Journal of Education and Practice
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.14, No.31, 2023

Examining the Conceptual and Procedural Knowledge Levels of Engineering Faculty Students on the Integral of a Two-Variable Function

Ümit Karabıyık
Necmettin Erbakan University, Department of Mathematics and Computer Sciences,
New Meram Street, Meram, Konya, Turkey
E-mail of the corresponding author: ukarabiyik@erbakan.edu.tr

Abstract
In this study, the focus will be on the concept of integral in two-variable functions, which holds great significance not only in mathematics but also in fields related to science and engineering in terms of learning and teaching. The integral of a two-variable function is a fundamental concept in mathematics with broad applications in science and engineering. This study addresses the vital role of this concept in shaping the foundational knowledge of engineering faculty students. The ability to comprehend and apply integrals is crucial not only for mathematical proficiency but also for success in various scientific and engineering disciplines. Within this scope, the study aims to examine the domain knowledge of engineering faculty students regarding the integral concept in terms of conceptual and procedural knowledge. This research was structured as a case study with twenty engineering faculty students selected according to critical case sampling. Participants were administered a questionnaire consisting of 10 questions to assess their knowledge of the integral of two-variable functions and in-depth insights into their conceptual and procedural knowledge of the topic were obtained. According to the results of data analysis, it was revealed that engineering faculty students possess procedural knowledge about the integral concept but have conceptual deficiencies and cannot explain their answers based on procedural knowledge. Furthermore, it was determined that conceptual knowledge is more inadequate compared to procedural knowledge. Therefore, developing content that aims to strengthen the integral topic both procedurally and conceptually in undergraduate-level Analysis courses is recommended.
Keywords: Engineering Student, Two-Variable Function, Integral, Procedural Knowledge, Conceptual Knowledge
DOI: 10.7176/JEP/14-31-02
Publication date: November 30th 2023

1. Introduction

One of the objectives of mathematics education is to ensure that students learn mathematical concepts to the best of their ability. Given the abstract nature of mathematical concepts, fully grasping these concepts can sometimes be challenging. Difficulties encountered in the process of learning mathematical concepts can further complicate the process of mathematical learning. Therefore, it is of utmost importance to identify and address students' difficulties in learning mathematics. In the field of mathematics, while understanding concepts is a significant goal, it is necessary to acknowledge that this is a challenging objective. Understanding students' learning difficulties in mathematics and uncovering the reasons behind these difficulties is a crucial step toward developing effective teaching methods to overcome them (Delong et al., 2005a).

Calculus, particularly for students, is a highly complex field that demands advanced thinking skills. Analysis, which is often challenging in terms of comprehending mathematical concepts, is frequently encountered by engineering faculty students. Calculus holds fundamental importance for various disciplines, making it essential for students to grasp the subject accurately. Understanding how a specific concept is perceived by students, identifying their mistakes, and recognizing possible misconceptions about the concept are of paramount importance. Such errors and misconceptions not only hinder the learning of the relevant topic but also impact overall learning processes. The concept of multivariable functions forms the foundation of mathematics and its applications (Trigueros and Martinez-Planell, 2010). Functions with multiple independent variables are referred to as multivariable functions. Many functions encountered in teaching and daily life are, in fact, examples of multivariable functions. The concept of integration is examined in two parts: definite and indefinite integration. In a section of widely accepted Calculus textbooks used by mathematicians (Thomas, Weir, Hass, and Giordano, 2009), indefinite integration is defined as the "inverse of differentiation." The concept of double integration in Analysis, like many other mathematical concepts, is an abstract concept that requires higher-level cognitive activity. Visualization is an effective method of teaching abstract concepts (Stylianou, 2002). This study emphasizes the significant importance of integral concepts related to two-variable functions in the context of learning and teaching, not only in a mathematical context but also in the fields of science and engineering. Subject domain knowledge (Shulman, 1986) is the knowledge of a specific subject, the underlying principles of the subject, concepts, and the relationships between these principles and concepts (Ball, 1988). Solid mathematical knowledge encompasses understanding mathematical topics, knowing procedures and concepts, and establishing relationships between topics, concepts, and procedures. The distinction between procedures and concepts plays a significant role in acquiring mathematical knowledge (Hiebert & Lefevre, 1986).

In the literature, mathematical knowledge, which has been categorized differently by various researchers (Hiebert & Lefevre, 1986), is generally divided into conceptual knowledge and procedural knowledge (Olkun & Toluk-Ucar, 2012). However, this distinction may not hold for all mathematical knowledge. Indeed, Hiebert and Lefevre (1986) suggested that some knowledge can be defined both conceptually and procedurally and that making this distinction, which they describe as conceptual and procedural knowledge, would assist in interpreting the learning process and better understanding students' achievements.

According to Van de Walle, Karp, and Bay-Williams (2012), procedural knowledge in mathematics pertains to knowledge of rules and symbols used during operations, while conceptual knowledge relates to fundamental ideas about topics and knowledge about the relationships among these concepts. It is emphasized that instruction lacking conceptual understanding leads to errors and a lack of love for mathematics (Van de Walle et al., 2012).

Conceptual knowledge goes beyond knowing the definition of a concept; it involves being able to facilitate transitions between concepts and relating new information to existing knowledge (Baki & Kartal, 2004). Conceptual knowledge is described as rich in terms of interrelationships between pieces of information, while procedural knowledge is defined as knowledge of symbols, the use of mathematical language, algorithms, rules, and procedural knowledge for problem-solving (Hiebert & Lefevre, 1986).

In another study that focused on the relationship between the area under a curve and integration, Mahir (2009) used the idea of Riemann sums, the integral-area relationship, and The Fundamental Theorem of Calculus proposed by Hiebert and Lefevre (1986) to examine the conceptual and procedural understanding of 62 students who completed a year-long analysis course. At the end of the study, Mahir revealed that students did not have sufficient conceptual understanding and that students who applied a conceptual approach and a procedural approach had different success levels. Furthermore, Mahir (2009) suggested that concept-based teaching could help students improve their conceptual understanding and emphasized that students with advanced conceptual understanding would also have higher procedural skills. In another study that emphasized conceptual and procedural understanding, Chapell and Kilpatrick (2003) examined the operational and conceptual interpretations of Calculus with students and instructors. Chapell and Kilpatrick (a.g.e.) found that students who received concept-based instruction significantly outperformed students who received operation-based instruction, thus supporting Mahir's (2009) proposal that "students with high conceptual understanding will also have high procedural skills".

One effective method of increasing conceptual understanding of integration is to engage with practical applications of integration. In secondary school mathematics programs and at the undergraduate and postgraduate levels in universities, problems related to area and volume often feature prominently in integral applications. In this context, Ergene (2014) conducted a detailed examination of the processes of solving volume problems with integration for students majoring in primary school mathematics teaching, mathematics teaching, mathematics, and mathematics engineering.

In another study that focused on obstacles encountered during the interpretation of definite integration using Riemann sums, Wagner (2017) continued her study with students who began their undergraduate studies in the physics department and students in their third year of study. Regarding difficulties arising from conceptual and procedural understanding of integration, there are also studies on the use of technology. Indeed, a study conducted by Swidan and Naftaliev (2019) concluded that the use of interactive diagrams and graphical approaches was effective in teaching indefinite integration. Based on research conducted with students taking an analysis course, Sevimli (2013) found that students who received instruction supported by Computer Algebra Systems (CAS) achieved higher success in transformational learning compared to students in regular instruction. They also found that students preferred graphical and numerical approaches when dealing with definite integration.

Understanding the relationship between conceptual and procedural knowledge and achieving a balance between these two types of knowledge is of great importance in the process of learning mathematics. There is a wealth of research in this area. When studies with teacher candidates and teachers are examined, it is often observed that these individuals may know mathematical rules and methods but may not fully understand why these rules are important and how they work. In studies related to various mathematical topics, it is observed that procedural knowledge is often not retained, conceptual knowledge is less emphasized compared to procedural knowledge, and balance is not achieved.

The literature review reveals that there are numerous similar studies related to our research topic. Delice and Sevimli (2011), in their research, examined the sequencing of concepts based on visuals in the teaching of definite and indefinite integrals. Their study focuses on the conceptual visualizations of the integral concept in instruction. Ulas and Biber (2020) investigated the mastery of pre-service mathematics and science teachers in their conceptual structures regarding the concept of derivatives during the academic year 2014–2015. Unver, Celik, and Guzel (2020) aimed to evaluate pre-service mathematics teachers' mechanisms for dealing with situations that require limited boundaries and to identify their misconceptions in their study. Unal (2021) conducted a study where he examined engineering students' problem-solving processes for integral problems in terms of conceptual and procedural knowledge using "Bloom's Taxonomy". In another study conducted by researchers in higher engineering education, students' learning outcomes are commonly assessed through tests and examinations, with the claim that these assessments indicate the achievement of learning goals set by teachers. Nevertheless, the students' learning experiences and their perceived gains in knowledge remain unclear. To obtain insights into their perceived learning gains, thirteen students from a Dutch technical university were interviewed. The outcomes of this study can provide valuable insights for teachers and curriculum leaders in designing courses and curricula (Uum&Pepin, 2022).

Yitmez, Yilmaz, and Dincer (2022) conducted research in which they examined the concept misconceptions related to the limits and continuity of multivariable functions among pre-service teachers. Considering the purpose of this research and the data collection tool, the study takes on the nature of a case study. In this study, the aim is to examine the domain knowledge of engineering faculty students in more detail regarding the multiple integrals of two-variable functions. This examination will particularly focus on understanding how procedural and conceptual knowledge are balanced. In pursuit of this goal, answers will be sought for the following questions: "What are the conceptual and procedural knowledge levels of engineering faculty students regarding the multiple integrals of two-variable functions? How do they comprehend these concepts, and what types of errors do they make in questions related to these concepts?" The sub-problems of the research and their objectives are outlined below.

• What is the relationship between the conceptual and procedural knowledge levels of engineering students regarding the multiple integrals of two-variable functions?

Objective: To analyze the relationship between students' conceptual and procedural knowledge levels and examine the impact of these two types of knowledge on the learning process.

• What is the fundamental definition of the integral concept in two-variable functions, and why is it important?

Objective: To explain the concept of multiple integrals in two-variable functions and understand its significance in the mathematical world.

• How are multiple integrals defined and calculated in non-rectangular bounded regions?

Objective: To elucidate the methods of calculating multiple integrals in non-rectangular bounded regions and explain the analysis of such regions.

• How can integration limits be determined to calculate a double integral, and what is the importance of changing the order of integration?

Objective: To understand how integration limits are determined for calculating double integrals and to comprehend the mathematical consequences of changing the order of integration.

• What are the properties of double integrals, and how are they used in calculating the area of a bounded region in the plane?

Objective: To understand the mathematical properties of double integrals and their application in calculating the area of a region in the plane.

• How is the average value of a two-variable function over a region calculated, and why is this value important?

Objective: To explain the concept of the average value of a two-variable function over a region and understand its mathematical and practical significance.

• How are multiple integrals used to calculate the volume of a region in the plane or space?

Objective: To elucidate the role of multiple integrals in volume calculations and understand their use in analyzing regions of different dimensions.

In this study, the domain knowledge of engineering faculty students regarding the double integrals of two-variable functions has been examined in terms of procedural and conceptual knowledge. Double integrals of two-variable functions are used in engineering and mathematics fields to calculate varying quantities in two or three dimensions, such as total mass, the angular momentum of an object with varying density, and the volume of a general curvilinear bounded region.

2. Methodology

This study is a case study aiming to examine the knowledge levels of engineering students regarding the topic of double integrals of two-variable functions in detail. A case study is a scientific method that involves the collection of systematic data to deeply investigate the functioning and operation of a specific system (Chmiliar, 2017). The focus of the research is on students' conceptual and procedural knowledge levels and the processes through which they acquire this knowledge. The examination is conducted in-depth on a specific case and emphasizes cause-and-effect relationships. Therefore, students were asked "how" and "why" questions to gain further insights into the underlying understanding beneath these questions.

2.1 Participant Group

The participants in our study consist of 20 second-year students from the Computer, Industrial, Electrical-Electronics, and Mechanical Engineering departments at Necmettin Erbakan University's Faculty of Engineering, who have completed the mathematics 1-2 courses with at least a (BA) grade. The criterion sampling method was used in selecting the participants. In addition to being conducted at Necmettin Erbakan University, this research preferred students from the local area to save time and space, and the convenience sampling method was used for accessibility. In the research, the names of the students who voluntarily participated were kept confidential, and the participants were identified with codes such as ES1, ES2, and so on. The reason for selecting these students is our desire to understand how the conceptual and procedural knowledge of students who have successfully passed these courses is structured since it is assumed that they are good in this regard. It was thought that more data-rich information could be obtained from more successful students; therefore, in this research, an attempt was made to reveal the conceptual and procedural knowledge of students who are known to be successful in the Integral topic. During the research process, by selecting students from various departments under the Faculty of Engineering, it was aimed to include different examples in the study. The frequencies of participant students according to their departments are shown below.

2.2 Data Collection Tools and Data Collection Process

The data used in the study were obtained from a written exam consisting of 10 questions administered to engineering faculty students. These written exam questions were reviewed and approved by two experienced faculty members specialized in mathematics education. The written exams aim to have students answer the questions in an original way based on their thinking and knowledge. These exams are particularly effective in assessing in-depth understanding and high-level cognitive learning abilities. Therefore, it was decided to use the written exam to assess how well engineering students are acquainted with the concept of integration. The responses of students to this written exam were analyzed in two different types, namely, conceptual, and procedural knowledge.

Making a clear distinction between conceptual and procedural knowledge is not always an easy task. As noted by Hiebert and Lefevre (1986), some knowledge can possess both conceptual and procedural characteristics. Therefore, it can be challenging to determine what type of knowledge a particular question measures. In a similar study in the literature, Kartal and Baki (2006) identified the characteristics used to define conceptual and procedural knowledge and used these characteristics to evaluate questions. In this study, especially in the context of questions related to the integration of two-variable functions, we determined the characteristics that define conceptual and procedural knowledge based on the literature (Baki & Kartal, 2004; Hiebert & Lefevre, 1986), as presented below. In this study, the characteristics we relied on to identify the required and used knowledge type in the responses provided are presented below.

- ***Characteristics Characterizing Procedural Knowledge***

Using previously learned mathematical knowledge (theorems, definitions, properties) at the level of knowledge (Baki & Kartal, 2004) Knowing the relevant formal language, algorithms, and rules (Hiebert & Lefevre, 1986) Performing operations step by step (Baki & Kartal, 2004; Hiebert & Lefevre, 1986)

- ***Characteristics that characterize conceptual knowledge***

Knowing the fundamental concepts related to the subject and their meanings (Baki & Kartal, 2004) Using previously learned mathematical knowledge (definitions, propositions, and theorems) at the level of understanding or application (Baki & Kartal, 2004) Perceiving the question as a whole and evaluating the given clues correctly and appropriately (Baki & Kartal, 2004).

2.3 Data Analysis

Details regarding the classification of data obtained from engineering students' written question solutions are provided below.

- ***Correct Answers:*** Those who provided correct answers, giving responses that include all elements of the correct answer.
- ***Partially Correct Answers:*** Those who provided partially correct answers, giving responses that include at least one but not all the elements of the correct answer.
- ***Incorrect Answers:*** Those who provide incorrect answers, giving responses that contain irrelevant information and wrong, or illogical answers.
- ***Blank Responses:*** Those who left the response blank.

The classification of student responses to each exam question was conducted individually, and their answers to each question were categorized using the definitions of categories provided above. This categorization process was repeated by the researcher at different times, and the opinions of two expert faculty members in their respective fields were consulted to ensure accuracy.

The responses in each category are expressed with the number of students, their frequencies, and percentages. This analysis provides valuable information for understanding student exam performance and improving educational programs. The classification criteria assist in the objective evaluation of student achievement, while expert opinions were utilized to enhance accuracy. These data can serve as a valuable resource for identifying students' strengths and weaknesses and enhancing educational strategies.

3. Findings

In the scope of this study, the data collected in line with the research questions have been subjected to a detailed examination under four main headings to comprehensively address the topic of the integral of two-variable functions. The research conducted under these subheadings represents a significant component of mathematical analysis, and the results of these studies provide valuable contributions to both theoretical mathematics and engineering applications. The findings under these four main headings in the research constitute an important source of information for mathematical thinking and problem-solving. The responses given for each research question and the findings obtained from them were analyzed meticulously individually. These analyses, when combined with the results of the interviews conducted with students, led to important conclusions.

The subheadings included in the scope of the research, namely "The Concept and Significance of Integrals of Two-Variable Functions", "Double Integrals in Non-Rectangular Bounded Regions", "Computing Double Integrals" and "Applications of Double Integrals" cover a significant and extensive area of mathematical analysis. The research conducted under these headings addresses the conceptual understanding, computation, and applications of double integrals, which are fundamental in both mathematical and engineering disciplines. In this context, under the title "The Concept and Significance of Integrals of Two-Variable Functions", we begin by examining why these integrals are important and their place within the mathematical analysis. Understanding the concept of integrals of two-variable functions is a fundamental step in solving more complex problems. Then, under the heading "Double Integrals in Non-Rectangular Bounded Regions", we investigate how these integrals are defined and computed, especially in mathematical modeling of real-world problems where non-rectangular regions are commonly encountered. The computation of these integrals in non-rectangular regions holds particular importance and is examined under this topic. Under the title "Computing Double Integrals", we discuss practical computation methods for this type of integral and fundamental steps such as determining integral limits. The process of computing integrals of two-variable functions is of great importance in solving complex problems mathematically. Finally, under the heading "Applications of Double Integrals", we explore how this type of integral can be applied to real-world problems. Under this topic, we observe how double integrals are used in the calculation of concepts such as area, volume, and more.

3.1 The Concept and Significance of Integrals of Two-Variable Functions

Q1. What is the concept of integrals of two-variable functions, and what role does this concept play in the world of mathematics?

When we examine the responses to the first research question, a distribution table indicating which types of answers engineering faculty students considered correct/partially correct/incorrect is presented below.

Response Types	Responses	Students	f	%
Correct	It is the integral of functions resulting from extending one-variable calculations to calculations with two-variable functions.	ES4, ES5,ES9,ES10	4	20
	It is the integral of a region on the plane.	ES1, ES6,ES11	3	15
	We find the volume of a region below a surface in space.	ES3,ES7	2	10
Partially Correct	It is the limit of Riemann sums of two-variable functions.	ES8, S15	2	10
	It is the integral of functions that are considered as having inputs x and y and an output z.	ES12,ES17	2	10
İncorrect	Its application in the mathematical world is less extensive compared to single-variable functions.	ES16	1	5
	Two-variable functions are used when calculating the length of a curve.	ES18	1	5

According to the table above, 9 students answered the question correctly. When we examined the written responses of these students in detail, it was determined that they had a conceptual understanding of the integral of two-variable functions. Additionally, students with codes ES3 and ES7 are understood to have procedural knowledge because they mentioned the use of this concept in mathematical volume calculations. Four students who partially answered the question correctly have been found to have a partial conceptual understanding based on their responses. Students with codes ES8 and ES15 based their answers on the concept of a single-variable integral only. Students with codes ES12 and ES17 only know about two-variable functions. Among the 2 students who answered the question incorrectly, ES16 students compared it to single-variable functions, which is a conceptually incorrect approach. ES18 student made a procedural mistake in their answer. Furthermore, 5 students did not answer this question in the study.

Q2. What do double integrals mean, and how are these integrals used to understand the mathematical properties of a region?

When we examine the answers to the second question, we can see a distribution of the types of responses that engineering faculty students considered as correct/partially correct/incorrect in the table below.

Response Types	Responses	Students	f	%
Correct	It is a way of taking an integral over a two-dimensional area. It allows you to calculate the volume beneath a surface.	ES1,ES5,ES9,ES13	4	20
	They are the limits of convergent Riemann sums.	ES4,ES8,ES2	3	15
	They are used to calculate quantities that vary in two or three dimensions.	ES7,ES11,ES15,ES19	4	20
Partially Correct	The integral of functions with two variables such as x and y is double.	ES6,S10	2	10
	It is used only in volume calculations.	ES20	1	5
İncorrect	The double nature of the integral is not related to the number of variables.	ES16	1	5
	It helps us find the area under a curve.	ES12,ES18	2	10

When we examined the responses of the 7 students who answered the second question correctly, it was observed that they had a conceptual understanding of what double integrals mean. Furthermore, when four students provided the answer 'It is used to calculate quantities that vary in two or three dimensions,' it indicates that these students have operational knowledge related to the applications of double integrals. Three students who partially answered the question seem to not fully grasp the required conceptual knowledge. Among the students who gave incorrect answers, ES16 stating that double integrals are not related to the number of variables provides an irrelevant response. ES12 and ES18 students, on the other hand, confused conceptual knowledge about the question, mixing single integrals with double integrals. Three students did not respond to the question.

Q3. How do we determine the integration boundaries and what should we pay attention to in the process of calculating multiple integrals?

When we examined the responses to question three, a table showing how the engineering faculty students evaluated the answers as correct, partially correct, or incorrect is presented below.

Response Types	Responses	Students	f	%
Correct	When calculating multiple integrals over a planar region, the x and y boundaries of the planar region are the integration limits.	ES5,ES,ES8,ES13	4	20
	We determine the integration limits for x and y by identifying the bounding curves of the integration region. For this, we use vertical and horizontal lines.	ES1,ES15,ES19	3	15
	We find the boundaries through the geometric shapes specified by the functions defining the shape	ES4,ES11,ES14	3	15
Partially Correct	The boundaries are given directly.	ES12,ES8	2	10
	We express the boundaries of the inner integral as a function of the outer variable.	ES18,ES3	2	10
İncorrect	The result remains unchanged when we switch the integration boundaries or their order.	ES16	1	5
	The integral is calculated over an interval determined by its two endpoints, and these endpoints specify the integration boundaries.	ES15	1	5

In the calculation of multiple integrals, determining integration boundaries is crucial. This is a critical step that ensures the integral reaches the correct result. Therefore, 10 students who provided correct answers have explained how integration boundaries are determined when calculating multiple integrals over a planar region. These explanations emphasize that integration boundaries are defined as x and y coordinates and that the bounding curves define these limits. Parallel lines drawn on the shape are used for this purpose, and the points where they enter and exit the shape provide the boundaries. Additionally, they mention that the functions that determine the shape also help in determining the boundaries. This conceptual knowledge demonstrates that the students have a good grasp of the subject. However, there are some important points to consider when determining integration limits in the calculation of multiple integrals. Expressing the limits of the inner integral as a function of the outer variable is a part of this process, and this conceptual knowledge is partially correct. According to the answers of two students who provided incorrect answers, the statement "Changing the limits or order of integration does not affect the result" is wrong. Changing the limits or order can indeed affect the integral result and lead to different outcomes. Multiple integrals are not calculated over an interval determined by two endpoints; these endpoints specify the limits of a single integral. This question was not answered by four students in the research.

3.2 Double Integrals in Non-Rectangular Unbounded Regions

The definition and computation of double integrals in non-rectangular regions form an important part of mathematical analysis. One of the focal points of this study is to examine the methods of integral calculations in such regions and investigate how integration order can be changed in non-rectangular regions. In this context, the following questions were posed to the students.

Q4. How do you define and compute double integrals in non-rectangular regions?

When we examine the answers given to the fourth question, you can find a table below showing how students evaluated these answers.

Response Types	Responses	Students	f	%
Correct	An unbounded region is divided by a rectangular grid. The norm of the division approaches zero, and the number of rectangles approaches infinity. It is the limit value to which Riemann sums converge.	ES5,ES6,ES11,ES15,ES16	5	25
	In the plane, a bounded region R is divided into rectangles, these divisions are multiplied by the value of the function, and the limit of the sum of the obtained volumes is calculated.	ES1,ES4,ES8, S19	4	20
Partially Correct	We draw a rectangular grid with equal areas that divides a non-rectangular bounded region into cells. The areas of these equal-area rectangles are multiplied by the z-value of the function, and the limit of the Riemann sums of the results is calculated.	ES13,ES15,ES20,ES17	4	20
Incorrect	Since the boundary of the region is curved, it is divided into rectangles, including the areas outside, and the limit of the Riemann sums of volumes is calculated.	ES12,ES1O	2	10
	The calculation is done in the same way as for rectangular regions.	ES18	1	5

The correct answers provided by the students contain important information about defining and calculating double integrals in non-rectangular regions. In their written responses, students generally express that when calculating double integrals in non-rectangular regions, they often consider these regions as bounded areas to make them more understandable and computable. They mention dividing this bounded region with a kind of rectangular grid and carefully placing this grid inside the region, striving to achieve finer subdivisions as the norms of these subdivisions approach zero. As the number of rectangles approaches infinity, they observe that the dimensions of each rectangle decrease. Then, they explain how they calculate the volumes of these regions by multiplying the area of each rectangle by a function value within that rectangle. Subsequently, they mention that they calculate the integral by summing up these volumes, which represents the limit at which Riemann sums converge. In other words, they calculate the integral by computing the areas of an infinite number of rectangles, ultimately yielding the value at which Riemann sums converge.

They emphasize that this process helps make integral calculations in non-rectangular regions more understandable and manageable. Breaking down regions into smaller pieces, approaching limits, and obtaining correct results. The explanations of students who partially answered the question indicate that they are missing essential steps in describing the definition and calculation of double integrals in non-rectangular regions. One student's response indicates a lack of understanding of the question, while two students' explanations reveal a deficiency in their conceptual understanding, as they use the phrase 'The region boundary is curved, so it is divided into rectangles, including the areas outside, and the volumes are calculated with the limit of Riemann sums.' Four students did not provide an answer to the question.

Q.5. How is the order of integration changed for double integrals in non-rectangular regions?

The findings related to the fifth question of the research are presented below.

Response Types	**Responses**	**Students**	**f**	**%**
Correct	The order of integration is crucial; sometimes, we may not be able to solve the integral in the given order. Therefore, we may need to change the order of integration.	ES1,ES5,ES11,ES15,ES19	5	25
	When changing the order of integration, both dx and dy change as well.	ES2,ES4,ES7,ES10,ES14	5	25
Partially Correct	The order of integration is estimated based on the solution of the integral.	ES6,ES8,ES18	3	15
Incorrect	We arrive at a solution by changing only the integration boundaries and the order of integration.	ES12,ES17,ES20	3	10

In non-rectangular regions, the order of integration for double integrals has a significant impact on the solution of the integral, as correctly indicated by the students in their answers. As they correctly pointed out, sometimes the given order of integration can make solving the integral difficult or even impossible. Therefore, it may be necessary to change the order of integration. When changing the order of integration, the terms dx and dy are also swapped, which means approaching the integral from a different perspective. Some students who partially answered the research question mentioned that the order of integration is based on a guess for the solution. While this may help solve the integral more efficiently in some cases, it cannot guarantee a complete solution. However, the statement "We arrive at a solution by changing only the integration limits and the order of integration" is incorrect and indicates a misunderstanding or an incomplete grasp of the integral calculation process by the answering students. Changing the order of integration is not solely about altering the limits but also depends on the structure of the integrand function and the nature of the integral itself. In conclusion, understanding and correctly applying the order of integration in double integrals in non-rectangular regions is a critical factor for successfully solving the integral.

3.3 Calculation of Double Integrals

The calculation of double integrals is an important topic in mathematics and its applications. Initially, to calculate double integrals in non-rectangular bounded regions, we typically divide the region into more manageable parts. Double integrals of continuous functions play a significant role in mathematical analysis and engineering applications. These integrals are often used in volume calculations and probability theory. They are particularly useful for solving multivariable problems, making them applicable in a wide range of scenarios. In this context, the calculation of double integrals and the use of continuous functions in such integrals form one of the cornerstones of mathematical analysis. To delve deeper into this topic and gain a better understanding of the importance of double integrals and continuous functions in this field, the following two questions were posed to the students.

Q6. What steps do we follow, and how do we perform the calculation of double integrals in non-rectangular bounded regions?

When examining the answers to the sixth question, the categorization of students' responses is provided in the table below.

Response Types	Responses	Students	f	%
Correct	First, we draw the region R using the graphs of the given basic functions in the xy-plane. Then, we determine the boundaries and proceed to calculate the integral.	ES2,ES3,ES7,ES1, ES15,ES19	6	30
	We draw the region R first. By keeping either x or y constant, we find the limits for the other variable. If we keep x constant, we draw a line parallel to the y-axis within the region, and the points where it enters and exits the region will give us the limits for y. After finding the limits, we integrate the function first with respect to y and then with respect to x, from the inside out.	ES1,ES8,ES9,ES3, ES14,ES7,ES20	7	35
Partially Correct	We first draw the non-rectangular region, then we calculate the double integral of the function in the appropriate order, step by step, with respect to the variables from the inside out.	ES5,ES11	2	10
Incorrect	When calculating the double integral of a function, the outer integral is written with the inner integral as the function and then computed.	ES6,ES16	2	10
	When calculating the double integral of a function, if we are integrating with respect to x first, we find the limits of y by drawing lines parallel to the x-axis in the region R. Then, we integrate with respect to y and proceed to obtain the result.	ES12,ES18	2	10

In the answers provided by the students, it is observed that they have explained the steps to calculate double integrals in non-rectangular bounded regions quite accurately. When examining the written responses, it can be seen that as the first step, they typically draw the region R in the xy-plane using the graphs of given functions, which helps them visually understand the boundaries of the region. After drawing R, they move on to the second step, where they fix one of the variables, either x or y, and determine the limits of the other variable to define the boundaries. For instance, if x is held constant, they draw the part of the region that is parallel to the y-axis, and the points where this curve enters and exits the region define the limits for y. In this way, they obtain the limits by defining the boundaries of x or y variables. Once the limits are found, they proceed to calculate the integral by integrating first concerning y or x and then integrating concerning the other variable to reach the result. These steps presented by the students explain the fundamental strategies used to calculate double integrals in non-rectangular bounded regions. These strategies represent critical steps for performing integral calculations correctly. When examining the written responses of students with codes ES5 and ES11, it can be said that they lack procedural knowledge in the process of following the steps to calculate double integrals mentioned in the question. When looking at the incorrect answers provided by students, two students have different approaches: ES6 and ES16 students express that the outer integral should be written as the function of the inner integral. This approach might indicate a misunderstanding of Fubini's Theorem. In double integrals, the inner and outer integrals are taken concerning different variables, and the order of integration can be changed depending on the problem and the structure of the region. ES12 and ES18 students state that the integral should be solved successively concerning x or y. This response ignores the fact that the order of integration depends not only on the structure of the region but also on the nature of the function. The order of integration is often changed based on which variable is more suitable to solve the problem. These incorrect answers seem to reflect a lack of complete understanding or a partial misunderstanding of the fundamental principles of double integrals. Correct sequencing and how the integral should be solved in double integrals are essential building blocks of integral calculations.

Q.7. Are there any useful properties of double integrals of continuous functions in calculations and applications?

The findings related to the seventh question are presented below.

Response Types	Responses	Students	f	%
Correct	Integrals can behave like sums. Therefore, they have properties of linearity and sum-difference. If a function f is replaced by its constant multiple cf, the Riemann sum for f will change to the Riemann sum for cf accordingly.	ES1,ES4,ES10,ES11,ES14	5	25
	If the region R is composed of non-overlapping regions like R1 and R2, then the sum of their double integrals is equal to the double integral of R.	ES3,ES6,ES8,ES13	4	20
Partially Correct	The double integrals of two-variable functions are preserved through algebraic operations.	ES20	1	5
Incorrect	The double integral of the sum of a two-variable function and a constant is equal to the sum of the double integral of the function and the double integral of the constant taken separately.	ES16,ES7,ES5	3	15
	The double integral of the product of two two-variable functions, f and g, is equal to the product of their double integrals taken separately.	ES15,ES12	2	10

When the answers to this question are examined, the statement 'Integrals can behave like sums. Therefore, there are product and sum-difference properties. If f is a function and c is a constant, when you replace f with c.f, the Riemann sum for f changes in the same way as the Riemann sum for c.f' emphasizes that the integral is sensitive to algebraic operations and that multiplying it by a constant does not affect the integral result. This accurately reflects the conceptual understanding of integration. Operationally, this statement reflects the ability to use various algebraic operations in integral calculations. Applying multiplication or addition operations to integrals requires operational knowledge. The statement 'R region, the composition of two non-overlapping regions such as R1 and R2, results in the sums of their double integrals being equal' indicates that the integral is not affected by the separateness of the regions. The composition of the region does not affect the integral result. This also accurately explains a conceptual feature of the integral. As for the incorrect answers given to the question, the statement that the double integral of the sum of a two-variable function and a constant is equal to the sum of their double integrals taken separately is an incorrect answer. Operationally, this statement reflects a wrong approach because the summation property of double integrals does not work on the sum of their limits but on the functions. This misunderstanding can lead to incorrect integral calculations. The statement that the double integral of the product of two two-variable functions is equal to the product of their double integrals taken separately is incorrect operationally and reflects an incorrect approach. The double integral of the product of two functions is not a process that can be calculated by separately calculating the double integrals of each function. In conclusion, while conceptual knowledge accurately reflects correct and critical information, operational knowledge reflects practical skills that affect integral calculations. In this context, misunderstood or incomplete operational knowledge by students can lead to errors in integral calculations.

3.4 Applications of Multiple Integrals

In the context of the applications of multiple integrals of two-variable functions used in mathematics and engineering, the detailed responses to the eighth, ninth, and tenth questions of the research are presented in the tables below.

Q8. Multiple integrals are used to calculate the area or volume of a region in the plane or space.

Response Types	Responses	Students	f	%
Correct	To find the area of a closed and bounded planar region R, you can calculate the multiple integral over R with the function f(x, y) = 1 in its definition, using the limit of Riemann sums.	ES5,ES9,ES11,ES15,ES19	5	25
	The first step is to draw the shape of the region for which we will calculate the area on the xy-plane. Then, the limits of the x and y variables are determined, and using these limits, the double integral is computed. This process provides us with the area.	ES1,ES4,ES10,ES13,ES14	5	25
	We find the volumes of three-dimensional objects using triple integrals. In a closed, bounded region D in space, the triple integral of F(x, y, z) = 1 will give us the volume. Calculation is done with three successive integrals, and the limits of the integral are determined with respect to the x, y, and z variables. Being able to visualize the object in space is crucial for this.	ES2, ES3,ES7,ES15	4	20
Partially Correct	In space, to find the volume of a region bounded by known surfaces with functions, we first determine the limits of the successive triple integrals, and then integrate with respect to the function F(x, y, z).	ES16,ES17,ES20	3	15
Incorrect	The double integral of a two-variable function f(x, y) gives the area of the region, while the integral of the three-variable function F(x, y, z) provides the volume in space.	ES6,ES8,ES12	3	15

Applications of multiple integrals are highly significant in calculating the area or volume of a closed and bounded region in the plane or space. By analyzing the correct answers provided by students, we can explain this topic in more detail:

To find the area of a closed and bounded planar region, it is calculated with the limit of Riemann Sums by taking $f(x, y) = 1$ as the definition of the double integral in the region R.' This statement explains how a double integral can be used to calculate the area of a planar region. By setting the function to be integrated as 1, it is stated that the limit of Riemann sums will give the area of the region. As the first step, the shape of the area to be calculated is drawn on the xy-plane. Then, the limits of the x and y variables are determined, and a double integral is calculated using these limits. This process results in the area.' This statement starts with the step of drawing the shape of the region and proceeds to explain how the area calculation is done by determining the limits of the integral concerning the x and y variables. We find the volumes of three-dimensional objects using triple integrals. In a closed and bounded region D in space, the triple integral of F(x, y, z) = 1 will give the volume. Calculation is done with three successive integrals. The limits of the integral will be determined according to the x, y, and z variables. It is important to be able to visualize the object in space for this.' This statement explains the use of triple integrals in calculating the volumes of three-dimensional objects. It emphasizes that the integral's limits are determined for the x, y, and z variables, and the volume is obtained through successive integrals. These responses highlight that students emphasize both the conceptual and procedural aspects of using multiple integrals for area and volume calculations. The integration process begins with defining the boundaries of the region or object and selecting the appropriate integration function. Then, the integral is calculated using these boundaries, resulting in the area or volume. Therefore, multiple integrals are recognized as essential tools widely used in mathematical and engineering applications, which is the fundamental message conveyed by the written responses.

Journal of Education and Practice www.iiste.org
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.14, No.31, 2023

Q.9. How do we calculate the average value over a given region when calculating the integral of a two-variable function, and in what context is this value important?

Response Types	Responses	Students	f	%
Correct	The average value of a single-variable function over a closed interval is the ratio of the integral of the function over that interval to the length of the interval. Similarly, for two-variable functions, the average value over a region is the ratio of the integral of the function over that region to the area of the region.	ES1,ES3,ES5,ES9,ES13,ES15	6	30
	The average height of the water in a pool, when the water is agitated, can be determined by allowing the water to come to rest at a constant height. In this case, the height is equal to the ratio of the volume of water in the pool to the area of the pool.	ES6, ES8,ES10,ES,11,ES20	5	25
Partially Correct	By taking the two-variable function as f(x, y) = 1, we can find its average value by dividing the double integral over the region R by the area of the region.	ES4, ES10	2	10
Incorrect	The average value of a two-variable function f(x, y) is the ratio of the integral of the function over the region to the area of the region.	ES18	1	5
	It is calculated by dividing the triple integral by the volume of the region.	ES17,ES20	2	10

The responses to this question from 11 students correctly demonstrate their understanding of how the average value on a region is calculated when finding the integral of two-variable functions and why it is important. The first response explains an extended version of how the average value calculation for single-variable functions is applied to two-variable functions. The second response, with a real-world example, explains how this concept can be used to find the average height. These answers show that students understand this topic from both a conceptual and operational perspective. The importance of the integral in this context is clearly understood when it is used to understand and model the properties of a region or volume. Therefore, students approach the topic both mathematically and with practical applications in mind. When we examine the response given by students with codes ES4 and ES10, it is clear that the statement "We can find the average value by taking f(x, y) = 1 and dividing the double integral over the region R by the area of R" does not accurately reflect the correct information. Students with codes ES17 and ES20 mention the concept of a triple integral instead of a double integral, indicating a complete misconception. A student with code ES18 appears to have confused the answer conceptually, mixing it with single-variable functions.

Q.10. How is the center of mass of a plate with a specific density that covers a given region R in the xy-plane calculated, and how is this calculation process related to multiple integrals?

Response Typs	Responses	Students	f	%
Correct	The mass calculation of a variable-density plate is performed by integrating the density function of the region R contained within this plate. First moments are calculated with respect to the axes. Mass center coordinates are determined by the ratios of the first moments to the mass.	ES7,ES11,ES19	3	15
Partially Correct	The mass of a material, represented by the density function d(x, y), is the mass per unit area. Mass is calculated using the double integral of this function.	ES1,ES5,ES20,ES8,ES15,ES17	6	30
Incorrect	It is calculated using the double integral of the density function represented by d(x, y) = 1.	ES6,ES12	2	10
	We do not take the density of the plate into account when calculating the integral because the density of the plate is constant everywhere.	ES16	1	5

Students who provided correct answers to this question demonstrate a solid understanding of the fundamental concepts used when calculating the center of mass of a plate with a given density. They have correctly stated that by taking the integral of the density function, they calculate the total mass of the plate. Additionally, when calculating the coordinates of the center of mass, they used the first moments and determined the center of mass by ratios of these moments to the total mass. Upon reviewing the given responses, it is evident that they accurately reflect the essential information regarding the center of mass calculations using multiple integrals, which is one of the practical applications of multiple integrals. These students have successfully explained the calculation process and understand the significance of multiple integrals in such applications. However, the statement provided by students said, 'The mass of a material is the mass per unit area represented by d(x, y). Mass is calculated with the double integral of this function,' indicates that they may not have a complete conceptual understanding of the center of mass calculation. When evaluating the response provided by student ES16, which states, 'To calculate the center of mass of the plate, we take the double integral of the plate's area. We do not take the density into account when taking this integral because we assume that the density is constant everywhere. Therefore, it is sufficient to only calculate the double integral of the area,' it becomes evident that this answer is incorrect. This is because the calculation of the center of mass is not solely based on taking the double integral of the plate's area. Density must also be considered because the density may vary in different regions of the plate. As a result, the integral of the product of the density for each region, followed by the integration of these densities, is used to calculate the center of mass. Hence, it should not be forgotten that density is associated with multiple integrals and should be considered in the calculation process. This situation indicates a misconceived understanding on the part of the student, and they may not have grasped the fundamental concepts related to multiple integrals correctly. The responses provided by students ES6 and ES12 are incorrect because the density function d(x, y) = 1 represents a constant density. It implies that the density is the same at every point on the plate. However, in practice, the density of plates can vary in different regions. Therefore, considering density is essential for the center of mass calculations. The density should be multiplied by the integral since it may vary in different regions. This incorrect response reflects a fundamental conceptual misunderstanding that density plays a significant role in the calculation process of the center of mass and should be treated accurately.

5. Conclusion, Discussion and Recommendations

In this study, the domain knowledge of twenty engineering students from various engineering faculty departments, whose levels of success were considered good according to certain criteria, was examined in terms of their both procedural and conceptual perspectives on double integrals of two-variable functions. The study aimed to investigate the students' knowledge of double integrals of two-variable functions from both procedural and conceptual angles. The research results indicate that the students possess procedural knowledge in this regard but have conceptual shortcomings. Particularly, the students' answers based on procedural knowledge reveal their inability to explain the subject conceptually. Therefore, it is recommended to develop course content at the undergraduate level that aims to strengthen the understanding of the integral concept both procedurally and conceptually. The findings demonstrate that students have procedural knowledge but lack conceptual understanding. These results highlight a significant issue in mathematics education, where students tend to learn mathematical concepts solely through procedural approaches and struggle to grasp them conceptually in-depth. A large portion of students prefer memorizing symbols and rules and applying them without understanding the underlying processes. This approach not only complicates the learning process but also hinders long-term retention. It is well-known that recalling rules without control is more difficult than understanding conceptual structures. Students often fail to establish a healthy relationship between conceptual and procedural knowledge.

According to the research results, it has been observed that students have procedural knowledge of double integrals and can perform integral calculations correctly. This indicates that students can successfully perform basic mathematical calculations. However, this procedural knowledge is limited to expressing calculation skills and does not include a conceptual understanding. It has been determined that students struggle to conceptually understand the subject and cannot conceptually explain their answers based on procedural knowledge. This suggests that students only learn mathematical concepts in terms of 'how to do it' but have difficulty understanding 'why it is done.' The lack of conceptual understanding limits mathematical thinking skills and can negatively affect student's ability to solve more complex problems. Furthermore, it was found that conceptual knowledge is less sufficient compared to procedural knowledge. This indicates that students learn the integral subject only through calculation practice and overlook conceptual aspects, such as why the subject is important or how it can be used in real-world problems. Therefore, it is important in mathematics education to strengthen not only procedural knowledge but also conceptual understanding. This will help students improve their mathematical thinking skills and solve complex problems more effectively. Li, Julaihi, and Eng (2017) emphasized in their study that identifying errors and misconceptions in the subject of integrals would enhance analytical success. Similarly, as another outcome of this research, it is believed that identifying engineering students' conceptual misconceptions about double integrals of multivariable functions would enhance their analytical achievements.

When the literature is reviewed, it is understood that calculus, integral calculus, is one of the most challenging concepts to comprehend. The topic of double integrals of multivariable functions is a highly demanding area for students. The difficulties students face in this regard arise from several factors, including conceptual misconceptions, the lack of appropriate teaching materials, and inadequacies in teaching the subject. Among these reasons, conceptual misconceptions, the lack of appropriate teaching materials, and inadequacies in teaching the subject can be listed. It is of great importance for future educators to use both procedural and conceptual knowledge effectively to help students better understand this challenging topic. This can be considered a significant step towards supporting students' conceptual learning and rectifying misconceptions.

For future research, to better understand the research objective, the effects of instructional practices can be examined in more detail using an experimental design that includes control and experimental groups. This approach could potentially strengthen and generalize the results of the research. Additionally, beyond focusing solely on multivariable functions, applications that relate concepts and procedural knowledge to multivariable function integrals, especially topics such as the Fundamental Theorem of Calculus and Riemann sums in analysis, can be developed to determine students' conceptual and procedural knowledge levels. In addition to focusing on multivariable function integrals, similar studies can be conducted that involve other important topics in analysis, such as Limits, Derivatives, Integrals, and their relationships. These studies may help students better understand the conceptual and procedural relationships among these different topics in analysis. These suggestions could contribute to future research in mathematics education having a deeper and more diversified perspective.

These future research directions aim to further enrich the understanding of integral concepts among engineering students and contribute to the continuous improvement of mathematics education in engineering programs.

- Explore and assess various teaching strategies and instructional methods aimed at enhancing both procedural and conceptual understanding of integrals in two-variable functions among engineering students. This could involve comparative studies to identify the most effective pedagogical approaches.
- Explore the effectiveness of incorporating technology, such as interactive simulations or computer-based tools, in teaching integral concepts. Investigate how technological interventions can positively impact students' learning experiences and outcomes.
- Extend the research to compare the understanding of integral concepts between engineering students and students in related disciplines. Understanding potential discipline-specific challenges can inform tailored instructional strategies.
- Explore how the depth of conceptual understanding of integral concepts correlates with students' ability to apply these concepts to real-world engineering problems. This could help bridge the gap between theoretical knowledge and practical applications.
- Supplement quantitative findings with qualitative analysis, such as interviews or focus group discussions, to gain a deeper understanding of students' perceptions and experiences regarding the integral concepts.

Based on the findings and results of this study, the following recommendations can be made for improving mathematics education, especially in the context of multivariable function integrals, and enhancing the success of engineering students:

- Strengthen Conceptual Understanding: Rather than focusing solely on procedural knowledge, it is necessary to enhance conceptual understanding in mathematics education. Concepts such as why the integral concept is important and how it can be applied to real-world problems should be explained more effectively to students.
- Develop Instructional Materials: Suitable instructional materials should be developed to support procedural knowledge and explain concepts. Providing students with concrete examples and applications can help them better understand the concepts.
- Teacher Training: Future teachers need to balance the use of procedural and conceptual knowledge to better explain mathematical concepts to students. Teacher education programs should be updated to equip teacher candidates with skills to support conceptual learning.
- Sensitivity to Student Needs: Every student has a different learning style. Teachers should plan their lessons in a way that is sensitive to students' needs and supports various learning styles.
- Addressing Conceptual Misconceptions: Special efforts should be made to identify and address common conceptual misconceptions that students often have. Identifying the frequent errors made by students can help teaching strategies focus on correcting these misconceptions.
- Application-Focused Lessons: Lessons related to multivariable function integrals should be application-oriented, focusing on real-world problems. This can help students understand how to apply theoretical knowledge in practice.
- Monitoring Student Progress: Regular assessment and evaluation methods should be used to monitor students' progress and identify their weaknesses promptly. This can provide students with additional support when needed.
- Collaboration and Group Work: Students should be given opportunities to collaborate and solve mathematical problems together. Group work can facilitate discussions of concepts and improve understanding.

These recommendations can guide educators and researchers who aim to develop strategies for balancing conceptual and procedural knowledge in mathematics education. By doing so, it becomes more likely that students will deepen their mathematical understanding and enhance their ability to solve more complex mathematical problems. Implementing these recommendations may particularly contribute to the success of engineering students in mathematics education and help them gain a deeper insight into mathematical concepts. Additionally, it can assist future mathematics teachers in delivering more effective education in this regard.

References

Ball, D. L. (1988). Research on teaching mathematics: Making subject matter knowledge part of the equation. (Report No. NCRTE-RR-88-2) East Lansing, MI: National Center for Research on Teacher Learning. (ERIC Document Reproduction Service No. ED301467)

Baki, A., & Kartal, T. (2004). Characterization of high school students' algebra knowledge in terms of conceptual and procedural knowledge. Turkish Journal of Educational Sciences, 2(1), 27-48.

Chapel, K. K., & Kilpatrick, K. (2003). Effects of concept-based instruction on students' conceptual understanding and procedural knowledge of calculus. PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 13(1), 17-37.

Delice, A., & Sevimli, E. (2011). An investigation of the sequence of topics in teaching integral concepts in terms of conceptual images: definite and indefinite integrals. Pamukkale University Journal of Education, 30(30), 51-62.

DeLong, M., Winter, D., & Yackel, C. (2005b). Mental maps and learning objectives: The FAST-SLO algorithm for creating student learning objectives. PRIMUS, 15(4), 307-338. https://doi.org/10.1080/10511970508984126

Ergene, O. (2014). Investigation of individual relations in the process of solving integral volume problems in the context of application community. Unpublished Master's Thesis. Istanbul: Marmara University.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Li, V. L., Julaihi, N. H., & Eng, T. H. (2017). Misconceptions and errors in learning integral calculus. Asian Journal of University Education, 13(2), 17-39.

Mahir, N. (2009). Conceptual and procedural performance of undergraduate students in integration. International Journal of Mathematical Education in Science and Technology, 40, 201-211.

Olkun, S., & Toluk-Ucar, Z. (2012). Activity-based mathematics education in primary education (5th ed.). Ankara: Egiten Kitap.

Sevimli, E. (2013). The effect of computer algebra system-supported instruction on the representational transformation processes of students with different cognitive styles in the integral topic. Unpublished Ph.D. Thesis. Istanbul: Marmara University.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15(2), 4-14.

Stylianau, D. A. (2002). On The interaction of visualization and analysis: The negotiation of visual representation in expert problem-solving. The Journal of Mathematical Behavior, 21(3).

Swidan, O., & Naftaliev, E. (2019). The role of the design of interactive diagrams in teaching–learning the indefinite integral concept. International Journal of Mathematical Education in Science and Technology, 50(3), 464-485.

Thomas, G., Weir, M., Hass, J., & Giordano, F. (2009). Thomas Calculus. Istanbul: Beta.

Trigueros, M., & Martínez-Planell, R. (2010). Geometrical representations in the learning of two-variable functions. Educational Studies in Mathematics, 73(1), 3-19.

Ulas, M. Y., & Biber, A. (2020). Conceptual structures of mathematics and science teacher candidates regarding the derivative concept. Bayburt Faculty of Education Journal, 15(30), 435-457.

Unal, S. E. D. S. (2021). Examining the integral problem-solving processes of engineering students in terms of conceptual and operational knowledge using Bloom's taxonomy. Educational Research (IJMCER), 3(6), 14-25.

Uum, M.S.J.V, & Pepin, B. (2022). Students' self-reported learning gains in higher engineering education. European Journal Of Engineering Education2023, Vol. 48, No. 1, 42–58. https://doi.org/10.1080/03043797.2022.2046708

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2012). Elementary and middle school mathematics. (S. Durmus, Trans.). Ankara: Nobel Academic Publishing.

Wagner, J. F. (2017). Students' obstacles to using Riemann sum interpretations of the definite integral. International Journal of Research in Undergraduate Mathematics Education, 4(3), 327–356. DOI: https://doi.org/10.1007/s40753-017-0060-7.

Yitmez, B. G., Yilmaz, S., & Dincer, B. (2022). An examination of the misconceptions about limits and continuity of multivariable functions of elementary mathematics education teacher candidates. Dokuz Eylul University Buca Faculty of Education Journal, (53), 611-637.

First A. Author: Ümit Karabıyık graduated from the Mathematics Education undergraduate program at Selçuk University Faculty of Education. He completed his master's and doctoral degrees in the field of mathematics. Currently, he serves as an assistant professor in the Department of Mathematics and Computer Sciences at Necmettin Erbakan University. In addition to mathematical analysis, he has a keen interest in mathematics education and continues his studies in this field. He particularly focuses on research related to mathematics and technology.