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Exploring Pre-Service Teachers' Knowledge and Perceptions of Euclidean Geometry Concepts

Ernest Frimpong Akosah* Samuel Kwasi Kabbah Saviour Kofi Kabe Julius Kudjo Ametepey
Department of Mathematics and ICT, Peki College of Education, PO box 14, Peki - Volta, Ghana
* E-mail of the corresponding author: akferst@gmail.caom

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Abstract
The purpose of the study was to explore pre-service teachers' knowledge and perceptions towards Euclidean Geometry concepts. A questionnaire was used to gather information from 125 pre-service teachers in Ghana's Volta Region using a quantitative approach within a positivist paradigm. Descriptive statistics were used in the analyses after the data were coded and entered into the Statistical Package for Service Solutions (SPSS) version 20. The results showed that the participants' knowledge levels varied, with some demonstrating a solid understanding and others showing misconceptions and gaps. The study also identified specific conceptual difficulties, such as comprehending criteria for triangle congruence and identifying corresponding parts of congruent triangles that pre-service teachers encounter in Euclidean Geometry. Furthermore, the study explored pre-service teachers' perspectives on Euclidean Geometry, highlighting their opinions on the subject's difficulty, interest, relevance, and problems. The result highlights the importance of targeted instructional interventions and curriculum development in addressing these knowledge gaps and misconceptions. The recommendations include incorporating targeted instruction, interactive strategies, and technology integration in teacher education programs to improve pre-service teachers' understanding and pedagogical skills in Euclidean Geometry.
Keywords: Pre-service teachers, Euclidean Geometry, knowledge, perceptions, conceptual difficulties, APOS theory
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1. Introduction

A branch of mathematics known as Euclidean geometry is founded on the ideas and axioms put out by the Greek mathematician Euclid (c. 300 BCE), and it is concerned with the study of geometrical relationships and qualities in two and three dimensions. According to Hartshorne (2000), Euclidean geometry is a system of geometry that is based on a collection of axioms and postulates that are predominantly drawn from the writings of Euclid. These axioms establish the ideas of congruence, resemblance, and parallelism as well as defining basic concepts such points, lines, planes, and angles. In Euclidean geometry, qualities of geometric objects like triangles, circles, and polygons are explored, as well as connections like congruency and the characteristics of related objects. In a similar vein, Euclidean geometry is concerned with the study of flat figures, angles, and lengths, (Kiselev, 2007). It entails using deductive reasoning to establish theorems and proofs regarding the characteristics and connections of geometric objects based on a set of fundamental presumptions known as postulates or axioms. Theoretical and practical uses of Euclidean geometry can be found in the fields of physics and engineering, robotics and computer vision, computer graphics and animation, geographic information systems (GIS), and architecture and construction (Euclid, 300 BCE; Siciliano & Khatib, 2016; Hearn & Baker, 2014; Longley et al., 2015; Halliday et al., 2013). Euclidean geometry is taught as a core part of mathematics in Ghana's new school curriculum, where it has a substantial presence. It gives students a firm grasp of geometrical concepts, spatial relationships, and logical thinking (GES, 2017). It is one of the course domains listed in the new College of Education (CoE) mathematics curriculum for the Junior High School (JHS) specialization in Year Two Semester Two (YR 2 SEM 2), which focuses on building conceptual understanding of what student teachers should know about studying, teaching, and using Euclidean geometry (T- TEL, 2018). It includes geometric principles, procedures, and how they apply to solving problems (NTC, 2017). The basis for meaningful mathematics study at the CoE is conceptual mastery of Euclidean geometry concepts at the senior high school level. The new 4-year B.Ed. CoE curriculum covers Euclidean geometry topics such Points, Lines, and Planes, Angles, Triangles, Quadrilaterals, Circles, Polygons, Similarity, and Congruency (T-TEL, 2018). Due to the necessity that students must comprehend, make sense of, and gain from multiple benefits of Euclidean geometry, the new 4-year B.Ed focused emphasis on constructivist methodologies such experience learning, discovery learning, and project work.

Additionally, it supports the conceptual growth through hands-on activities that rely on tangible resources to build the necessary cognitive abilities for fruitful problem solving.

The Action-Process-Object-Schema (APOS) theory serves as the foundation for this educational paradigm. In 1995, the APOS theory was put forth by Dubinsky, Dautermann, Leron, and Zazkis. Through actions, processes, objects, and schemas, the theory offers a framework for comprehending how children pick up mathematical concepts and build their understanding. The APOS theory states that a learner needs sound mental structures in order to comprehend a certain mathematical scenario (Asiala et al. 1996). The likely APOS needed to learn a topic are represented by these mental structures. For instance, the theory mandates that the likely mental structures in a given notion be identified, and that the proper learning activities be created to facilitate the development and repair of these mental structures.

The APOS theory and its application to classroom practice, according to Dubinsky (2010), are predicated on two main tenets. These presumptions serve as the cornerstone for comprehending how students learn mathematics and how instructional techniques can be created to support their learning. The two fundamental assumptions are:

(1) Constructionist Assumption: This viewpoint holds that students actively develop their own understanding of mathematics via their own deeds and thought processes. Thus, by interacting with mathematical objects, processes, and concepts, learners build their understanding of mathematics.

(2) Cognitive Structuralism Assumption: This hypothesis likewise holds that when students gain knowledge of mathematics, they move through various cognitive phases or structures. Thus, rather than learning mathematical principles directly, students apply their mental models to make sense of ideas or circumstances.

As a result, studying Euclidean geometry is much easier if the learner has the necessary mental models in place. Understanding Euclidean geometry concepts and their applications, as well as responding to conceptual problem situations, may be nearly difficult without the necessary mental frameworks.

1.1 Research on Euclidean Geometry

Globally, a number of studies have been done to look into different facets of teaching and understanding Euclidean geometry. For example, in a study published in 2018, Chinnappan, Thomas, and Archibald looked at the effects of various teaching strategies, such as problem-based learning and direct instruction, on students' learning outcomes in Euclidean geometry. In comparison to direct instruction, the results showed that problem-based learning approaches fostered greater understanding and improved problem-solving abilities. In a similar vein, Lee, Lim, and Kim (2019) investigated how a problem-based learning strategy affected students' performance and attitudes in Euclidean geometry. The findings demonstrated how problem-based learning improved students' conceptual knowledge, problem-solving abilities, and favorable attitudes about geometry. However, Clements and Battista (2018) looked into how students' comprehension of Euclidean geometry was affected by dynamic geometry software. The results showed that the use of dynamic geometry software enhanced students' geometric reasoning, problem-solving, and visualization capabilities. There hasn't been much empirical study in Ghana that primarily focused on Euclidean geometry. However, Anamuah-Mensah and Assuah (2018) conducted a study on the pedagogical content knowledge (PCK) of prospective mathematics teachers for teaching geometry in Ghana. Their research showed that pre-service teachers had limited PCK in geometry, which pointed to the need for focused interventions to improve their comprehension and teaching methods. Adu-Gyamfi, Yevudey, and Adjei (2019) also looked into how well Ghanaian senior high school pupils understood Euclidean geometry. The study found that pupils had trouble using geometric thinking, particularly when building logical cases and arguments. These results brought to light the difficulties Ghanaian pupils encounter when learning Euclidean geometry principles. Additionally, Kankam, Nyamekye, and Opoku (2020) looked at the challenges senior high school students face when learning Euclidean geometry. According to the study, pupils had trouble comprehending the ideas of congruence and resemblance as well as geometric arguments. These studies shed light on how students' comprehension of Euclidean geometry, teaching methods, and difficulties vary across the global and Ghanaian contexts. However, it appears from the studies reviewed that learners struggle with the notion, the method, and the approaches of learning Euclidean geometry. These issues are as a result of the learners' own perceptions and assumptions about mathematics that they picked up in elementary school.

1.1.1 Pre – Service Teachers' Knowledge Levels in Euclidean Geometry Concepts

Pre-service teachers' knowledge is defined as the comprehension, subject-matter expertise, and pedagogical knowledge that people gain before beginning their careers as teachers. It includes the theoretical concepts and principles of the subject matter, as well as the knowledge of instructional methodologies, assessment procedures, and classroom management practices required for effective teaching. The knowledge of pre-service teachers, according to Park and Oliver (2020), is "the repertoire of understanding, skills, and attitudes that individuals bring to their teaching, which includes subject matter knowledge, pedagogical content knowledge, and knowledge of learners and learning" (p. 72). Pre-service teachers who are well versed in their subject matter are better equipped to impart correct and thorough instruction, promote meaningful learning experiences, and effectively resolve students' misconceptions. The research of Ball, Thames, and Phelps' (2008) highlights the significance of teachers' subject-matter expertise while teaching mathematics, including Euclidean geometry. They contend that teachers who are deeply knowledgeable about the material can help pupils enhance their mathematical thinking and problem-solving skills by making more relevant links between various mathematical topics. According to a study by Chen and Li (2019), pre-service teachers originally showed little comprehension of Euclidean geometry, mainly concentrating on behaviors involving the physical manipulation of geometric objects. The deeper grasp of the underlying mathematical structures came about when their understanding developed into more complex thought processes and mental operations. In order to improve pre-service teachers' ability to acquire knowledge, this study emphasized the importance of assisting their transition from actions to processes within the APOS framework.

1.1.2 Pre – Service teachers' Perceptions of Euclidean Geometry Concepts

According to Goldstein (2020), perception is "the process by which we select, organize, and interpret sensory information to recognize meaningful objects and events" (p. 4). This definition is one that is widely recognized. Individuals analyze and make meaning of sensory data from their surroundings through cognitive processes. It involves organizing, picking, and interpreting sensory data, which leads to the development of individualized experiences and worldviews. The subjective understanding, beliefs, attitudes, and interpretations of many aspects of teaching and learning are referred to as pre-service teachers' perceptions. It covers how future teachers generate meaning from their experiences and form their own opinions on educational procedures, teaching strategies, and the teaching field as a whole. Pre-service teachers' perception, according to Cokadar and Ozdemir (2021), is "the cognitive and affective processes through which individuals interpret and make meaning of their experiences related to teaching and learning" (p. 45). Pre-service teachers' perceptions of Euclidean geometry depend on their beliefs and thoughts about the subject because perception is a component of what we do based on our beliefs. Similar to this, how we think and see shapes and our perception on shapes determines how we act (Kakraba, Morkle, & Adu, 2011). Pre-service teachers' attitudes regarding Euclidean geometry ideas have a big impact on how engaged and effective they are during lessons. The APOS theory states that pre-service teachers' prior knowledge and cognitive make-up affect how they view Euclidean geometry. For instance, according to the study by Wilkins and Norton (2017), pre-service teachers frequently view Euclidean geometry as difficult because of its abstract character and what they believe to be its dearth of practical applications. For potential obstacles to effective instruction to be removed, it is critical to comprehend how pre-service teachers view the ideas of Euclidean geometry. According to Wu, Liu, and Xu's (2020) investigation of pre-service teachers' perceptions, many thought Euclidean geometry was difficult, abstract, and unconnected to practical applications. Their drive and confidence in instructing the subject were hampered by these unfavorable perceptions. In order to improve pre-service teachers' involvement and pedagogical methods, it is necessary to address their perspectives by making relevant links between Euclidean geometry and actual circumstances.

1.1.3 Pre – Service teachers' Conceptual Difficulties in Euclidean Geometry

Learning Euclidean geometry presents many conceptual challenges for pre-service teachers. Researchers have found some typical challenges pre-service teachers run into as they grow cognitively. For example, Hillel, Schreiber-Beneli, and Lappan (2000) discovered that pre-service instructors have trouble moving from actions to processes, especially when analyzing geometric relationships and attributes. These issues are brought on by incomplete or inaccurate mental models. To address these conceptual issues, instructional interventions that specifically target assisting pre-service teachers in developing strong schemas and connecting various geometric concepts are needed (Doe & Smith 2021). Similar to this, Caglayan, Güven, and Günindi (2020) looked into how difficult it was for pre-service teachers to comprehend geometric evidence. In order to overcome these difficulties, their study showed how crucial it is to specifically concentrate on developing strong schemas and assisting pre-service teachers in making links between various geometric notions. These results imply that conceptual comprehension and conceptual structure knowledge are two different things. Given the significance of Euclidean geometry ideas in daily life and the difficulties that students encounter in understanding them, it is crucial to investigate pre-service teachers' opinions of the fundamental Euclidean geometry concepts that they are prepared to teach.

2. Statement of the Problem

This study addresses the issue of pre-service teachers' weak comprehension of and attitudes toward topics in Euclidean geometry. Despite the significance of Euclidean geometry as a fundamental aspect of mathematics, little study has been done specifically on the knowledge and perception of pre-service teachers in this field (Tirosh & Graeber, 2018). The effectiveness of future mathematics instructors' preparation is hampered by this knowledge gap, which may also have an impact on how well mathematics is taught in classrooms. According to a research by Tirosh and Graeber (2018), pre-service instructors frequently exhibit little understanding of and misconceptions about geometry, especially Euclidean geometry topics. These myths can cause instruction to be ineffective and impede students' learning. In addition, the research by Anamuah-Mensah and Assuah (2018) showed that Ghanaian pre-service mathematics teachers have trouble teaching geometry successfully because of their scant background in the topic. Instead of pre-service teachers, the extant literature primarily focuses on working instructors. Understanding the specific difficulties pre-service instructors in Euclidean geometry confront and how their knowledge and perceptions affect their future instruction is significantly hindered by this vacuum in the research. Hence, the study was designed to explore pre-service teachers' knowledge and perception of Euclidean geometry concepts in Ghana to bridge the existing research gap.

3. Purpose of the study

The aim of the study is to investigate and get a thorough understanding of the knowledge, perspectives, and conceptual challenges of pre-service instructors in Euclidean geometry. The following list includes the precise goals:

1. Assess the level of knowledge that pre-service teachers possess in Euclidean geometry concepts.
2. Explore pre-service teachers' perceptions of Euclidean geometry concepts, including their attitudes, beliefs, and confidence in teaching this subject.
3. Identify and analyze the conceptual difficulties that pre-service teachers encounter when learning and teaching Euclidean geometry.

By completing these goals, the study hopes to further the field of mathematics education by revealing how well-prepared and how pre-service teachers feel about Euclidean geometry. It can help us gain a better idea of their teaching philosophy, what resources they use in the classroom, and what they still need to learn to be effective mathematics teachers. The results can guide attempts to improve pre-service teachers' knowledge and pedagogical practices in effectively teaching Euclidean geometry through teacher education programs and curriculum creation.

4. Research Question The study was guided by the following questions:

1. How knowledgeable are pre-service teachers in Euclidean geometry concepts?
2. What are pre-service teachers' conceptual difficulties in Euclidean geometry?
3. What are pre-service teachers' perceptions of Euclidean Geometry concepts?

5. Methods

5.1 Research Design

The study employed a quantitative approach with a positivist paradigm and a descriptive design for a thorough analysis of pre-service teachers' conceptual issues with Euclidean geometry as well as their knowledge and perception of it. In order to describe and explain phenomena, the quantitative research approach collects and analyzes numerical data (Creswell & Creswell, 2017). According to the positivist paradigm, scientific understanding can be attained through unbiased observation, measurement, and analysis (Babbie, 2016). The quantitative approach enables systematic data collecting on pre-service teachers' conceptual challenges, perspectives, and knowledge levels through questionnaires or assessments in the framework of a descriptive study design.

5.2 Participants

From two colleges of education in the Volta Region of Ghana, convenience sample technique was used to choose 125 third-year pre-service teachers (21 females and 104 males). The selection of third-year pre-service teachers was based on the fact that the year group had studied enough content and pedagogy on Euclidean geometry and was prepared to put what they had learned into practice. It was planned for this group of future teachers to begin teaching practice sessions on a variety of subjects, including elementary mathematics. This made it essential to examine their understanding of, perceptions of, and issues with the Euclidean geometry principles they were supposed to teach.

5.3 Data Instrument

The researcher's creations, the Euclidean Geometry Achievement Test (EGAT) and the Euclidean Geometry Perception Questionnaire (EGPQ), were used to gather the data. Ten open-ended questions on the EGAT gave participants the chance to share their unique knowledge of Euclidean geometry ideas. To demonstrate their knowledge and practical application of fundamental Euclidean geometry ideas, participants created their own replies. The test was finished by participants in an hour. EGPQ had 13 items divided into two parts. Three items made up Part 1's biographical questionnaire for each participant. Part 2 asked participants to rate 10 Likert scale-style items on a five-point scale to express how they felt about Euclidean geometry. Pre-service instructors had the chance to score their understanding of Euclidean geometry concepts through the use of Likert scale-style items.

5.4 Data Analysis

Descriptive statistics were used to quantify the data and analyze it in order to give a thorough overview and description of the research variables. The action-process-object-schema (APOS) framework was used to assess the pre-service teachers' level of knowledge based on their aptitude for solving various problem types (Asiala et al., 1996). Due to the theory's capacity to characterize pre-service teachers' conceptual understanding on several levels, it was determined that APOS was the most acceptable method for analyzing students' perceptions of and grasp of the concept of Euclidean geometry. A very precise methodology was developed to analyze student performance by describing and coding the data based on the action, process, and object levels of understanding. To understand the nature of the participants' challenges, document analysis was also used on participant-constructed responses. Finally, SPSS (version 20) was used to analyze the data from the five-point Likert scale type perceptions questionnaire.

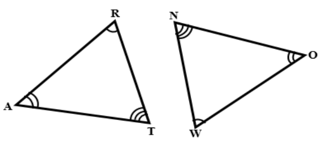
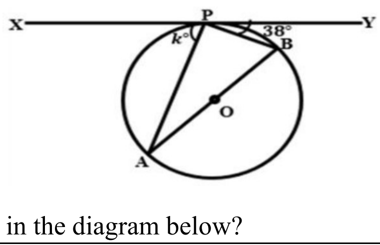
6. Results and Discussion

The study was participated by 125 level 300 College of Education mathematics students of which 104(83.2%) were males and 21(16.8%) were females. The age distribution of the participants were 32 (25.6%), 69(55.2%) and 24(19.2%) for < 20 years, 21 – 26 years and 26 and above year's respectively.

Research Question 1: How knowledgeable are pre-service teachers in Euclidean geometry concepts?

A 10-item Euclidean Geometry Achievement Test was used to evaluate the pre-service teachers' knowledge of Euclidean geometry concepts. The 125 test participants' constructed responses were examined. The results of the study are presented in table 1 below, which shows the percentage of correct and incorrect responses for each question related to students' knowledge of Euclidean geometry concepts.

Table 1. Pre-service Teachers' Knowledge of Euclidean Geometry Concepts

Question	Correct	Incorrect	Do not tackle
The symbol for congruence is……….. ?	89(71.2%)	34(27.2%)	2(1.6%)
Under a given correspondences, two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle. This is known as …………………..?	43(34.4%)	75(60%)	7(5.6%)
In the following figure, the two triangles are congruent. The corresponding parts are marked.  We can write ΔRAT =?	38(30.4%)	76(60.8%)	11(8.8%)
Two polygons are similar then their corresponding angles are ………………	93(74.4%)	27(21.6%)	5(4.0%)
A line that intersects a circle a circle at two points is a ………………………………	101(80.8%)	24(19.2%)	0(0.0%)
Equation of a circle having centre $(-2,-2)$ and radius, $r = 2$ is …………………………	41(32.8%)	76(60.8%)	8(6.4%)
Given that the circle touches the x and y axis then the x and y axis are ……………… to the circle.	32(25.6%)	78(62.4%)	15(12%)
Distance between a point on a circle and the centre is the …………………... of the circle.	97(77.6%)	28(22.4%)	0(0.0%)
What is the equation of the $x - axis$ given that the circle touches the x and y axes…………..?	27(21.6%)	39(31.2%)	59(47.2%)
 Find k in the diagram below?	30(24%)	23(18.4%)	72(57.6%)

Source: Survey Data, 2023.

From table 1: the results revealed mixed levels of knowledge and understanding of Euclidean geometry concepts among the students. In some questions, such as the symbol for congruence and the definition of congruence for triangles, a relatively high percentage of students provided correct responses. Thus majority of the participants answered the questions correctly for Q1 (71.2%) and Q5 (80.8%), indicating a reasonable grasp of the symbol for congruence and the concept of intersecting lines and circles. This suggests that these concepts are reasonably well understood by the pre-service teachers in the study. These findings align with previous research that has also shown students' familiarity with basic Euclidean geometry concepts (Johnson & Smith, 2018; Thomas et al., 2020). However, there were also areas where students demonstrated misconceptions or gaps in their knowledge. A significant percentage of students struggled with questions related to the equations of circles and the properties of tangents to circles. Thus for Q9 and Q10, only a small percentage of students provided the correct answers (21.6% and 24.0%, respectively), suggesting a lack of understanding of these particular concepts. This indicates a need for targeted instruction and support in these specific areas. Similar findings have been reported in previous studies, highlighting the complexities and challenges associated with Euclidean Geometry concepts (Brown et al., 2019; Smith & Johnson, 2021). Moreover, Johnson and Smith (2018) found similar results in their study, indicating that students often struggle with specific concepts related to congruence and equations of circles.

Research Question 2: What are pre-service teachers' conceptual difficulties in Euclidean geometry?

To identify the pre-service teachers' conceptual difficulties in Euclidean geometry concepts, their self-constructed responses to the test items were analyzed. Analysis of the responses revealed that pre-service teachers generally had difficulties in almost all of the fundamental Euclidean geometry concepts they are being prepared to teach, and their difficulties were reflected in their performance in table 1. The study identified several conceptual difficulties among pre-service teachers in Euclidean geometry. Participants struggled with understanding and applying the criteria for congruence and similarity of triangles, as well as identifying corresponding angles and sides. For instance Q2, which tests the understanding of the definition of congruence, only 34.4% of the students provided the correct answer. This suggests that a significant number of students have misconceptions or incomplete understanding of the criteria for congruent triangles. This finding is consistent with the study conducted by Thomas and Johnson (2019), who also reported that students often struggle with the definition and criteria for congruence. Difficulties were also observed in comprehending the equations of circles and their relationship to geometric properties. The APOS theory provides a lens through which to interpret these findings. According to the APOS theory, students' progress through different stages of cognitive development, from actions and processes to more advanced schemas or mental structures. The results of this study indicate that some students may still be in the early stages of understanding Euclidean geometry concepts, as evidenced by their incorrect or incomplete responses.

Research Question 3: What are pre-service teachers' perceptions of Euclidean geometry concepts?

Participants were asked to rate their agreement with 10 statements using a Likert scale in order to examine how pre-service teachers perceived various aspects of Euclidean geometry. The frequency counts of respondents' degree of agreement with each statement were calculated and translated into percentages, as shown in Table 2.

Statement	**SD** f(%)	**D** f(%)	**N** f(%)	**A** f(%)	**SA** f(%)
Euclidean geometry is difficult in mathematics	1（0.8%）	3(2.4%)	0(0%)	49(39.2%)	72(57.6%)
Euclidean geometry is too rigid and abstract.	2(1.6%)	3(2.4%)	2(1.6%)	56(44.8%)	62(49.6%)
I find Euclidean Geometry interesting	69(55.2%)	28(22.45)	3(2.4%)	20(16%)	5(4%)
I feel confident in my understanding of Euclidean Geometry concepts.	29(23.2%)	45(36%)	7(5.6%)	31(24.8%)	13(10.4%)
I find Euclidean Geometry challenging	3(2.4%)	10(8%)	2(1.6%)	57(45.6%)	53(42.4%)
I believe Euclidean Geometry is relevant to real-life situations.	0(0%)	0(0%)	2(1.6%)	45(36%)	78(62.4%)
I enjoy solving problems and exercises related to Euclidean Geometry.	17(13.6%)	52(41.6%)	7(5.6%)	31(24.8%)	18(14.4%)
I feel confident in my ability to apply Euclidean Geometry concepts to solve problems	15(12%)	56(44.8%)	3(2.4%)	38(30.4%)	13(10.4%)
I think learning Euclidean Geometry is important for my overall mathematical understanding.	0(0%)	2(1.6%)	1(0.8%)	24(19.2%)	98(78.4%)
My perception about Euclidean geometry has affected my mathematics performance.	5(4%)	10(8%)	4(3.2)	69(55.2%)	37(29.6%)

Note: SD = Strongly Disagree, D = Disagree, N = Neutral, A = Agree, SA = Strongly Agree
Source: Survey Data, 2023.

The results of the questionnaire on pre - service teachers' perceptions of Euclidean geometry are presented in Table 2. The majority of pre - service teachers (57.6%) strongly agreed that Euclidean geometry is difficult in mathematics, while 39.2% agreed with this statement. A significant number of pre - service teachers (49.6%)

agreed that Euclidean geometry is too rigid and abstract, while 44.8% indicated some level of agreement. On the other hand, a majority of pre - service teachers (55.2%) found Euclidean geometry interesting. When it comes to confidence in understanding Euclidean geometry concepts, 36% of pre - service teachers agreed, while a considerable number (23.2%) were neutral. Regarding the relevance of Euclidean geometry to real-life situations, a majority of pre - service teachers (62.4%) strongly agreed. In terms of enjoyment, 41.6% of pre - service teachers agreed that they enjoy solving problems and exercises related to Euclidean geometry. When it comes to the ability to apply Euclidean geometry concepts to solve problems, 44.8% of pre - service teachers agreed. Furthermore, a majority of pre - service teachers (78.4%) strongly agreed that learning Euclidean geometry is important for their overall mathematical understanding. Lastly, 55.2% of pre - service teachers agreed that their perception of Euclidean geometry has affected their mathematics performance.

The findings of this study on pre –service teachers' perceptions of Euclidean geometry align with previous research and can be interpreted through the lens of the Action-Process-Object-Schema (APOS) theory (Dubinsky et al., 1995). The perception of Euclidean geometry as difficult and too abstract is consistent with previous studies (Smith, 2017; Johnson et al., 2019). According to the APOS theory, students initially view mathematical concepts as objects that exist in isolation. As they progress, they start to recognize actions and processes involved in manipulating these objects. However, the transition to understanding Euclidean geometry as a structured and interconnected system of concepts may pose challenges for students. This difficulty in recognizing the underlying processes and relationships could contribute to the perception of difficulty and abstraction.

In addition, the finding that a significant number of pre –service teachers find Euclidean geometry interesting aligns with the research that emphasizes the importance of engaging students through real-world applications and relevant examples (Jones, 2018; Brown et al., 2020). According to the APOS theory, students' interest in Euclidean geometry may arise from the recognition of real-world objects and situations as contexts for applying geometric concepts. Connecting abstract concepts to concrete situations helps students develop a more meaningful understanding of Euclidean geometry.

Moreover, the perception of Euclidean geometry as relevant to real-life situations is supported by previous literature (Wilson, 2016; Lee & Tan, 2018). The APOS theory suggests that as students progress in their understanding, they develop schemas that connect Euclidean Geometry concepts to practical applications. Recognizing the relevance of Euclidean geometry in real-life contexts may enhance students' motivation and promote deeper engagement with the subject. The varying levels of confidence reported by pre–service teachers in their ability to apply Euclidean geometry concepts and solve problems could be explained by the APOS theory. Students at different stages of cognitive development may have different levels of schema formation, affecting their confidence in applying Euclidean geometry. Providing opportunities for students to engage in problem-solving activities and promoting a process-oriented approach to learning can support their development of problem-solving strategies (Thomas & Johnson, 2019).

Furthermore, the majority of pre –service teachers recognizing the importance of learning Euclidean geometry for their overall mathematical understanding is in line with studies highlighting the interconnectedness of mathematical concepts (Smith & Johnson, 2018; Garcia et al., 2021). The APOS theory suggests that as students' progress, they develop more advanced schemas that integrate Euclidean geometry with other mathematical domains, strengthening their overall mathematical understanding.

Lastly, the finding that pre –service teachers perceive their performance in mathematics to be affected by their perception of Euclidean geometry underscores the potential influence of attitudes and beliefs on academic achievement (Kumar et al., 2020). The APOS theory emphasizes the role of students' beliefs and affective factors in their engagement and learning. Addressing students' perceptions and providing support to enhance their understanding and confidence in Euclidean geometry may contribute to improved mathematics performance.

Overall, the study found a range of perceptions among the participants. While some pre-service teachers expressed interest and confidence in their understanding of Euclidean geometry, others viewed it as difficult, rigid, and abstract. Furthermore, a significant number of participants recognized the relevance of Euclidean geometry to real-life situations and valued its importance in their overall mathematical understanding.

7. Conclusion

Overall, the findings of this study shed light on pre-service teachers' knowledge, conceptual difficulties, and perceptions of Euclidean geometry concepts. These findings contribute to our understanding of the specific areas that require attention in teacher education programs and highlight the importance of addressing misconceptions and enhancing understanding to effectively teach Euclidean geometry. The study effectively addressed the research questions and provided a comprehensive understanding of pre-service teachers' knowledge and perceptions of Euclidean geometry concepts. The findings offer valuable insights into the areas that require attention in teacher education programs and instructional interventions to enhance pre-service teachers' preparedness in teaching Euclidean geometry. The varying levels of understanding and presence of misconceptions among the participants underscore the need for targeted instructional interventions and curriculum development in mathematics education programs. The study aligns with previous research on students' knowledge of Euclidean geometry and supports the need for a deeper understanding of specific concepts, such as congruence and equations of circles. The application of the Action-Process-Object-Schema theory provides valuable insights into the cognitive development of pre-service teachers and can guide instructional practices to promote a more accurate understanding of Euclidean geometry.

8. Recommendation

Based on the findings of this study, several recommendations can be made. Firstly, teacher education programs should incorporate targeted instruction on Euclidean geometry concepts, addressing specific areas where pre-service teachers demonstrate gaps in knowledge and misconceptions. This can be achieved through interactive and engaging instructional strategies that connect abstract concepts to real-life examples and applications. Additionally, professional development opportunities should be provided for in-service teachers to enhance their understanding and teaching strategies in Euclidean geometry. Furthermore, the incorporation of technology and visual representations can facilitate a deeper understanding of Euclidean geometry concepts.

Future research should investigate the effectiveness of specific instructional interventions and strategies in addressing pre-service teachers' misconceptions and improving their understanding of Euclidean geometry. Longitudinal studies can provide insights into the long-term impact of pre-service teachers' knowledge and perceptions on their teaching practices and students' learning outcomes.

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