects covered by the questionnaire. The themes were scored on a scale of 1 to 4, with 1 representing the least preferred reaction and 4 the most preferred response. The graph and Table 7 below indicate the results when taking into account the general information gathered through the questionnaire of ten (10) items from the 37 respondents on the perspectives of students on the use of the number rule.

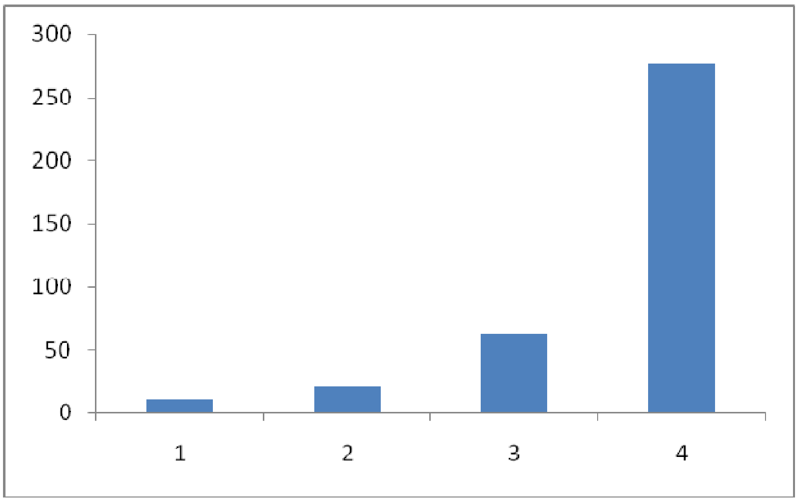

Fig. 1: Bar Graph Showing the Responses of the Views of Students Taught with Number Rule
KEY: 1 strongly disagree (SD), 2 disagree (D), 3 agree (A), 4 strongly agree (SA)

If you look at the graph in fig. 1 above, it is clear that the majority of students gave very positive responses to each item in the questionnaire, which is number 4. This indicates a strong agreement with the use of the number rule, which is followed by response number 3, which also indicates that the use of the number rule on the themes expressed above is very helpful to the learning of addition and subtraction of integers. However, a small percentage of respondents claimed that the tool could not accomplish the questionnaire's intended goal, as was depicted graphically on the bar graph.

Table 8: Responses of the Students Taught with Number Rule on the Questionnaire Items on the use of the Number Rule (*N* = 37)

Statement	SD f(%)	D f(%)	A f(%)	SA f(%)
Number rule is a tool to the approach of learning addition and subtraction of integers	1(2.7)	3(8.1)	4(10.8)	29(78.4)
Number rule is comfortable to use to learn integers	1 (2.7)	3 (8.1)	7(18.9)	26 (70.3)
Number rule helps explore properties of integer	2 (5.4)	3(8.1)	3(8.1)	29 (78.4)
Number rule promote group work	3(8.1)	3(8.1)	3(8.1)	28(75.6)
Number rule makes integer lesson more interesting	1(2.7)	1(2.7)	5(13.5)	30(81.1)
Number rule is a useful tool for learning integers	1(2.7)	3(8.1)	6(16.2)	27 (70.3)
Number rule improves performance in integer lessons	0(0.0)	1(2.7)	10(27.0)	26(70.3)
Number rule makes lesson very easy to understand	1(2.7)	1(2.7)	6(16.2)	29(78)
Number rule makes lesson more practical	1(2.7)	1(2.7)	8(21.6)	27(72.9)
Number rule enables understanding in a short time	0(0)	2(5.4)	10(27.0	25(68)
Total	11(2.9)	21(5.7)	62(16.8)	276(74.6)

Table 8 shows that, overall, while 276 replies, or 74.6%, expressed strong agreement with the application of the number rule in teaching addition and subtraction of integers, 11 out of 370 responses, or 2.9%, stated they strongly disagreed. As you can see from Table 7, 29 people (78.4%) strongly agree that using the number rule is a suitable strategy for learning integers, whereas just 1 person (2.7%) strongly disagrees. Additionally, 2(5.4%) strongly disagrees with 29(78.4%), who said that using the number rule as a tool enabled them to study the characteristics of integers. 28 respondents (75.7%) strongly agreed that the number rule improves group work, whereas 3 respondents (8.1%) strongly disagreed. Regarding the value of the number rule in teaching students how to add and subtract integers There are 27 (70.3%) strongly agreeing, 6 (16.2%) agreeing, 3 (8.8%) disagreeing, and 1 (2.7%) severely disagreeing. This viewpoint also supports the value of the number rule in teaching integer addition and subtraction. The same pattern of responses was observed for the remaining items, demonstrating that students had a favorable opinion of the application of the number rule to the teaching of integer addition and subtraction. In light of this, the research question, "How do students see the use of number rule in the teaching and learning of addition and subtraction of integers?" can be answered as follows: students think the application of number rule is beneficial when teaching addition and subtraction of integers. To confirm whether the views of the students taught using the number rule were positive or not, further analysis of the means and standard deviations of the students' responses in accordance with the themes of the questionnaire items was conducted.

Table 9. Means and Standard Deviations of the Students' Response to the Questionnaire Items by the Students Taught with Number Rule

	Mean	Standard deviation
Number rule is a tool to the approach of learning addition and subtraction of integers	3.57	0.94
Number rule is comfortable to use to learn integers	3.49	0.94
Number rule helps explore properties of integer	3.51	1.03
Number rule promote group work	3.46	1.08
Number rule makes integer lesson more interesting	3.65	0.87
Number rule is a useful tool for learning integers	3.51	0.94
Number rule improves performance in integer lessons	3.60	0.78
Number rule makes lesson very easy to understand	3.62	0.88
Number rule makes lesson more practical	3.57	0.88
Number rule enables understanding in a short time	3.54	0.82

According to Table 9, the lowest mean, which is 3.46 and is closer to 3.50, and the highest mean, which is 3.65 and is roughly 4.0, both indicate a positive response. Since responses 3 and 4 imply that the respondent agrees and strongly agrees, respectively, it is evident that the majority of students have a favorable opinion on the application of the number rule. The tool (Number rule) boosts understanding, improves performance, encourages group effort, and makes learning addition and subtraction of integers more realistic, according to the questionnaire items, mean, and standard deviations. The research literature has repeatedly concluded that students who were taught using manipulative materials understood mathematical concepts better, and these findings are consistent with the earlier finding regarding the students who were taught using the conventional method and the students who were taught using the number rule (manipulatives). Chang (2008), for instance, looked at the studies of researcher Jennifer Kaminski and discovered that using concrete examples helps kids comprehend math. According to the findings of Ojose's 2009 study, manipulative materials have an impact on all students, regardless of their sexual orientation, socioeconomic position, academic standing, or disability.

Conclusion

Similar studies conducted in China, England, Japan, and the United States support the notion that the use of manipulative materials will improve the effectiveness of mathematics instruction and student understanding (Canny, 1984; Clements & Battista, 1990). Additionally, when students illustrate concepts with manipulatives, they "provide learners essential tools for building comprehension, sharing information, and demonstrating reasoning" (Fennell & Rowan, 2001, p. 289). One of the most crucial ways to express mathematical concepts and ideas is through the use of manipulative objects in mathematics teaching and learning. According to research on mathematics instruction, using manipulatives will help pupils better comprehend maths (Cotter, 2000; Clements & Battista, 1990). Students were encouraged to cooperate and share resources through the use of manipulatives.

This kind of cooperation allowed students to solve issues collectively, talk about mathematical ideas and concepts, communicate their mathematical thinking, create presentations, and solve problems without only following the teacher's instructions. The constructivist premise is that an individual's constructive action produces knowledge. It is not a thing separate from the mind you want to possess. Reality is whatever each person creates in their own imagination. The development of meaning as a result of life experiences is the source of knowledge. Experience is the only source of knowledge, not that of others. As active "makers of meanings," the students actively contributed to the creation of new knowledge. The teacher's job was to interact with the students and try to comprehend what they produce using the materials. This proposes that number rules should be used as manipulatives to teach and learn addition and subtraction of integers in group settings at the junior high school level. A few areas are recommended for additional research in light of the subject matter of this study. Additional research is required on a number of additional subjects, including fraction addition and subtraction, integer multiplication and division, and addition and subtraction of fractions in the mathematics curriculum for elementary schools. Further research can be done to ascertain the competency level of trainees from the colleges of education as well as both pre-service and in-service mathematics teachers with regard to the use of number rule in teaching mathematics because the use of number rule requires knowledge and skills on the part of the teacher. To obtain a nationwide reflection of the effect of number rule on learners' accomplishment in mathematics instruction in addition and subtraction of integers and learning, this study should be duplicated in other regions of Ghana and in a greater number of institutions. In addition, the study could be carried out at second cycle institutions to see if the scenario is the same. For future studies, Since the result justify the importance of number rule to mathematics achievement in integer addition and subtraction, it is recommended that teacher trainees be taught how to use it and in service teachers be encouraged to use number rule in teaching addition and subtraction of integers. Also, it is recommended that the findings serve as resource material for mathematics educators

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