Journal of Education and Practice www.iiste.org
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.14, No.6, 2023

Students' Understanding of the Real Nature of Mathematics and Its Relationship to Their Attitude Towards the Discipline

Rainilyn L. Duque
College of Science, Bulacan State University, Malolos City Bulacan, Philippines
rainilyn.leonardo@bulsu.edu.ph

Abstract
This research aims to determine students' understanding of the nature of mathematics and how it affects their attitude towards the discipline. Students' understanding of the nature of mathematics was assessed using the developed instrument of the researcher. Analysis of the results from this survey resulted to classroom implications that recognize an alternative perspective to mathematics vis-à-vis teaching methodologies that aims to promote understanding of the nature of mathematics. This study found out that students have very limited understanding of the nature of mathematics despite majoring on the discipline. This explains their inch-deep appreciation and understanding of the importance of mathematics. It was further concluded that explicit but seamless teaching of the nature of mathematics may improve students' appreciation, interest and understanding of the fullness of the discipline. It also has the potential to improve students' attitude towards mathematics which affects their motivation and interest in the discipline.
Keywords: mathematics education, nature of mathematics
DOI: 10.7176/JEP/14-6-15
Publication date: February 28th 2023

1. Introduction

Tracing the history of mathematics as a discipline, one can discover the unresolved debate of its nature and the appropriate method to deliver its content to students. Unlike other disciplines, mathematics has not undergone remarkable changes. Other disciplines had increasingly put less emphasis on the traditional and classical conception of the content as given and inert. Over the years, the approach to teaching has shifted to recognized the undeniable role and contribution of human agency in the creation and continuous discovery of concepts in the discipline. However, a similar metamorphosis in mathematics has not taken place because of the clashing and opposing ideas of what and how mathematics should be approached. The real nature of the discipline is hardly understood which explains the inch-deep learning of the concepts and appreciation of its use. As a result, mathematics teaching and consequently mathematics learning had stayed stagnant. Hersh (1979) shared the same sentiment and emphasized that the problem in mathematics teaching and learning is not what is the best way to teach it, but what is mathematics is about – nature of mathematics.

The nature of mathematics is well accounted for through the philosophy of mathematics which should include the history, origin and practices of mathematics (external), and the justification of knowledge (internal). The nature of mathematics has significant practical outcomes and implications to the teaching and consequently, learning of mathematics. According to Thompson (1984), there is a strong indication that the teacher's professed conception of what mathematics is about and consequently, the manner by how they teach the content to students is heavily influenced by their views, personal beliefs and preferences about mathematics.

There are two opposing stands on the nature of mathematics: the absolutist and constructivist views. The absolutist view to mathematics regards mathematics as a unique branch of knowledge that offers certain truth that is permanent, incorrigible and regarded as above all other knowledge. The absolutist view characterizes mathematics as objective, logical and void of human involvement which has made it incorrigible and resistant from changes and conceptual shifts that other disciplines has undergone.

This view of mathematics was put to challenge during the early twentieth century when several antinomies and controversies due to contradictions were found (Kline, 1980; Kneebone, 1963; Wilder, 1965). Absolutists tends to offer no explanations of how humans can come to know, grow and use mathematics in any significant sense. They argue that humans play no role in creating mathematics because according to them humans need only to discover mathematics.

On the other hand, it is constructivism which became the most influential forms of non-absolutism. Constructivism unlike absolutism describes both what "knowing" is and how one "comes to know". It is a theory on knowledge that describes it as temporary, nonobjective, developmental and constructed internally which is further refined through social and cultural mediation. (Fosnot, 1996).

In line with the ideas discussed above, it is suggested in this study that mathematical knowledge therefore is better regarded with unique stability and remarkable universality while also realizing its ability to grow, change and improve.

2. Methodology

2.1. Pre-survey Phase

2.1.1. Students' Mathematics Conception Questionnaire

The study conducted a survey of students' understanding of the nature of mathematics while also assessing the students' attitude towards the discipline. A survey questionnaire which aimed to assess whether a student hold more of an absolutist or constructivist view of mathematics was developed to realize the objectives of this study. The questionnaire development included the following phase: planning the test, face validation, first trial run validation, item analysis, second trial run validation, final survey run, and evaluation of the test. Students assessed their mathematics conception using the scale from 6 for Strongly Agree to 1 for Strongly Disagree.

The first draft of questionnaire included 40 items which are constructed based on literature study and interviews to mathematics experts. This draft was pilot tested to 120 college students taking up BS Mathematics in a state university. The result of this survey was analyzed and the Cronbach Alpha was obtained to help in further sifting the questionnaire items. The second draft of the questionnaire included 26 items divided into two parts: nature of mathematical knowledge and beliefs in learning of mathematics. Further analysis of the characteristics indexes of the questionnaire was done. The final form of the questionnaire has 26 items with two items that had been reworded for better understanding on the part of the students. The analysis and results of the final form of the questionnaire checked the construct validity and its item reliability which also includes its content, convergent and discriminant validity. Table 1 describes the Cronbach alpha index of the constructs *Nature of Mathematics Understanding and Beliefs in Learning Mathematics.*

Table 1. *Reliability Indexes of the Constructs of Students' Mathematics Conception Inventory*

Reliability Statistics			
Construct	Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
Nature of Mathematics Understanding	.827	.864	13
Beliefs in Learning Mathematics	.910	.912	13

Tables 1 shows the descriptive statistics for the constructs *Nature of Mathematics Understanding* and *Beliefs in Learning Mathematics.* It can be seen from the table that the mean of the two constructs namely, nature of mathematics and beliefs in learning mathematics, are above the midpoint of 4.00. It can also be observed from the table that the standard deviations ranging from 0.6 to 1.32 are indicative of a narrow spread around the mean. To test the construct validity of the items, exploratory and confirmatory factor analyses using principal axis were performed. The reliabilities of factors were assessed using Cronbach's alpha. All items demonstrated high loadings which indicates that the items are a good fit to the aim of the survey instrument. Furthermore, all of the measures included in the questionnaire showed excellent internal consistency, 0.827 and 0.910, thereby exceeding the reliability estimate ($\alpha = .70$) recommended by Nunnally (1967). Table 2 shows each item mean and standard deviation for the respective constructs.

Table 2. *Mean and Standard Deviation for Students' Mathematics Conception Inventory*

Construct	Question	Mean	SD
Nature of Mathematics Understanding	1	5.283	.7386
	2	5.367	.7123
	3	5.283	.7612
	4	5.167	.8061
	5	5.450	.7231
	6	4.350	1.4593
	7	5.083	.8693
	8	5.550	.5945
	9	5.350	.7324
	10	5.400	.6431
	11	5.167	.9051
	12	4.150	1.3879
	13	5.167	.8061
Beliefs in Learning Mathematics	1	4.88	.904
	2	4.73	.880
	3	4.68	1.033
	4	4.47	.965
	5	4.68	1.000
	6	4.75	1.035
	7	4.90	1.145
	8	4.83	.960
	9	5.02	.948

Construct	Question	Mean	SD
	10	4.48	1.200
	11	4.90	1.003
	12	4.50	1.157
	13	5.02	.930

All items under each construct are a good fit and reliable as shown by the item characterization discussed above. Hence, the survey instrument is valid and reliable in assessing students understanding of the nature of mathematics and their general mathematics conception.

2.1.2 Mathematics Attitudinal Questionnaire

A Students' Mathematics Attitudinal Questionnaire was developed and validated for the purpose of this study. The questionnaire is scaled as 6 for Strongly Agree to 1 for Strongly Disagree. It is divided into three dimensions namely: affect, behavior and cognition with 10 items for each dimension, hence the survey instrument has a total of 30 items. The development and validation of this attitudinal questionnaire followed similar processes done in the Students' Mathematics Conception Inventory.

Table 3. *Reliability Index for Students' Mathematics Attitude Questionnaire*

Reliability Statistics			
Construct	Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
Affect Dimension	.726	.724	10
Behavior Dimension	.868	.875	10
Cognition Dimension	.856	.858	10

As shown in Table 3 each construct in the Mathematics Attitudinal Questionnaire obtained a high Cronbach's alpha which indicates that the items in the questionnaire are reliable and measure accurately the level of attitude of the students towards mathematics. The mean of each item in the questionnaire ranges from 2.98 to 5.52 while the standard deviation ranges from 0.887 to 1.522 which all indicate sufficiently good fit.

2.2. Survey Phase

A total of 60 third year BS Mathematics students were surveyed using the developed and validated Mathematics Conception Inventory and Attitudinal Questionnaire. Table 4 reveals the descriptive statistics of the results of the survey.

Table 4. *Descriptive Statistics of the Survey Result*

Construct	Mean	Standard Deviation
Nature of Mathematics Understanding	4.21	.341
Beliefs in Mathematics Learning	4.82	.561
Overall	4.51	.451

It can be observed from Table 4 that the mean for both constructs are below average which indicates that students viewed mathematics more as an absolute knowledge, beyond human influence and unchanging. The item with the lowest mean, *"Existing mathematical knowledge can be changed, revised or expanded"*, proves that students hold absolutist conception of mathematics. This can be accounted to their lack of knowledge of the new discoveries in mathematics contradicting or expanding existing mathematical knowledge. Table 5 presents the correlation done among students understanding of the nature of mathematics and their attitude towards the discipline.

Table 5. *Association of Students' Understanding of the Nature of Mathematics and their Attitude towards Mathematics*

	Correlation Coefficient	**Test Statistic**
Understanding of the Nature of Mathematics vs Attitude Towards Mathematics	0.0291	2.13

Table 5 shows that the relationship between students' understanding of the nature of mathematics and their attitude towards the discipline using the computed Pearson test statistic was 2.13 which was greater than the tabular value of 1.96. Therefore, there is sufficient evidence to support that students' understanding of the nature of mathematics is related to their attitude towards mathematics.

Results and Discussion

The philosophical views in mathematics that regard it as strictly absolute or strictly constructive had deterred the growth of the discipline. Students have very limited understanding of the nature of mathematics despite the fact that they are majoring on the discipline. This explains their inch-deep appreciation and understanding of the importance of mathematics. Students lack awareness of the new discoveries in mathematics which explains their belief that mathematics is stagnant and unchanging. It was also found out in this study that students' understanding of the nature of mathematics and their general conception of how one learns mathematics influence their attitude towards the discipline.

Conclusion

Through the lens of this study, mathematics educators are encouraged to explicitly teach the nature of mathematics and include classroom activities which will foster greater understanding of the characterization of mathematics, which in turn, will improve students' appreciation of the discipline and its practical use.

Furthermore, the results of this study suggest that teachers incorporate in a seamless way, rich historical account of mathematical concepts so as to provide students the necessary background, understanding and appreciation of the discipline. This will provide students better understanding of the nature, developmental stages and purpose of mathematics which in turn enlightens students of the kind of mathematics they deal with at present.

References

Fosnot, C. (1996). *Constructivism: Theory, perspectives and practice.* New York: Teachers College Press.
Hersh, R. (1979). Some proposals for reviving the philosophy of mathematics. *Advances in Mathematics,* 31, pp. 31-50
Kline, M. (1980). *Mathematics: The loss of certainty.* New York: Oxford University Press.
Kneebone, G.T. (1963). *Mathematical logic and the foundations of mathematics.* London, Van Nostrand.
Thompson, A.G. (1984). The relationship between teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics,* 15, pp. 105-127.
Wilder, R. L. (1965). *Introduction to the foundations of mathematics.* New York. John Wiley & Sons.